3S INSTITUTE
JEE ADVANCED | JEE MAIN | NEET | OLYMPIADS | MHT-CET | FOUNDATION

08-April-2024 (Morning Batch) : JEE Main Paper

MATHEMATICS

1. D

Sol. $I_n = \int_0^1 (1-x^k)^n \cdot 1dx$

$I_n = (1-x^k)^n \cdot x - nk\int_0^1 (1-x^k)^{n-1} \cdot x^{k-1} \cdot dx$

$I_n = nk\int_0^1 [(1-x^k)^n - (1-x^k)^{n-1}]dx$

$I_n = nkI_n - nkI_n$

$\frac{I_n}{I_{n-1}} = \frac{nk}{nk+1}$

$\frac{I_{21}}{I_{20}} = \frac{21k}{1+21k}$

$= \frac{147}{148} \Rightarrow k = 7$

2. C

Sol. $(8)^{2x} - 16 \cdot (8)^x + 48 = 0$

Put $8^x = t$

$t^2 - 16 + 48 = 0$

$\Rightarrow t = 4$ or $t = 12$

$\Rightarrow 8^x = 4 \qquad 8^x = 12$

$\Rightarrow x = \log_8 x \qquad x = \log_8 12$

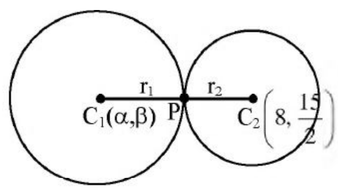
sum of solution $= \log_8 4 + \log_8 12$

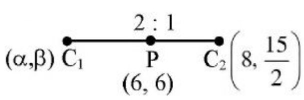
$= \log_8 48 = \log_8(6.8)$

$= 1 + \log_8 6$

3. B

Sol.





$\because \frac{16+\alpha}{3} = 6$ and $\frac{15+\beta}{3} = 6$

$\Rightarrow (\alpha, \beta) \equiv (2, 3)$

Also, $C_1C_2 = r_1 + r_2$

$\Rightarrow \sqrt{(2-8)^2 + \left(3 - \frac{15}{2}\right)^2} = 2r_2 + r_2$

$\Rightarrow r_2 = \frac{5}{2} \Rightarrow r_1 = 2r_2 = 5$

$\therefore (\alpha+\beta) + 4(r_1^2 + r_2^2)$

$= 5 + 4\left(\frac{25}{4} + 25\right) = 130$

4. A

Sol. $P(x, y, z), Q(x, y, O); x^2 + y^2 + z^2 = \gamma^2$

$\overline{OQ} = x\hat{i} + y\hat{j}$

$\cos\phi = \frac{z}{\sqrt{x^2+y^2+z^2}}$

$\Rightarrow \sin\phi = \frac{x^2+y^2}{x^2+y^2+z^2}$

distance of P from x-axis $\sqrt{y^2+z^2}$

$\Rightarrow \sqrt{\gamma^2 - x^2} \Rightarrow \gamma\sqrt{1 - \frac{x^2}{\gamma^2}}$

$\Rightarrow \gamma\sqrt{1 - \cos^2\theta \sin^2\phi}$

5. A

Sol. $f(x) = (x-2)^{2/3}(2x+1)$

$f'(x) = \frac{2}{3}(x-2)^{-1/3}(2x+1) + (x-2)^{2/3}(2)$

$f'(x) = 2 \times \frac{(2x+1)+(x-2)}{3(x-2)^{1/3}}$

$$\frac{3x-1}{(x-2)^{1/3}}=0$$

Critical points $x=\frac{1}{3}$ and $x=2$

6. C

Sol. $\int_0^a f(x)dx = e^{-a}+4a^2+a-1$

$f(a) = -e^{-a} + 8a + 1$

$f(x) = -e^{-x} + 8x + 1$

Now $y = C_1 f(x) + C_2$

$$\frac{dy}{dx}=C_1 f'(x)=C_1(e^{-x}+8) \quad ....(1)$$

$$\frac{d^2y}{dx^2}=-C_1e^{-x} \Rightarrow -e^{x}\frac{d^2y}{dx^2}$$

Put in equation (1)

$$\frac{dy}{dx}=-e^{x}\frac{d^2y}{dx^2}(e^{-x}+8)$$

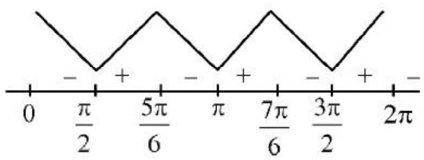
$$(8e^{x}+1)\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$$

7. B

Sol. $f(x)=4\cos^3(x)+3\sqrt{3}\cos^2(x)-10;\ x\in(0,\ 2\pi)$

$\Rightarrow f'(x)=12\cos^2 x[-\sin(x)]+3\sqrt{3}(2\cos(x))[-\sin(x)]$

$\Rightarrow f'(x)=-6\sin(x)\cos(x)[2\cos(x)+\sqrt{3}]$



local maxima at $x=\frac{5\pi}{6},\ \frac{7\pi}{6}$

8. B

Sol. $A^3 - 4A^2 + A + 21I = 0$

$tr(A) = 4 = 5 + 6 \Rightarrow b = -1$

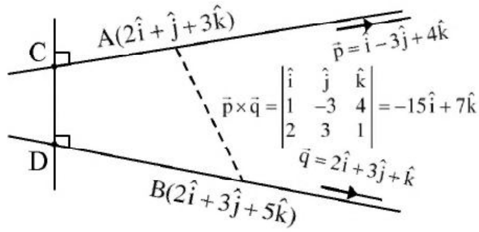
$|A| = -21$

$-16 + a = -21 \Rightarrow a = -5$

$2a + 3b = -13$

9. B

Sol.



