MATHEMATICS

1. B

Sol. Image of point (–4, 5)

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=-2\left(\frac{ax_1+by_1+c}{a^2+b^2}\right)$$

Line : x + 2y – 2 = 0

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=-2\left(\frac{ax_1+by_1+c}{a^2+b^2}\right)$$

$$=\frac{-8}{5}$$

$$x=-4-\frac{8}{5}=-\frac{28}{5}$$

$$y=-\frac{16}{5}+5=\frac{9}{5}$$

2. B

Sol. $\vec{r}=k(\vec{b}+\vec{c})$

$$\vec{r}\cdot\vec{a}=3$$

$$\vec{r}\cdot\vec{a}=k(\vec{b}\cdot\vec{a}+\vec{c}\cdot\vec{a})$$

$$3=k(2+6-15+3-2+3\lambda)$$

$$3=k(-6+3\lambda) \quad ....(1)$$

$$\vec{r}=k(5\hat{i}+2\hat{j}-(5-\lambda)\hat{k})$$

$$|\vec{r}|=k\sqrt{25+4+25+\lambda^2-10\lambda}=1 \quad ....(2)$$

$$k=\frac{3}{-6+3\lambda}=\frac{1}{-2+\lambda} \quad \text{put in (2)}$$

$$4+\lambda^2-4\lambda=54+\lambda^2-10\lambda$$

$$6\lambda=50$$

$$3\lambda=25$$

3. C

Sol. $R_1\to R_1-R_2,\ R_2\to R_2-R_3$

$$\begin{vmatrix}\alpha-a & b-\beta & 0\\ 0 & \beta-b & c-\gamma\\ a & b & \gamma\end{vmatrix}=0$$

$$(\alpha-a)(\gamma(\beta-b)-b(c-\gamma))-(b-\beta)(-a(c-\gamma))=0$$

$$\gamma(\alpha-a)(\beta-b)-b(\alpha-a)(c-\gamma)+a(b-\beta)(c-\gamma)$$

$$\frac{\gamma}{\gamma-c}+\frac{b}{\beta-b}+\frac{a}{\alpha-a}=0$$

4. 3

Sol. $T_2+T_6=\frac{70}{3}$

$$ar+ar^5=\frac{70}{3}$$

$T_3\cdot T_5=49$

$ar^2\cdot ar^4=49$

$a^2r^6=49$

$$ar^3=+7,\ a=\frac{7}{r^3}$$

$$ar(1+r^4)=\frac{70}{3}$$

$$\frac{7}{r^2}(1+r^4)=\frac{70}{3},\ r^2=t$$

$$\frac{1}{t}(1+t^2)=\frac{10}{3}$$

$3t^2-10t+3=0$

$$t=3,\ \frac{1}{3}$$

Increasing G.P. $r^2=3,\ r=\sqrt{3}$

$T_4+T_6+T_8$

$=ar^3+ar^5+ar^7$

$=ar^3(1+r^2+r^4)$

$=7(1+3+9)=91$

5. D

Sol. AA, MM, TT, H, I, C, S, E

(1) All distinct

$^{8}C_5 \rightarrow 56$

(2) 2 same, 3 different

$^{3}C_1 \times {}^{7}C_3 \rightarrow 105$

(3) 2 same I[st] kind, 2 same 2[nd] kind, 1 different

$^{3}C_2 \times {}^{6}C_1 \rightarrow 18$

Total $\rightarrow$ 179

6. B

Sol. $Z = \dfrac{1 + i\cos\theta}{1 - 2i\cos\theta}$

$$Z = -\bar{Z} \Rightarrow \frac{1 + i\cos\theta}{1 - 2i\cos\theta} = -\left(\overline{\frac{1 + i\cos\theta}{1 - 2i\cos\theta}}\right)$$

$$(1 + i\cos\theta)(\overline{1 - 2i\cos\theta}) = -(1 - 2i\cos\theta)(\overline{1 + i\cos\theta})$$

$$(1 + i\cos\theta)(1 + 2i\cos\theta) = -(1 - 2i\cos\theta)(1 - i\cos\theta)$$

$$1 + 3i\cos\theta - 2\cos^2\theta = -(1 - 3i\cos\theta - 2\cos^2\theta)$$

$$2 - 4\cos^2\theta = 0$$

$$\Rightarrow \cos^2\theta = \frac{1}{2} \Rightarrow \theta = -\frac{\pi}{4}, \ -\frac{3\pi}{4}, \ \frac{\pi}{4}, \ \frac{3\pi}{4}, \ \frac{5\pi}{4}, \ \frac{7\pi}{4}$$

sum $= 3\pi$

7. B

Sol. $\Delta = \begin{vmatrix} 1 & 4 & -1 \\ 7 & 9 & \mu \\ 5 & 1 & 2 \end{vmatrix} = 0$

$$\Rightarrow (18 - \mu) - 4(14 - 5\mu) - (7 - 45) = 0 \Rightarrow \mu = 0$$

$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$ (For infinite solution)

$$\Delta_x = \begin{vmatrix} \lambda & 4 & -1 \\ -3 & 9 & \mu \\ -1 & 1 & 2 \end{vmatrix} = 0$$

$$\lambda(18 - \mu) - 4(-6 + \mu) - 1(-3 + 9) = 0$$

$$18\lambda + 24 - 6 = 0 \Rightarrow \lambda = -1$$

8. C

Sol. $\left.\begin{aligned} \bar{r}_1 &= (\lambda\hat{i} + 4\hat{j} + 3\hat{k}) + \alpha(2\hat{i} + 3\hat{j} + 4\hat{k}) \\ \bar{r}_2 &= (2\hat{i} + 4\hat{j} + 7\hat{k}) + \beta(2\hat{i} + 3\hat{j} + 4\hat{k}) \end{aligned}\right\}$

$\bar{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\bar{a}_1 + \lambda\hat{i} + 4\hat{j} + 3\hat{k}$

