MATHEMATICS

1. C

Sol. Doutful points : $-1,\ 0,\ 1,\ \sqrt{2},\ \sqrt{3},\ 2$

at $x=\sqrt{2},\ \sqrt{3}$

$f(x)=(2x^2 \underset{\text{Cont.}}{+} x-[x])+[\underset{\text{Cont}}{x^2}]=\text{Discount}$

at $x=-1$;

$\text{RHL} \Rightarrow \left.\begin{array}{l} f(x)=(2-1-(-1))+0=2 \\ f(-1)=2-1-(-1)+1=3 \end{array}\right\}\text{Dis.}$

at $x=2$

$\text{LHL} \Rightarrow \left.\begin{array}{l} f(x)=8+2-1+3=12 \\ f(2)=8+2-2+4=12 \end{array}\right\}\text{Cont.}$

at $x=0$

$\text{LHL} \Rightarrow \left.\begin{array}{l} 0+0-(-1)+0=1 \\ f(=0) \end{array}\right\}\text{Cont.}$

at $x=1$

$\left.\begin{array}{l} \text{LHL} \Rightarrow 2+1-0+0=3 \\ \quad f(1)=3-1+1=3 \\ \text{RHL} \Rightarrow 2+1-1+1=3 \end{array}\right\}\text{Cont.}$

2. C

Sol. $C \equiv x^2+y^2+gx+gy=0$ .....(1)

$2x+2yy'+g+gy'=0$

$g=-\left(\dfrac{2x+2yy'}{1+y'}\right)$

Put in (1)

$x^2+y^2-\left(\dfrac{2x+2yy'}{1+y'}\right)(x+y)=0$

$(x^2-y^2-2xy)y'=x^2-y^2+2xy$

3. D

Sol. $S_1: x^2+y^2 \le 25$ ....(1)

$S_2: \text{Im of } \dfrac{z+(1-\sqrt{3}i)}{(1-\sqrt{3}i)} \ge 0$

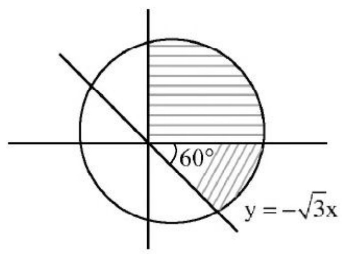
$\text{Im of } \left(\dfrac{x+iy}{1-\sqrt{3}i}+1\right) \ge 0$

$\text{Im of } \left(\dfrac{(x+y)(1+\sqrt{3}i)}{4}\right) \ge 0$

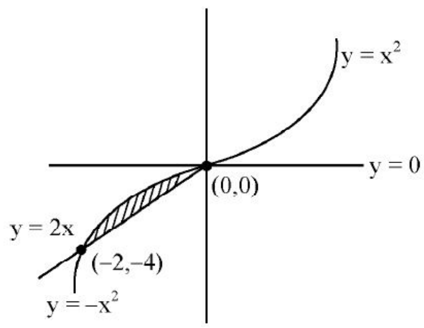
$\Rightarrow \sqrt{3}x+y \ge 0$ ....(2)

$S_3: x \ge 0$ ....(3)

$\text{Area} = \dfrac{5}{12}(\pi(5)^2)$



4. D



$A=\int_{-2}^{0} -x^2-2x = \dfrac{4}{3}$

5. C

Sol. B B H J O

$\boxed{B}$ ______ $4! = 24$

$\boxed{H}$ ______ $\frac{4!}{2!} = 12$

$\boxed{J}$ ______ $\frac{4!}{2!} = 12$

O B B H J

O B B J H $\rightarrow$ 50^{th} rank

6. B

Sol. Let's assume $\vec{v} = \vec{a} + \vec{b} + \hat{i}$

$= 5\hat{i} + 3\hat{j} + \hat{k}$

and $\vec{c} = \hat{i} = \vec{p}$

So,

$\vec{p} \times \vec{v} = \vec{a} \times \vec{p}$

$\vec{p} \times \vec{v} + \vec{p} \times \vec{a} = \vec{0}$

$\vec{p} \times (\vec{v} + \vec{a}) = \vec{0}$

$\Rightarrow \vec{p} = \lambda(\vec{v} + \vec{a})$

$\vec{c} + i = \lambda(7\hat{i} + 8\hat{j})$

$\vec{a} \cdot \vec{c} + \vec{a} \cdot \hat{i} = \lambda \vec{a} \cdot (7\hat{i} + 8\hat{j})$

$-29 + 2 = \lambda(14 + 40)$

$\lambda = -\frac{1}{2}$

$\vec{c} \cdot (-2\hat{i} + \hat{j} + \hat{k}) + \hat{i} \cdot (-2\hat{i} + \hat{j} + \hat{k})$

$= \lambda(7\hat{i} + 8\hat{j}) \cdot (-2\hat{i} + \hat{j}\hat{k})$

$= -\frac{1}{2}(-14 + 8) + 2 = 5$

7. C

Sol. $|\vec{c} - \vec{a}| = |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a} \cdot \vec{c}$

$= |\vec{c}|^2 + 4 - 0$

$\because \vec{a} = \vec{b} \times \vec{c}$

$|\vec{a}| = |\vec{b} \times \vec{c}|$

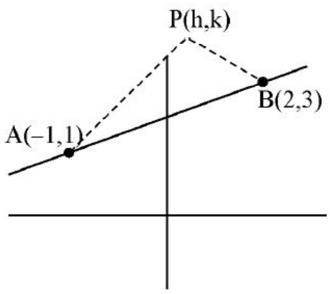
$2 = 3|\vec{c}|\sin\alpha$ $\qquad \alpha \in \left[0, \frac{\pi}{3}\right]$

$|\vec{c}|_{min} = \frac{2}{3} \times \frac{2}{\sqrt{3}}$ $\qquad \operatorname{cosec}\alpha \in \left[\frac{2}{\sqrt{3}}, \infty\right)$

$\Rightarrow 27|\vec{c} - \vec{a}|^2_{min} = 27\left(\frac{16}{27} + 4\right) = 124$

8. A

Sol.



$$\frac{1}{2}\begin{vmatrix} h & k & 1 \\ -1 & 1 & 1 \\ 2 & 3 & 1 \end{vmatrix} = 10$$

$-2x + 3y = 25$

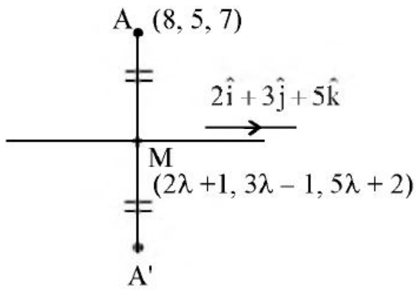
$-\frac{6}{5}x + \frac{9}{5}y = 15$

$a = -\frac{6}{5}, \ b = \frac{9}{5}$

$5a = -6, \ 2b = \frac{18}{5}$

9. C

Sol.



