MATHEMATICS

1. B

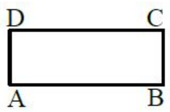
Sol. $f'(x) = 3x^2 \sin\left(\frac{1}{x}\right) - x\cos\left(\frac{1}{x}\right)$

$$f''(x) = 6x\sin\left(\frac{1}{x}\right) - 3\cos\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right) - \frac{\sin\left(\frac{1}{x}\right)}{x}$$

$$f''\left(\frac{2}{\pi}\right) = \frac{12}{\pi} - \frac{\pi}{2} = \frac{24-\pi^2}{2\pi}$$

2. A

Sol.



$$\text{Area} = \frac{1}{2}\left|\overline{BD} \times \overline{AC}\right|$$

$$\overline{BD} = \frac{5}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{2}{3}\hat{k}$$

$$\overline{AC} = \hat{i} - j - 2\hat{k}$$

3. C

Sol. Divide Nr & Dr by cos x

$$\int_0^{\pi/4} \frac{\tan^2 x \sec^2 x dx}{(1+\tan^3 x)^2} dx$$

Let $1 + \tan^3 x = t$

$$\tan^2 x \sec^2 x dx = \frac{dt}{3}$$

$$\frac{1}{3}\int_1^2 \frac{dt}{t^2} = \frac{1}{6}$$

4. C

Sol. Mean $(\overline{x}) = 0$

$$\Rightarrow \frac{\Sigma x_i}{20} = 10$$

$\Sigma x_i = 10 \times 20 = 200$

If 8 replaced by 12, then

$\Sigma x_i = 200 - 8 + 12 = 204$

$\therefore$ Correct mean $(\overline{x}) = \frac{\Sigma x_i}{20}$

$= \frac{204}{20} = 10.2$

$\because$ Standard deviation = 2

$\therefore$ Variance = $(S.D.)^2 = 2^2 = 4$

$$\Rightarrow \frac{\Sigma x_i^2}{20} - \left(\frac{\Sigma x_i}{20}\right)^2 = 4$$

$$\Rightarrow \frac{\Sigma x_i^2}{20} - (10)^2 = 4$$

$$\Rightarrow \frac{\Sigma x_i^2}{20} = 104$$

$\Rightarrow \Sigma x_i^2 = 2080$

Now, replaced '8' observations by '12'

Then, $\Sigma x_i^2 = 2080 - 8^2 + 12^2 = 2160$

$\therefore$ Variance of removing observations

$$\Rightarrow \frac{\Sigma x_i^2}{20} - \left(\frac{\Sigma x_i}{20}\right)^2$$

$$\Rightarrow \frac{2160}{20} - (10.2)^2$$

$\Rightarrow 108 - 104.04$

$\Rightarrow 3.96$

Correct standard deviation

$= \sqrt{3.96}$

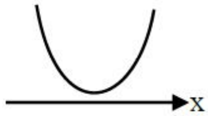
5. C

Sol. $f(x)=\dfrac{(x+5)(x-3)}{x^2-4x+9}$

Let $g(x) = x^2 - 4x + 9$

$D < 0$

$g(x) > 0$ for $x \in R$



$\therefore \begin{bmatrix} f(-5)=0 \\ f(3)=0 \end{bmatrix}$

So, f(x) is many-one.

again,

$yx^2 - 4xy + 9y = x^2 + 2x - 15$

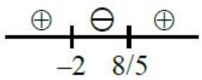
$x^2 (y - 1) - 2x(2y + 1) + (9y + 15) = 0$

for $\forall x \in R \Rightarrow D \geq 0$

$D = 4(2y+1)^2 - 4(y-1)(9y+15) \geq 0$

$5y^2 + 2y + 16 \leq 0$

$(5y-8)(y+2) \leq 0$



Note : If function is defined from f : R $\to$ R then only correct answer is option (3)

$\Rightarrow$ Bonus

6. A

Sol. n(3) $\Rightarrow$ multiple of 3

102, 105, 108, ... , 699

$T_n = 699 = 102 + (n - 1)(3)$

$n = 200$

$n(3) = 200$

$\therefore$ n(4) $\Rightarrow$ multiple of 4

100, 104, 108, ..., 700

$Tn = 700 = 100 + (n - 1)(4)$

$n = 151$

$n(4) = 151$

$n(3 \cap 4) \Rightarrow$ multiple of 3 & 4 both

108, 120, 132, ..., 696

$T_n = 696 = 108 + (n - 1)(12)$

$n = 50$

$n(3 \cap 4) = 50$

$n(3 \cup 4) = m(3) + n(4) - n(3 \cap 4)$

$= 200 + 151 - 50$

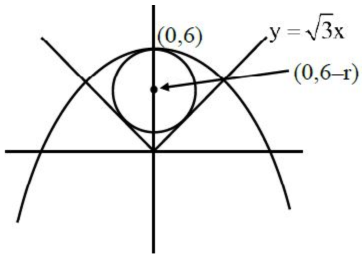
$= 301$

$n(\overline{3 \cup 4}) =$ Total $-(3 \cup 4) =$ neither a multiple of 3 nor multiple of 4

$= 601 - 301 = 300$

7. A

Sol.



Equation of circle

$x^2 + (y - (6 - r))^2 = r^2$

touches $\sqrt{3}x - y = 0$

$p = r$

$\dfrac{|0-(6-r)|}{2} = r$

$|r - 6| = 2r$

$r = 2$

$\therefore$ Circle $x^2 + (y - 4)^2 = 4$

(2, 4) Satisfies this equation

8. A

Sol. $A_r = \begin{vmatrix} r & 1 & \dfrac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & \dfrac{n(3n-1)}{2} \end{vmatrix}$

$2A_{10} - A = \begin{vmatrix} 20 & 1 & \dfrac{n^2}{2}+\alpha \\ 40 & 2 & n^2-\beta \\ 56 & 3 & \dfrac{n(3-1)}{2} \end{vmatrix} - \begin{vmatrix} 8 & 1 & \dfrac{n^2}{2}+\alpha \\ 16 & 2 & n^2-\beta \\ 22 & 3 & \dfrac{n(3n-1)}{2} \end{vmatrix}$

$$\Rightarrow \begin{vmatrix} 0 & 1 & \frac{n^2}{2}+\alpha \\ 0 & 2 & n^2-\beta \\ -2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$$

$$\Rightarrow -2((n^2-\beta)-(n^2+2\alpha))$$

$$\Rightarrow -2(-\beta-2\alpha) \Rightarrow 4\alpha+2\beta$$

9. B

Sol. $\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$ & $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$

$$S.D=\frac{|(\bar{a}_2\cdot\bar{a}_1)\cdot(\bar{b}_1\cdot\bar{b}_2)|}{|\bar{b}_1\times\bar{b}_2|}$$

$a_1 = 3, -15, 9$ $\quad b_1 = 2, -7, 5$

$a_2 = -1, 1, 9$ $\quad b_2 = 2, 1, -3$

$a_2 - a_1 = -4, 16, 0$

$$\bar{b}_1\times\bar{b}_2=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix}=\hat{i}(16)-\hat{j}(-16)+\hat{k}(16)$$

$$16(\hat{i}+\hat{j}+\hat{k})$$

$$|\bar{b}_1\times\bar{b}_2|=16\sqrt{3}$$

$$\therefore (\bar{a}_2-\bar{a}_1)\cdot(\bar{b}_1-\bar{b}_2)=16[-4+16]=(16)(12)$$

$$S.D=\frac{(16)(12)}{16\sqrt{3}}=4\sqrt{3}$$

10. A

Sol.