$\bar{a}_2 = 2\hat{i} + 4\hat{j} + 7\hat{k}$

Shortest dist. $= \dfrac{|\bar{b} \times (\bar{a}_2 - \bar{a}_1)|}{|b|} = \dfrac{13}{\sqrt{29}}$

$$\frac{|(2\hat{i} + 3\hat{j} + 4\hat{k}) \times ((2 - \lambda)\hat{i} + 4\hat{k})|}{\sqrt{29}} = \frac{13}{\sqrt{29}}$$

$$|-8\hat{j} - 3(2 - \lambda)\hat{k} + 12\hat{i} + 4(2 - \lambda)\hat{j}| = 13$$

$$|12\hat{i} - 4\lambda\hat{j} + (3\lambda - 6)\hat{k}| = 13$$

$$144 + 16\lambda^2 + (3\lambda - 6)^2 = 169$$

$$16\lambda^2 + (3\lambda - 6)^2 = 25 = \lambda \Rightarrow = 1$$

9. C

Sol. $\dfrac{\dfrac{3(\sqrt{5}+1)}{4} + 5\left(\dfrac{\sqrt{5}-1}{4}\right)}{5\left(\dfrac{\sqrt{5}+1}{\sqrt{5}+4}\right) - 3\left(\dfrac{\sqrt{5}-1}{4}\right)} = \dfrac{8\sqrt{5}-2}{2\sqrt{5}+8}$

$$= \frac{4\sqrt{5}-1}{\sqrt{5}+4} \times \frac{\sqrt{5}-4}{\sqrt{5}-4}$$

$$= \frac{20 - 16\sqrt{5} - \sqrt{5} + 4}{-11}$$

$$= \frac{17\sqrt{5} - 24}{11} \Rightarrow a = 17, \ b = 27, \ c = 11$$

a + b + c = 52

10. C

Sol. $\sec^2 y \dfrac{dy}{dx} + 2x \sin y \sec y = x^3 \cos y \sec y$

$$\sec^2 y \frac{dy}{dx} + 2x \tan y = x^3$$

$$\tan y = t \Rightarrow \sec^2 y \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} + 2xt + x^3, \ \text{If} = e^{\int 2x dx} = e^{x^2}$$

$$te^{x^2} = \int x^3 \cdot e^{x^2} dx + c$$

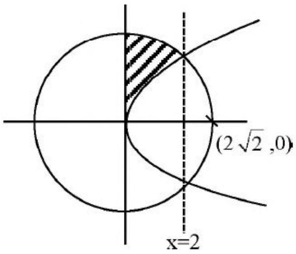
$$x^2 = Z \Rightarrow t \cdot e^z = \frac{1}{2}\int e^Z \cdot ZdZ = \frac{1}{2}[e^Z \cdot Z - e^Z] + c$$

$$2\tan y = (x^2 - 1) + 2ce^{-x^2}$$

$$y(1) = 0 \Rightarrow c \Longrightarrow y(\sqrt{3}) = \frac{\pi}{4}$$

11. B

Sol.



Required area = Ar(circle from 0 to 2) – ar(para from 0 to 2)

$$= \int_0^2 \sqrt{8 - x^2} dx - \int_0^2 \sqrt{2x} dx$$

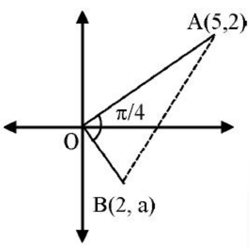
$$= \left[\frac{x}{2}\sqrt{8 - x^2} + \frac{8}{2}\sin^{-1}\frac{x}{2\sqrt{2}}\right]_0^2 - \sqrt{2}\left[\frac{x\sqrt{x}}{3/1}\right]_0^2$$

$$= \frac{2}{2}\sqrt{8 - 4} + \frac{8}{2}\sin^{-1}\frac{2}{2\sqrt{2}} - \frac{2\sqrt{2}}{3}(2\sqrt{2} - 0)$$

$$\Rightarrow 2 + 4 \cdot \frac{\pi}{4} - \frac{8}{3} = \pi - \frac{2}{3}$$

12. D

Sol.



$$m_{OA} = \frac{2}{5}$$

$$m_{OB} = \frac{a}{2}$$

$$\tan\frac{\pi}{4} = \left|\frac{2}{5} - \frac{a}{2}\right|$$

$$1 = \left|\frac{4 - 5a}{10 + 2a}\right|$$

$4 - 5a = \pm(10 + 2a)$

$4 - 5a = 10 + 2a$ $\quad$ $4 - 5a = -10 - 2a$

$\Rightarrow 7a + 6 = 0$ $\quad$ $3a = 14$

$\Rightarrow a = -\frac{6}{7}$ $\quad$ $a = +\frac{14}{3}$

$$-\frac{6}{7} \times \frac{14}{3} = -4$$

13. B

Sol. $(\vec{a} + \vec{b}) \times \vec{c} - \vec{c} \times (-2\vec{a} + 3\vec{b}) = 0$

$(\vec{a} + \vec{b}) \times \vec{c} + (-2\vec{a} + 3\vec{b}) \times \vec{c} = 0$

$\Rightarrow (\vec{a} + \vec{b}) - \vec{a} + 3\vec{b}) \times \vec{c} = 0$

$\Rightarrow \vec{c} = \lambda(4\vec{b} - \vec{a})$

$\Rightarrow = \lambda(44\hat{i} - 4\hat{j} + 4\hat{k} - 4\hat{i} + \hat{j} - \hat{k})$

$= \lambda(40\hat{i} - 3\hat{j} + 3\hat{k})$

Now

$(8\hat{i} - 2\hat{j} + 2\hat{k} + 33\hat{i} - 3\hat{j} + 3k) \cdot \lambda(40\hat{i} - 3\hat{j} + 3\hat{k}) = 1670$

$\Rightarrow (41\hat{i} - 5\hat{j} + 5\hat{k}) \cdot (40\hat{i} - 3\hat{j} + 3\hat{k}) \times \lambda = 1670)$

$\Rightarrow (1640 + 15 + 15)\lambda = 1670 \Rightarrow \lambda = 1$

so $\vec{c} = 40\hat{i} - 3\hat{j} - 3\hat{k}$

$\Rightarrow |\vec{c}|^2 = 1600 + 9 + 9 = 1618$

14. A

Sol. $f'(x) = 6x^2 - 18ax + 12a^2 = 0$ $\langle$ α, α^2

$\alpha + \alpha^2 = 3a$ & $\alpha \times \alpha^2 = 2a^2$

$\downarrow$

$(\alpha + \alpha^2)^3 = 27a^3$

$\Rightarrow 2a^2 + 4a^4 + 3(3a)(2a^2) = 27a^3$

$\Rightarrow 2 + 4a^2 + 18a = 27a$

$\Rightarrow 4a^2 - 9a + 2 = 0$

$\Rightarrow 4a^2 - 8a - a + 2 = 0$

$\Rightarrow (4a-1)(a-2)=0$

$\Rightarrow a = 2$ so $6x^2 - 36x + 48 = 0$

$\Rightarrow x^2 - 6x + 8 = 0$ (1)

If we take $a=\frac{1}{4}$ then $\alpha=\frac{1}{2}$ which is not possible.