$\overline{AM}.(2\hat{i} + 3\hat{j} + 5\hat{k}) = 0$

$(2\lambda - 7)(2) + (3\lambda - 6)(3) + (5\lambda - 5)(5) = 0$

$38\lambda = 57$

$\lambda = \frac{3}{2}$

$M\left(4, \frac{7}{2}, \frac{19}{2}\right)$

A'(0, 2, 12)

10. C

Sol. $T_{r+1} = {}^{12}C_r \left(\frac{3^{1/5}}{x}\right)^{12-r} \left(\frac{2x}{5^{1/3}}\right)^r$

$$T_{r+1} = \frac{{}^{12}C_r (3)^{\frac{12-r}{3}} (2)^r (x)^{2r-12}}{(5)^{r/3}}$$

r = 6

$$T_7 = \frac{{}^{12}C_6 (3)^{6/5} (2)^6}{5^2} = \left(\frac{9 \times 11 \times 7}{25}\right) 2^8 . 3^{1/5}$$

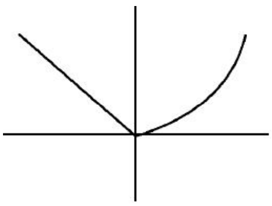
$25\alpha = 693$

11. A

Sol. f(g(x)) = |g(x) – 1|

$$fog\begin{bmatrix} |e^x - 1| & x \geq 0 \\ |x+1-1| & x \leq 0 \end{bmatrix}$$

$$fog\begin{bmatrix} e^x - 1 & x \geq 0 \\ -x & x \leq 0 \end{bmatrix}$$



12. A

Sol. $S_1 : x^2 + y^2 - 2x - 2y + 1 = 0$

$S_2 : x^2 + y^2 + 2x - 3 = 0$

Common chord $= S_1 - S_2 = 0$

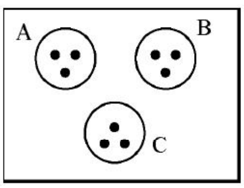
$-4x - 2y + 4 = 0$

$2x + y = 2 \Rightarrow P(0, 2)$

$d^2_{(c.p)} = (1-0)^2 + (2-1)^2 = 2$

13. A

Sol.



$$\frac{9!}{(3!3!3!)3!} \times 3!$$

14. D

Sol. $D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & 5 \\ 1 & 2 & m \end{vmatrix} = 0 \Rightarrow m = 2$

$$D_3 = \begin{vmatrix} 1 & 1 & 4 \\ 2 & 5 & 17 \\ 1 & 2 & n \end{vmatrix} = 0 \Rightarrow n = 7$$

15. B

Sol. D > 0

$b^2 > 4ac$

b = 3 : (a, c) = (1, 1)(1, 2)(2, 1)

b = 4 : (a, c) = (1, 1)(1, 2)(2, 1)(1, 3)(3, 1)

b = 5 : (a, c) = (1, 1)(1, 2)(2, 1) (1, 3)(3, 1)(1, 4) (4, 1) (1, 5)(5, 1)(1, 6)(6, 1)(2, 3)(3, 2)(2, 2)

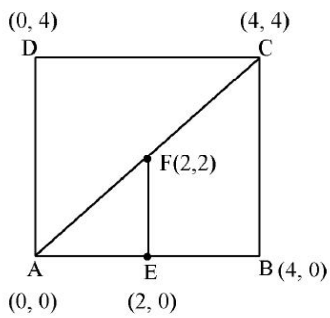
b = 6 : (a, c) = (1, 1)(1, 2)(2, 1) (1, 3)(3, 1)(1, 4) (4, 1) (1, 5)(5, 1)(1, 6)(6, 1)(2, 3) (3, 2) (2, 4) (4, 2) (2, 2)

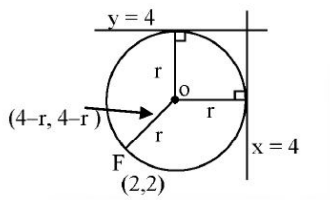
fav. cases = 38

$$\text{Prob.} : \frac{38}{6 \times 6 \times 6}$$

16. B

Sol.





$OF^2 = r^2$

$(2 - r)^2 + (2 - r)^2 = r^2$

$r^2 - 8r + 8 = 0$

17. D

Sol. $I = \int_0^1 1 \cdot (1 - x^{10})^{20} dx$

$x^{10} = t$

$x = t^{1/10}$

$$dx = \frac{1}{10}(t)^{-9/10}dt$$

$$I = \int_0^1 (1-t)^{20}\frac{1}{10}(t)^{-9/10}dt$$

$$I = \frac{1}{10}\int_0^1 t^{-9/10}(1-t)^{20}dt$$

$$a = \frac{1}{10} \quad b = \frac{1}{10} \quad c = 21$$

18. D

Equation co-factor of A_{21}

$(2\alpha^2 - 3\alpha) = \alpha$

$\alpha = 0,\ 2\,(\text{accept})$

Now, $2\alpha^2 - \alpha\beta = 3\alpha$

$\alpha = 2 \quad \beta = 1$

$|AB| = |A\ \text{cof}\ (A)| = |A|^3$

$$A = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \\ -1 & 2 & 4 \end{vmatrix} = 6 - 2(9) + 3(6) = 6$$