	A	B
Manufactured	60%	40%
Standard quality	80%	90%

P(Manufactured at B/found standard quality) =?

A : Found S.Q

B : Manufacture B

C : Manufacture A

$$P(E_1)=\frac{40}{100}$$

$$P(E_2)=\frac{60}{100}$$

$$P(A/E_1)=\frac{90}{100}$$

$$P(A/E_2)=\frac{80}{100}$$

$$\because P(E_1/A)=\frac{P(A/E_1)P(E_1)}{P(A/E_1)P(E_1)+P(A/E_2)P(E_2)}=\frac{3}{7}$$

$$\therefore 126P=54$$

11. C

Sol. by newton's theorem

$a_{n+2}-(t^2-5t+6)a_{n+1}+a_n=0$

$$\therefore a_{2023}+a_{2023}=(t^2-5t+6)a_{2024}$$

$$\therefore \frac{a_{2023}+a_{2023}}{a_{2024}}=t^2-5t+6$$

$$\because t^2-5t+6=\left(t-\frac{5}{2}\right)^2-\frac{1}{4}$$

$$\therefore \text{ minimum value }=-\frac{1}{4}$$

12. D

Sol. x = {1, 2, 3, ...20}

$R_1 = \{(x, y) : 2x - 3y = 2\}$

$R_2 = \{(x, y) : -5x + 4y = 0\}$

$R_1 = \{(4, 2), (7, 4), (10, 6), (13, 8), (16, 10), (19, 12)\}$

$R_2 = \{(4, 5), (8, 10), (12, 15), (16, 20)\}$

in R_1 6 element needed

in R_2 4 element needed

So, total 6 + 4 = 10 element

13. A

Sol. equation of line is

y + 9 = m(x – 4)

$$\therefore A=\left(\frac{9+4m}{m},\ 0\right)$$

B = (0, –9 – 4m)

$$\therefore OA+OB=\frac{9+4m}{m}+9+4m$$

$$\because m>0$$

$$=13+\frac{9}{m}+4m$$

$$\because \frac{4m+\frac{9}{m}}{2} \geq \sqrt{36} \Rightarrow 4m+\frac{9}{m} \geq 12$$

$\therefore OA + OB \geq 25$

14. D

Sol. $f(x) = x^x \; ; x > 0$

$\ell ny = x\ell nx$

$$\frac{1}{y}\frac{dy}{dx} = \frac{x}{x} + \ell nx$$

$$\frac{dy}{dx} = x^x(1+\ell nx)$$

for strictly increasing

$$\frac{dy}{dx} \geq 0 \Rightarrow x^x(1+\ell nx) \geq 0$$

$\Rightarrow \ell nx \geq -1$

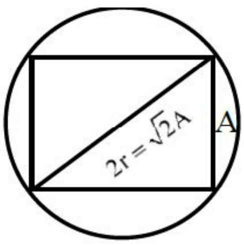
$x \geq e^{-1}$

$$x \geq \frac{1}{e}$$

$$x \in \left[\frac{1}{e}, \infty\right)$$

15. B

Sol. $$\because r = \frac{\Delta}{s} = \frac{\sqrt{3}a^2}{4.\frac{3a}{2}} = \frac{a}{2\sqrt{3}} = \frac{12}{2\sqrt{3}} = 2\sqrt{3}$$



$\therefore \mathbf{A} = r\sqrt{2} = 2\sqrt{6}$

Area = m = A^2 = 24

Perimeter $= n = 4A = 8\sqrt{6}$

$\therefore m + n^2 = 24 + 384$

$= 408$

16. C

Sol. $\because$ no. of triangles having no side common with a n sided polygon $= \frac{{}^nC_1 \cdot {}^{n-1}C_2}{3}$

$$= \frac{{}^8C_1 \cdot {}^4C_2}{3} = 16$$

17. B

Sol. $$\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{\tan^{-1}x}}{1+x^2}$$

$$\text{I.F.} = e^{\int \frac{1}{1+x^2}dx} = e^{\tan^{-1}x}$$

$$y \cdot e^{\tan^{-1}} = \int \left(\frac{e^{\tan^{-1}x}}{1+x^2}\right) e^{\tan^{-1}x} \cdot dx$$

Let $\tan^{-1}x = z$ $\qquad \therefore \frac{dx}{1+x^2} = dz$

$$\therefore y \cdot e^x = \int e^{2z}dz = \frac{e^{2z}}{2} + C$$

$$y \cdot e^{\tan^{-1}} = \frac{e^{2\tan^{-1}x}}{2} + C$$

$$\Rightarrow y = \frac{e^{\tan^{-1}x}}{2} + \frac{C}{e^{\tan^{-1}x}}$$

$$\because y(1) = 0 \Rightarrow 0 = \frac{e^{\pi/4}}{2} + \frac{C}{e^{\pi/4}} \Rightarrow C = \frac{-e^{\pi/2}}{2}$$

$$\therefore y = \frac{e^{\tan^{-1}x}}{2} - \frac{e^{\pi/2}}{2e^{\tan^{-1}x}}$$

$$\boxed{\therefore y(0) = \frac{1-e^{\pi/2}}{2}}$$

18. C

Sol. $$\frac{dy}{dx} + \frac{y}{x\ell nx} = \frac{3}{2x^2}$$

$$\therefore \text{I.F.} = e^{\int \frac{1}{x\ell nx}dx} = e^{\ell n(\ell n(x))} = \ell nx$$

$$\therefore y\ell nx = \int \frac{3\ell nx}{2x^2}dx$$

$$= \frac{3\ell nx}{2}\int x^{-2}dx - \int\left(\frac{3}{2x}\cdot\int x^{-2}dx\right)dx$$

$$= \frac{3\ell nx}{2}\left(-\frac{1}{x}\right) - \int\frac{3}{2x}\left(-\frac{1}{x}\right)dx$$

$$y\cdot\ell nx = \frac{-3\ell nx}{2x} - \frac{3}{2x} + C$$

$$\because y(e^{-1}) = 0$$

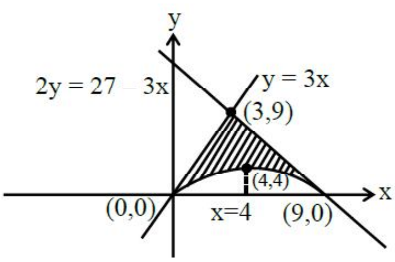
$$\therefore 0(-1) = \frac{3e}{2} - \frac{3e}{2} + C \qquad \Rightarrow C = 0$$

