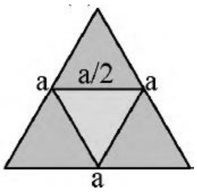
3S INSTITUTE
JEE ADVANCED | JEE MAIN | NEET | OLYMPIADS | MHT-CET | FOUNDATION

06-April-2024 (Evening Batch) : JEE Main Paper

MATHEMATICS

1. A

Sol.



Area of first $\Delta = \frac{\sqrt{3}a^2}{4}$

Area of second $\Delta = \frac{\sqrt{3}a^2}{4} \frac{a^2}{4} = \frac{\sqrt{3}a^2}{16}$

Area of third $\Delta = \frac{\sqrt{3}a^2}{64}$

sum of area $= \frac{\sqrt{3}a^2}{4}\left(1 + \frac{1}{4} + \frac{1}{16}......\right)$

$Q = \frac{\sqrt{3}a^2}{4} - \frac{1}{\frac{3}{4}} = \frac{a^2}{\sqrt{3}}$

perimeter of 1^{st} $\Delta = 3a$

perimetr of 2^{nd} $\Delta = \frac{3a}{2}$

perimeter of 3^{rd} $\Delta = \frac{3a}{4}$

$P = 3a\left(1 + \frac{1}{2} + \frac{1}{4} + ...\right)$

$P = 3a.2 = 6a$

$a = \frac{P}{6}$

$Q = \frac{1}{\sqrt{3}} \cdot \frac{P^2}{36}$

$P^2 = 36\sqrt{3}Q$

2. C

Sol. Given : $4x \le 5y$

then

R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 2), (2, 3), (2, 4)

(2, 5), (3, 3), (3, 4), (3, 5), (4, 4), (4, 5), (5, 4), (5, 5)}

i.e. 16 elements.

i.e. $m = 16$

Now to make R a symmetric relation add

{(2, 1)(3, 2)(4, 3)(3, 1)(4, 2)(5, 3)(4, 1)(5, 2)(5, 1)}

i.e. $n = 9$

So $m + n = 25$

3. A

Sol. Total method = 5^3

faverable = ${}^5C_2\,(2^3 - 2) = 60$

probability $= \frac{60}{125} = \frac{12}{25}$

4. C

Sol. $\frac{dy}{dx} = \frac{(2+\alpha)x - \beta y + 2}{\beta x - y(2\alpha + \beta) + 4\alpha}$

$\beta xdy - (2\alpha + \beta)ydy + 4\alpha dy = (2+\alpha)xdx - \beta ydx + 2dx$

$\beta(xdy + ydx) - (2\alpha + \beta)ydy + 4\alpha dy = (2+\alpha)xdx + 2dx$

$\beta xy - \frac{(2\alpha + \beta)y^2}{2} + 4\alpha y = \frac{(2+\alpha)x^2}{2}$

$\Rightarrow \beta = 0$ for this to be circle

$(2+\alpha)\frac{x^2}{2} + \alpha y^2 + 2x - 4\alpha y = 0$

coeff. of $x^2 = y^2$ $\rangle$ $2 + a = 2a$

$\Rightarrow \boxed{\alpha = 2}$

i.e. $2x^2 + 2y^2 + 2x - 8y = 0$

$x^2 + y^2 + x - 4y = 0$

$$rd = \sqrt{\frac{1}{4}+4} = \frac{\sqrt{17}}{2}$$

5. C

Sol. let P(x, y)

$$\frac{(x-2)^2+(y-1)^2}{(x-1)^2+(y-3)^2} = \frac{25}{16}$$

$9x^2 + 9y^2 + 14x - 118y + 170 = 0$

$a^2 + 2b + 3c + 4d + e$

$= 81 + 18 + 0 + 56 - 118$

$= 155 - 118$

$= 37$

6. B

Sol. $$\lim_{n\to\infty} \frac{\sum_{r=1}^{n-1}(r^2-r)(n-r)}{\sum_{r=1}^{n} r^3 - \sum_{r=1}^{n} r^2}$$

$$\lim_{n\to\infty} \frac{\sum_{r=1}^{n-1}(-r^3+r^2(n+1)-nr)}{\left(\frac{n(n+1)}{2}\right)^2 - \frac{n(n+1)(2n+1)}{6}}$$

$$\lim_{n\to\infty} \frac{\left(\frac{((n-1)n)}{2}\right)^2 + \frac{(n+1)(n-1)n(2n-1)}{6} - \frac{n^2(n-1)}{2}}{\frac{n(n+1)}{2}\left(\frac{n(n+1)}{2} - \frac{2n+1}{3}\right)}$$

$$\lim_{n\to\infty} \frac{\frac{n(n-1)}{2}\left(\frac{-n(n-1)}{2} + \frac{(n+1)(2n-1)}{3} - n\right)}{\frac{n(n+1)}{2}\frac{3n^2+3n-4n-2}{6}}$$

$$\lim_{n\to\infty} \frac{(n-1)(-3n^2+3n+2(2n^2+n-1)-6)}{(n+1)(3n^2-n-2)}$$

$$\lim_{n\to\infty} \frac{(n-1)(n^2+5n-8)}{(n+1)(3n^2-n-2)} = \frac{1}{3}$$

7. C

Sol. $$\frac{{}^{n+1}C_r}{{}^{n}C_r} = \frac{55}{35}$$

$$\frac{(n+1)!}{(r+1)!(n-r)}! \frac{r!(n-r)!}{n!} = \frac{11}{7}$$

$$\frac{(n+1)}{r+1} = \frac{11}{7}$$

$7n = 4 + 11r$

$$\frac{{}^{n}C_r}{{}^{n-1}C_{r-1}} = \frac{(r-1)!(n-r)!}{(n-1)!} = \frac{5}{3}$$

$$\frac{n}{r} > \frac{5}{3}$$

$3n = 5r$

By solving $r = 6$ $n = 10$

$2n + 5r = 50$

8. B

Sol. $17m = m + (m-4) + (m-4\times 2)...+...(m-4\times 24)$

$17m = 25m - 4\ (1 + 2\ ...24)$

$$8m = \frac{4\cdot 25\cdot 25}{2} = 150$$

9. A

Sol. $$\frac{z_1 - 2z_2}{\frac{1}{2} - z_1\bar{z}_2} \times \frac{\bar{z}_1 - 2\bar{z}_2}{\frac{1}{2} - \bar{z}_1 z_2}$$