15. A

Sol.

X	Y	Z
5 one & 4 five	4 one & 5 five	3 one & 6 five

$$P=\frac{4/9}{5/9+4/9+3/9}=\frac{4}{12}=\frac{1}{3}$$

16. A

Sol. $\displaystyle\int_{\alpha}^{\log_e 4}\frac{dx}{\sqrt{e^x-1}}=\frac{\pi}{6}$

Let $e^x - 1 = t^2$

$e^x\,dx = 2t\,dt$

$$=\int\frac{2dt}{t^2+1}$$

$= 2\tan^{-1} t$

$$=2\tan^{-1}\left(\sqrt{e^x-1}\right)\Big|_{\alpha}^{\log_e^4}$$

$$=2\left[\tan^{-1}\sqrt{3}-\tan^{-1}\sqrt{e^{\alpha}-1}\right]=\frac{\pi}{6}$$

$$=\frac{\pi}{3}-\tan^{-1}\sqrt{e^{\alpha}-1}=\frac{\pi}{12}$$

$$\Rightarrow \tan^{-1}\sqrt{e^{\alpha}-1}=\frac{\pi}{4}$$

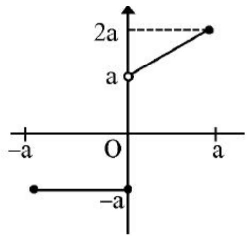
$e^{\alpha}=2$ $\qquad$ $e^{-\alpha}=\frac{1}{2}$

$$x^2-\left(2+\frac{1}{2}\right)x+1=0$$

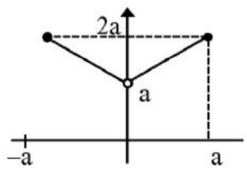
$2x^2 - 5x + 2 = 0$

17. A

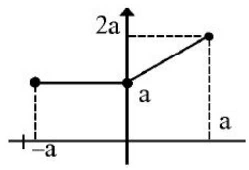
Sol. $y = f(x)$



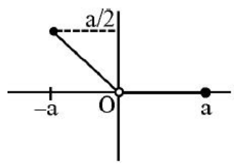
$y = f|x|$



$y = |f(x)|$



$$g(x)=\frac{f(|x|)-|f(x)|}{2}$$



18. C

Sol. A = {2, 3, 6, 8, 9, 11} (a, b)R (c, d)

B = {1, 4, 5, 10, 15} 3ad – 7bc

Reflexive : (a, b) R(a, b)

$\Rightarrow$ 3ab – 7ba = –4ab always even so it is reflexive.

Symmetric : If 3ad – 7bc = Even

Case-I : odd odd

Case-II : even even

(c, d) R(a, b) $\Rightarrow$ 3bc – 3ab

Case-I : odd odd

Case-II : even even

so symmetric relation Transitive :

Set (3, 4)R (6, 4) Satisfy relation

Set (6, 4)R(3, 1) Satisfy relation

but (3, 4)R(3, 1) does not satisfy relation so not transitive.

19. D

Sol. $\lim\limits_{x\to 0} f(x) = f(0) = 3$

$$\lim_{x\to 0^+} \frac{\sqrt{ax+b^2x^2}-\sqrt{ax}}{b\sqrt{a}x\sqrt{x}} = 3$$

$$\lim_{x\to 0^+} \frac{b^2}{b\sqrt{a}x^{3/2}\left(\sqrt{ax+b^2x^2}+\sqrt{ax}\right)}$$

$$\lim_{x\to 0^+} \frac{b^2}{b\sqrt{a}\left(\sqrt{a+b^2x}+\sqrt{a}\right)}$$

$$\frac{b}{\sqrt{a}\cdot 2\sqrt{a}} \Rightarrow \frac{b}{2a} = 3 \Rightarrow \frac{b}{a} = 6$$

20. A

Sol. $\left(\sqrt{a}x^2 + \frac{1}{2x^3}\right)^{10}$

General term $= {}^{10}C_r\left(\sqrt{a}x^2\right)^{10-r}\left(\frac{1}{2x^3}\right)^r$

20 – 2r – 3r = 0

r = 4

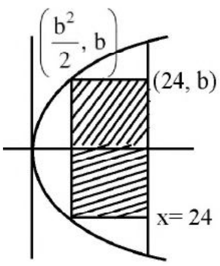
$${}^{10}C_4 a^3 \cdot \frac{1}{16} = 105$$

$a^3 = 8$

$a^2 = 4$

21. 128

Sol.



$$A = 2\left(24 - \frac{b^2}{2}\right)\cdot b$$

$$\frac{dA}{db} = 0 \qquad \Rightarrow b = 4$$

A = 2(24 – 8)4

= 128

22. 6

Sol. $\alpha = \lim\limits_{x\to 0^+} e^{\sqrt{x}} \frac{(e^{\sqrt{\tan x}-\sqrt{x}}-1)}{\sqrt{\tan x}-\sqrt{x}}$

