MATHEMATICS

1. C

Sol. $\frac{x+6}{3}=\frac{y}{2}=\frac{z+1}{1}=\lambda$ ....(1)

$x=3\lambda-6,\ y=2\lambda,\ z=\lambda-1$

$\frac{x-7}{4}=\frac{y-9}{3}=\frac{z-4}{2}=\mu$ ....(2)

$x=4\mu+7,\ y=3\mu+9,\ z=2\mu+4$

$3\lambda-6=4\mu+7\Rightarrow 3\lambda-4\mu=13$ ....(3) × 2

$2\lambda=3\mu+9\Rightarrow 2\lambda-3\mu=9$ ....(4) × 3

$6\lambda-8\mu=26$

$6\lambda-9\mu=27$

$-\quad+\quad-$

$\mu=-1$

$\Rightarrow 3\lambda-4(-1)=13$

$3\lambda=9$

$\lambda=3$

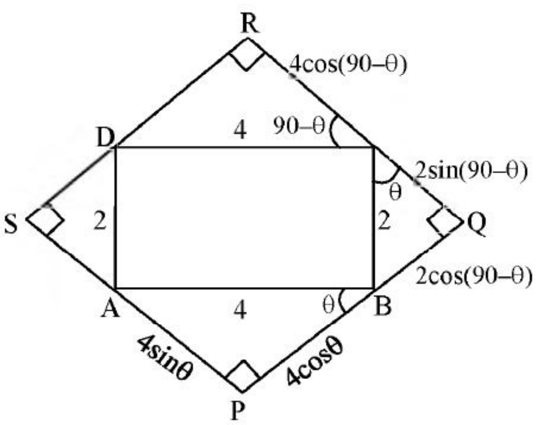
int. point (3, 6, 2) ; (7, 8, 9)

$d^2 = 16 + 4 + 49 = 69$

Ans. $d^2 + 6 = 69 + 6 = 75$

2. A

Sol.



$\text{Area}=(4\cos\theta+2\sin\theta)(2\cos\theta+4\sin\theta)$

$=8\cos^2\theta+16\sin\theta\cos\theta+4\sin\theta\cos\theta+8\sin^2\theta$

$=8+20\sin\theta\cos\theta$

$=8+10\sin 2\theta$

Max Area $=8+10=18(\sin 2\theta=1)\theta=45^\circ$

$(a+b)^2=(4\cos\theta+2\sin\theta+2\cos\theta+4\sin\theta)^2$

$=(6\cos\theta+6\sin\theta)^2$

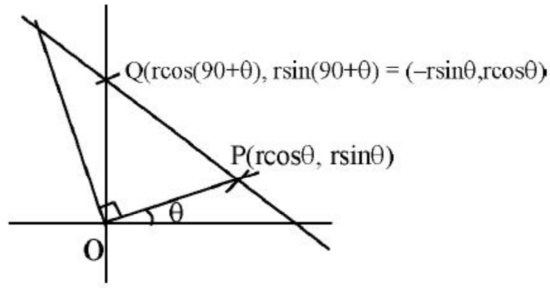
$=36(\sin\theta+\cos\theta)^2$

$=36(\sqrt{2})^2$

$=72$

3. C

Sol.



$3x + 4y = 12$

$3(r\cos\theta)+4(r\sin\theta)=12$

$r(3\cos\theta+4\sin\theta)=12$ .....(1)

$3(-r\sin\theta)+4(r\cos\theta)=12$

$r(-3\sin\theta+4\cos\theta)=12$ ....(2)

$\left(\frac{12}{r}\right)^2+\left(\frac{12}{r}\right)^2=(3\cos\theta+4\sin\theta)^2+(-3\sin\theta+4\cos\theta)^2$

$2\left(\frac{12}{r}\right)^2=9+16$

$\frac{2\times144}{r^2}=25\Rightarrow 288=25r^2$

$\Rightarrow\frac{288}{25}=r^2$

$\Rightarrow \sqrt{2}\left(\frac{12}{5}\right) = r$

$\ell = OP^2 + PQ^2 + QO^2$

$\ell = r^2 + r^2 + r^2(\cos\theta + \sin\theta)^2 + r^2(\sin\theta + \cos\theta)^2$