19. D

Sol. $$y = \frac{2\cos\theta + 2\cos^2\theta - 1}{4\cos^3\theta - 3\cos\theta + 8\cos^2\theta - 4 + 5\cos\theta + 2}$$

$$y = \frac{(2\cos^2\theta + 2\cos\theta - 1)}{(2\cos^2\theta + 2\cos\theta - 1)(2\cos\theta + 2)}$$

$$y = \frac{1}{2}\left(\frac{1}{1+\cos\theta}\right)$$

$$\Rightarrow \theta = \frac{\pi}{2} \quad y = \frac{1}{2}$$

$$y'' = \frac{1}{2}\left[\frac{\cos\theta(1+\cos\theta)^2 - \sin\theta(2)(1+\cos\theta)(-\sin\theta)}{(1+\cos\theta)^4}\right]$$

$$\Rightarrow \theta = \frac{\pi}{2} \qquad y = 1$$

20. A

Sol. $$k = 4\underbrace{\left(4^x + \frac{1}{4^x}\right)}_{\geq 2} + \underbrace{\left(4^{2x} + \frac{1}{4^{2x}}\right)}_{\geq 2}$$

$k \geq 10$

21. 5

Sol. $$\frac{1}{3} + k + \frac{1}{6} + \frac{1}{4} = 1 \qquad \Rightarrow k = \frac{1}{4}$$

$$\mu = \frac{\alpha}{3} + \frac{1}{4} - \frac{3}{4}$$

$$\boxed{\mu = \frac{\alpha}{3} - \frac{1}{2}}$$

$$\sigma = \sqrt{\left(\alpha^2 + \frac{1}{3} + \frac{1}{4} + 9\frac{1}{4}\right) - \left(\frac{\alpha}{3} - \frac{1}{2}\right)^2}$$

$$\sigma = \sqrt{\frac{2\alpha^2}{9} + \frac{\alpha}{3} + \frac{9}{4}}$$

$$\sigma = \mu + 2$$

$$\sigma = (\mu+2)^2 \Rightarrow \frac{2\alpha^2}{9} + \frac{\alpha}{3} + \frac{9}{4} = \frac{\alpha^2}{9} + \frac{9}{4} + \alpha$$

$$\frac{\alpha^2}{9} - \frac{2\alpha}{3} = 0$$

$\alpha = 0,$ (reject) or $\alpha = 6$

($\because$ $x = 0$ is already given)

$\Rightarrow \alpha + \mu = 2\mu + 2$

$= 5$

22. 18

Sol. $$IF = e^{\int \frac{2x}{(1+x^2)^2}dx} = e^{\frac{-1}{1+x^2}}$$

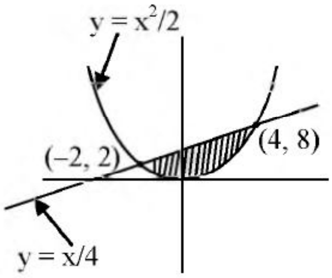
$$y \cdot e^{\frac{-1}{1+x^2}} = \int x \cdot e^{\frac{1}{1+x^2}} \cdot e^{\frac{-1}{1+x^2}}dx$$

$$y \cdot e^{\frac{-1}{1+x^2}} = \frac{x^2}{2} + c$$

$(0,\ 0) \Rightarrow \boxed{C = 0}$

$$y(x)=\frac{x^2}{2}e^{\frac{1}{1+x^2}}$$

$$f(x)=\frac{x^2}{2}$$

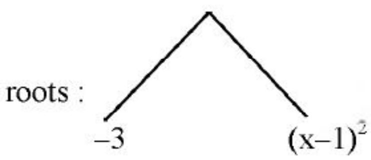


$$A=\int_{-2}^{4}(x+4)-\frac{x^2}{2}dx=18$$

23. 2

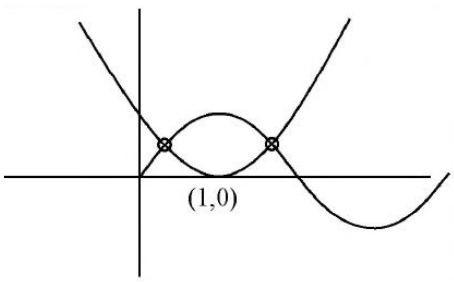
Sol. $\sin^2x-(x^2-2x-2)\sin x-3(x-1)^2=0$

$\sin^2x-(x-1)^2)\sin x-3(x-1)^2=0$



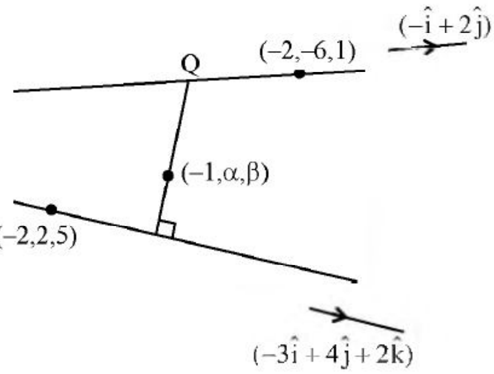
$\sin x=-3$ (reject) or $(x-1)^2$

$\sin x=(x-1)^2$



24. 25

Sol.



$P(-3\lambda-2,\ 4\lambda+2,\ 2\lambda+5)$

$Q(-\mu-2,\ 2\mu-6,\ 1)$

$$\text{DRS of PQ}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 2 & 0\\ -3 & 4 & 2\end{vmatrix}$$

$$=(4\hat{i}+2\hat{j}+2\hat{k})$$

OR

(2, 1, 1)

$$\frac{3\lambda-\mu}{2}=\frac{2\mu-4\lambda-8}{1}=\frac{-2\lambda-4}{1}$$

$$\Rightarrow\mu=\lambda+2\ \ \&\ \ 7\lambda=\mu-8$$

$\boxed{\lambda=-1}$ $\boxed{\mu=1}$

Q : (–3, –4, 1)

$$L_{PQ}=\frac{x+3}{2}=\frac{y+4}{1}=\frac{z-1}{1}$$

$$(-1,\ \alpha,\ \beta)\Rightarrow 1=\frac{\alpha+4}{1}=\frac{\beta-1}{1}$$

$$\Rightarrow\alpha=-3,\ \beta=2$$

$$(\alpha-\beta)^2=25$$

25. 76

Sol. $S=1+\frac{x}{2\sqrt{3}}+\frac{x^3}{18}+\frac{x^3}{36\sqrt{3}}+\frac{x^4}{180}+....\infty$