$|z_1|^2\ 2z_1\bar{z}_2 - 2\bar{z}_1 z_2 + 4|z_2|^2$

$$= 4\left(\frac{1}{4} - \frac{\bar{z}_1 z_2}{2} - \frac{z_1\bar{z}_2}{2} + |z_1|^2|z_2|^2\right)$$

$z_1\bar{z}_1 + 2z_2\cdot 2z_2 - z_1\bar{z}_1 2z_2\, 2\bar{z}_2 - 1 = 0$

$(z,\ \bar{z}_1 - 1)(1 - 2z_2\cdot 2\bar{z}_2) = 0$

$(|z_1|^2 - 1)\ (|2z_2|^2 - 1) = 0$

10. C

Sol. Let $y = \left(\frac{1}{x}\right)^{2x}$

$$\ell ny = 2x\ell n\left(\frac{1}{x}\right)$$

$\ell ny = -2x\ell nx$

$\frac{1}{y}\frac{dy}{dx} = -2(1+\ell nx)$

for $x > \frac{1}{e}$ f" is decreasing

so, $e < \pi$

$\left(\frac{1}{e}\right)^{2e} > \left(\frac{1}{\pi}\right)^{2\pi}$

$e^{\pi} > \pi^{e}$

11. D

Sol. $|(\vec{a}\times\vec{b}\times\vec{c})| = |\vec{a}\times\vec{b}||\vec{c}|\frac{\sqrt{3}}{2}$

$|\vec{c}-\vec{a}| = 2\sqrt{2}$

$|c|^2 + |a|^2 - 2\vec{c}\cdot\vec{a} = 8$

$|z|^2 + 38 - 12|z| = 8$

$|z|^2 - 12|d + 30 = 0$

$|z| = \frac{12 \pm \sqrt{144-120}}{2}$

$= \frac{12 \pm 2\sqrt{6}}{2}$

$|z| = 6+\sqrt{6}$

$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{\ell} & \hat{j} & \hat{k} \\ 6 & 1 & -1 \\ 1 & 1 & 0 \end{vmatrix}$

$\hat{\ell} - \hat{j} + 5\hat{k}$

$|\vec{a}\times\vec{b}| = \sqrt{27}$

$|(\vec{a}\times b)\times z| = \sqrt{27}(6+\sqrt{6})\frac{\sqrt{3}}{2}$

$\frac{9}{2}(6+\sqrt{6})$

12. C

Sol. NAGPUR

A $\rightarrow$ 5! = 120	
G $\rightarrow$ 5! = 120	240
NA $\rightarrow$ 4! = 24	264
NG $\rightarrow$ 4! = 24	288
NP $\rightarrow$ 4! = 24	312
NRAGPU = 1	313
NRAGUP	314
NRAPGU	315

13. D

Sol. $g(x) = h(e^x)\cdot e^{h(x)}$

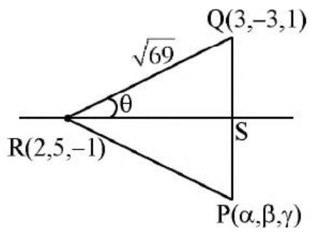
$g'(x) = h(e^x)\cdot e^{h(x)}\cdot h'(x) + e^{h(x)}\, h'(ex)\cdot e^x$

$g'(0) = h(1)e^{h(0)}\, h'(0) + e^{h(0)}\, h'(1)$

$= 2 + 2 = 4$

14. D

Sol.



$RQ = \sqrt{1+64+4} = \sqrt{69}$

$\overrightarrow{RQ} = \hat{\ell} - 8\hat{j} + 2\hat{k}$

$\overrightarrow{RS} = \hat{\ell} + \hat{j} - \hat{k}$

$\cos\theta = \frac{\overrightarrow{RQ}\cdot\overrightarrow{RS}}{|\overrightarrow{RQ}||\overrightarrow{RS}|} = \left|\frac{1-8-2}{\sqrt{69}\sqrt{3}}\right| = \frac{9}{3\sqrt{23}}$

$\cos\theta = \frac{3}{\sqrt{23}} = \frac{RS}{RQ} = \frac{RS}{\sqrt{69}}$

$RS = 3\sqrt{3}$

$\sin\theta = \frac{\sqrt{14}}{\sqrt{23}} = \frac{QS}{\sqrt{69}}$

$QS = \sqrt{42}$

$\text{area} = \frac{1}{2}\cdot 2QS\cdot RS = \sqrt{42}\cdot 3\sqrt{3}$

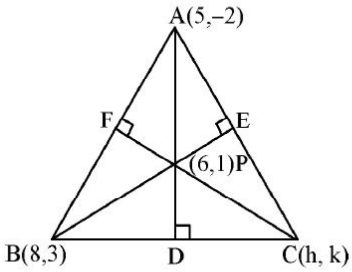
$\lambda = 9\sqrt{14}$

$\lambda^2 = 81.14 = 14k$

$k = 81$

15. A

Sol.



Slope of AD = 3

Slope of $BC = -\frac{1}{3}$

equation of BC = 3y + x – 17 = 0

slope of BE = 1

Slope of AC = –1

equation of AC is x + y – 3 = 0

point C is (–4, 7)

16. D

Sol. $\sin 5x \in [-1,\ 1]$

$-\sin 5x \in [-1,\ 1]$

$7 - \sin 5x \in [6,\ 8]$

$\frac{1}{7-\sin 5x} \in \left[\frac{1}{8},\ \frac{1}{6}\right]$

17. B

Sol. $\vec{a} \times (\hat{i}+\hat{j}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 1 & 0 \end{vmatrix}$

$= \hat{i} - \hat{j} + \hat{k}$

$(\vec{a} \times (\hat{i} \times \hat{j})) \times \hat{i} = \hat{k} + \hat{j}$

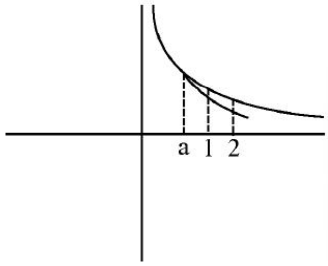
$((\vec{a} \times (\hat{i} \times \hat{j})) \times i) \times \hat{i} = \hat{j} - \hat{k}$

projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$= \frac{1+1}{\sqrt{2}} = \sqrt{2}$