Shortest distance $(CD)=\left|\frac{\overline{AB}\cdot\vec{p}\times\vec{q}}{|\vec{p}\times\vec{q}|}\right|$

$$=\left|\frac{(0\hat{i}+2\hat{j}+2\hat{k})\cdot(-15\hat{i}+7\hat{j}+9\hat{k})}{\sqrt{355}}\right|$$

$$=\frac{0+14+18}{\sqrt{355}}=\frac{32}{\sqrt{355}}$$

$\therefore m+n=32+355=387$

10. D

Sol. $x+y=24,\ x,\ y\in N$

$AM > GM \Rightarrow xy \le 144$

$xy \ge 108$

Favorable pairs of (x, y) are

(13, 11), (12, 12), (14, 10), (15, 9), (16, 8), (17, 7), (18, 6), (6, 18), (7, 17), (8, 16), (9, 15), (10, 14), (11, 13) i.e. 13 cases

Total choices for x + y = 24 is 23

Probability $=\frac{13}{23}=\frac{m}{n}$

$n - m = 0$

11. A

Sol. $\sin x=\frac{-3}{5},\ \pi<x<\frac{3\pi}{2}$

$\tan x=\frac{3}{4}\cos x=-\frac{4}{5}$

$80(\tan^2 x - \cos x)$

$$=80\left(\frac{9}{16}+\frac{4}{5}\right)=45+64=109$$

12. B

Sol. $I(x)=\int\frac{6dx}{\sin^2 x(1-\cot x)^2}=\int\frac{6\operatorname{cosec}^2 x dx}{(1-\cot x)^2}$

Put 1 – cot x = t

$\operatorname{cosec}^2 x\,dx = dt$

$$I = \int \frac{6dt}{t^2} = \frac{-6}{t} + c$$

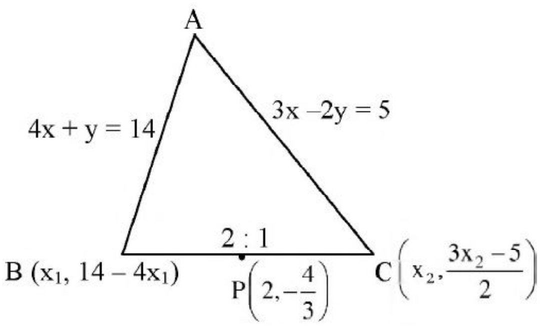
$$I(x) = \frac{-6}{1-\cot x}c,\ c = 3$$

$$I(x) = 3 - \frac{6}{1-\cot x},\ I\left(\frac{\pi}{12}\right) = 3 - \frac{6}{1-(2+\sqrt{3})}$$

$$I\left(\frac{\pi}{12}\right) = 3 + \frac{6}{\sqrt{3}+1} = 3 + \frac{6(\sqrt{3}-1)}{2} = 3\sqrt{3}\sqrt{2}$$

13. C

Sol.



$2x_2 + x_1 = 6,\ 3x_2 - 4x_1 = -13$

$x_2 = 1,\ x_1 = 4$

So, C(1, –1), B(4, –2)

$$m = \frac{-1}{3}$$

Equation of BC : $y + 1 = \frac{-1}{3}(x-1)$

$3y + 3 = -x + 1$

$x + 3y + 2 = 0$

14. B

Sol. $N = 2310 = 231 \times 10$

$= 3 \times 11 \times 7 \times 2 \times 5$

A = {2, 3, 5, 7, 11}

$$f(x) = \left[\log_2\left(x^2 + \left[\frac{x^3}{5}\right]\right)\right]$$

$f(2) = [\log_2 (5)] = 2$

$f(3) = [\log_2 (14)] = 3$

$f(5) = [\log_2 (25 + 25)] = 5$

$f(7) = [\log 2\ (117)] = 6$

$f(11) = [\log_2 387] = 8$

Range of f : B = {2, 3, 5, 6, 8}

No. of one-one functions = 5! = 120

15. D

Sol. $|z+2| = 1,\ \operatorname{Im}\left(\frac{z+1}{z+2}\right) = \frac{1}{5}$

Let $z + 2 = \cos\theta + \sin\theta$

$$\frac{1}{z+2} = \cos\theta - i\sin\theta$$

$$\Rightarrow \frac{z+1}{z+2} = 1 - \frac{1}{z+2} = 1 - (\cos\theta - i\sin\theta)$$

$= (1-\cos\theta) + i\sin\theta$

$$\operatorname{Im}\left(\frac{z+1}{z+2}\right) = \sin\theta,\ \sin\theta = \frac{1}{5}$$

$$\cos\theta = \pm\sqrt{1-\frac{1}{25}} = \pm\frac{2\sqrt{6}}{5}$$

$$|\operatorname{Re}(\overline{z+2})| = \frac{2\sqrt{6}}{5}$$

16. B

Sol. $a + 5b = 42,\ a,\ b \in N$

a = 42 – 5b, b = 1, a = 37

b = 2, a = 32

b = 3, a = 27

⋮

b = 8, a = 2

R has "8" elements ⇒ m = 8

$$\sum_{n=1}^{8}(1 - i^{n!}) = x + iy$$

for $n \geq 4,\ i^{n!} = 1$

$\Rightarrow (1 - i) + (1 - i^{2!}) + (1 - i^{3!})$

= 1 – I + 2 + 1 + 1

= 5 – I = x + iy

m + x + y = 8 + 5 – 1 = 12

17. B

Sol. $f(x) = \cos x - x + 1$

$f'(x) = -\sin x - 1$

f is decreasing $\forall x \in R$

$f(x) = 0$

$f(0) = 2,\ f(\pi) = -\pi$

f is strictly decreasing in $[0, \pi]$ and $f(0) \cdot f(\pi) < 0$

$\Rightarrow$ only one solution of f(x) = 0

S1 is correct and S2 is incorrect.

18. C

Sol. $\vec{a} = \alpha t\hat{i} + 6\hat{j} - 3\hat{k}$

$\vec{b} = t\hat{i} - 2\hat{j} - 2\alpha t\hat{k}$

so $\vec{a} \cdot \vec{b} < 0,\ \forall t \in R$

$\alpha t^2 - 12 + 6\alpha t < 0$

$\alpha t^2 + 6\alpha t - 12 < 0,\ \forall t \in R$

$\alpha < 0$, and $D < 0$

$36\alpha^2 + 48\alpha < 0$

$12\alpha(3\alpha + 4) < 0 \quad \frac{-4}{3} < \alpha < 0$

also for a = 0, $\vec{a} \cdot \vec{b} < 0$

hence a $\alpha \in \left(\frac{-4}{3}, 0\right]$

19. C

Sol. $(1 + y^2) e^{\tan x} dx + \cos^2 x (1 + e^{2 \tan x}) dy = 0$

$$\int \frac{\sec^2 x e^{\tan x}}{1 + e^{2\tan x}} dx + \int \frac{dy}{1 + y^2} = C$$