Put $\frac{x}{\sqrt{3}}=t$, where $x=\sqrt{3}-\sqrt{2}$

$$S=1+\frac{1}{2}+\frac{t^2}{6}+\frac{t^3}{12}+\frac{t^4}{20}+.....$$

$$S=1+t\left(1-\frac{1}{2}\right)+t^2\left(\frac{1}{2}-\frac{1}{3}\right)+t^3\left(\frac{1}{3}-\frac{1}{4}\right)+t^4\left(\frac{1}{4}-\frac{1}{5}\right)$$

$$S=\left(1+t+\frac{t^2}{2}+\frac{t^3}{3}+\frac{t^3}{4}+....\right)-\left(\frac{t}{2}+\frac{t^2}{3}+\frac{t^3}{4}+\frac{t^4}{5}+....\right)$$

$$S=\left(t+\frac{t^2}{2}+....\right)-\frac{1}{t}\left(t+\frac{t^2}{2}+\frac{t^3}{3}+.....\right)+2$$

$$S=2+\left(1-\frac{1}{t}\right)(-\log(1-t))=\left(\frac{1}{t}-1\right)\log(1-t)+2$$

$$S=2\left(\frac{\sqrt{3}}{\sqrt{3}-\sqrt{2}}-1\right)\log\left(1-\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}\right)$$

$$S=2+\left(\frac{\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)\log e\frac{\sqrt{2}}{\sqrt{3}}$$

$$S=2+\frac{\left(\sqrt{6}+2\right)}{}\log e\frac{2}{3}=2+\left(\sqrt{\frac{3}{2}}+1\right)\log e\frac{2}{3}$$

$a = 2,\ b = 3$

$11a + 18b = 11 \times 2 + 18 \times 3 = 76$

26. 170

Sol. $2x^2+x-2=0 \to a,\ b$

$2x^2-x-2=0 \to \frac{1}{a},\ \frac{1}{b}$

$$\lim_{x\to\frac{1}{a}}\frac{\left(1-\cos 2\left(s-\frac{1}{a}\right)\left(x-\frac{1}{b}\right)\right)}{4\left(x-\frac{1}{b}\right)}\times\frac{4\left(x-\frac{1}{b}\right)^2}{a^2\left(x-\frac{1}{a}\right)^2}$$

$$=16\times\frac{2}{a^2}\left(\frac{1}{a}-\frac{1}{b}\right)^2$$

$$=\frac{32}{a^2}\left(\frac{17}{4}\right)=\frac{17.8}{a^2}=\frac{17\times8\times16}{\left(-1+\sqrt{117}\right)^2}$$

$$=\frac{136.16}{18.2\sqrt{7}}\times\frac{18+2\sqrt{7}}{18+2\sqrt{7}}$$

$$=\frac{136}{256}\left(18+2\sqrt{7}\right)\cdot16$$

$$=153+17\sqrt{17}=\alpha+\beta\sqrt{17}$$

$$\alpha+\beta=153+17=170$$

27. 1

Sol. $f(t)=\int_0^{\pi}\frac{2x}{1-\cos^2 t\sin^2 x}dx$ ....(1)

$$=2\int_0^{\pi}\frac{(\pi-x)dx}{1-\cos^2 t\sin^2 x} \quad ....(2)$$

$$2f(t)=2\int_0^{\pi}\frac{\pi}{1-\cos^2 t\sin^2 x}dx$$

divide & by $\cos^2 x$

$$f(t)=\pi\int_0^{\pi}\frac{\sec^2 xdx}{\sec^2 x-\cos^2 t\tan^2 x}$$

$$f(t)=2\pi\int_0^{\pi/2}\frac{\sec^2 xdx}{\sec^2 x-\cos^2 t\tan^2 t}$$

$\tan x = z$

$\sec^2 xdx = dz$

$$f(t)=2\pi\int_0^{\infty}\frac{dz}{1+\sin^2 t\cdot z^2}$$

$$=\frac{\pi^2}{\sin t}$$

Then $\int_0^{\pi/2}\frac{\pi^2}{f(t)}dt$

$$=\int_0^{\pi/2}\sin tdt$$

$= 1$

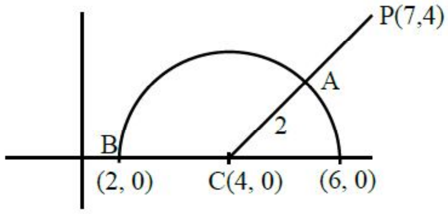
28. 1600

Sol. $(x-7)^2+(y-4)^2$

$$y=\sqrt{8x-x^2-12}$$

$y^2 = -(x-4)^2 + 16 - 12$

$(x-4)^2 + y^2 = 4$



$m = 9$

$M = 41$

$M^2 - m^2 = 41^2 - 9^2 = 1600$

29. 10

Sol. $y^2 = 4(x - 9)$

slope of tangent $= \frac{-1}{2}$

Point of contact $P\left(9 + \frac{1}{\left(-\frac{1}{2}\right)^2}, \frac{2\times 1}{\frac{-1}{2}}\right)$

$P(13, -4)$

center of circle $C(7, 4)$

distance $CP = \sqrt{(13-7)^2 + (-4-4)^2}$

$= 10$

30. 3

Sol. **Case I :** $x \geq -5$

$x^2 + 5x + 2x + 12 = 0$

$x^2 + 7x + 12 = 0$

$x = -3, -4$

Case II : $-7 < x < -5$

$-x^2 - 5x + 2x + 14 - 2 = 0$

$-x^2 - 3x + 12 = 0$

$x = \frac{-3 \pm \sqrt{9+48}}{2}$

$= \frac{-3 \pm \sqrt{57}}{2}$

$x = \frac{-3-\sqrt{57}}{2}, \frac{-3+\sqrt{57}}{2}$ (rejected)

Case III : $x \leq -7$

$-x^2 - 5x - 2x - 14 - 2 = 0$

$x^2 + 7x + 16 = 0$

$D = 49 - 64 < 0$

No solutions

No. of solutions = 3

PHYSICS

Section - A (Single Correct Answer)

31. A

Sol. As $\lambda_{red} > \lambda_{yellow} > \lambda_{violet}$

Light ray with longer wavelength bends less.