= 1

$$\beta = \lim_{x\to 0}(1+\sin x)^{\frac{1}{2}\cot x}$$

$= e^{1/2}$

$$x^2 - (1+\sqrt{e}) + \sqrt{e} = 0$$

$$ax^2 + bx - \sqrt{e} = 0$$

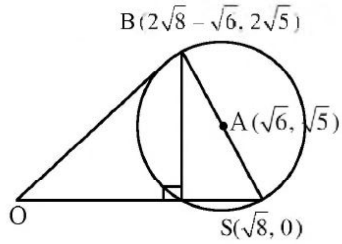
On comparing

$$a = -1,\ b = \sqrt{e}+1$$

$$12\ell n(a+b) = 12\times\frac{1}{2} = 6$$

23. 40

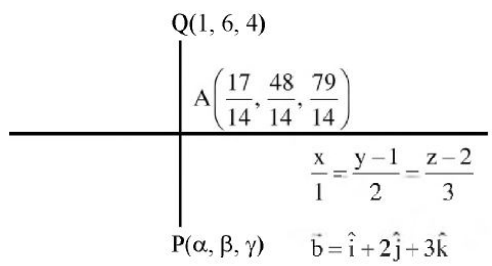
Sol.



$$\text{Area} = \frac{1}{2}(OS)h = \frac{1}{2}\sqrt{8}2\sqrt{5} = \sqrt{40}$$

24. 11

Sol.



A(t, 2t + 1, 3t + 2)

$$\overrightarrow{QA} = (t-1)\hat{i} + (2t-5)\hat{j} + (3t-2)\hat{k}$$

$$\overrightarrow{QA} \cdot \vec{b} = 0$$

$(t - 1) + 2(2t - 5) + 3(3t - 2) = 0$

$14t = 17$

$$\alpha = \frac{20}{14} \quad \beta = \frac{12}{14} \quad \gamma = \frac{102}{14}$$

$$2\alpha + \beta + \gamma = \frac{154}{14} = 11$$

25. 1505

Sol. 2, 5, 11, 20, ...

$$\text{General term } = \frac{3n^2 - 3n + 4}{2}$$

$$T_{10} = \frac{3(100) - 3(10) + 4}{2}$$

$= 137$

10 terms with c.d. = 3

$$\text{sum } = \frac{10}{2}(2(137) + 9(3))$$

$= 1505$

26. 2

Sol. $|x + 1|\,|x + 3| - 4|x + 2| + 5 = 0$

case-1

$(x + 1)(x + 3) + 4(x + 2) + 5 = 0$

$x^2 + 4x + 3 + 4x + 8 + 5 = 0$

$x^2 + 8x + 16 = 0$

$(x + 4)^2 = 0$

$x = -4$

case-2

$-3 \le x \le -2$

$-x^2 - 4x - 3 + 4x + 8 + 5 = 0$

$-x^2 + 10 = 0$

$x = \pm\sqrt{10}$

case-3

$-2 \le x \le -1$

$-x^2 - 4x - 3 - 4x - 8 + 5 = 0$

$-x^2 - 8x - 6 = 0$

$x^2 + 8x + 6 = 0$

$$x = \frac{-8 \pm 2\sqrt{10}}{2} = -4 \pm \sqrt{10}$$

case-4

$x \ge -1$

$x^2 + 4x + 3 - 4x - 8 + 5 = 0$

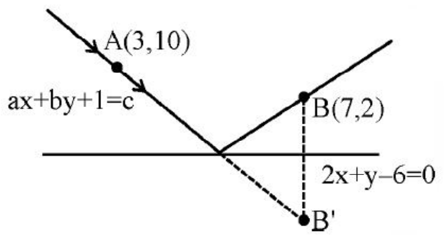
$x^2 = 0$

$x = 0$

No. of solution = 2

27. 1

Sol.



$$\text{For B' } \frac{x - 7}{2} = \frac{y - 2}{1} = -2\left(\frac{14 + 2 - 6}{5}\right)$$

$$\frac{x - 7}{2} = \frac{y - 2}{1} = -4$$

$x = -1 \qquad y = -2 \qquad B'(-1, -2)$

incident ray AB'

$M_{AB'} = 3$

$y + 2 = 3(x + 1)$

$3x - y + 1 = 0$

$a = 3\ b = -1$

$a^2 + b^2 + 3ab = 9 + 1 - 9 = 1$

28. 33

Sol. $a, b, c \in N \qquad a < b < c$

$$\bar{x} = \text{mean} = \frac{9 + 25 + a + b + c}{5} = 18$$

$a + b + c = 56$

$$\text{Mean deviation } = \frac{\Sigma |x_i - \bar{x}|}{n} = 4$$

$= 9 + 7 + |18 - a| + |18 - b| + |18 - c| = 20$

$= |18 - a| + |18 - b| + |18 - c| = 4$

$$\text{Variance} = \frac{\Sigma |x_i - \bar{x}|^2}{n} = \frac{136}{5}$$

$= 81 + 49 + |18 - a|^2 + |18 - b|^2 + |18 - c|^2 = 136$

$= (18 - a)^2 + (18 - b)^2 + (18 - c)^2 = 6$

Possible values $(18 - a)^2 = 1$, $(18 - b)^2 = 1$

$(18 - c)^2 = 4$

$a < b < c$

so $18 - a = 1$ $18 - b = -1$ $18 - c = -2$

$a = 17$ $b = 19$ $c = 20$

$a + b + c = 56$

$2a + b - c$ 34 = 19 – 20 = 33

29. 4

Sol. $a|x| = |y|e^{yx-\beta}$, a, $b \in N$

$xdy - ydx + xy(xdy + ydx) = 0$

$$\frac{dy}{y} - \frac{dx}{x} + (xdy + ydx) = 0$$

$$\ell n|y| - \ell n|x| + xy = c$$

$y(1) = 2$

$$\ell n|2| - 0 + 2 = c$$

$$c = 2 + \ell n2$$

$$\ell n|y| - \ell n|x| + xy = 2 + \ell n2$$

$$\ell n|x| = \ell n\left|\frac{y}{2}\right| - 2 + xy$$

$$|x| = \left|\frac{y}{2}\right|e^{xy-2}$$

$2|x| = |y|e^{xy-2}$

$\alpha = 2$ $\beta = 2$ $\alpha + \beta = 4$

30. 7

Sol. $$\int \frac{1}{\sqrt[5]{(x-1)^4(x+3)^6}}dx = A\left(\frac{\alpha x - 1}{\beta x + 3}\right)^B + C$$

$$I = \int \frac{1}{(x-1)^{4/5}(x+3)^{6/5}}dx$$

$$I = \int \frac{1}{\left(\frac{x-1}{x+3}\right)^{4/5}(x+3)^2}dx$$

$$\left(\frac{x-1}{x+3}\right) = t \Rightarrow \frac{4}{(x+3)}dx = dt \qquad t^{-4/5+1}$$

$$I = \frac{1}{4}\int \frac{1}{t^{1/4}}dt = \frac{1}{4}\ \frac{t^{1/5}}{1/5} + c$$

$$I = \frac{5}{4}\left(\frac{x-1}{x+3}\right)^{1/5} + C$$

$$A = \frac{5}{4} \quad \alpha = \beta = 1 \quad B = \frac{1}{5}$$

$$\alpha + \beta + 20AB = 2 + 20 \times \frac{5}{4} \times \frac{1}{5} = 7$$

PHYSICS

Section - A (Single Correct Answer)