$\Rightarrow \tan^{-1}(e^{\tan x}) + \tan^{-1} y = C$

for x = 0, y = 1, $\tan^{-1}(1) + \tan^{-1} 1 = C$

$$C = \frac{\pi}{2}$$

$$\tan^{-1}(e^{\tan x}) + \tan^{-1} y = \frac{\pi}{2}$$

Put $x = \pi,\ \tan^{-1} e + \tan^{-1} y = \frac{\pi}{2}$

$\tan^{-1} y = \cot^{-1} e$

$$y = \frac{1}{e}$$

20. B

Sol. $H: \frac{y^2}{b^2} - \frac{x^2}{a^2} = 1,\ e = \sqrt{3}$

$e = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{3} \quad \Rightarrow \frac{a^2}{b^2} = 2$

$$a^2 = 2b^2$$

length of L.R. $= \frac{2a^2}{b} = 4\sqrt{3}$

$$a = \sqrt{6}$$

$P(\alpha, 6)$ lie on $\frac{y^2}{3} - \frac{x^2}{6} = 1$

$12 - \frac{\alpha^2}{6} = 1 \Rightarrow \alpha^2 = 66$

Foci $= (0, \pm be) = (0, 3)$ & $(0, -3)$

Let d_1 & d_2 be focal distance of $P(\alpha, 6)$

$d_1 = \sqrt{\alpha^2 + (6 + be)^2},\ d_2 = \sqrt{\alpha^2 + (6 - be)^2}$

$d_1 = \sqrt{66 + 81},\ d_2 = \sqrt{66 + 9}$

$\beta = d_1 d_2 = \sqrt{147 \times 75} = 105$

$\alpha^2 + \beta = 66 + 105 = 171$

21. 7

Sol. $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 3 & -3 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 6 & -3 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 3 & -6 \\ 6 & -3 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -9 \\ 9 & -9 \end{bmatrix}$$

$$A^5 = \begin{bmatrix} 0 & -9 \\ 9 & -9 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -9 & -9 \\ 9 & -18 \end{bmatrix}$$

$$A^6 = \begin{bmatrix} -9 & -9 \\ 9 & -18 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -27 & 0 \\ 0 & -27 \end{bmatrix}$$

$$A^7 = \begin{bmatrix} -27 & 0 \\ 0 & -27 \end{bmatrix}\begin{bmatrix} -54 & 27 \\ -27 & -27 \end{bmatrix} = \begin{bmatrix} 3^6 \times 2 & -27^2 \\ 27^2 & 3^6 \end{bmatrix}$$

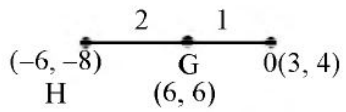
$3^7 = 3^n \Rightarrow n = 7$

22. 16

Sol. $2x + 3y - 1 = 0$

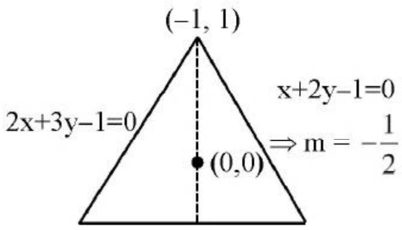
$x + 2y - 1 = 0$

$ax + by - 1 = 0$



$$\left(\frac{6-6}{3}, \frac{8-8}{3}\right)$$

$= (0, 0)$



$ax + by - 1 = 0$

$$\left(\frac{1-0}{-1-0}\right)\left(\frac{-a}{b}\right) = -1$$

$\Rightarrow -a = b$

$\Rightarrow ax - ay - 1 = 0$

$$ax - a\left(1 - \frac{2x}{3}\right) - 1$$

$$x\left(a + \frac{2a}{3}\right) = \frac{a}{3}$$

$$x = \frac{a+3}{5a}$$

$$2\left(\frac{a+3}{5a}\right) = 3y - 1 = 0$$

$$y = \frac{1 - \frac{2a+6}{5a}}{3} = \frac{3a-6}{3 \times 5a}$$

$$y = \frac{a-2}{5a}$$

$$\frac{\left(\frac{a-2}{5a}\right)}{\left(\frac{a+3}{5a}\right)} = 2 \Rightarrow a - 2 = 2a + 6$$

$a = -8$

$b = 8$

$-8x + 8y - 1 = 0$

$|a - b| = 16$

23. 17

Blue balls	0	1	2	3	4	5
Prob.	$\frac{{}^5C_0 \cdot {}^4C_1}{{}^9C_3}$	$\frac{{}^5C_1 \cdot {}^4C_2}{{}^9C_3}$	$\frac{{}^5C_2 \cdot {}^4C_1}{{}^9C_3}$	$\frac{{}^5C_3 \cdot {}^4C_0}{{}^9C_3}$	0	0

$$7\bar{x} = \frac{{}^5C_1\,{}^4C_2 + {}^5C_2 \cdot {}^4C_1 \times 2 + {}^5C_3 \cdot {}^4C_0 \times 3}{{}^9C_3} \times 7$$

$$\frac{30+80+30}{84} \times 7$$

$$= \frac{140}{12} = \frac{70}{6} = \frac{35}{3}$$

yellow	0	1	2	3	4
		${}^5C_2\,{}^4C_1$	${}^5C_1\,{}^4C_2$	${}^5C_0\,{}^4C_3$	0

$$\bar{y} = \frac{40+60+12}{84} \times 4 = \frac{112}{21} = \frac{16}{3}$$

24. 36

Sol. 2, 3, 4, 5, 7

total number of three digit numbers not divisible by 3 will be formed by using the digits

(4, 5, 7)

(3, 4, 7)

(2, 5, 7)

(2, 4, 7)

(2, 4, 5)

(2, 3, 5)

number of ways = 6 × 3! = 36

25. 103

Sol. $S = 1 + 2 + 4 + 7 + ... + T_n$

$S = 1 + 2 + 4 + ...$

$T_n = 1 + 1 + 2 + 3 + ... + (T_n - T_{n-1})$

$$T_n = 1 + \left(\frac{n-1}{2}\right)[2 + (n-2) \times 1]$$

$$T_n = 1 + 1 + \frac{n(n-1)}{2}$$

$$n = 100 \quad T_n = 1 + \frac{100 \times 99}{2} = 4950 + 1$$

$$n = 101 \quad T_n = 1 + \frac{101 \times 100}{2} = 5050 + 1 = 5051$$

$$n = 102 \quad T_n = 1 + \frac{102 \times 101}{2} = 5151 + 1 = 5152$$

$$n = 103 \quad T_n = 1 + \frac{103 \times 102}{2} = 5254$$

$$n = 104 \quad T_n = 1 + \frac{104 \times 103}{2} = 5357$$

26. 96

Sol. $$f(\theta) = \frac{\sin^2\theta + 3\cos^2\theta}{\sin^4\theta + \cos^2\theta}$$