$$\therefore y = \frac{-3\ell nx}{2x} - \frac{3}{2x}$$

$$\boxed{\therefore y(e) = \frac{-3}{2e} - \frac{3}{2e} = \frac{-3}{e}}$$

19. D

Sol. $y = 3x,\ 2y = 27 - 3x\ \&\ y = 3x - x\sqrt{x}$



$$A = \int_0^3 3x(3x - x\sqrt{x})dx + \int_3^9\left(\frac{27-3x}{2} - (3x - x\sqrt{x})\right)dx$$

$$A = \int_0^3 x^{3/2}dx + \int_3^9 \frac{27}{2} - \frac{9x}{2} + x^{3/2}dx$$

$$A = \left[\frac{2x^{5/2}}{5}\right]_0^3 + \frac{27}{2}[x]_3^9 - \frac{9}{2}\left[\frac{x^2}{2}\right]_3^9 + \left[\frac{2x^{5/2}}{5}\right]_3^9$$

$$A = \frac{2}{5}(3^{3/2}) + \frac{27}{2}(6) - \frac{9}{4}(72) + \frac{2}{5}(9^{5/2} - 3^{5/2})$$

$$A = \frac{2}{5}(3^{5/2}) + 81 - 162 + \frac{2}{5}\times 3^5 - \frac{2}{5}\times 3^{5/2}$$

$$A = \frac{486}{5} - 81 = \frac{81}{5}$$

$10A = 162$

20. C

Sol. $f:(-\infty,\ \infty) - \{0\} \to R$

$$f'(1) = \lim_{a\to\infty} a^2 f\left(\frac{1}{a}\right)$$

$$\lim_{a\to\infty}\frac{a(a+1)}{2}\tan^{-1}\left(\frac{1}{a}\right) + a^2 - 2\ell n(a)$$

$$\lim_{a\to\infty} a^2\left(\frac{\left(1+\frac{1}{a}\right)}{2}\tan^{-1}\left(\frac{1}{a}\right) + 1 - \frac{2}{a^2}\ell n(a)\right)$$

$$f(x) = \frac{1}{2}(1+x)\tan^{-1}(x) + 1 - 2x^2\ell n(x)$$

$$f'(x) = \frac{1}{2}\left(\frac{1+x}{1+x^2} + \tan^{-1}(x) + 4x\ell n(x)\right) + 2x$$

$$f'(1) = \frac{1}{2}\left(1 + \frac{\pi}{4}\right) + 2$$

$$f'(1) = \frac{5}{2} + \frac{\pi}{8}$$

Ans. 3

21. 55

Sol. $\alpha\beta\gamma = 45,\ \alpha\beta\gamma \in R$

$x(\alpha,\ 1,\ 2) + y(1,\ \beta,\ 2) + z(2,\ 3,\ \gamma) = (0,\ 0,\ 0)$

$x,\ y,\ z \in R,\ xyz \neq 0$

$\alpha x + y + 2z = 0$

$x + \beta y + 3z = 0$

$2x + 2y + \gamma z = 0$

$xyz \neq 0 \Rightarrow$ non-trivial

$$\begin{vmatrix} \alpha & 1 & 2 \\ 1 & \beta & 3 \\ 2 & 2 & \gamma \end{vmatrix} = 0$$

$$\Rightarrow \alpha(\beta\gamma - 6) - 1(\gamma - 6) + 2(2 - 2\beta) = 0$$

$$\Rightarrow \alpha\beta\gamma - 6\alpha - \gamma + 6 + 4 - 4\beta = 0$$

$$\Rightarrow 6\alpha + 4\beta + \gamma = 55$$

22. 75

Sol. $P(x, y)$ & $x \ge 3$

Slope of line at $P(x, y)$ will be $\frac{dy}{dx} = \frac{1}{2}\left(\frac{y+5}{x-3}\right)$

$\Rightarrow 2\frac{dy}{(y+5)} = \frac{1}{(x-3)}dx$

$\Rightarrow 2\ell n(y+5) = \ell n(x-3) + C$

Passes through (4, –2)

$\Rightarrow 2\ell n(3) = \ell n(1) + C$

$\Rightarrow C = 2\ell n(3)$

$\Rightarrow 2\ell n(y+5) = \ell n(x-3) + 2\ell n(3)$

$\Rightarrow 2\left(\ell n\left(\frac{y+5}{3}\right)\right) = \ell n(x-3)$

$\Rightarrow \left(\frac{y+5}{3}\right)^2 = (x-3)$

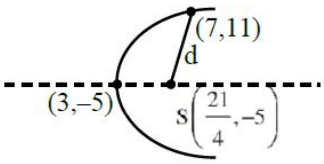
$\Rightarrow (y+5)^2 = 9(x-3)$

↓

Parabola

$4a = 9$

$a = \frac{9}{4}$



$d = \sqrt{\left(\frac{7}{4}\right)^2 + 6^2}$

$d = \frac{\sqrt{625}}{4}$

$d = \frac{25}{4}$

$12d = 75$

23. 65

Sol. $I_K = \int 1.(1-x^7)^K dx$

$I_K = (1-x^7)^K x|_0^1 + 7K\int_0^1 (1-x^7)^{K-1} x^6 \cdot x dx$

$I_K = -7K\int_0^1 (1-x^7)^{K-1}((1-x^7)-1)dx$

$I_K = -7KI_K + 7KI_{K-1}$

$\Rightarrow \frac{I_K}{I_{K+1}} = \frac{7K+8}{7K+7}$

$r_K = \frac{7K+8}{7K+7}$

$\Rightarrow 7(r_K - 1) = \frac{1}{K+1}$

$\sum_{K=1}^{10}(K+1) = 11(6) - 1 = 65$

24. 221

Sol. $4x^4 + 8x^3 - 17x^2 - 12x + 9$

$= 4(x - x_1)(x - x_2)(x - x_3)(x - x_4)$

Put $x = 2i$ & $-2i$

$64 - 64i + 68 - 24i + 9 = (2i - x_1)(2i - x_2)(2i - x_3)$

$(2i - x4)$

$= 141 - 88i$ ......(1)

$64 + 64i + 68 + 24i + 9 = 4(-2i - x_1)(-2i - x_2)$

$(-2i - x_3)(-2i - x_4)$

$= 141 + 88i$ ....(2)