32. C

Sol. $KE_{max} = h\nu - \phi_0 = eV$

33. A

Sol. $F_C = \frac{mv^2}{r}$

$\frac{Kq_1q_2}{r^2} = \frac{mv^2}{r}$

$mv^2r^2 = Kq_1q_2r$

$\frac{L^2}{m} = Kq_1q_2r$

$L \propto \sqrt{r}$

34. C

Sol. $i_g = \frac{10}{400+100} = 20\times 10^{-3}\,A$

For ammeter

Let shunt resistance = S

$i_gR = (i - i_g)S$

$20 \times 10^{-3} \times 100 = 10\,S$

$S = 20 \times 10^{-2}\,\Omega$

35. A

Sol. Static charge is developed due to air friction. This can result in combustion. So, metallic chains is used to discharge excess charge.

36. C

Sol. n = number of molecule per unit volume

d = diameter of the molecule

$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$ (By Theory)

37. A

Sol. $\vec{P} = \cos(kt)\hat{i} - \sin(kt)\hat{j}; |\vec{P}| = 1$

$\because \vec{P} = m\vec{v}$

$\therefore \hat{P} = \hat{v}$

$\Rightarrow \hat{v} = \cos(kt)\hat{i} - \sin(kt)\hat{j}$

$\hat{a} = \frac{-k\sin(kt)\hat{i} - k\cos(kt)\hat{j}}{k}$

$\Rightarrow \hat{a} = -\sin kt\hat{i} - \cos kt\hat{j}$

$\because \hat{F} = \hat{a} - \sin kt\hat{i} - \cos kt\hat{j}$

$$\cos\theta = \frac{\hat{F}.\hat{P}}{|\hat{F}||\hat{P}|} = -\frac{\sin kt\cos t + \sin kt\cos t}{1\times 1} = 0$$

$$\Rightarrow \theta = \frac{\pi}{2}$$

38. C

Sol. $\vec{F}_1 = q\vec{E}$ (Theory)

$$\vec{F}_2 = q\left(\vec{V}\times\vec{B}\right)$$

39. B

Sol. Given : R = 9m,

120 revolution in 3 min

$$\omega = \frac{120\,\text{Rev.}}{3\,\text{min.}} = \frac{120\times 2\pi\text{rad}}{3\times 60\,\text{sec}} = \frac{4\pi}{3}\text{rad/s}$$

$$a_{\text{centripetal}} = \omega^2 R = \left(\frac{4\pi}{3}\right)^2 \times 9 = 16\pi^2\ \text{m/s}^2$$

40. C

Sol. Voltage across inductor $V_L = IX_L$

$31.4 = I[L\omega]$

$31.4 = I[L(2\pi f)]$

$31.4 = I[10\times 10^{-3}(2\times 3.14)\times 50 \Rightarrow I = 10\ \text{A}$

41. B

Sol. $\because$ [V] = [b]

$\therefore$ Dimension of b = $[L^3]$

$$\&[P] = \left[\frac{a}{V^2}\right]$$

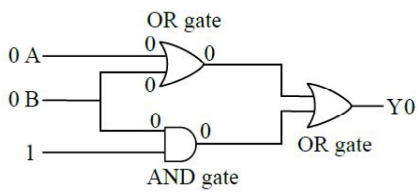
$[a] = [PV^2] = [ML^{-1}T^{-2}][L^6]$

Dimension of a = $[ML^5T^{-2}]$

$$\therefore ab^{-1} = \frac{[ML^5T^{-2}]}{[L^3]} = [ML^2T^{-2}]$$

42. B

Sol.



43. C

Sol. P = costant $\Rightarrow$ FV = constant

$$\Rightarrow \quad m\frac{dV}{dt}V = \text{constant}$$

$$\int_0^V VdV = (C)\int_0^t dt$$

$$\left(\frac{V^2}{2}\right) = Ct$$

$V \propto t^{1/2}$

$$\frac{ds}{dt} \propto t^{1/2}$$

$$\int_0^S ds = K\int_0^t t^{1/2}dt$$

$$S = K\times\frac{2}{3}t^{3/2}$$

$S \propto t^{3/2}$

$\therefore$ displacement is proportional to $(t)^{3/2}$

44. B

Sol. Infrared is the least energetic thus having biggest wavelength (λ) & gamma rays are most energetic thus having smallest wavelength (λ).

45. C

Sol. $P \propto T^3$

PT^{-3} = constant

$\because \frac{PV}{T} = nR =$ constant from ideal gas equation

(P) $(PV)^{-3}$ = constant

$P^{-2}\,V^{-3}$ = cosntant ...(1)

Process equation for adiabatic process is

PV^{y} = constant ...(2)

Comparing equation (1) and (2)

$$\frac{C_P}{C_V} = y = \frac{3}{2}$$

46. C

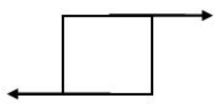
Sol. (A) stress $= \frac{F_{\text{restoring}}}{A}$

If A = 1

Stress = $F_{\text{restoring}}$

(A)-(III)

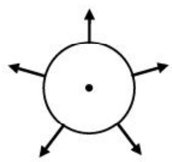
(B)



$\Rightarrow$ Shear stress

(B)-(IV)

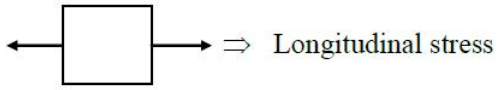
(C)



$\Rightarrow$ Volumetric stress

(C)-(I)

(D)



(D)-(II)

47. C

Sol. 20 VSD = 19 MSD

$$1\text{VSD} = \frac{19}{20}\text{MSD}$$

L.C. = 1 MSD – 1 VSD

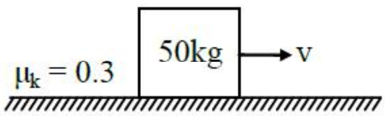
$$0.1\text{ mm} = 1\text{ MSD} - \frac{19}{20}\text{MSD}$$

$$0.1 = \frac{1}{20}\text{MSD}$$

1 MSD = 2mm

48. B

Sol.