18. C

Sol.



area $\int_1^2 \left(\frac{1}{x} - \frac{a}{x^2}\right) dx$

$\left[\ell nx + \frac{a}{x}\right]_1^2$

$\ell n2 + \frac{a}{2} - a = \log_e 2 - \frac{1}{7}$

$\frac{-a}{2} = -\frac{1}{7}$

$a = \frac{2}{7}$

$7a = 2$

$7a - 3 = -1$

19. A

Sol. $\int \frac{\sec^2 x dx}{a^2 \tan^2 x + b^2}$

let tan x = t

$\sec^2 dx = dt$

$\int \frac{dt}{a^2 t^2 + b^2}$

$\frac{1}{a^2} \int \frac{dt}{t^2 + \left(\frac{b}{a}\right)^2}$

$\frac{1}{a^2} \frac{1}{\frac{b}{a}} \tan^{-1}\left(\frac{t}{b} a\right) + c$

$\frac{1}{ab} \tan^{-1}\left(\frac{\alpha}{b} \tan x\right) + c$

on comparing $\frac{a}{b}=3$

ab = 12

a = 6, b = 2

maximum value of

6 sin x + 2 cos x is $\sqrt{40}$

20. C

Sol. |A| = 3

$|adj(-4\ adj(-3adj\ (3adj((2A)^{-1}))))|$

$|-4adj(-3adj(3adj(\ 2A)^{-1})|^2$

$4^6\ |adj(-3adj(3adj(2A)^{-1}))|^2$

$2^{12}\cdot 3^{12}|3adj(2A)^{-1}|^8$

$2^{12}\cdot 3^{12}\cdot 3^{24}\ |adj(2A)^{-1}|^8$

$2^{12}\cdot 3^{36}\ |(2A)^{-1}|^{16}$

$$2^{12}\cdot 3^{36}\frac{1}{|2A|^{16}}$$

$$2^{12}\cdot 3^{36}\frac{1}{2^{48}|A|^{16}}$$

$$2^{12}\cdot 3^{36}\frac{1}{2^{48}\cdot 3^{16}}$$

$$\frac{3^{20}}{2^{36}}=2^{-36}\cdot 3^{20}$$

m = –36 n = 20

m + 2n = 4

21. 17

Sol. $\left[\frac{x}{2}+3\right]$ is discontinuous at x = 2, 4, 6, 8

$\sqrt{x}$ is discontinuous at x = 1, 4

F(x) is discontinuous at x = 1, 2, 6, 8

$\sum a=1+2+6+8=17$

22. 61

Sol. Given $\frac{2b^2}{a}=9$ and $\frac{a}{e}=\pm\frac{4}{\sqrt{3}}$

equation of tangent $y-\sqrt{3}x+\sqrt{3}=0$

be equation of tangent

Let slope $=S=\sqrt{3}$

Constant $=-\sqrt{3}$

By condition of tangency

$\Rightarrow 6=6a^2-9a$

$\Rightarrow a=2,\ b^2=9$

Equation of Hyperbola is

$\frac{x^2}{4}-\frac{y^2}{9}=1$ and for tangent

Point of contact is $(4,\ 3\sqrt{3})=(x_0,\ y_0)$

Now $e=\sqrt{1+\frac{9}{4}}=\frac{\sqrt{13}}{2}$

Again product of focal distances

$m=(x_0e+a)\ (x_0e-a)$

$m+4e^2=20e^2-a^2$

(There is a printing mistake in the equation of directrix $x=\pm\frac{4}{\sqrt{3}}$.

Corrected equation is $x=\pm\frac{4}{\sqrt{13}}$ for directrix, as eccentricity must be greater than one, so question must be bonus)

23. 3660

Sol. $S(x)=(1+x)+2(1+x)^2+3(1+x)^3+.....+60(1+x)^{60}$ $(1+x)S=(1+x)^2+........59(1+x)^{60}+60(1+x)^{61}$

$$-xS=\frac{(1+x)(1+x)^{60}-1}{x}-60(1+x)^{61}$$

Put x = 60

$$-60S=\frac{61((61)^{60}-1)}{60}-60(61)^{61}$$

on solving 3660

24. 23

Sol. $\int_0^3[x^2]dx+\int_0^3\left[\frac{x^2}{2}\right]dx$

$$=\int_0^1 0dx+\int_1^{12}1dx+\int_{\sqrt{2}}^{\sqrt{3}}2dx$$

$$+\int_{\sqrt{3}}^{2} 3dx + \int_{2}^{\sqrt{5}} 4dx + \int_{\sqrt{5}}^{\sqrt{6}} 5dx$$

$$+\int_{\sqrt{6}}^{\sqrt{7}} 6dx + \int_{\sqrt{7}}^{\sqrt{6}} 7dx + \int_{\sqrt{8}}^{3} 8dx$$

$$+\int_{0}^{\sqrt{2}} 0dx + \int_{\sqrt{2}}^{2} 1dx$$

$$+\int_{2}^{\sqrt{6}} 2dx + \int_{\sqrt{6}}^{\sqrt{8}} 3dx + \int_{\sqrt{8}}^{3} 4dx = 31 - 6\sqrt{2} - \sqrt{3} - \sqrt{5}$$

$$-2\sqrt{6} - \sqrt{7}$$

a = 31 b = –6 c = –2

a + b + c = 31 – 6 – 2 = 23

25. 71

Sol. $a = 1 - \frac{^{3}C_5}{^{12}C_5}$

$$b = 3 \cdot \frac{^{9}C_4}{^{12}C_5}$$

$$c = 3 \cdot \frac{^{9}C_3}{^{12}C_5}$$

$$d = 1 \cdot \frac{^{9}C_2}{^{12}C_5}$$

u = 0 · a + 1 · b + 2 · c + 3 · d = 1.25

$$\sigma = 0 \cdot a + 1 \cdot b + 4 \cdot c + 9d - u^2$$

$$\sigma^2 = \frac{105}{176}$$

Ans. 176 – 150 = 71

26. 39

Sol. $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$$\frac{2}{3} = \frac{8^2 + c^2 - 7^2}{2 \times 8 \times c}$$