$\frac{125}{16}m = \frac{141^2 + 88^2}{16}$

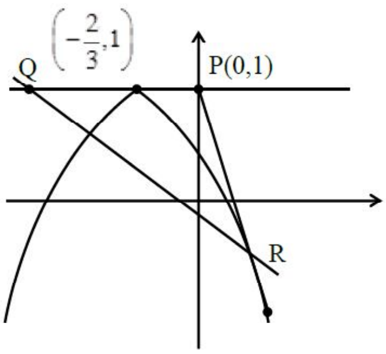
$m = 221$

25. 68

Sol. $9x^2 + 12x + 4 = -18(y - 1)$

$(3x + 2)^2 = -18(y - 1)$

$\left(x + \frac{2}{3}\right)^2 = -2(y-1)$



(0, 1)

y = mx + 1

$$\left(x+\frac{2}{3}\right)^2=-2(y-1)$$

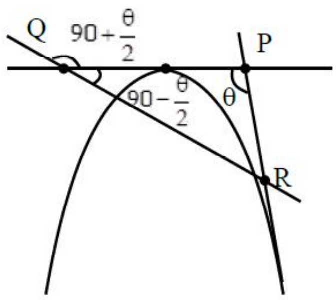
$(3x + 2)^2 = -18mx$

$9x^2 + (12 + 18m)x + 4 = 0$

$4(6 + 9m)^2 = 4(36)$

6 + 9m = 6, – 6

$$m=0,\ \frac{-4}{3}$$



$$\tan\theta=-\frac{4}{3}$$

$$\frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}=\frac{-4}{3}$$

$$\left(\tan\frac{\theta}{2}-2\right)\left(2\tan\frac{\theta}{2}+1\right)=0$$

$$\tan\frac{\theta}{2}=2,\ \frac{-1}{2}$$

$$m_{QR}=\tan\left(90+\frac{\theta}{2}\right)$$

$$=-\cot\frac{\theta}{2}$$

$$m_1=\frac{-1}{2}\qquad m_2=\frac{-1}{-1/2}=2$$

$$16(m_1^2+m_2^2)=16\left(\frac{1}{4}+4\right)$$

= 4 + 64 = 68

26. 806

Sol. $^nC_1\,x^{n-1}\,y = 135$ .....(i)

$^nC_2\,x^{n-2}\,y^2 = 30$ .....(ii)

$$^nC_3x^{n-3}y^3=\frac{10}{3}\quad .....(iii)$$

By $\frac{(i)}{(ii)}$

$$\frac{^nC_1}{^nC_2}\frac{x}{y}=\frac{9}{2}\quad .....(iv)$$

By $\frac{(ii)}{(iii)}$

$$\frac{^nC_2}{^nC_3}\frac{x}{y}=9\quad .....(v)$$

By $\frac{(iv)}{(v)}$

$$\frac{^nC_1\,^nC_3}{^nC_2\,^nC_2}=\frac{1}{2}$$

$$\frac{2n^2(n-1)(n-2)}{6}=\frac{n(n-1)}{2}\frac{n(n-1)}{2}$$

4n – 8 = 3n – 3

$\Rightarrow \boxed{n=5}$

put in (v)

$$\frac{x}{y}=9$$

x = 9y

put in (i)

$$^5C_1x^4\left(\frac{x}{9}\right)=135$$

$x^5 = 27 \times 9$

$$\Rightarrow x=3,\ y=\frac{1}{3}$$

$6(n^3 + x^2 + y)$

$$=6\left(125+9+\frac{1}{3}\right)$$

= 806

27. 6

Sol. $T_r = 3T_{r-1} + 6^r$, r = 2, 3, 4, … n

$T_2 = 3.T_1 + 6^2$

$T_2 = 3.6 + 6^2$ ….(1)

$T_3 = 3T_2 + 6^3$

$T_3 = 3T_2 + 6^3$

$T_3 = 3(3.6 + 6^2) + 6^3$

$T_3 = 3^2.6 + 3.6^2 + 6^3$ ….(2)

$T_r = 3^{r-1}.6 + 3^{r-2}.6^2 + …. + 6^r$

$$T_r = 3^{r-1}\cdot 6\left[1+\frac{6}{3}+\left(\frac{6}{3}\right)^2+.....+\left(\frac{6}{3}\right)^{r-1}\right]$$

$$T_r = 3^{r-1}\cdot 6(1+2+2^2+....+2^{r-1})$$

$$T_r = 6\cdot 3^{r-1}\cdot\frac{(1-2^r)}{(-1)}$$

$$T_r = 6.3^{r-1}.(2^r-1)$$

$$T_r = \frac{6\cdot 3^r}{3}\cdot(2^r-1)$$

$$T_r = 2\cdot(6^r-3^r)$$

$$S_n = 2\Sigma(6^r-3^r)$$

$$S_n = 2\cdot\left[\frac{6\cdot(6^n-1)}{5}-\frac{3\cdot(3^n-1)}{2}\right]$$

$$S_n = 2\left[\frac{12(6^n-1)-15(3^n-1)}{10}\right]$$

$$S_n = \frac{3}{5}[4.6^4-5.3^n+1]$$

$\therefore n^2-12n+39=3$

$n^2 - 12n + 36 = 0$

n = 6

28. 47

Sol. $\cot^{-1}3+\cot^{-1}4+\cot^{-1}5+\cot^{1}n=\frac{\pi}{4}$

$$\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{n}=\frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{46}{48}\right)+\tan^{-1}\frac{1}{n}=\frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{23}{24}\right)+\tan^{-1}\frac{1}{n}=\frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{1}{n}\right)=\tan^{-1}1-\tan^{-1}\frac{23}{24}$$

$$\tan^{-1}\frac{1}{n}=\tan^{-1}\left(\frac{1-\frac{23}{24}}{1+\frac{23}{24}}\right)$$

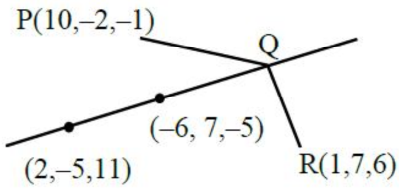
$$\tan^{-1}\frac{1}{n}=\tan^{-1}\left(\frac{\frac{1}{24}}{\frac{47}{24}}\right)$$

$$\tan^{-1}\frac{1}{n}=\tan^{-1}\frac{1}{47}$$

n = 47

29. 13

Sol.



$$\text{Line } \frac{x+6}{-8}=\frac{y-7}{12}=\frac{z+5}{-16}$$

$$\frac{x+6}{2}=\frac{y-7}{-3}=\frac{z+5}{4}=\lambda$$

$Q(2\lambda-6,\ 7-3\lambda,\ 4\lambda-5)$

$\overline{QR}(2\lambda-7,\ 7-3\lambda,\ 4\lambda-11)$

$\overline{QR}\cdot$ dr's of line $=0$

$4\lambda-14+9\lambda+16\lambda-44=0$

Q(–2, 1, 3)

$PQ=\sqrt{144+9+16}=\sqrt{169}=13$

30. 46

Sol. $\vec{a}\times\vec{b}+\vec{a}\times\vec{c}=(1,\ 8,\ 13)$

$\vec{a}\times(\vec{a}\times\vec{b})+\vec{a}\times(\vec{a}\times\vec{c})+\vec{a}\times(\vec{b}\times\vec{c})$

$= \vec{a}\times(\hat{i}+8\hat{j}+13\hat{k})$

$(\vec{a}\cdot\vec{b})\vec{a} - a^2\vec{b} + (\vec{a}\cdot\vec{c})\vec{a} - a^2\vec{c} + (\vec{a}\cdot\vec{b})\vec{c} = \vec{a}\times(\hat{i}+8\hat{j}+13\hat{k})$