$F_k = \mu_k N = 0.3 \times 50 \times 9.8 = 147$ N

49. D

Sol. $T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$

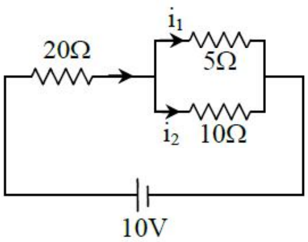
$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}\left(\frac{M_2}{M_1}\right)^{1/2}$$

$$\frac{6}{24} = \frac{(r_1)^{3/2}}{(4.2\times10^4)^{3/2}}\left(\frac{M}{M/4}\right)^{1/2}$$

$r_1 = 1.05 \times 10^4$ km

50. B

Sol.



$$\frac{i_1}{i_2} = \frac{10}{5} = \frac{2}{1}$$

$$\frac{P_1}{P_2} = \frac{i_1^2 R_1}{i_2^2 R_2} = \left(\frac{2}{1}\right)^2 \times \frac{5}{10} = \frac{2}{1}$$

Section - B (Numerical Value)

51. 500

Sol. $\mu_0 ni = B$ n = number of turns per unit length

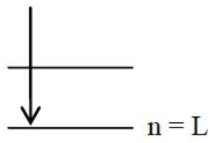
$$\mu_0\left(\frac{m}{l}\right)i = B$$

$$m = \frac{Bl}{\mu_0 i} = \frac{6.28\times10^{-3}\times0.5}{12.56\times10^{-7}\times5}$$

$m = 500$

52. 6588

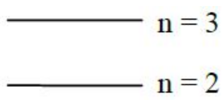
Sol. Lyman Series



Shortest, $\frac{hc}{\lambda} = -13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\lambda \uparrow E \uparrow$; $\frac{hc}{\lambda_0} = -13.6(1)$

Balmer Series :



$$\frac{hc}{\lambda_1} = -13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\frac{hc}{\lambda_1} = -13.6\left(\frac{1}{4} - \frac{1}{9}\right)$$

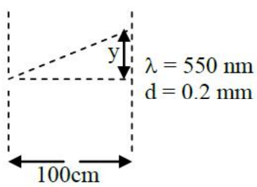
$$\frac{hc}{\lambda_1} = -13.6\times\left(\frac{5}{36}\right)$$

$$\Rightarrow \frac{-13.6\lambda_0}{\lambda_1} = -13.6\times\frac{5}{36}$$

$$\lambda_1 = \frac{\lambda_0\times36}{5} = \frac{915\times36}{5} = 6588$$

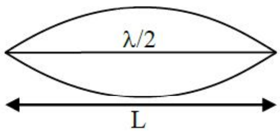
53. 275

Sol.



$$y = \frac{\lambda D}{d} = \frac{550 \times 10^{-9} \times 100 \times 10^{-2}}{0.2 \times 10^{-3}} = 275$$

54. 60

Sol. 

$$f_0 = 400 \text{ Hz} ; \ v = \sqrt{\frac{T}{\mu}} = \text{ constant}$$

$$\frac{\lambda}{2} = L; \ \ v = f_0 \lambda$$

$$\frac{v}{2f_0} = L \Rightarrow v = 2Lf_0$$

$$L' = \frac{v}{2f'} = \frac{2Lf_0}{2f'}$$

$$= \frac{Lf_0}{f'} = \frac{90 \times 400}{600} = 60$$

55. 2

Sol. $$\frac{\frac{1}{2} I\omega^2}{\frac{1}{2} I\omega^2 + \frac{1}{2} mv^2} = \frac{\left(\frac{1}{2}\right)\left(\frac{2}{3} mR^2\right)\omega^2}{\left(\frac{1}{2}\right)\left(\frac{2}{3} mR^2\right)\omega^2 + \frac{1}{2} m (R\omega)^2}$$

$$= \frac{\frac{2}{3}}{\frac{2}{3} + 1} = \frac{2}{5}$$

$x = 2$

56. 1000

Sol. $$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\frac{F_1}{\pi (7)^2} = \frac{10}{\pi \times (0.7)^2}$$

$F_1 = 1000$ N

57. 16

Sol. $$E_P = \frac{2KP}{r^3} = E$$

$$E_R = \frac{KP}{(2r)^3} = \frac{E}{16}$$

$x = 16$

58. 16

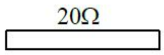
Sol. $$H_{max} = \frac{u^2 \sin^2 \theta}{2g}$$

$$\frac{H_{1max}}{H_{2max}} = \frac{u_1^2}{u_2^2}$$

$$\frac{64}{H_{2max}} = \frac{u^2}{(u/2)^2}$$

$H_{2max} = 16$ m

59. 5

Sol. 

$\Rightarrow$ 10 equal part

Each part has resistance = 2Ω

2 parts are connected in parallel so, R = 1Ω

Now, there will be 5 parts each of resistance 1Ω, they are connected in series.

$R_{eq} = 5R$, $R_{eq} = 5\Omega$

60. 4

Sol. $I = 3t + 8$

$\varepsilon = 12$ mV

$$|\varepsilon| = L\left|\frac{dI}{dt}\right|$$

$12 = L \times 3$

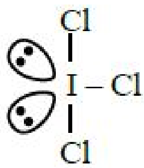
$L = 4$ mH

CHEMISTRY

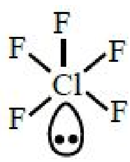
Section - A (Single Correct Answer)

61. (C)

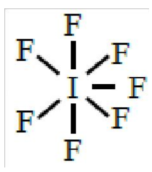
Sol. A. I – Cl (IV) linear

B. 

(I) T-shape

C. 