C = 9

$$\cos C = \frac{7^2 + 8^2 - 9^2}{2 \times 7 \times 8} = \frac{2}{7}$$

49 cos 3C + 42

49(4 cos^3 C – 3 cos C) + 42

$$49\left(4\left(\frac{2}{7}\right)^3 - 3\left(\frac{2}{7}\right)\right) + 42$$

$$= \frac{32}{7}$$

m + n = 32 + 7 = 39

27. 43

Sol. $\bar{a}_1 = \lambda\hat{i} + 2\hat{j} + \hat{k}$

$$\bar{a}_2 = -2\hat{i} - 5\hat{j} + 4\hat{k}$$

$$\vec{p} - = 3\hat{i} - \hat{j} + \hat{k}$$

$$\vec{q} - = -3\hat{i} + 2\hat{j} + 4\hat{k}$$

$$(\lambda + 2)\hat{i} + 7\hat{j} - 3\hat{k} = \bar{a}_1 - \bar{a}_2$$

$$\vec{p} \times \vec{q} - = -6\hat{i} - 15\hat{j} + 3\hat{k}$$

$$\frac{44}{\sqrt{30}} = \frac{|-\lambda - 12 - 105 - 9|}{\sqrt{(-6)^2 + (-15)^2 + 3^2}}$$

$$\frac{44}{\sqrt{30}} = \frac{|6\lambda + 126|}{3\sqrt{30}}$$

$$132 = |6\lambda + 126|$$

$$\lambda = 1,\ \lambda = -43$$

$$|\lambda| = 43$$

28. 4

Sol. $\frac{\alpha^{10} + \beta^{10} + \sqrt{2}(\alpha^9 + \beta^9)}{2(\alpha^8 + \beta^8)}$

$$\frac{\alpha^8(\alpha^2 + \sqrt{2}\alpha) + \beta^8(\beta^2 + \sqrt{2}\beta)}{2(\alpha^8 + \beta^8)}$$

$$\frac{8\alpha^8 + 8\beta^8}{2(\alpha^8 + \beta^8)} = 4$$

29. 38

Sol. $D = D_1 = D_2 = D_3 = 0$

$$D_3 = \begin{vmatrix} 2 & 7 & 3 \\ 3 & 2 & 4 \\ 1 & \mu & -1 \end{vmatrix} = 0 \Rightarrow \mu = -39$$

$$D = \begin{vmatrix} 2 & 7 & \lambda \\ 3 & 2 & 5 \\ 1 & -39 & 32 \end{vmatrix} = 0 \Rightarrow \lambda = -1$$

$\lambda - \mu = 38$

30. 3

Sol. $(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$

$\Rightarrow d((e^y + 1)\sin x) = 0$

$(e^y + 1)\sin x = C$

It passes through $\left(\frac{\pi}{2}, 0\right)$

$\Rightarrow c = 2$

Now, $x = \frac{\pi}{6}$

$\Rightarrow e^y = 3$

PHYSICS

Section - A (Single Correct Answer)

31. D

Sol. For longest wavelength in Paschen's series:

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For longest $n_1 = 3$

$n_2 = 4$

$$\frac{1}{\lambda} = R\left[\frac{1}{(3)^2} - \frac{1}{(4)^2}\right]$$

$$\frac{1}{\lambda} = R\left[\frac{1}{9} - \frac{1}{16}\right]$$

$$\frac{1}{\lambda} = R\left[\frac{16-9}{16\times 9}\right]$$

$$\Rightarrow \lambda = \frac{16\times 9}{7R} = \frac{16\times 9}{7\times 1.097\times 10^7}$$

$\lambda = 1.876 \times 10^{-6}$ m

32. D

Sol. 1st law of thermodynamics

$\Delta Q = \Delta U + W$

$\Rightarrow \quad +48 = nC_v\Delta T + W$

$\Rightarrow \quad 48 = (1)\left(\frac{3R}{2}\right)(2) + W$

$\Rightarrow \quad W = 48 - 3 \times R$

$\Rightarrow \quad W = 48 - 3 \times (8.3)$

$\Rightarrow \quad W = 23.1$ Joule

33. A

Sol. 1 MSD $= \frac{1\text{cm}}{20} = 0.05$ cm

$1\text{VSD} = \frac{49}{50}\text{MSD} = \frac{49}{50} \times 0.05 \text{ cm} = 0.049 \text{ cm}$

LC = 1 MSD – 1 VSD = 0.001 cm

For mark on paper, $L_1 = 8.45$ cm + 26 × 0.001 cm

= 84.76 mm

For mark on paper through slab,

$L_2 = 7.12$ cm + 41× 0.001 cm = 71.61 mm

For powder particle on top surface,

ZE = 4.05 cm + 1 × 0.001 cm = 40.51 mm

∴ actual $L_1 = 84.76 - 40.51 = 44.25$ mm

actual $L_2 = 71.61 - 40.51 = 31.10$ mm

$$L_2 = \frac{L_1}{\mu}$$

$$\Rightarrow \mu = \frac{L_1}{L_2} = \frac{44.25}{31.10} = 1.42$$

34. A

Sol. Intensity $= \frac{1}{2}\varepsilon_0 E_0^2 c$

$$= \frac{1}{2} \times 9 \times 10^{-12} \times (600)^2 \times 3 \times 10^8$$

$$= \frac{9}{2} \times 36 \times 3 = 486 \text{ w/m}^2$$

35. D

Sol. At surface: mg = 300 N

$$m = \frac{300}{g_s}$$

At Depth $\frac{R}{4}$: $g_d = g_s\left[1 - \frac{d}{R}\right]$

$$g_d = g_s\left[1 - \frac{R}{4R}\right]$$

$$g_d = \frac{3g_s}{4}$$

weight at depth = m × g_d

$$= m \times \frac{3g_s}{4} = \frac{3}{4} \times 300 = 225N$$

36. C

Sol. Energy = hc/λ;

$$E = \frac{1240}{\lambda(nm)} eV$$

$$\sigma = \frac{1240}{\lambda(nm)}$$

$$\lambda = \frac{1240}{6} = 207nm$$

37. B

Sol. m = 800 kg

r = 300 m

θ = 30°

$\mu_s = 0.2$

$$V_{max} = \sqrt{Rg\left[\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right]}$$

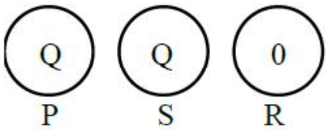
$$= \sqrt{300 \times g \times \left[\frac{\tan 30° + 0.2}{1 - 0.2 \times \tan 30°}\right]}$$

$$= \sqrt{300 \times 10 \times \left[\frac{0.57 + 0.2}{1 - 0.2 \times 0.57}\right]}$$

V_{max} = 51.4 m/s

38. B

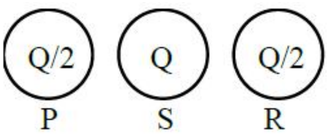
Sol.