$$f(\theta) = 1 + \frac{2\cos^2\theta}{\sin^4\theta + \cos^2\theta}$$

$$f(\theta) = \frac{2\cos^2\theta}{\cos^4\theta - \cos^2\theta + 1} + 1$$

$$f(\theta) = \frac{2}{\cos^2\theta + \sec^2\theta - 1} + 1$$

$$f(\theta)|_{min} = 1$$

$$f(\theta)_{max} = 3$$

$$S = \frac{64}{1 - 1/3} = 96$$

27. 5

Sol. $$\alpha = \sum_{r=0}^{n} (4r^2 + 2r + 1) \cdot {}^nC_r$$

$$\alpha = 4\sum_{r=0}^{n} r^2 \cdot \frac{n}{r} \cdot {}^{n-1}C_{r-1} + 2\sum_{r=0}^{n} r \cdot \frac{n}{r} \cdot {}^{n-1}C_{r-} + \sum_{r=0}^{n} {}^nC_r$$

$$+4n\sum_{r=0}^{n} {}^{n-1}C_{r-1} + 2n\sum_{r=0}^{n} {}^{n-1}C_{r-1} + \sum_{r=0}^{n} {}^nC_r$$

$$\alpha = 4n(n-1) \cdot 2^{n-2} + 4n \cdot 2^{n-1} + 2n \cdot 2^{n-1} + 2^n$$

$$\alpha = 2^{n-2}[4n(n-1) + 8n + 4n + 4]$$

$$\alpha = 2^{n-2}[4n^2 + 8n + 4]$$

$$\alpha = 2n(n+1)^2$$

$$\beta = \sum_{r=0}^{n} \frac{{}^nC_r}{r+1} + \frac{1}{n+1}$$

$$= \sum_{r=0}^{n} \frac{{}^{n+1}C_{r+1}}{n+1} + \frac{1}{n+1}$$

$$= \frac{1}{n+1}(1 + {}^{n+1}C_1 + ... + {}^{n-1}C_{n+1})$$

$$= \frac{2^{n+1}}{n+1}$$

$$\frac{2\alpha}{\beta} = \frac{2^{n+1}(n+1)^2}{2^{n+1}} \cdot (n+1) = (n+1)^3$$

$140 < (n + 1)^3 < 281$

$n = 4 \Rightarrow (n + 1)^3 = 125$

$n = 5 \Rightarrow (n + 1)^3 = 216$

$n = 6 \Rightarrow (n + 1)^3 = 343$

$\therefore n = 5$

28. 569

Sol. $$\vec{a} = 9\hat{i} - 13\hat{j} + 25\hat{k}$$

$$\vec{b} = 3\hat{i} + 7\hat{j} - 13\hat{k}$$

$$\vec{c} = 17\hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{b} + \vec{c} = 20\hat{i} + 5\hat{j} - 12\hat{k}$$

$$\vec{b} - \vec{c} = -14\hat{i} + 9\hat{j} - 14\hat{k}$$

$$(\vec{r} - (\vec{b} + c)) \times \vec{a} = 0$$

$$r - (\vec{b} + \vec{c}) = \lambda\vec{a}$$

$\vec{r} = \lambda\vec{a} + \vec{b} + \vec{c}$

But $\vec{r}\cdot(\vec{b}-\vec{c}) = 0$

$\Rightarrow (\lambda\vec{a} + \vec{b} + \vec{c})\cdot(\vec{b}-\vec{c}) = 0$

$\Rightarrow \lambda\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{b} + \vec{c}\cdot\vec{b} - \lambda\vec{a}\cdot\vec{c} - \vec{b}\cdot\vec{c} - \vec{c}\cdot\vec{c} = 0$

$$\lambda = \frac{\vec{c}\cdot\vec{c} - \vec{b}\cdot\vec{b}}{\vec{a}\cdot\vec{b} - \vec{a}\cdot\vec{c}} = \frac{294-227}{-389-204} = \frac{-67}{593}$$

$$\therefore \vec{r} = \vec{b} + \vec{c} - \frac{67}{593}\vec{a}$$

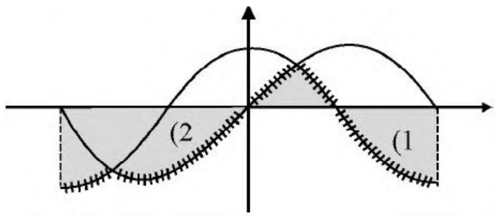
$\Rightarrow 593\vec{r} + 67\vec{a} = 593(\vec{b}+\vec{c})$

$\Rightarrow |\vec{b}+\vec{c}|^2 = 569$

29. 16

Sol. $y = \min\{\sin x, \cos x\}$

x–axis $\quad x - \pi \quad x = \pi$



$$\int_0^{\pi/4} \sin x = (\cos x)_{\pi/4}^{0} = 1 - \frac{1}{\sqrt{2}}$$

$$\int_{-\pi}^{-3\pi/4} (\sin x - \cos x) = (-\cos x - \sin x)_{-\pi}^{-3\pi/4}$$

$$= (\cos x + \sin x)_{-3\pi/4}^{-\pi}$$

$$= (-1+0) - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$$

$$= -1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$

$$\int_{\pi/4}^{\pi/2} \cos x dx = (\sin x)_{\pi/4}^{\pi/2} = 1 - \frac{1}{\sqrt{2}}$$

A = 4

$A^2 = 16$

30. 55

Sol. $$\lim_{x\to 0} 2\left(\frac{1 - \left(1-\frac{x^2}{2!}\right)\left(1-\frac{4x^2}{2!}\right)\left(1-\frac{9x^2}{2!}\right)....\left(1-\frac{100x^2}{2!}\right)}{x^2}\right)$$

By expansion

$$\lim_{x\to 0}\frac{2\left(1-\left(1-\frac{x^2}{2}\right)\right)\left(1-\frac{1}{2}\cdot\frac{4x^2}{2}\right)\left(1-\frac{1}{3}\cdot\frac{9x^2}{2}\right)....\left(1-\frac{1}{10}\cdot\frac{100x^2}{2}\right)}{x^2}$$

$$\lim_{x\to 0}\frac{2\left(1-\frac{x^2}{2}\right)\left(1-\frac{2x^2}{2}\right)\left(1-\frac{3x^2}{2}\right).....\left(1-\frac{10x^2}{2}\right)}{x^2}$$

$$\lim_{x\to 0}\frac{2\left(1-1+x^2\left(\frac{1}{2}+\frac{2}{2}+\frac{3}{2}+.....+\frac{10}{2}\right)\right)}{x^2}$$

$$2\left(\frac{1}{2}+\frac{2}{2}+\frac{3}{2}+.....+\frac{10}{2}\right)$$

$$1+2+.....+10 = \frac{10\times 11}{2} = 55$$

PHYSICS

Section - A (Single Correct Answer)