$\Rightarrow -26\vec{a} - 29\vec{b} + 13\vec{a}29\vec{c} + 13\vec{b} + 26\vec{c} = \vec{a}\times(\hat{i}+8\hat{j}+13\hat{k})$

$\Rightarrow -13\vec{a} - 16\vec{b} - 3\vec{c} = \vec{a}\times(\hat{i}+8\hat{j}+13\hat{k})$

$\Rightarrow -13\vec{a}\cdot\vec{b} - 16b^2 - 3\vec{b}\cdot\vec{c} = \{\vec{a}\times 8\hat{j}+13\hat{k}\}\cdot\vec{b}$

$$\Rightarrow (-13)(-26) - 16(50) - 3\vec{b}\cdot\vec{c} = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 8 & 13 \\ 3 & 4 & -5 \end{vmatrix}$$

$\Rightarrow -462 - 3\vec{b}\cdot\vec{c} = -396$

$\Rightarrow \vec{b}\cdot\vec{c} = -22$

Hence $24 - \vec{b}\cdot\vec{c} = 46$

PHYSICS

Section - A (Single Correct Answer)

31. D

Sol. $T = 2\pi\sqrt{\dfrac{m}{k}}$

$T^2 \propto \dfrac{m}{k}$

$\dfrac{2\Delta T}{T}\% = \dfrac{\Delta m}{m}\% - \dfrac{\Delta k}{k}\%$

$\dfrac{\Delta k}{k}\% = \dfrac{\Delta m}{m}\% - \dfrac{2\Delta T}{T}\%$

$\dfrac{\Delta k}{k}\% = (-1)\% - 2(2)\% = |-5\%| = 5\%$

32. D

Sol. $K_i = \dfrac{1}{2}m(100)^2$

$K_f = \dfrac{1}{2}m(40)^2$

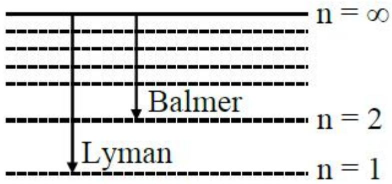
$\%\text{loss} = \dfrac{|K_f - K_i|}{K_i}\times 100$

$$= \dfrac{\left|\dfrac{1}{2}m(40)^2 - \dfrac{1}{2}m(100)^2\right|}{\dfrac{1}{2}m(100)^2}\times 100$$

$$= \dfrac{|1600 - 100\times 100|}{100} = 84\%$$

33. A

Sol.



$\dfrac{1}{\lambda} = Rz^2\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

$$\dfrac{\dfrac{1}{\lambda_L} = Rz^2\left(\dfrac{1}{1^2}\right)}{\dfrac{1}{\lambda_B} = Rz^2\left(\dfrac{1}{2^2}\right)}$$

$\dfrac{\lambda_B}{\lambda_L} = 4:1$

34. B

Sol. $\dfrac{1}{2}mv_e^2 = \dfrac{GMm}{R_E}$

$g = \dfrac{GM}{R_E^2}$

$K = mg\,R_E$

35. D

Sol. $\dfrac{\varepsilon_m\times\mu_m}{\varepsilon_0\times\mu_0} = \dfrac{\dfrac{1}{v^2}}{\dfrac{1}{c^2}}$

$\varepsilon_r\times\mu_r = \dfrac{c^2}{v^2}$

$\varepsilon_r\times 2 = \dfrac{(3\times 10^8)^2}{(1.5\times 10^8)^2}$

$\varepsilon_r\times 2 = 4$

$\varepsilon_r = 2$

36. B

Sol. (Theory)

Photoelectric effect prove particle nature of light.

37. B

Sol. MSR = 1mm, CSR = 42, pitch = 1 mm

$$LC = \frac{\text{pitch}}{\text{No. of CSD}} = \left(\frac{1}{100}\right) = 0.01 \text{ mm}$$

Diameter = MSR + LC × CSD

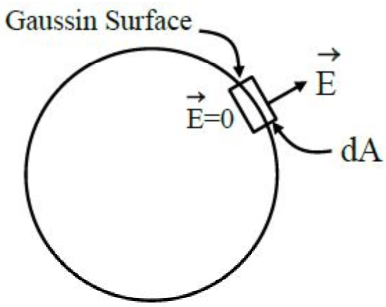
Diameter = 1 + (0.01) × 42 mm

Diameter = 1.42 mm = x/50

$\therefore$ x = 71

38. C

Sol.



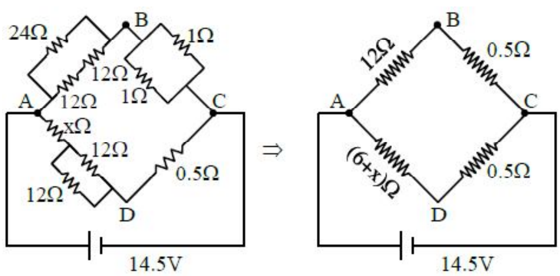
By Gauss law $\int \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$

$$EdA = \frac{\sigma \times dA}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

39. C

Sol.



In case of balanced Wheatstone Bridge

$$\frac{V_{AB}}{V_{AD}} = \frac{V_{BC}}{V_{CD}} \Rightarrow \frac{12}{6+x} = \frac{0.5}{0.5}$$

x = 6 Ω

40. D

Sol. dQ = du + dW

$CdT = C_V dT + PdV$ ......(1)

$\therefore$ $PV^2 = RT$

P = constant

P(2VdV) = RdT

$$PdV = \frac{RdT}{2V}$$

Put in equation (1)

$$C = C_V + \frac{R}{2V}$$

41. B

Sol. $[\vec{\tau}] = [\vec{r}\times\vec{F}] = [ML^2T^{-2}]$

[F] = [qVB]

$$\Rightarrow B = \left(\frac{F}{qV}\right) = \left[\frac{MLT^{-2}}{ATLT^{-1}}\right] = [MA^{-1}T^{-2}]$$

[M] = [I × A] = [AL^2]

$$B = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$$

$$\Rightarrow [\mu] = \left[\frac{Br^2}{Idl}\right] = \left[\frac{MT^{-2}A^{-1}\times L^2}{AL}\right]$$

$$= [MLT^{-2}A^{-2}]$$

42. C

Sol. Statement-I

$$I_m = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}} \text{ at resonance } X_L = X_C$$

Thus, $I_m = \frac{V_m}{R}$

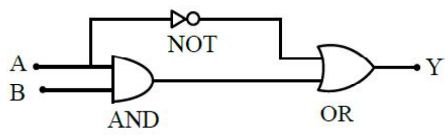
$\because$ Impendence is minimum therefore I is maximum at resonance.

Statement-II

$I = \left(\frac{V}{R}\right)$ in purely resistive circuit.

43. B

Sol.