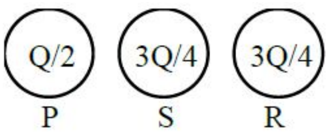
$F_{PS} \propto Q^2$

F_{PS} = 16 N

Now If P & R are brought in contact then



Now If S & R are brought in contact then



New force between P & S is :

$$F_{PS} \propto \frac{Q}{2} \times \frac{3Q}{4}$$

$$F_{PS} \propto \frac{3Q^2}{8} = \frac{3}{8} \times 16 = 6N$$

39. A

Sol. $(Emf)_{induced} = -L\frac{di}{dt}$

In magnitude form,

$$|Emf_{ind}| = \left|(-)L\frac{di}{dt}\right|$$

$$\Rightarrow 0.1 = \frac{(L)[+2-(-2)]}{0.2}$$

$$\Rightarrow \boxed{L = \frac{0.1 \times 0.2}{4} = 5\ mH}$$

40. C

Sol. $\frac{1}{f} = \left(\frac{\mu_{lens}}{\mu_{air}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\Rightarrow \frac{1}{+20} = \left(\frac{\mu}{1} - 1\right)\left(\frac{1}{+15} - \frac{1}{(-30)}\right)$$

$$\Rightarrow \frac{1}{20} = (\mu - 1)\left(\frac{3}{30}\right)$$

$$\Rightarrow \mu - 1 = \frac{1}{2}$$

$$\Rightarrow \boxed{\mu = 1 + \frac{1}{2} = \frac{3}{2} = 1.5}$$

41. D

Sol. Degree of freedom(f) = 5 + 2(3N – 5)

f = 5 + 2(3 × 2 – 1) = 7

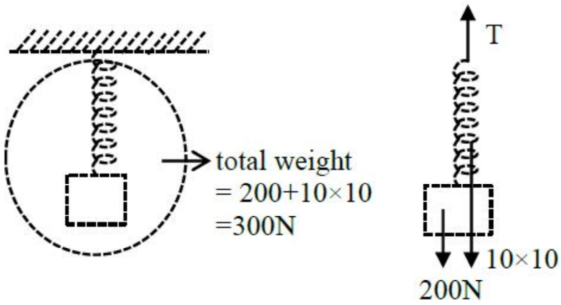
energy of one molecule $= \frac{f}{2}K_BT$

energy of 10 molecules

$$= 10\left(\frac{f}{2}K_BT\right) = 10\left(\frac{7}{2}K_BT\right) = 35K_BT$$

42. B

Sol.



Chain block system is in equilibrium so

T = 200 + 100 = 300 N.

43. C

Sol. $\frac{hc}{\lambda} - \phi = e.V_s$

$\Rightarrow \frac{1240}{300} eV - 2.13eV = eVs$

$\Rightarrow$ 4.13 eV – 2.13 eV = eVs.

$\Rightarrow$ So, V_s = 2volt

44. A

Sol. Power (P) = V.I

$\Rightarrow$ 110 = (220) (I)

$\Rightarrow$ I = 0.5 A

Now, $I = \frac{n \cdot e}{t}$

$\Rightarrow 0.5 = \left(\frac{n}{t}\right)(1.6 \times 10^{-19})$

$\Rightarrow \frac{n}{t} = \frac{0.5}{1.6 \times 10^{-19}}$

$\Rightarrow \boxed{\frac{n}{t} = 31.25 \times 10^{17}}$

45. A

Sol. Kinetic energy (K) $= \frac{P^2}{2m}$

$\Rightarrow$ $P = \sqrt{2mK}$

If $K_f = 36\ K_i$

So, $P_f = 6\ P_i$

% increase in momentum $= \frac{P_f - P_i}{P_i} \times 100\%$

$= \frac{6P_i - P_i}{P_i} \times 100\% = 500\%$

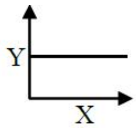
46. A

Sol. There are two liquid-air surfaces in bubble so

$\Delta P = 2\left(\frac{2S}{R}\right) = \frac{4S}{R}$

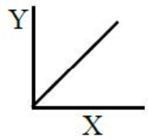
47. D

Sol. (A) Graph between Magnetic susceptibility and magnetising field is :



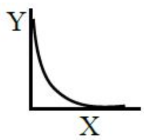
(B) magnetic field due to a current carrying wire for x < a :

$B = \frac{\mu_0 ir}{2\pi a^2}$

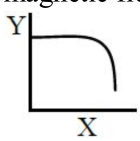


(C) magnetic field due to a current carrying wire for x > a :

$B = \frac{\mu_0 i}{2\pi a}$



(D) magnetic field inside solenoid varies as:



48. C

Sol. 4^{th} division coincides with 3rd division then

0.004 cm = 4VSD – 3MSD

49MSD = 50 VSD

$1MSD = \frac{1}{N} cm$

$0.004 = 4\left\{\frac{49}{50} MSD\right\} - 3MSD$

$0.004 = \left(\frac{196}{50} - 3\right)\left(\frac{1}{N}\right)$

$$N = \frac{46}{50} \times \frac{1000}{4} = \frac{46 \times 1000}{200} = 230$$

49. C

Sol. $\Delta Q = mS\Delta T$

$$s = \frac{\Delta Q}{m\Delta T}$$

$$[s] = \left[\frac{ML^2T^{-2}}{MK}\right]$$

$[s] = [L^2\ T^{-2}\ K^{-1}]$

Statement-(I) is correct

$$PV = nRT \Rightarrow R = \frac{PV}{nT}$$

$$[R] = \frac{[ML^{-1}T^{-2}][L^3]}{[mol][K]}$$

$[R] = [ML^2T^{-2}\ mol^{-1}K^{-1}]$

Statement-II is incorrect

50. A

Sol. $t_1 = \dfrac{u + \sqrt{u^2 + 2gh}}{g}$

$$t_2 = \frac{-u + \sqrt{u^2 + 2gh}}{g}$$

$$t = \frac{\sqrt{2gh}}{g}$$

$$t_1 t_2 = \frac{(u^2 + 2gh) - u^2}{g^2} = \frac{2gh}{g^2} = t^2 \quad \Rightarrow t = \sqrt{t_1 t_2}$$