(II)Square pyramidal

D. 

(III)Pentagonal bipyramidal

62. (A)

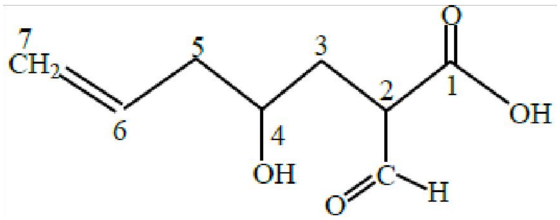
Sol. Fe^{+2} ions undergoes hydrolysis, therefore while preparing aqueous solution of ferrous sulphate and ammonium sulphate in water dilute sulphuric acid is added to prevent hydrolysis of ferrous sulphate.

63. (C)

Sol.:

64. (C)

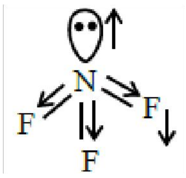
Sol.:



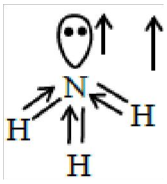
2-formly-4-hydroxyhept-6-enoic acid

65. (A)

Sol.



Resultant dipole moment = 0.80×10^{-30} Cm



Resultant dipole moment = 4.90×10^{-30} cm

66. (A)

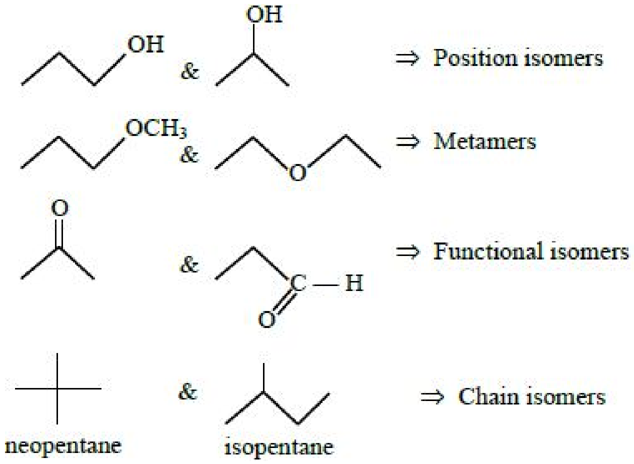
Sol. $BaCl_2$, NaCl are soluble but on adding HCl(g) to $BaCl_2$, NaCl solutions, Sodium or Barium chlorides may precipitate out, as a consequence of the law of mass action.

67. (B)

Sol. Tetrahedral complex does not show geometrical isomerism.

68. (D)

Sol.:



69. (D)

Sol. W = ZIt

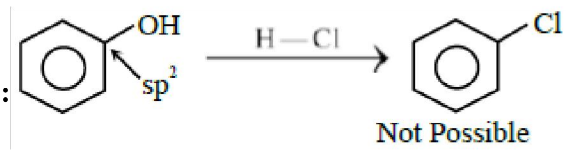
W = ZQ

$$Q = \frac{W}{Z}$$

W = ZQ = (electrochemical equivalent)

70. (B)

Sol.:



71. (A)

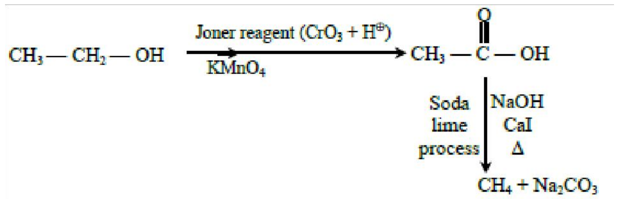
Sol. $r_{Na} > r_{Na^+}$

So, Statement (I) is correct but size of anions are greater than size of neutral atoms.

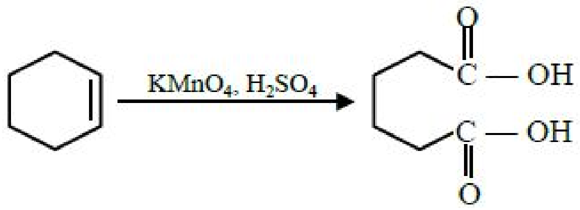
So statement (II) is incorrect.

72. (A)

Sol.



73. (D)



74. (C)

Sol. $M \mid M^{+2} \parallel X / X^{2-}$

$$E^0_{cell} = E^0_{M/M^{+2}} + E^0_{X/X^{-2}}$$

$= -0.46 + 0.34 = -0.12V$

As E^0_{cell} is negative so anode becomes cathode and cathode become anode. Spontaneous reaction will be

$M^{+2} + X^{2-} \rightarrow M + X$

75. (B)

Sol. $$C_nH_{2n+2} + \frac{3n+1}{2}O_2 \rightarrow nCO_2 + (n+1)H_2O$$

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

4gm	11gm
0.25mole	0.25mole

0.25 mole CH_4 gives 0.25 mole (or 11gm) CO_2

76. (A)

$TiCl_4 \Rightarrow Ti^{+4}$	$e^0t_2^0$
$MnO_4^- \Rightarrow Mn^{+7}$	$e^0t_2^0$
$FeO_4^{2-} \Rightarrow Fe^{+6}$	$e^2t_2^0$
$FeCl_4^{2-} \Rightarrow Fe^{+2}$	$e^3t_2^3$
$CoCl_4^{2-} \Rightarrow Co^{+2}$	$e^4t_2^3$

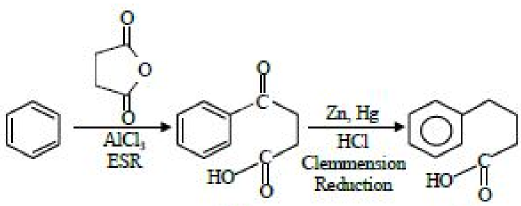
77. (C)

Sol. The ions Ti^{+2}, V^{+2} Cr^{+2} are strong reducing agents and will liberate hydrogen from a dilute acid, eg.

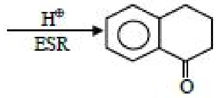
$$2Cr^{+2}_{(aq)} + 2H^+_{(aq)} \longrightarrow 2Cr^{+3}_{(aq)} + H_2(g)$$

78. (B)

Sol.