44. B

Sol. $V_{rms} = \sqrt{\frac{3RT}{M_w}}$

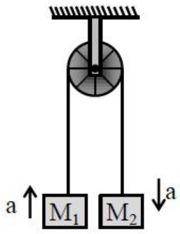
$$\Rightarrow \frac{V_{O_2}}{V_{He}} = \sqrt{\frac{M_{w,He}}{M_{w,O_2}}}$$

$$=\sqrt{\frac{4}{32}}=\frac{1}{2\sqrt{2}}$$

$$\frac{V_{He}}{V_{O_2}}=\frac{2\sqrt{2}}{1}$$

45. A

Sol. $a=\left(\frac{M_2-M_1}{M_1+M_2}\right)g$



$$\frac{g}{\sqrt{2}}=\left(\frac{M_2-M_1}{M_1+M_2}\right)g$$

$$(M_1+M_2)=\sqrt{2}M_2-\sqrt{2}M_1$$

$$\frac{M_1}{M_2}=\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)$$

46. C

Sol. $KE=\frac{p^2}{2m}$

Same momentum, so less mass means more KE.
So m/2 will have max. KE.

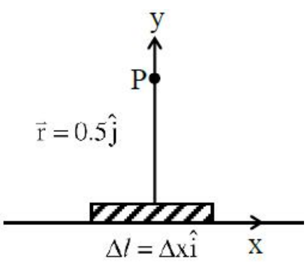
47. B

Sol. $\text{Average speed}=\frac{\text{total distance}}{\text{time taken}}$

$$=\frac{\frac{80\times t}{2}+80\times 3t}{4t}=70\text{ km/hr}$$

48. A

Sol.



$$\overrightarrow{dB}=\frac{\mu_0 I}{4\pi}\frac{\left(\vec{dl}\times\vec{r}\right)}{r^3}(\text{Tesla})$$

$$=\frac{10^{-7}\times 10\times\left(\frac{1}{2}\times\frac{1}{100}\right)\left(+\hat{k}\right)}{\left(\frac{1}{2}\right)^3}=4\times 10^{-8}T\left(+\hat{k}\right)$$

49. D

Sol. $mg-F_B-F_v=ma$

$a=0$ for constant velocity

$mg-F_B=F_v$

$$F_v=mg-v\rho_0 g=mg-\frac{m}{\rho}\rho_0 g=mg\left(1-\frac{\rho_0}{\rho}\right)$$

50. D

Sol. $eV_s=h\nu-\phi$

$0.5\text{ V}=2.48-\phi$

work function $(\phi)=2.48\text{ V}-0.5\text{ V}=1.98\text{ V}$

Section - B (Numerical Value)

51. 13

Sol. By conservation of angular momentum

$I_1\omega_1=I_2\omega_2$

$$\left(\frac{2}{5}MR^2\right)\frac{2\pi}{T_1}=\frac{2}{5}M\left(\frac{3}{4}R\right)^2\frac{2\pi}{T_2}$$

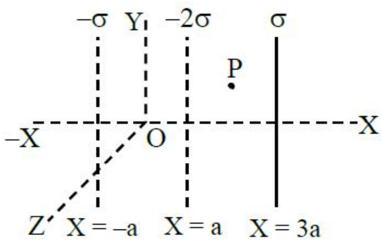
$$\frac{1}{T_1}=\frac{9}{16T_2}$$

$$\frac{1}{T_2}=\frac{9}{16}\times T_1=\frac{9}{16}\times 24\text{hr}$$

$$=\frac{27}{2}\text{hr}=13\text{hr }30\text{ mins}$$

52. 2

Sol.

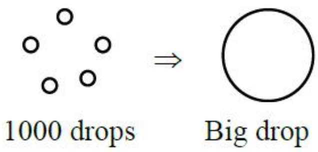


$$\vec{E}_P=\left(\frac{\sigma}{2\varepsilon_0}+\frac{2\sigma}{2\varepsilon_0}+\frac{\sigma}{2\varepsilon_0}\right)\left(-\hat{i}\right)$$

$$= -\frac{2\sigma}{\varepsilon_0}\hat{i}$$

53. 1

Sol.



$$1000\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$10r = R$

$R = 10r$

$$\frac{\text{S.E. of 1000 drops}}{\text{S.E. of Big drop}} = \frac{1000\left(4\pi r^2\right)T}{4\pi R^2 T}$$

$$= \frac{1000 \times r^2}{(10r)^2} = 10 = \frac{10}{x}$$

$\therefore \quad x = 1$

54. 250

Sol. For DC voltage

$$R = \frac{V}{I} = \frac{100}{5} = 20\Omega$$

for AC voltage

$X_L = 20\sqrt{3}\ \Omega$

$R = 20\ \Omega$

$$Z = \sqrt{X_L^2 + R^2} = \sqrt{3 \times 400 + 400} = 40\Omega$$

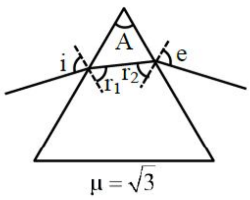
Power = i^2_{rms} R

$$= \left(\frac{V_{rms}}{Z}\right)^2 \times R = \left(\frac{\frac{200}{\sqrt{2}}}{40}\right)^2 \times 20 = 250W$$

55. 60

Sol. For δ_{min}

$i = e$



$$r_1 = r_2 = \frac{A}{2}$$

$$\frac{\delta_{min}}{A} = 1$$

$$\frac{2i - A}{A} = 1$$

$2i = 2A$

$i = A$

Snell's law

$1 \times \sin i = \mu \sin r$

$$\sin i = \mu \sin\left(\frac{A}{2}\right)$$

$$\sin A = \mu \sin\left(\frac{A}{2}\right)$$

$$2\sin\frac{A}{2}\cos\frac{A}{2} = \sqrt{3}\sin\left(\frac{A}{2}\right)$$

$$\cos\left(\frac{A}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\therefore \frac{A}{2} = 30^\circ$$

$\therefore A = 60^\circ$

56. 16

Sol. We know $R = \frac{\rho l}{A}, R \propto \frac{l}{r^2}$

As we starch the wire, its length will increase but its radius will decrease keeping the volume constant

$V_i = V_f$

$$\pi r^2 l = \pi \frac{r^2}{4} l_f$$

$l_f = 4l$

$$\frac{R_{new}}{R_{old}} = \left(\frac{4l}{\frac{r^2}{4}}\right)\frac{r^2}{l} = 16$$

$R_{new} = 16R$

$\therefore \quad x = 16$

57. 16

Sol. We know

$$r = 0.529\frac{n^2}{Z} \Rightarrow 8.48 = 0.529\frac{n^2}{1}$$

$n^2 = 16 \Rightarrow n = 4$

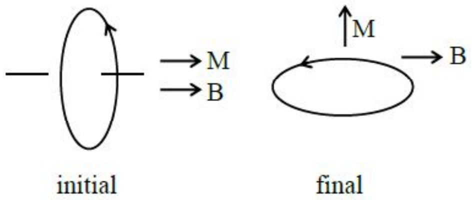
We know

$$E \propto \frac{1}{n^2}$$

$$E_{n^{th}} = \frac{E}{16}$$

x = 16

58. 5

Sol. 