51. 122

Sol. $10.2\ eV = hc/\lambda$

$$\lambda = \frac{1245 eV - nm}{10.2 eV} = 122.06\ nm$$

52. 10

Sol. for maximum current, circuit must be in resonance.

$$f_0 = \frac{1}{2\pi\sqrt{L \times C}}$$

$$f_0 = \frac{1}{2\pi\sqrt{100 \times 10^{-3} \times 2.5 \times 10^{-9}}}$$

$$= \frac{1}{2\pi\sqrt{25 \times 10^{-11}}}$$

$$= \frac{1}{2\pi \times 5} \times 10^5 \times \sqrt{10}\ Hz$$

$$= \frac{100}{10} \times 10^3\ Hz$$

$f_0 = 10 \times 10^3$ Hz

53. 3

Sol. $x^2 = 1 + t^2$

$$2x\frac{dx}{dt} = 2t$$

$xv = t$

$$x\frac{dv}{dt} + v\frac{dx}{dt} = 1$$

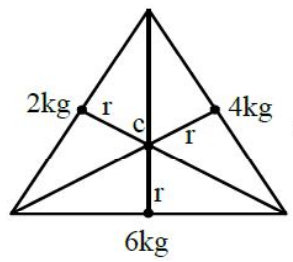
$x.a + v^2 = 1$

$$a = \frac{1 - v^2}{x} = \frac{1 - t^2/x^2}{x}$$

$$a = \frac{1}{x^3} = x^{-3}$$

54. 4

Sol.



$r = \dfrac{1}{\sqrt{3}}$

Moment of inertia about C and perpendicular to the plane is :

$I = r^2\ [2 + 4 + 6]$

$$= \frac{1}{3} \times 12$$

$I = 4$ kg-m^2

55. 100

Sol. $W = \Delta U = U_f - U_i$

$$W = (-\vec{\mu}.\vec{B})_f - (-\vec{\mu}.\vec{B})_i$$

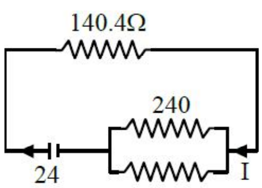
$$= 0 + (\vec{\mu}.\vec{B})_i$$

$= (100 \times 5 \times 10^{-3} \times 1 \times 10^{-3}) \times 0.2$ J

$= 1 \times 10^{-4}$ J $= 100$ μJ

56. 160

Sol.



$$Req = 140.4 + \frac{240 \times 10}{240 + 10}$$

$$Req = 140.4 + \frac{2400}{250}$$

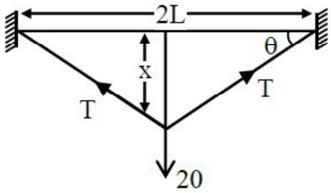
Req. = 150Ω

∴ Current in ammeter = 24/150

= 160 mA

57. 1

Sol.



In vertical derection

$2T \sin\theta = 20$

using small angle approximation $\sin\theta = \theta$

$\theta = 1/100$

∴ $T = 10/\theta$

$T = 1000N$

Change in length

$$\Delta L = 2\sqrt{x^2 + L^2} - 2L$$

$$= 2L\left[1 + \frac{x^2}{2L^2} - 1\right]$$

$$\Delta L = \frac{x^2}{L}$$

∴ Modulus of elasticity = stress/ strain

$$2 \times 10^{11} = \frac{10^3}{A \times \frac{x^2}{L}} \times 2L$$

∴ $A = 1 \times 10^{-4}\ m^2$

58. 8

Sol. $I_{max} = \left(\sqrt{I} + \sqrt{4I}\right)^2 = 9I$

$$I_{min} = \left(\sqrt{4I} - \sqrt{I}\right)^2 = I$$

∴ $I_{max} - I_{min} = 8I$

59. 740

Sol. The difference in frequency in open organ pipe

$$f = \frac{nv}{2L}$$

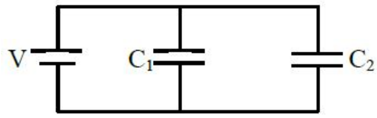
$$\Delta f = \frac{6v}{2 \times 0.6} - \frac{5v}{2 \times 0.9}$$

$v = 333$ m/s

$\Delta f = 740$ Hz

60. 80

Sol.



$C_{eq} = C_1 + C_2$

$$C_1 = \frac{2 \in_0 A}{2 \times d} = 10\mu F$$

$$C_2 = \frac{3 \in_0 A}{2d} = 15\mu F$$

$C_{eq} = 25\ \mu F$

Now the charge on $C_1 = 10V\ \mu c$

$C_2 = 1.5\ V\ \mu C.$

Now force between the plates

$$\left[F = \frac{Q^2}{2A \in_0}\right]$$

$$\frac{100V^2 \times 10^{-12}}{2 \times 2 \times 10^{-4} \in_0} + \frac{225V^2 \times 10^{-12}}{2 \times 2 \times 10^{-4} \times \in_0} = 8$$

$325\ V^2 = 8 \times 4 \times 10^{-4} \times 8.85$

$$V^2 = \frac{32 \times 8.85 \times 10^{-4}}{325}$$

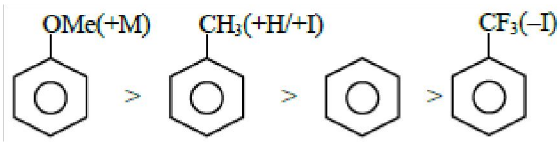
$$\therefore V = \sqrt{\frac{283.2 \times 10^{-4}}{325}}$$

$$\boxed{V = 0.93 \times 10^{-2}}$$

CHEMISTRY

Section - A (Single Correct Answer)