31. B

Sol. $w_g + w_{Fr} + w_s = \Delta KE$

$$5 \times 10 \times 5 - 0.5 \times 5 \times 10 \times x - \frac{1}{2}Kx^2 = 0 - 0$$

$250 = 25x + 50x^2$

$2x^2 + x - 10 = 0$

$x = 2$

32. C

Sol. Z in LHS = 92

Z in RHS = 56 + 36 = 92

A in LHS = 236

A in RHS = 141 + 92 = 233

So 3 neutrons are released.

33. B

Sol. $$E = \frac{KQ}{R^2}$$

$$E = \frac{Q}{4\pi\varepsilon_0 R^2}$$

$$\varepsilon_0 = \frac{Q}{4\pi R^2 E}$$

Now, $$\varepsilon_0 E^2 = \frac{Q}{4\pi R^2 E}\cdot E^2 = \frac{Q}{4\pi R^2}\cdot E$$

$$\left[\varepsilon_0 E^2\right] = \left[\frac{QE}{R^2}\right] = \frac{[Q][E]}{[R^2]} = \frac{[Q]}{[R^2]}\frac{[W]}{[Q][R]}$$

$$= \frac{[W]}{[R^3]} = \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-1}$$

34. A

Sol. **For lens 1** : $f_1 = 10$, $u = -30$, $v = ?$

$$v = \frac{uf}{u+f} = \frac{-30 \times 10}{-30 + 10} = 15$$

For lens 2 : $f_1 = -10$, $u = 10$, $v = ?$

$$v = \frac{uf}{u+f} = \frac{10 \times -10}{10 - 10} = \infty$$

For lens 3 : f = 30, u = – ∞, v = ?
So v will be 30.

35. B

Sol. y = 2 cos 2π(330 t – x) m
y = Acos (ωt – kx)
by comparing ω = 2π × 330
2πf = 2π × 330
f = 330

36. C

Sol. $I_1\omega = I_2\omega_2$

$$\frac{MR^2}{2}\omega = \frac{3}{2}\left(\frac{MR^2}{2}\right)\omega_2$$

$$\omega_2 = \frac{2}{3}\omega$$

37. D

Sol. v_1 = volume immersed in water.
v_2 = volume immersed in oil.
$v_1 \rho_w g + v_2 \rho_o g = (v_1 + v_2) \rho_c g$

$$v_1 + \frac{v_2\rho_o}{\rho_w} = (v_1 + v_2)\frac{\rho_c}{\rho_w}$$

$= v_1 + 0.8\ v_2 = 0.9\ v_1 + 0.9\ v_2$
$= 0.1\ v_1 = 0.1\ v_2$
$v_1 : v_2 = 1 : 1$

38. D

Sol. $$\lambda = \frac{RT}{\sqrt{2}\pi d^2 N_A P}$$

$$KE = \frac{f}{2}nRT$$

39. C

Sol. $$v = \sqrt{\frac{GM}{R}}$$

$$\frac{v_A}{v_B} = \sqrt{\frac{R_B}{R_A}} = \sqrt{\frac{R}{4R}} = \frac{1}{2}$$

$$v_B = 2v_A = 6v$$

40. C

Sol. $$B_1 2\pi\frac{a}{2} = \mu_o\frac{I}{4}$$

$$B_1 = \frac{\mu_o I}{4\pi a}$$

$$B_2 2\pi 2a = \mu_o I$$

$$B_2 = \frac{\mu_o I}{4\pi a}$$

41. B

Sol. $$\frac{u^2\sin 2\theta}{g} = \frac{u^2\sin^2\theta}{2g}$$

4sin θ cosθ = sin2θ
4 = tanθ

42. C

Sol. $$P = \frac{v^2}{R}, R = \frac{\rho l}{A}$$

$$P \propto \frac{1}{l}$$

$$\frac{P_1}{P_2} = \frac{t_2}{t_1} = \frac{15}{20} = \frac{l_2}{l_1}$$

$$l_2 = \frac{3}{4}l_1$$

43. A

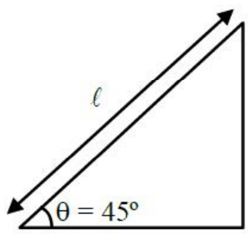
Sol. $$\frac{A\varepsilon_0}{d} = \frac{A\varepsilon_0}{\left(0.2 + \frac{d}{k}\right)}$$

$$0.6 = 0.2 + \frac{0.6}{k}$$

$$k = \frac{3}{2}$$

44. A

Sol.



Case-1 : No friction
a = g sinθ

$$l = \frac{1}{2}(g\sin\theta)t_1^2$$

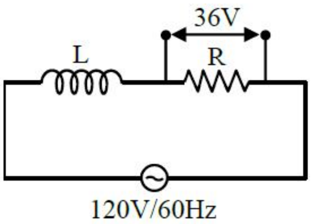
Case-2 : With friction
a = g sinθ - μg cosθ

$$l = \frac{1}{2}(g\sin\theta - \mu g\cos\theta)t_2^2$$

$$\sqrt{\frac{2l}{g\sin\theta - \mu g\cos\theta}} = n\sqrt{\frac{2l}{g\sin\theta}}$$

$$\mu = 1 - \frac{1}{n^2}$$

45. A

Sol. 