31. A

Sol. $KE = \dfrac{P^2}{2m}$

$P \propto \sqrt{m}$

Hence, $P_A : P_B : P_C$

$= \sqrt{400} : \sqrt{1200} : \sqrt{1600} = 1 : \sqrt{3} : 2$

32. B

Sol. Pressure $= \dfrac{1}{C} = \dfrac{F}{A}$

$$\Rightarrow \frac{360}{10^{-4}\times 3\times 10^{8}} = \frac{2.4\times 10^{-4}}{A}$$

$\Rightarrow A = 2 \times 10^{-2}\ m^2 = 0.02\ m^2$

33. A

Sol. λ is same for both

$P = h/\lambda$ same for both

$P = \sqrt{2mK}$

Hence,

$$K \propto \frac{1}{m} \quad \Rightarrow \frac{KE_p}{KE_e} = \frac{m_e}{m_p} = \frac{1}{1836}$$

34. C

Sol. $\dfrac{(C_v)_{mono}}{(C_v)_{dia}} = \dfrac{\frac{3}{2}R}{\frac{5}{2}R} = \dfrac{3}{5}$

35. B

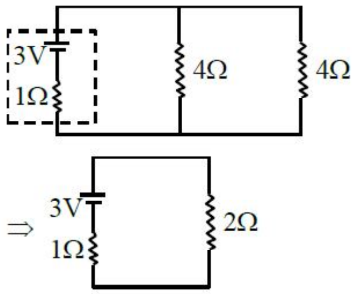
Sol. $a \times 10^b$

if $a \le 5$ order is b

$a > 5$ order is $b + 1$

36. A

Sol.



$i = \dfrac{3}{1+2} = 1A$

$v = E - ir$

$= 3 - 1 \times 1 = 2V$

37. B

Sol. $\Delta mc^2 = 18 \times 10^8$

$\Delta m \times 9 \times 10^{16} = 18 \times 10^8$

$\Delta m = 2 \times 10^{-8}$ kg = 20 μg

38. D

Sol. A, C only

39. A

Sol. $x_{min} = \pi \times r_{min}$

$= \pi \times \dfrac{60}{100} m$

$x_{second} = 30 \times 2\pi \times r_{second}$

$= 30 \times 2\pi \times \dfrac{75}{100}$

$x = x_{second} - x_{min}$

$= 139.4$ m

40. C

Sol. $\dfrac{\Delta Y}{Y} = \dfrac{\Delta m}{m} + \dfrac{\Delta l}{l}$

$= \dfrac{5}{500} + \dfrac{0.02}{2} = 0.01 + 0.01$

$\dfrac{\Delta Y}{Y} = 0.02 \Rightarrow \% \dfrac{\Delta Y}{Y} = 2\%$

41. C

Sol. For adiabatic process

$TV^{\gamma-1}$ = constant

$T_a \cdot V_a^{\gamma-1} = T_d \cdot V_d^{\gamma-1}$

$\left(\dfrac{V_a}{V_d}\right)^{\gamma-1} = \dfrac{T_d}{T_a}$

$T_b \cdot T_b^{\gamma-1} = T_c \cdot T_c^{\gamma-1}$

$\left(\dfrac{V_b}{V_c}\right)^{\gamma-1} = \dfrac{T_c}{T_b}$

$\dfrac{V_a}{V_d} = \dfrac{V_b}{V_c} \quad \left(\begin{matrix} \because T_d = T_c \\ T_a = T_b \end{matrix}\right)$

42. C

Sol. $\dfrac{\pi r_1^2}{T_A} = \dfrac{L}{2m_1}$ .....(1)

$\dfrac{\pi r_2^2}{T_B} = \dfrac{3L}{2m_2}$ ......(2)

$\Rightarrow \dfrac{T_A}{T_B} = 3.\dfrac{m_1}{m_2}.\left(\dfrac{r_1}{r_2}\right)^2$

$\left(\dfrac{T_A}{T_B}\right)^2 = \left(\dfrac{r_1}{r_2}\right)^3 \Rightarrow \left(\dfrac{r_1}{r_2}\right)^2 = \left(\dfrac{T_A}{T_B}\right)^{\frac{4}{3}}$

$\Rightarrow \dfrac{1}{27}.\left(\dfrac{m_2}{m_1}\right)^3 = \left(\dfrac{T_A}{T_B}\right)$

43. B

Sol. In resonance Z = R

I = V/R

R → halved

⇒ I → 2I

I becomes doubled.

44. C

Sol. By truth table

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	0

45. C

Sol. Potential at surface will be same

$$\frac{Kq_1}{a} = \frac{Kq_2}{b}$$

$$\frac{q_1}{q_2} = \frac{a}{b}$$

46. B

Sol. $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$

47. C

Sol. $F = \frac{\Delta P}{\Delta t} = \frac{mv - 0}{0.1}$

$$= \frac{150\times 10^{-3}\times 20}{0.1} = 30N$$

48. A

Sol. Initial momentum is zero.

Hence $|P_A| = |P_B|$

$\Rightarrow \quad m_A v_B = m_B V_B$

$$\frac{(KE)_A}{(KE)_B} = \frac{\frac{1}{2}m_A v_A^2}{\frac{1}{2}m_B v_B^2} = \frac{v_A}{v_B}$$

$$\frac{(KE)_B}{(KE)_A} = \frac{v_B}{v_A}$$

49. A

Sol. $\sin\theta_c = \frac{\mu_R}{\mu_d} = \frac{\mu_2}{\mu_1}$

$$\sin 45° = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \frac{\mu_1}{\mu_2} = \frac{\sqrt{2}}{1}$$

50. D

Sol. Given 9MSD = 10VSD

mass = 8.635 g

LC = 1 MSD – 1 VSD

LC = 1 MSD - 9/10 MSD

LC = 1/10 MSD

LC = 0.01 cm

Reading of diameter = MSR + LC × VSR

= 2 cm + (0.01) × (B) = 2.02 cm

Volume of sphere

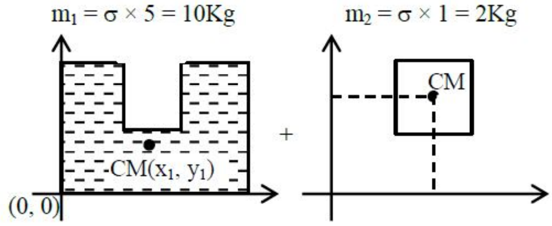
$$= \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{2.02}{2}\right)^3 = 4.32\ cm^3$$

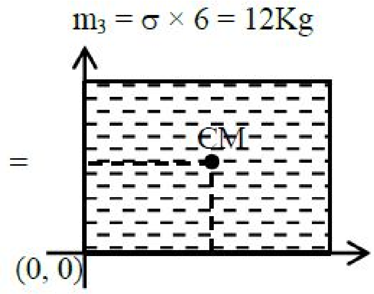
$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{8.635}{4.32} = 1.998 \sim 2.00g$$