We know

$W_{ext} = \Delta U + \Delta KE$

$\left(P.E. = -\vec{M}\cdot\vec{B}\right)$

$= -\vec{M}\cdot\vec{B}_f + \vec{M}\cdot\vec{B}_i + 0$

= – MB cos 90 + MB cos 0

= MB

= NIAB

$$= 200 \times 100 \times 10^{-6} \times \frac{5}{2} \times 10^{-4} \times 1 = 5\mu J$$

59. 12

Sol. We know

$v_{max} = \omega A$ at mean position

$$= \frac{2\pi}{T} A = \frac{2\pi}{\pi} \times 0.06 = 0.12 \text{ m/sec}$$

v_{max} = 12 cm/sec

60. 4

Sol. $\vec{B}\times\vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = 15\hat{i} - 21\hat{j} + 33\hat{k}$

$$\vec{A}\cdot\left(\vec{B}\times\vec{C}\right) = \left(-x\hat{i} - 6\hat{j} - 2\hat{k}\right)\cdot\left(15\hat{i} - 21\hat{j} + 33\hat{k}\right)$$

0 = – 15x + 126 – 66

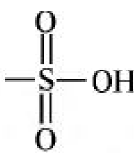
15x = 60

x = 4

CHEMISTRY

Section - A (Single Correct Answer)

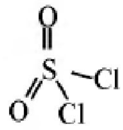
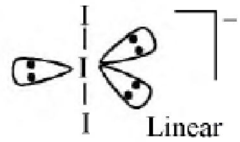
61. (B)

Sol. 

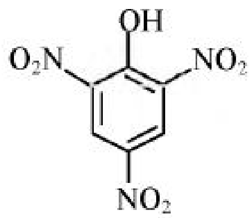
Group present in sulphonic acids

62. (B)

Sol.

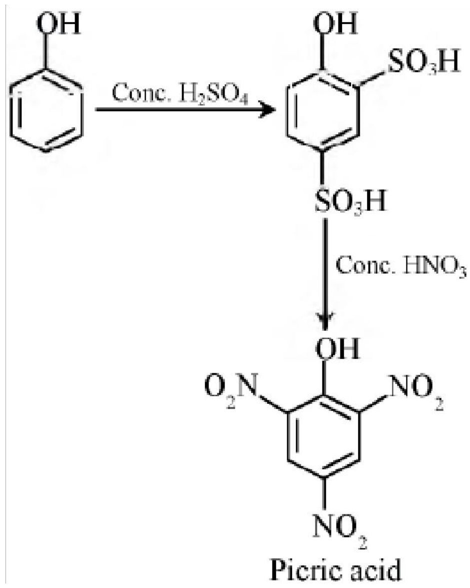
(A)	SO_2Cl_2	sp^3	 Tetrahedral
(B)	NO		Paramagnetic
(C)	NO_2^-		Diamagnetic
(D)	I_3^-	sp^3d	 Linear

63. (A)

Sol. 

picric acid

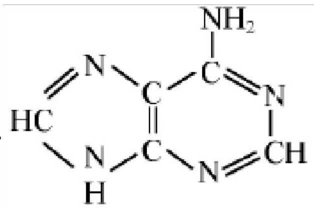
(2, 4, 6-trinitrophenol)



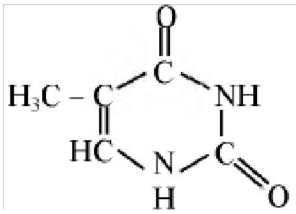
64. (D)

Sol. Metamer $\Rightarrow$ Isomer having same molecular formula, same functional group but different alkyl/aryl groups on either side of functional group.

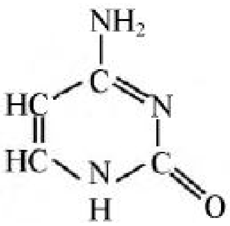
65. (C)

Sol. 

$\Rightarrow$ Adenine



$\Rightarrow$ Thymine



$\Rightarrow$ Cytosine

Are bases of DNA molecule. As DNA contain four bases, which are adenine, guanine, cytosine and thymine.

66. (D)

Sol. $sp^3 \rightarrow$ Tetrahedral

$dsp^2 \rightarrow$ Square planar

$sp^3d \rightarrow$ Trigonal Bipyramidal

$sp^3d^2 \rightarrow$ Octahedral

67. (D)

Sol. Statement - I $\Rightarrow$ Correct

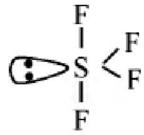
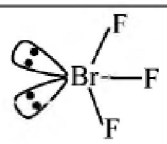
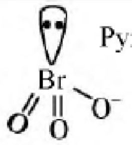
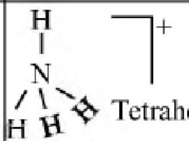
Statement - II $\Rightarrow$ False

Ga is used to measure high temperature

68. (B)

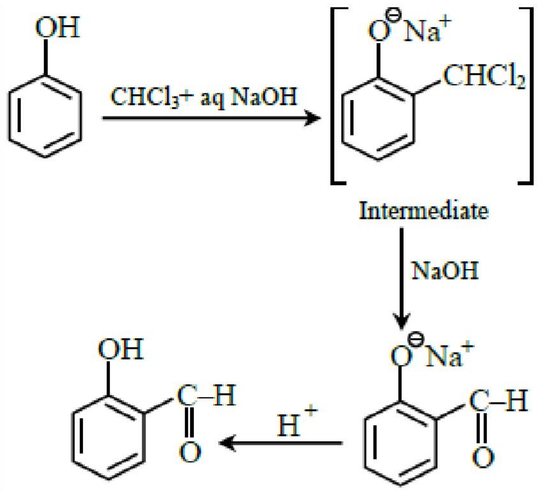
Sol. Option (B) is incorrect because aniline is immisible in water.

69. (B)

(A)	SF_4	sp^3d hybridisation	
(B)	BrF_3	sp^3d hybridisation	 Bent T-Shape
(C)	BrO_3^-	sp^3 hybridisation	 Pyramidal
(D)	NH_4^+	sp^3 hybridisation	 Tetrahedral

70. (D)

Sol.:



71. (B)

Sol.: Graphite is conductor

72. (B)

As ligand field increases, light of more energy is absorbed

Energy $\propto$ wave number $(\bar{\upsilon})$

73. (C)

Sol.: Theory based question

74. (D, A)

Sol.: (A) $Be + e^- \rightarrow Be^-$, E.A = –ive

(B) $N + e^- \rightarrow N^-$ E.A = –ive

(C) $O + e^- \rightarrow O^-$

$O^- + e^- \rightarrow O^{-2}$ E.A = –ive

(D) $Na + e^- \rightarrow Na^-$ E.A = +ive

(E) $A\ell + e^- \rightarrow A\ell^-$ E.A = +ive

Ans. A,B and C only

75. (C)

Sol.: Cm is actinite

76. (B)

Sol.: $\text{Molality} = \dfrac{1000 \times M}{1000 \times d - M \times (Mw)_{solute}}$

$$3 = \frac{1000 \times x}{1000 \times 1.12 - (x \times 40)}$$

x = 3

77. (A)

Sol.: $H-\overset{\overset{O}{||}}{C}-H$ has low steric hindrance at carbonyl carbon and high partial positive charge at carbonyl carbon.