$36 = I_{rms} R$

$$36 = \frac{120}{\sqrt{X_L^2 + R^2}} \times R$$

$$R = 90\Omega \Rightarrow 36 = \frac{120 \times 90}{\sqrt{X_L^2 + 90^2}}$$

$$\sqrt{X_L^2 + 90^2} = 300$$

$X_L^2 = 81900$

$X_L = 286.18$

$\omega L = 286.18$

$L = 286.18 / 376.8$

$L = 0.76\ H$

46. B

Sol. Least count $= \frac{1}{100} mm = 0.01\ mm$

zero error = +0.05 mm

Reading = 4 × 1 mm + 60 × 0.01 mm – 0.05 mm = 4.55 mm

47. D

Sol. De Broglie wavelength of proton & electron = λ

$\because \lambda = \frac{h}{p}$

$\therefore\ p_{proton} = p_{electron}$

$\because KE = \frac{p^2}{2m}$

$\therefore\ KE_{proton} < K_{Eelectron}$

$[K_p < K_e]$

48. B

Sol. Least count of vernier calipers $\frac{1}{20N}$cm.

$\because$ Least count = 1 MSD – 1 VSD

let x no. of divisions of main scale coincides with N division of vernier scale, then

$$1\ VSD = \frac{x \times 1mm}{N}$$

$$\therefore \frac{1}{20N} cm = 1mm - \frac{x \times 1mm}{N}$$

$$\frac{1}{2N} mm = 1mm - \frac{x}{N} mm$$

$$x = \left(1 - \frac{1}{2N}\right) N$$

$$x = \frac{2N - 1}{2}$$

49. A

Sol. B.E. = ΔmC^2

$(5\ M_p + 7M_n - M_o)C^2$

50. A

For Isobaric process

$w = P\Delta v = nR\Delta T = 100\ J$

$Q = \Delta u + w$

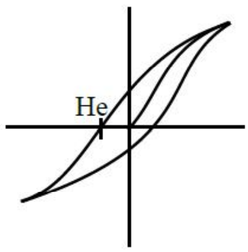
$$\Delta Q = \frac{F}{2} nR\Delta T + nR\Delta T$$

$$\left(\frac{f}{2} + 1\right) nR\Delta T$$

$$\left(\frac{5}{2} + 1\right) 100 = 350J$$

Section - B (Numerical Value)

51. 10

Sol. 

$$H_e = \frac{\mu_0 ni}{\mu_0}$$

$$5 \times 10^3 = \frac{150}{30} \times 100 \times i$$

$$\frac{50}{5} = i$$

$I = 10$

52. 40

Sol. m = mass of small drop

M = mass of bigger drop

$$V_t = \frac{2}{9}\frac{R^2(\rho-\sigma)g}{\eta}$$

$8 \propto m = M$

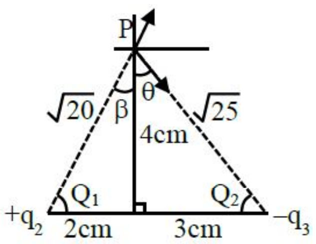
$8r^3 = R^3 \Rightarrow R = 2R$

as $V_t \times R^2$ ∵ Radius double so V_t becomes 4 time

∴ 4 × 10 = 40 cm/s

53. 5

Sol.



$$\frac{Kq_2}{20}\cos\beta = \frac{Kq_3}{25}\cos\theta$$

$$\frac{Kq_2}{20}\frac{4}{\sqrt{20}} = \frac{Kq_3}{25}\frac{4}{\sqrt{25}}$$

$$\frac{q_2}{q_1} = \frac{20}{25}\sqrt{\frac{20}{25}} = \frac{8}{5\sqrt{x}}$$

$$\Rightarrow \sqrt{x} = \frac{8\times 25\sqrt{25}}{5\times 20\sqrt{20}}$$

x = 5

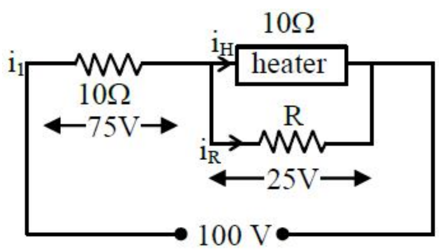
54. 5

Sol. $R_{heater} = \frac{V^2}{P} = \frac{(100)^2}{1000} = 10\Omega$

For heater $P = \frac{V^2}{R} \Rightarrow V = \sqrt{PR}$

$V = \sqrt{62.5\times 10}$

V = 25 v



$$i_1 = \frac{75}{10} = 7.5A,\ i_H = \frac{25}{10} = 2.5A$$

$i_R = i_1 - i_H = 5$

V = IR

$$R = \frac{25}{5} = 5\Omega$$

55. 22

Sol. $C = 2\mu f$; E = $110\sqrt{2}$ sin (100 t)

$$X_C = \frac{1}{\omega c} = \frac{1}{100\times 2\times 10^6}$$

$$= \frac{10000}{2} = 5000\Omega$$

$$i_o = \frac{110\sqrt{2}}{5000}$$

$$i_{rms} = \frac{110\sqrt{2}}{5000\sqrt{2}}$$

$$= \frac{110}{5}mA = 22mA$$

56. 2

Sol. d = 1 mm, D = 1m, λ = 500 nm

$$10\left(\frac{\lambda D}{d}\right) = \frac{2\lambda D}{a}$$

$$a = \frac{d}{5}$$

$$= \frac{10\times 10^{-4}m}{5} = 2\times 10^{-4}$$

57. 6

Sol. Total energy = K.E. + P.E.

at x = 0.04 m, T.E. = 0.5 + 0.4 = 0.9 J

T.E = 1 $m\omega^2A^2$ = 0.9

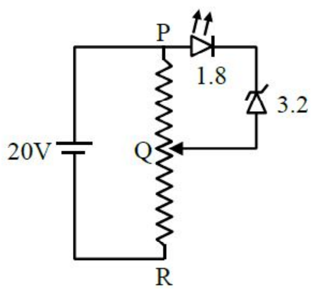
$$= \frac{1}{2}\times 0.2\left(2\pi\times\frac{25}{\pi}\right)^2\times A^2 = 0.9$$

⇒ A = 0.06 m

A = 6 cm

58. 5

Sol.