(A) (B)



79. (A)

Sol. A. size order $T\ell$ > In > Al > Ga > B

B. Electronegativity order B > Al < Ga < In < $T\ell$

D. B, Al are more stable in +3 oxidation state

So, only C, E statements are correct.

80. (A)

Sol. Coagulation of egg give primary structure of protein, which is known as denaturation of protein

<u>Section - B (Numerical Value Type)</u>

81. (6535)

Sol. 6C(graphite)+3H_2(g) $\rightarrow$ $C_6H_6(\ell)$; ΔH = 48.5 kJ/mol

C(graphite) + O_2(g) $\rightarrow$ CO_2(g); ΔH = –393.5 kJ/mol

$$H_2^{(g)} + \frac{1}{2}(g) \rightarrow H_2O(\ell); \Delta H = -286\,kJ/mol$$

equation –(1) × 1 + (2) × 6 + (3) × 3

– 48.5 –6 × 393.5 – 3 × 286

= – 3267.5 kJ for 1 mol

= – 6535 kJ for 2 mol

Ans. 6535 kJ

82. (6)

Sol. $4FeCr_2O_4 + 8Na_2CO_3 + 7O_2 \rightarrow$

$8Na_2CrO_4 + 2Fe_2O_3 + 8CO_2$

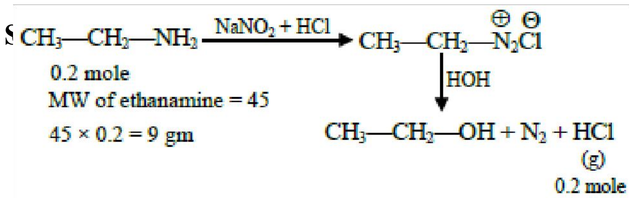
A B

Spin only magnetic moment

For Na_2CrO_4	$\mu_B = 0$
For Fe_2O_3	$\mu_B = 5.9$

sum = 5.9

83. (9)



84. (6)

Sol. n = 4

ℓ	m_ℓ
0	0
1	–1, 0, +1
2	–2, –1, 0, +1, +2, +3

So number of orbital associated with

$n = 4, |m_\ell| = 1$ are 6

Now each orbital contain one e^- with $m_s = -\frac{1}{2}$

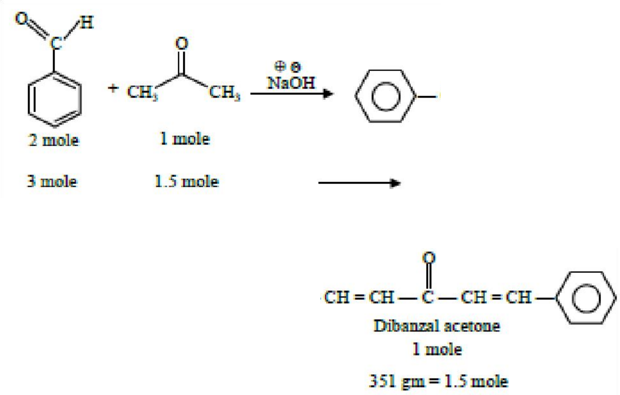
85. (50)

Sol.: R_f of A $= \frac{5}{12.5}$ $\quad$ R_f of C $= \frac{10}{12.5}$

Ratio $= \frac{R_{f(A)}}{R_{f(C)}} = \frac{1}{2} = 0.5$ or 50×10^{-2}.

86. (318)

Sol. Claisen Schmidt reaction



mw of benzaldehyde = 106

106 × 3 = 318 gm. Benzaldehyde is required to give 1.5 mole (or 351 gm) product.

87. (315)

Sol. $2A(g) + B(g) \rightarrow C(g)$

r_1 1.5 atm $\quad$ 0.7 atm

r_2 0.5 atm $\quad$ 0.2 atm $\quad$ 0.5 atm

$\because r = K[P_A]^2[P_B]$

$r_1 = K[1.5]^2[0.7]$

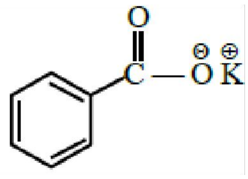
$r_2 = K[0.5]^2[0.2]$

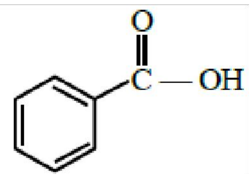
$\frac{r_1}{r_2} = 9\times\frac{7}{2} = 31.5 = 315\times10^{-1}$

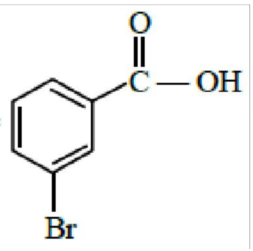
Ans. 315

88. (4)

Sol.

A =


B =


C =


π bonds = 4

89. (19)

Sol.: Mass of $CH_3COOH = V \times d$

= 5 ml × 1.2 g/ml

= 6 gm

$^{n}CH_3COOH = \frac{6}{60} = 0.1$ mol

$^{m}CH_3COOH \approx M_{CH_3COOH} = \frac{0.1}{1} = 0.1M$

$CH_3COOH \rightleftharpoons CH_3COO^- + H^+$

C

$C - C\alpha \quad\quad C\alpha \quad\quad C\alpha$

$K_a = \frac{C\alpha^2}{1-\alpha}$

$1-\alpha \approx 1 \Rightarrow K_a = C\alpha^2$

$\alpha = \sqrt{\frac{Ka}{C}} = \sqrt{\frac{6.25\times10^{-5}}{0.1}} = 25\times10^{-3}$

V.f.(i) = 1 + α (n – 1) = 1 + α (2 – 1) = 1 + α

= 1 + 25 $\times10^{-3}$ = 1.025

$\Delta T_f = iK_f m$

$= (1.025)(1.86)(0.1)$

$= 0.19$

$= 19 \times 10^{-2}$

90. (6)

Sol. H_2, CO_2, BF_3, CH_4, SiF_4, BeF_2

are symm. molecule so dipole moment is zero

☯ ☯ ☯