78. (C)

Sol.: $\Delta n_g = 0$ $\qquad K_p = \dfrac{(n_{HI})^2}{n_{H_2} n_{I_2}} \left(\dfrac{P_T}{n_T}\right)^{\Delta n_g}$

$n_{HI} = n_{H_2} = n_{I_2}$ so $K_p = 1$

$1 = x \times 10^{-1}$ $\qquad x = 10$

79. (C)

Sol.: Iodoform – Antiseptic

CCl_4 – Fire extinguisher

CFC – Refrigerants

DDT – Insecticide

80. (B)

Sol.: Solution is already infinitely dilute, hence no change in molar conductivity upon addition of water

Section - B (Numerical Value Type)

81. (76)

Sol.: $HX \rightleftharpoons H^+ + X^-$ $K_a = 1.2 \times 10^{-5}$.

0.03 M

0.03 – x x x

$$K_a = 1.2 \times 10^{-5} = \frac{x^2}{0.03 - x}$$

$0.03 - x \approx 0.03$ (K_a is very small)

$$\frac{x^2}{0.03} = 1.2 \times 10^{-5}$$

$x = 6 \times 10^{-4}$

Final solution: 0.03 – x + x + x

$= 0.03 + x = 0.03 + 6 \times 10^{-4}$

$\Pi = (0.03 + (6 \times 10^{-4})) \times 0.083 \times 300$

$= 76.19 \times 10^{-2} \approx 76 \times 10^{-2}$

82. (6)

Sol.: Spin only magnetic moment of Mn in $KMnO_4$ = 0 Spin only value of manganese product fromed during titration of $KMnO_4$ aganist oxalic acid in acidic medium is = 6

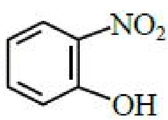
83. (3)

Sol.: $K = \dfrac{1}{t_{99.9\%}} \ell n\left(\dfrac{100}{0.1}\right) = \dfrac{1}{t_{90\%}} \ell n\left(\dfrac{100}{10}\right)$

$$t_{99.9\%} = t_{90\%} \frac{\ell n(10^3)}{\ell n 10}$$

$$t_{99.9\%} = t_{90\%} \times 3$$

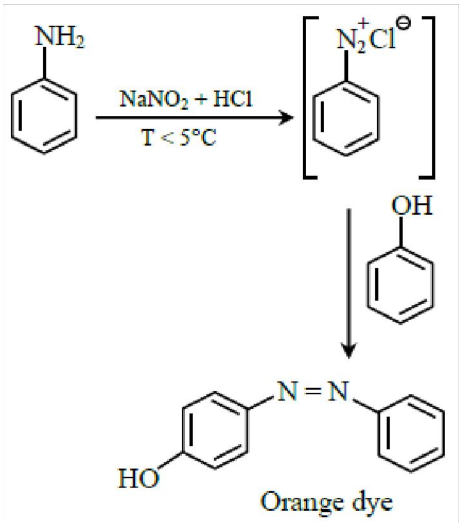
84. (5)

Sol.: CH_3OH, H_2O,  HF, NH_3

Can show H-bonding.

85. (20)

Sol.:



Reaction suggests that 1 mole of aniline give 1 mole of orange dye.

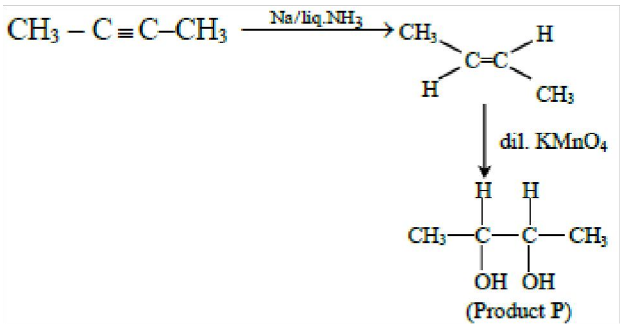
so $(\text{mol})_{\text{aniline}} = (\text{mole})_{\text{orange dye}}$

$$\frac{9.3\text{g}}{93\text{g mol}^{-1}} = \frac{\text{mass of orange dye}}{199\text{g mol}^{-1}}$$

mass of orange dye = 19.9 g $\simeq$ 20 g

86. (2)

Sol.:



87. (661)

Sol.: $\lambda = \dfrac{h}{mv}$, $\lambda = \dfrac{hv}{mv^2}$

$$\frac{mv^2}{h} = \frac{v}{\lambda} = v(\text{frequency})$$

Given $\frac{1}{2}mv^2 = 2.18 \times 10^{-18}$ J

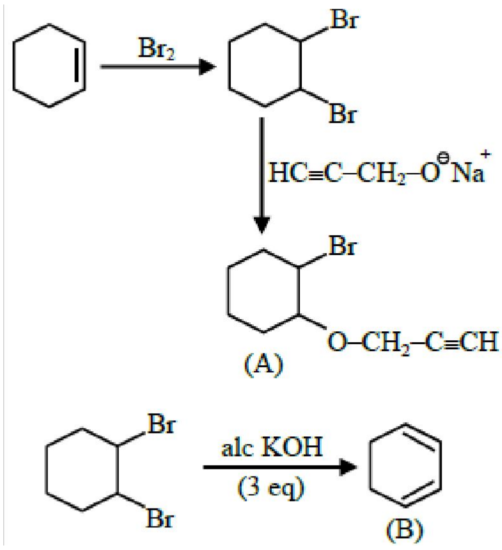
$h = 6.6 \times 10^{-34}$

$$v = \frac{4.36 \times 10^{-18}}{6.6 \times 10^{-34}} = 660.60 \times 10^{13}\ \text{Hz}$$

$\approx 661 \times 10^{13}$ Hz

88. (8)

Sol.:



89. (877)

Sol.: CrO Basic oxide

Cr_2O_3 Amphoteric oxide

In CrO, Cr exist as Cr^{+2} and have μ only = 4.90

In Cr_2O_3, Cr exist as Cr^{+3} and have μ only = 3.87

Sum of spin only magnetic moment

= 4.90 + 3.87 = 8.77

$\mu_{\text{only}} = 877 \times 10^{-2}$

Ans. 877

90. (274)

Sol.: $\Delta U = q + w (q = 0)$

$$nCv\Delta T = -P_{ext}(V_2 - V_1)$$

$V_2 = 2V_1$

$$\frac{nRT_2}{P_2} = \frac{2nRT_1}{P_1}$$

$P_1 = 5$, $T_1 = 298$

$$P_2 = \frac{5T_2}{2 \times 298}$$

$$n\frac{5}{2}R(T_2 - T_1) = -\left(\frac{nRT_2}{P_1} - \frac{nRT_1}{P_1}\right)$$

Put $T_1 = 298$

and $P_2 = \dfrac{5T_2}{2 \times 298}$

Solve and we get $T_2 = 274.16$ K

$T_2 \approx 274$ K

☯ ☯ ☯