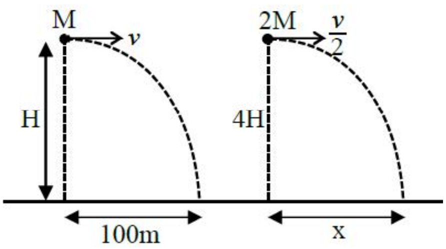
$PR = 20$ cm

$$V_{PQ} = \frac{1}{4} \times R_{PR}$$

$$l_{min}(PQ) = \frac{1}{4} \times 20 = 5cm$$

59. 100

Sol.



$$100 = v\sqrt{\frac{2H}{g}}; \quad x = \frac{v}{2}\sqrt{\frac{2(4H)}{g}} = v\sqrt{\frac{2H}{g}}$$

$\Rightarrow \quad x = 100$

60. 2

Sol. $a_c = \omega^2 x$

$$\frac{vdv}{dx} = \omega^2 x$$

$$\int_0^v vdv = \int_1^3 \omega^2 x dx$$

$$\frac{v^2}{2} = \omega^2\left[\frac{x^2}{2}\right]$$

$$\frac{v^2}{2} = \frac{\omega^2}{2}\left[3^2 - 1^2\right]$$

$$v = 2\sqrt{2}\omega$$

$x = 2$

CHEMISTRY

Section - A (Single Correct Answer)

61. (C)

Sol.:

$$PO_4^{3-} + (NH_4)_2MoO_4 \xrightarrow{H^+} (NH_4)_3PO_4.12MoO_3 \downarrow$$

Or HPO_4^- Ammonium Molybdate   Canary yellow ppt. (Ammonium phopho molybdate)

62. (D)

Sol.: Rate of formation of B is

$$\frac{d[B]}{dt} = k_1[A] - k_2[B]$$

$0 = k_1[A] - k_2[B]$

$$\left(\frac{k_1}{k_2}\right)[A] = [B]$$

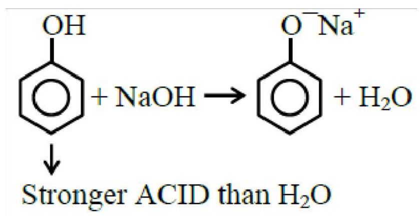
63. (B)

Sol.: Antibonding molecular orbitals are formed by destructive interference of wave functions.

(ABMO) $\sigma^* = \psi_A - \psi_B$

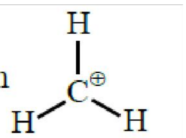
64. (D)

Sol.:



65. (A)

Sol.: Carbocation



Trigonal planar

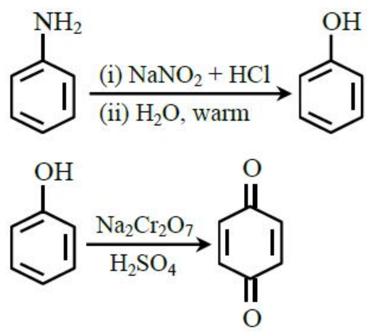
66. (C)

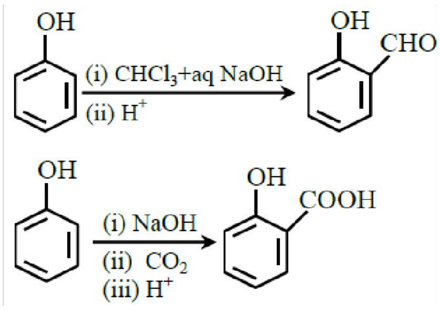
Sol.: $SN^2 \rightarrow$ Inversion

$SN^1 \rightarrow$ Racemisation

67. (D)

Sol.:





68. (D)

Sol.: (A) Bayer's test $\rightarrow$ Unsaturation

(B) Ceric ammonium nitrate test $\rightarrow$ Alcoholic-OH group

(C) Phthalein dye test $\rightarrow$ Phenol

(D) Schiff's test $\rightarrow$ Aldehyde

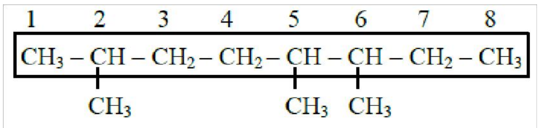
69. (D)

Sol.: (D) Due to inert pair effect lower oxidation state is more stable.

(E) Nitrogen belongs to 2nd period and cannot expand its octet.

70. (C)

Sol.:



2,5,6-Trimethyloctane

71. (B)

Sol.: $Cr_2O_7^{2-} \underset{H^+}{\overset{OH^-}{\rightleftharpoons}} 2CrO_4^{2-}$

72. (A)

Sol.: Buffer solution is a mixture of either weak acid / weak base and its respective conjugate.

Blood is a buffer solution of carbonic acid H_2CO_3 and bicarbonate HCO_3^-

73. (A)

Sol.: CH_3CH_2COOH, CH_3COOH, $CH_3CH_2CH_2COOH$, $HCOOH$

The correct order is :

$HCOOH > CH_3COOH > CH_3CH_2COOH > CH_3CH_2CH_2COOH$

74. (A)

Sol.: Hinsberg test given by 1° amine only.

75. (D)

Sol.:

Anodic reaction	$[T\ell_{(s)} \rightarrow T\ell^+_{(aq)} + e^-]^2$
Cathodic Reaction	$Cu^{+2}_{(aq)} + 2e^- \rightarrow Cu_{(s)}$
Overal Redox Reaction	$2T\ell_{(s)} + Cu^{+2}_{(aq)} \rightarrow 2T\ell^+_{(aq)} + Cu_{(s)}$

$$E_{cell} = E^o_{cell} - \frac{0.0591}{2}\log\frac{[T\ell^+]^2}{[Cu^{+2}]}$$

E_{cell} increases by increasing concentration of $[Cu^{+2}]$

76. (B)

Sol.: As electronegativity increases non-metallic nature increases.

Along the period ionisation energy increases.

High electronegativity difference results in ionic bond formation.