Section - B (Numerical Value)

51. 15

Sol. $m_1 = \sigma \times 5 = 10$ Kg





$\Rightarrow \quad m_1 x_1 + m_2 x_2 = m_3 x_3$

$10x_1 + 2(1.5) = 12(1.5)$

$\Rightarrow \quad x_1 = 1.5$ cm

$\Rightarrow \quad m_1 y_1 + m_2 y_2 = m_3 y_3$

$10y_1 + 2(1.5) = 12 \times 1$

$\Rightarrow \quad y_1 = 0.9$ cm

$$\frac{x_1}{y_1} = \frac{15}{0.9} = \frac{15}{9}$$

$n = 15$

52. 4

Sol. For the given condition of moving undeflected, net force should be zero.

qE = qVB

E = VB

$$= \sqrt{\frac{2\times KE}{m}} \times B$$

= 4 N/C

53. 3

Sol. $\in = NBlv$

$$i = \frac{\in}{R} = \frac{NBlv}{R}$$

$$F = N(ilB) = \frac{N^2B^2l^2v}{R}$$

$$W = F \times l = \frac{N^2B^2l^3}{R}\left(\frac{l}{t}\right)$$

$A = l^2$

$$W = \frac{(10\times10)(0.5)^2\times(3.6\times10^{-3})^2}{100\times1}$$

$W = 3.24 \times 10^{-6}$ J

54. 748

Sol. $R = R_0(1 + \alpha\Delta T)$

$$\frac{\Delta R}{R_0} = \alpha\Delta T$$

Case-I

$0°C \to 100°C$

$$\frac{10.2-10}{10} = \alpha(100-0) \quad .....(1)$$

Case-II

$0°C \to t°C$

$$\frac{10.95-10}{10} = \alpha(t-0) \quad ....(2)$$

$$\Rightarrow \frac{t}{100} = \frac{0.95}{0.2} = 475°C$$

$t = 475 + 273 = 748$ K

55. 12

Sol. $\phi = \vec{E}\cdot\vec{A}$

$$= \left(\frac{2\hat{i}+6\hat{j}+8\hat{k}}{\sqrt{6}}\right)\cdot 4\left(\frac{2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}}\right)$$

$$= \frac{4}{6}\times(4+6+8) = 12\text{Vm}$$

56. 7

Sol. $\rho gh = \frac{4S}{R}$

$$\Rightarrow R = \frac{4\times0.28}{8\times10^3\times10\times4\times10^{-4}}$$

$$\Rightarrow \frac{0.28}{8}\text{m} = \frac{28}{8}\text{cm}$$

$\Rightarrow R = 3.5$ cm

Diameter = 7 cm

57. 16

Sol. For closed organ pipe

$$f_c = (2n+1)\frac{v}{4l} = \frac{15v}{4l}$$

For open organ pipe

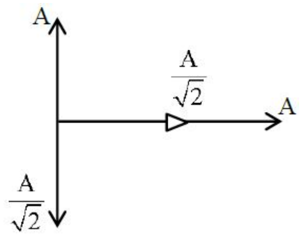
$$f_o = (n+1)\frac{v}{2l} = \frac{8v}{2l}$$

$$\frac{f_c}{f_o} = \frac{15}{16} = \frac{a-1}{a}$$

$\Rightarrow \quad a = 16$

58. 3

Sol.



$$\vec{R} = \left(A+\frac{A}{\sqrt{2}}\right)\hat{i} + \left(A-\frac{A}{\sqrt{2}}\right)\hat{j}$$

59. 6

Sol. $\sin\theta \simeq \theta \simeq \frac{2\lambda}{b}$

$$= \frac{2\times600\times10^{-9}}{4\times10^{-4}} = 3\times10^{-3}\text{rad}$$

Total divergence = $(3 + 3) \times 10^{-3} = 6 \times 10^{-3}$ rad

60. 156

Sol. $v = \sqrt{\frac{4KZe^2}{mr_{min}}}$

$$= \sqrt{\frac{4\times9\times10^9\times80}{6.72\times10^{-27}\times4.5\times10^{-14}}}\times1.6\times10^{-19}$$

$= 9.759 \times 10^{25} \times 1.6 \times 10^{-19}$

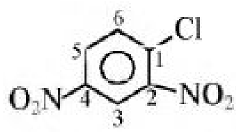
$= 156 \times 10^5$ m/s

CHEMISTRY

Section - A (Single Correct Answer)

61. (B)

Sol.Statement-I:

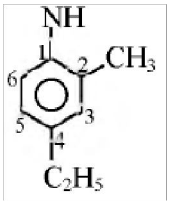


IUPAC name

$\Rightarrow$ 1-chloro-2, 4-dinitrobenzene

$\Rightarrow$ statement-I is incorrect

Statement-II:

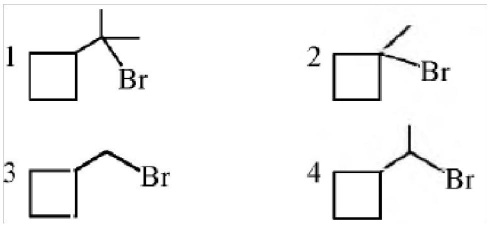


$\Rightarrow$ 4-ethyl-2-methylaniline

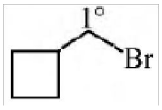
$\Rightarrow$ statement-II is correct

62. (C)

Sol.:



fastest SN^2 reaction give



Rate of SN^2 is Me – x > 1° – x > 2° – x > 3° – x

63. (B)

Sol.: $C_6H_{12}O_{6(s)} + 6O_{2(g)} \longrightarrow 6CO_{2(g)} + 6H_2O_{(\ell)}$

$\frac{900}{180}$

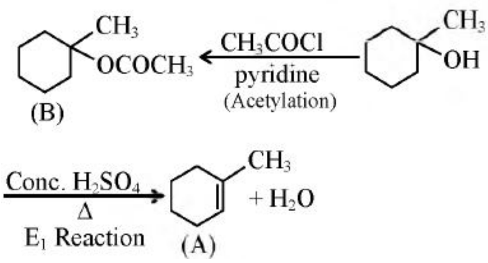
= 5 mol 30 mol

Mass of O_2 required

= 30 ×32 = 960 gm

64. (A)

Sol.:



65. (A)

Sol.: The relative stability of +1 oxidation state progressively increases for heavier elements due to inert pair effect.