61. (B)



62. (B)

Sol.: 3 moles are present in 1 litre solution

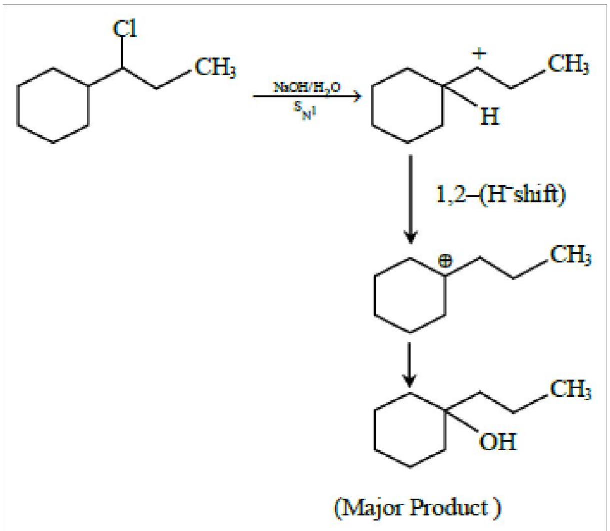
$$\text{molality} = \frac{3 \times 1000}{1.25 \times 1000 - [3 \times 58.5]} = 2.79\,m$$

63. (C)

Sol.: Direct NCERT Based

64. (B)

Sol.:



65. (A)

Sol.: $Ag^+ + I^- \rightarrow AgI$ Yellow ppt.

$Ag^+ + Cl^- \rightarrow AgCl$ White ppt

$Ag^+ + Br^- \rightarrow AgBr$ Pale yellow ppt

66. (A)

Sol.: Applied external potential should be greater than E^0_{cell} in opposite direction

67. (D)

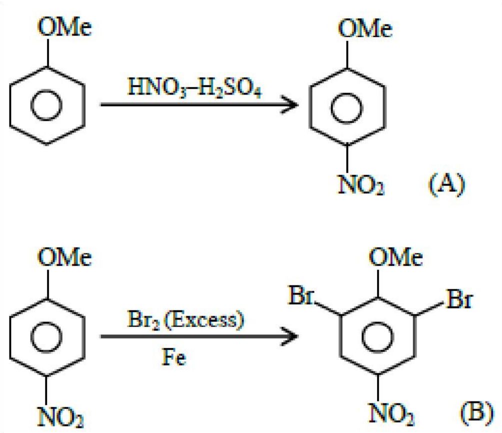
Sol.: Unpaired electron

Sc[Ar]$4s^2 3d^1$	1
Cr[Ar]$4s^1 3d^5$	6
V[Ar]$4s^2 3d^3$	3
Ti : [Ar] $4s^2\ 3d^2$	2
Mn: [Ar]$4s^2 3d^5$	5

68. (B)

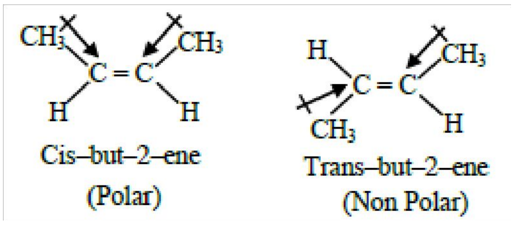
Sol.: Fact Based

69. (B)

Sol.: 

70. (B)

Sol.:



Cis–but–2–ene has higher Dipole moment than trans–but–2–ene

71. (C)

Sol.:

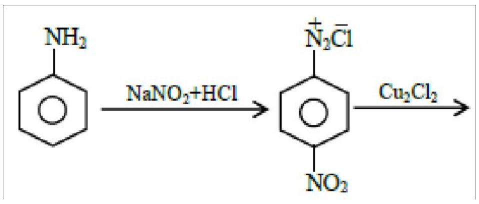
	Hybridisation		Hybridisation
PF_5	sp^3d	SF_6	sp^3d^2
BrF_5	sp^3d^2	$[Co(NH_3)_6]^{+3}$	d^2sp^3

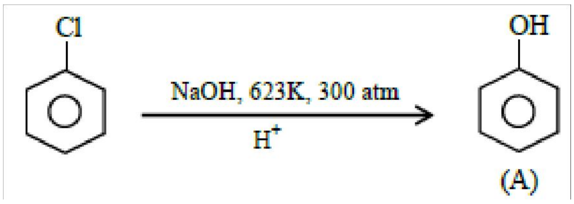
Both statement (A) and (B) are false.

72. (D)

Sol.: Due to inert pair effect; $T\ell^{+3}$ and Pb^{+4} can behave as oxidising agents.

73. (B)

Sol.: 



74. (C)

Sol.: Non polar compounds are having higher value of R_f than polar compound.

75. (A)

Sol.: (A) Down the group; radius increases

(B) EN does not decrease gradually from C to Pb.

(C) Correct.

(D) Correct.

(E) Range of oxidation state of carbon ; –4 to +4

76. (D)

Sol.: $A \rightarrow (IV)$

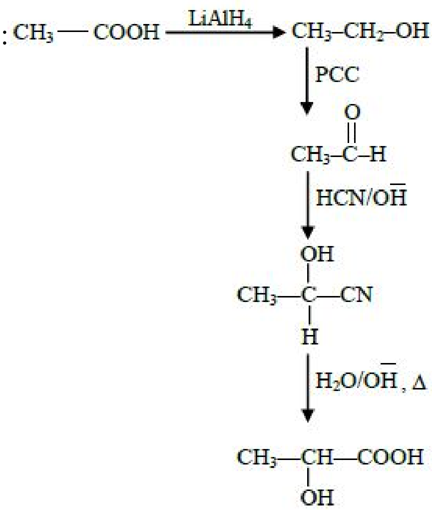
$B \rightarrow (I)$

$C \rightarrow (II)$

$D \rightarrow (III)$

77. (D)

Sol.:

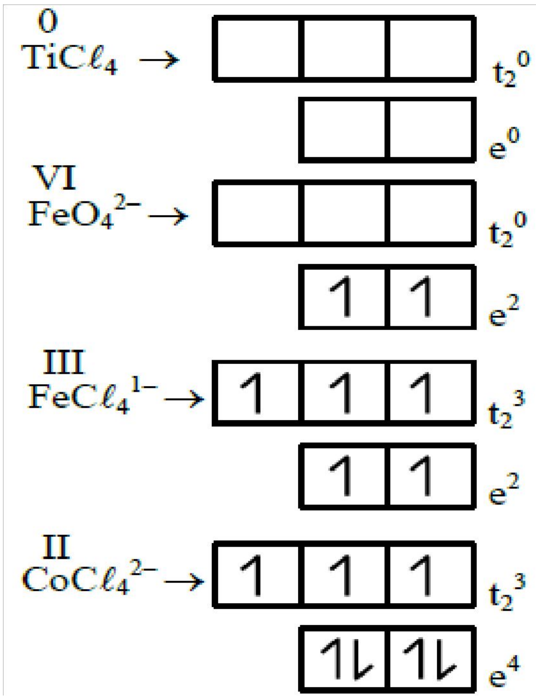


78. (C)

Sol.: Dibromo bis(trimethylphosphine) platinum (II)

79. (D)

Sol.:



80. (D)

Sol.: $CO(g) + \frac{1}{2}O_2(g) \rightleftharpoons CO_2(g)$

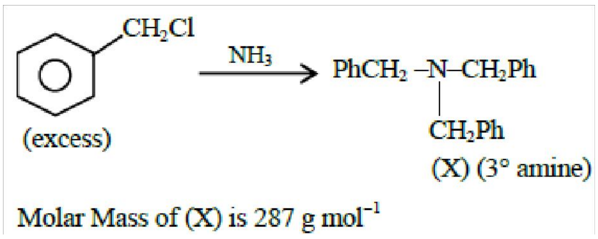
$$\Delta n_g = 1 - \left(1 + \frac{1}{2}\right) = -\frac{1}{2}$$

$$\frac{K_P}{K_C} = (RT)^{\Delta n_g} = \frac{1}{\sqrt{RT}}$$

Section - B (Numerical Value Type)

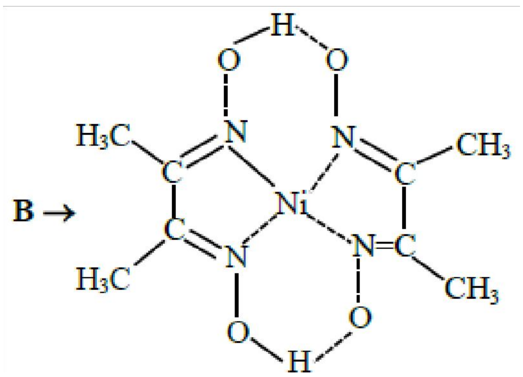
81. (287)

Sol.:



82. (12)

Sol.: B →



$3NiS + 2HNO_3 + 6HCl$

$\rightarrow 3NiCl_2 + 2NO + 3S + 4H_2O$

$$NiCl_2 + 2NH_4OH + \begin{matrix} H_3C-C=N-OH \\ | \\ H_3C-C=N-OH \end{matrix}$$

$\rightarrow NH_4Cl + H_2O + (B)$

83. (543)

Sol.: $\Delta T_f = 273.15 - 270.65 = 2.5$ K

$$\Delta T_f = K_{fm} \Rightarrow 2.5 = 1.86 \times \frac{n}{0.1}$$

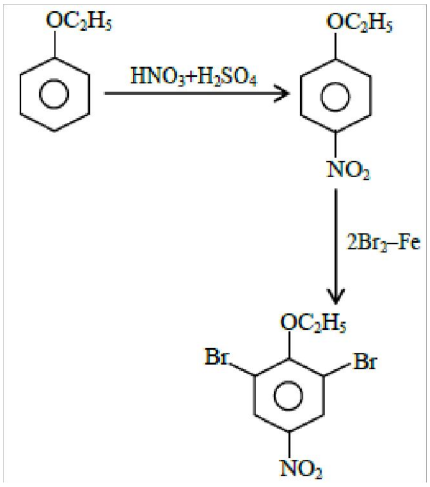
$\Rightarrow n = 0.1344$ moles

$\Rightarrow w = 0.1344 \times 32 = 4.3$ g

$$\text{Volume} = \frac{4.3}{0.792} = 5.43 \text{ ml} = 543 \times 10^{-2} \text{ ml}$$

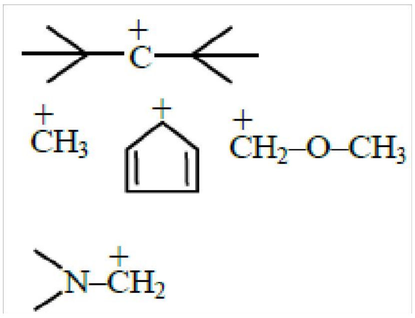
84. (15)

Sol.:



85. (5)

Sol.:



86. (200)

Sol.: $\Delta G = 0$

$$T = \frac{\Delta H}{\Delta S} = \frac{400}{0.2} = 2000\text{K}$$

87. (6)

Sol.: Central atom utilising sp^2 hybrid orbitals

$SO_2, C_2H_4, BCl_3, HCHO, C_6H_6, BF_3$

88. (17)

Sol.: $$\frac{(t_{1/2})_I}{(t_{1/2})_{II}} = \frac{K_2}{K_1} = \frac{5}{2}$$

$$\therefore K_1t_1 = \ell n \frac{1}{1-\frac{2}{3}} = \ell n3$$

$$K_2t_1 = \ell n \frac{1}{1-\frac{4}{5}} = \ell n5$$

$$\Rightarrow \frac{K_1}{K_2} \times \frac{t_1}{t_2} = \frac{0.477}{0.699}$$

$$\Rightarrow \frac{t_1}{t_2} = \frac{0.477}{0.699} \times \frac{5}{2} = 1.7 = 17 \times 10^{-1}$$

89. (34)

90. (0)

Sol.: For 3d transition series:

Oxidising power : $V^{+5} < Cr^{+6} < Mn^{+7}$

V^{+5} : [Ar] $4s^0 3d^0$

Number of unpaired electron = 0

$\mu = 0$

☯ ☯ ☯