Oxides of metals are generally basic and that of non-metals are acidic in nature

77. (A)

Sol.: Nitrogen present in pyridine can not be estimated by Kjeldahl method as the nitrogen present in pyridine can not be easily converted into ammonium sulphate.

78. (C)

Sol.: Anodic half cell

Gas – gas ion electrode

$$\frac{1}{2}H_{2(g)} \rightarrow H^+_{(aq)} + e^-$$

Cathodic Reaction

Metal-metal insoluble salt anion electrode

$$Ag^+_{(aq)} + e^- \rightarrow Ag_{(s)}$$

$$AgCl_{(s)} \rightleftharpoons Ag^+_{(aq)} + Cl^-_{(aq)}$$

$$AgCl_{(s)} + e^- \rightarrow Ag_{(s)} + Cl^-_{(aq)}$$

Overall redox reaction

$$\frac{1}{2}H_{2(g)} + AgCl_{(s)} \rightarrow H^+_{(aq)} + Cl^-_{(aq)} + Ag_{(s)}$$

Cell Representation

Pt $|H_{2(g)}|kCl_{(sol)}|AgCl_{(s)}|Ag$

79. (A)

Sol.:

$$MnO_2 + 4KOH + O_2 \xrightarrow{\text{fused}} 2K_2MnO_4 + 2H_2O$$

Dark green

Electrolytic oxidation in alkaline medium:

At anode:

$$MnO_4^{2-} \rightarrow MnO_4^- + e^-$$

80. (C)

Sol.: (A) $[Cr(NH_3)_6]^{3+}$

$Cr^{3+} : 3d^3$

n = 3 (unpaired electrons)

$\mu \simeq 3.87\,B.M.(II)$

(B) $[NiCl_4]^{2-}$

$Ni^{2+} : 3d^8$

n = 2

$\mu \simeq 2.83\,B.M.(IV)$

(C) $[CoF_6]^{3-}$

$Co^{3+} : 3d^6$

n = 4

$\mu \simeq 4.90\,B.M.(I)$

(D) $[Ni(CN)_4]^{2-}$

$Ni^{2+} : 3d^8$

n = 0

$\mu = 0\,B.M.(III)$

Section - B (Numerical Value Type)

81. (38)

Sol.: $H_2O(\ell) \rightleftharpoons H_2O(g)$

$\Delta H^0_{vap} = 40.79\,kJ/mole$

$\Delta H^0_{vap} = \Delta U^0_{vap} + \Delta n_g RT$

$$40.79 = \Delta U^0_{vap} + \frac{1\times 8.3\times 373.15}{1000}$$

$\Delta U^0_{vap} = 40.79 - 3.0971$

=37.6929

$\Delta U^0_{vap} \simeq 38$

82. (2)

Sol.: C_2

$(12e^-) : \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2 \left[\pi 2p_x^2 = \pi 2p_y^2\right]$

$$B.O. = \frac{8-4}{2} = 2$$

O_2

$(16e^-) : \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2pz^2$

$\left[\pi 2p_x^2 = \pi 2p_y^2\right]\left[\pi^* 2p_x^1 = \pi^* 2p_y^1\right]$

$$B.O. = \frac{10-6}{2} = 2$$

Be_2

$(8e^-) : \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2$

$$B.O. = \frac{4-4}{2} = 0$$

Li_2

$(6e^-) : \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2$

$$B.O. = \frac{4-2}{2} = 1$$

Ne_2

$(20e^-) : \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2pz^2$

$\left[\pi 2p_x^2 = \pi 2p_y^2\right]\left[\pi^* 2p_x^2 = \pi^* 2p_y^2\right]\sigma^* 2p_z^2$

$$B.O. = \frac{10-10}{2} = 0$$

N_2

$(14e^-) : \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2 \left[\pi 2p_x^2 = \pi 2p_y^2\right]\sigma 2p_z^2$

$$B.O. = \frac{10-4}{2} = 6$$

He_2

$(4e^-) : \sigma 1s^2, \sigma^* 1s^2$

$$B.O. = \frac{2-2}{2} = 0$$

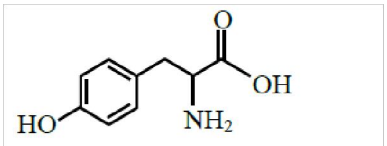
83. (1)

Sol.: $CH_3 - CH_2 - CH - CH_3$ (with Cl on the third carbon)

$$\underset{\displaystyle \mathrm{Cl}}{CH_3 - CH_2 - \underset{|}{CH} - CH_3}$$

84. (9)

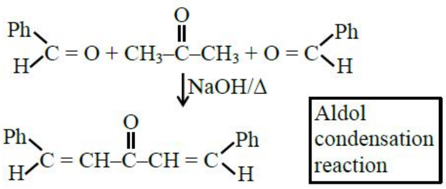
Sol.: Tyrosine



Number of carbon atoms = 9

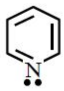
85. (9)

Sol.:



86. (1)

Sol.:



87. (74)

Sol.:Molality of urea is 4.44 m, that means 4.44 moles of urea present in 1000 gm of water

$$\therefore \quad X_{urea} = \frac{4.44}{4.44 + \frac{1000}{18}}$$

$= 0.0740$

OR

74×10^{-3}

$X = 74$

88. (2)

Sol.: $Co^{+3} : 3d^6 t_{2g}^{2,2,2} e_g^{0,0}$

Unpaired e^- = 0

$Ni^{+2} : 3d^8 \quad e^{2,2} \quad t_2^{2,1,1}$

Unpaired e^- = 2

89. (1724)

Sol.: $\bar{\nu}(\text{wave no.}) = \frac{1}{\lambda} = \frac{1}{5800 \times 10^{-8}\,\text{cm}} = 17241$

OR

$1724 \times 10\,\text{cm}^{-1} \Rightarrow x = 1724$

90. (22)

Sol.: Mass percent of Alcohol

$$= \frac{\text{Mass of ethyl alcohol}}{\text{Total mass of solution}} \times 100$$

$$= \frac{1 \times 46}{1 \times 46 + 9 \times 18} \times 100 = \frac{4600}{208}$$

$= 22.11$

Or 22

☯ ☯ ☯