$\therefore$ Stability of $A\ell^{+1} < Ga^{+1} < In^{+1} < T\ell^{+1}$

66. (D)

Sol.: Cobalt nitrate test

$MCO_3 \rightarrow MO \xrightarrow[+\Delta]{Co(NO_3)_2} CoO.\ MO$

Flame test

$MCO_3 \rightarrow MCl_2 \rightarrow M^{2+}$

Borax Bead test

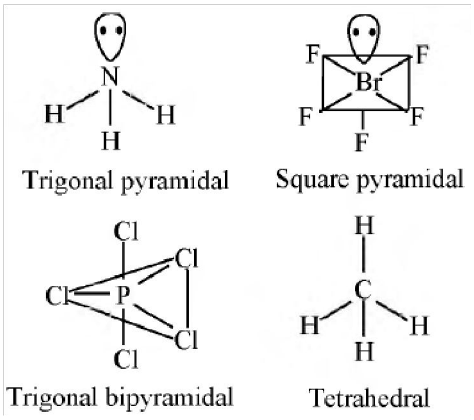
$MSO_4 \xrightarrow[\Delta]{Na_2B_4O_7} M(BO_2)_2 \rightarrow MBO_2 \rightarrow M$

Charcoal cavity test

$MSP_4 \xrightarrow[\Delta]{Na_2CO_3} MCO_3 \rightarrow MO \rightarrow M$

67. (C)

Sol.:



68. (C)

Sol.: $X \rightleftharpoons Y$ $k_1 = 1$

$Y \rightleftharpoons Z$ $k_2 = 2$

$Z \rightleftharpoons \omega$ $k_3 = 4$

$X \rightleftharpoons \omega$ $k_1.k_2.k_3$

$k = 1 \times 2 \times 4$

$k = 8$

69. (C)

Sol.: In the reaction of $S_2O_3^{2-}$ with I_2, oxidation state of sulphur changes to +2 to +2.5

In the reaction of $S_2O_3^{2-}$ with Br_2, oxidation state of sulphur changes from +2 to +6.

$\therefore$ Both I_2 and Br_2 are oxidant (oxidising agent) and Br_2 is stronger oxidant than I_2.

70. (C)

$[\overset{+3}{C}o(NH_3)_5Cl]Cl_2 + excessAgNO_3 \rightarrow 2AgCl$ (2 moles)

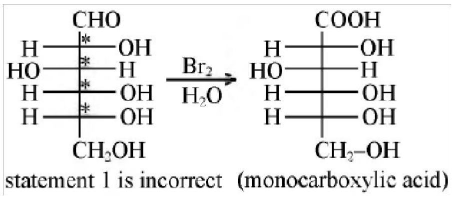
$x + 0 - 1 - 2 = 0$

$x = +3$

$n = 5$

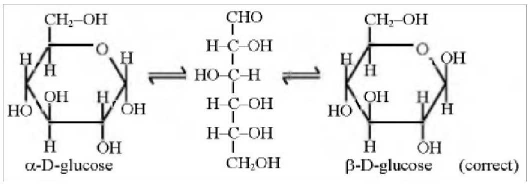
$\therefore \quad x + n = 8$

71. (A)

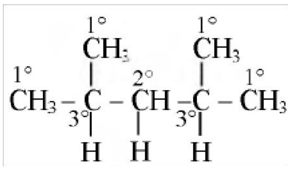
Sol.: (A) 

(B) Correct

(C) c.c. is D (correct)

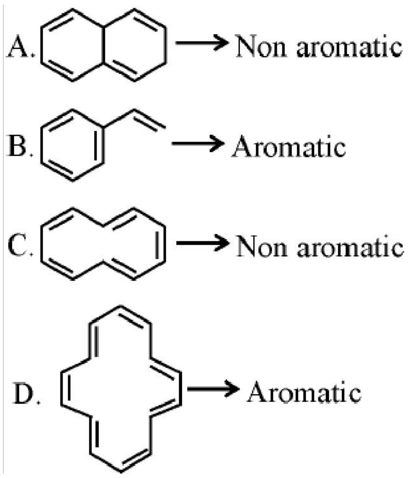
(D) 

72. (B)

Sol.: 

only one 2° carbon is present in this compound.

73. (A)

Sol.: 

74. (B)

Sol.: F_2 do not disproportionate because fluorine do not exist in positive oxidation state however Cl_2, Br_2 & I_2 undergoes disproportionation.

75. (C)

Sol.: $N(CH_3)_3$ and $P(CH_3)_3$ both are Lewis base and acts as ligand, However, $P(CH_3)_3$ has a π-acceptor character.

76. (C)

Sol.: Elements with highest electronegativity → F, O
Elements with largest atomic size → Fr, Ra
Elements which shows properties of both metal and non-metals i.e. metalloids → Ge, As
Elements with highest negative electron gain enthalpy → Cl, S.

77. (D)

$2Fe^{3+} + 2I^- \longrightarrow 2Fe^{2+} + I_2$

$2Fe^{2+} + S_2O_8^{2-} \longrightarrow 2Fe^{3+} + 2SO_4^{2-}$

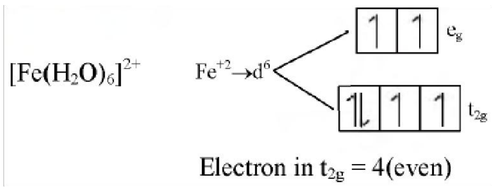
Fe^{+3} oxidises I^- to I_2 and convert itself into Fe^{+2}.

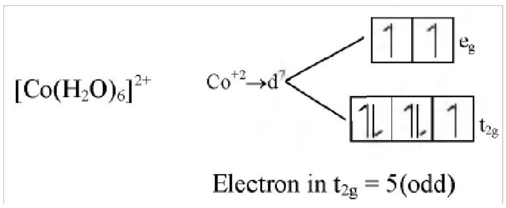
This Fe^{+2} reduces $S_2O_8^{2-}$ to SO_4^{2-} and converts itself into Fe^{+3}.

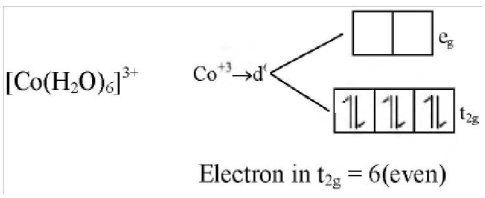
78. (A)

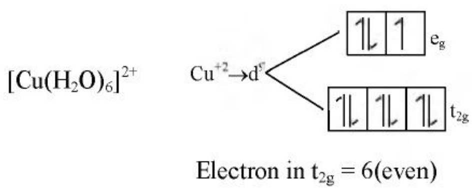
Sol.: $Fe_4[Fe(CN)_6]_3 .xH_2O$ → Prussian Blue
$[Fe(CN)_5NOS]^{4-}$ → Violet
$[Fe(SCN)]^{2+}$ → Blood Red
$(NH_4)_3PO_4.12MoO_3$ → Yellow

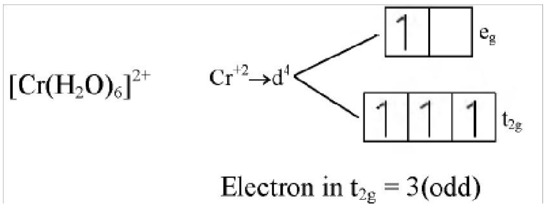
79. (B)

Sol.: 



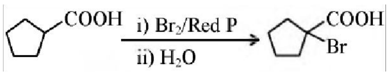






80. (A)

Sol.: HVZ Reaction



Section - B (Numerical Value Type)

81. (5)

Sol.: $\lambda = 1.5 \times 4pm$

$= 6 \times 10^{-12}$ meter

$\lambda v = C$

$6 \times 10^{-12} \times v = 3 \times 10^{8}$

$v = 5 \times 10^{19}$ Hz

82. (55)

Sol.: $\omega = -nRT \ln\left(\frac{V_2}{V_1}\right)$

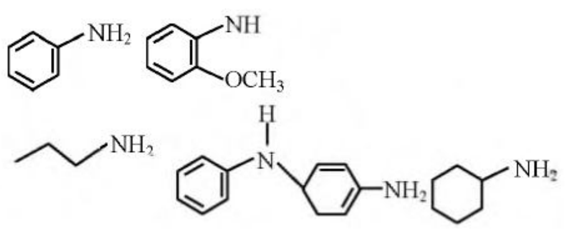
$= -1 \times 0.8206 \times 291.15 \ln\left(\frac{100}{10}\right)$

$= -55.0128$

Work done by system ≈ 55 atm lit.

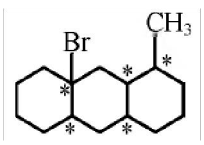
83. (5)

Sol.: Primary amine give an ionic solid upon reaction with Hinsberg reagent which is soluble in NaOH.



84. (32)

Sol.:



Total chiral centre = 5

No. of optical isomers = $2^5 = 32$.

85. (0)

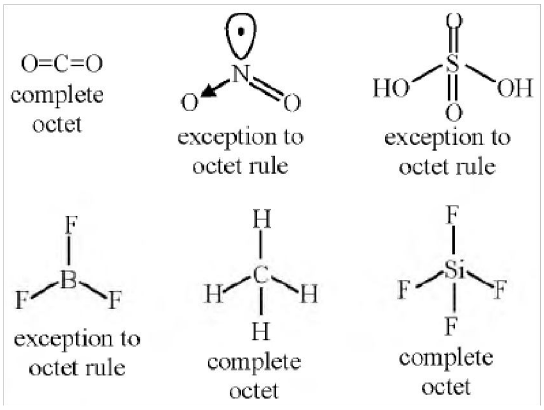
Sol.: Metal having least metallic radii among Sc, Ti, V, Cr, Mn & Zn is Cr.

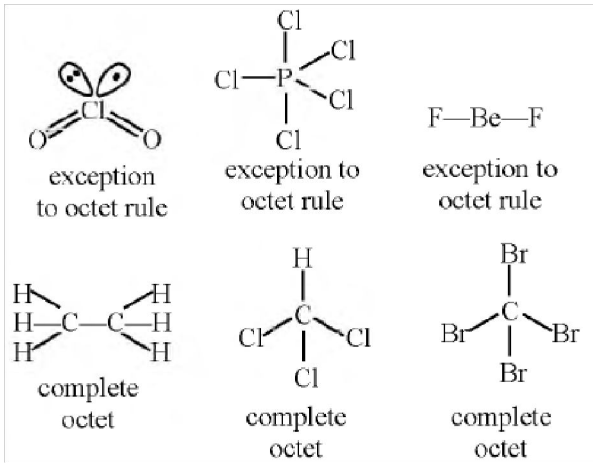
Spin only magnetic moment of CrO_4^{2-}.

Here Cr^{+6} is in d^0 configuration (diamagnetic).

86. (6)

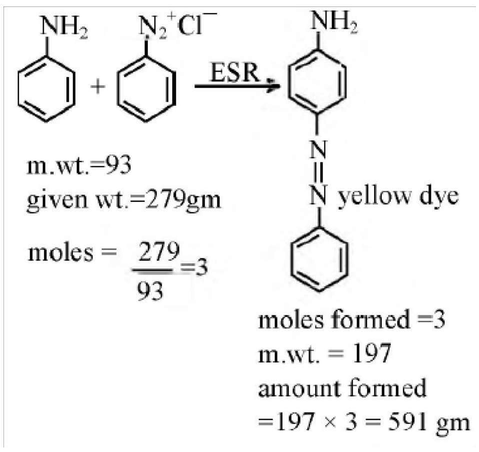
Sol.:





87. (591)

Sol.:

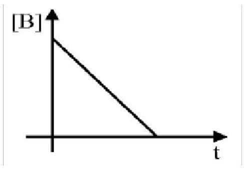


88. (1)

Sol.: For 1st order reaction

75% life = 2 × 50% life

So order with respect to A will be first order.

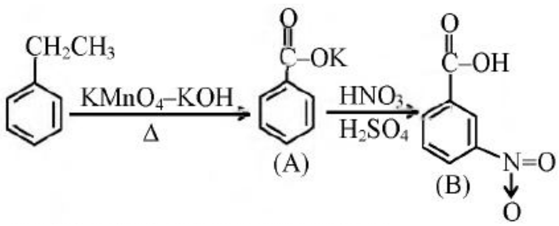


So order with respect to B will be zero.

Overall order of reaction = 1 + 0 = 1

89. (5)

Mojor product B is $\rightarrow$



Total number of π bonds in B are 5

90. (5)

Sol.: $AB_2 \rightarrow A^{+2} + 2B$

$i = 1 + (3 - 1)\alpha$

$i = 1 + 2\alpha$

$\Delta T_b = k_b \, im$

$$0.52 = 0.52\,(1 + 2\alpha)\frac{\frac{10}{200}}{\frac{100}{1000}}$$

$$1 = (1 + 2\alpha)\frac{10}{20}$$

$2 = 1 + 2\alpha$

$\alpha = 0.5$

Ans: $\alpha = 5 \times 10^{-1}$

☯ ☯ ☯