$= 2r^2 + r^2(1 + \sin 2\theta + 1 - 2\sin 2\theta)$

$= 2r^2 + 2r^2$

$= 4r^2$

$= 4\left(\frac{288}{25}\right) = \frac{1152}{25} = 46.08$

$[\ell] = 46$

4. B

Sol. $\frac{dy}{dx} + 2y = \sin 2x,\ y(0) = \frac{3}{4}$

$I.F. = e^{\int 2dx} = e^{2x}$

$y.e^{2x} = \int e^{2x} \sin 2x dx$

$y.e^{2x} = \frac{e^{2x}(2\sin 2x - 2\cos 2x)}{4+4} + C$

$x = 0,\ y = \frac{3}{4} \Rightarrow \frac{3}{4}.1 = \frac{1(0-2)}{8} + C$

$\frac{3}{4} = -\frac{1}{4} + C$

$1 = C$

$y = \frac{2\sin 2x - 2\cos 2x}{8} + 1.e^{-2x}$

$x = \frac{\pi}{8},\ y = \frac{1}{8}\left(2\sin\frac{\pi}{4} - 2\cos\frac{\pi}{4}\right) + e^{-2\left(\frac{\pi}{8}\right)}$

$y = 0 + e^{-\frac{\pi}{4}}$

5. D

Sol. $f(x) = \sin x + 3x - \frac{2}{\pi}(x^2 + x)x \in \left[0,\ \frac{\pi}{2}\right]$

$f'(x) = \cos x + 3 - \frac{2}{\pi}(2x+1) > 0\ f(x) \uparrow$

$f''(x) = -\sin x + 0 - \frac{\pi}{2}(2)$

$= -\sin x - \frac{4}{\pi} < 0\ f'(x) \downarrow$

$0 < x < \frac{\pi}{2}$

$\Rightarrow -\frac{2}{\pi}\left(\underset{+1}{0} < \underset{+1}{2x} < \underset{+1}{\pi}\right)$

$-\underset{+3}{\frac{2}{\pi}} > \frac{-2}{\pi}\underset{+3}{(2x+1)} > \frac{2}{\pi}\underset{+3}{(\pi+1)}$

$3 - \underset{(+ve)}{\frac{2}{\pi}} > 3 - \frac{2}{\pi}(2x+1) > 3 - \underset{(+ve)}{\frac{2}{\pi}}(\pi+1)$

6. C

Sol. $11x + y + \lambda z = -5$

$2x + 3y + 5z = 3$

$8x - 19y - 39z = \mu$

for infinite sol.

$D = \begin{vmatrix} 11 & 1 & \lambda \\ 2 & 3 & 5 \\ 8 & -19 & -39 \end{vmatrix} = 0$

$\Rightarrow 11(-117 + 95) - 1(-78 - 40) + \lambda(-38 - 24)$

$\Rightarrow 11(-22) + 118 - \lambda(62) = 0$

$\Rightarrow 62\lambda = 118 - 242$

$\Rightarrow \lambda - \frac{-124}{62} = -2$

$D_1 = \begin{vmatrix} -5 & 1 & -2 \\ 3 & 3 & 5 \\ \mu & -19 & -39 \end{vmatrix} = 0$

$\Rightarrow -5(-117 + 95) - 1(-117 - 5\mu) - 2(-57 - 3\mu) = 0$

$\Rightarrow -5(-22) + 117 + 5\mu + 114 + 6\mu = 0$

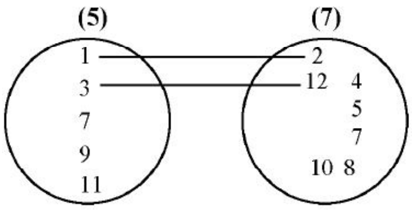
$\Rightarrow 11\mu = -110 - 231 = -341$

$\Rightarrow \mu = -31$

$\lambda^4 - \mu = (-2)^4 - (-31) = 16 + 31 = 47$

7. A

Sol.



A = {1, 3, 7, 9, 11}

B = {2, 4, 5, 7, 8, 10, 12} f(1) + f(3) = 14

(i) 2 + 12

(ii) 4 + 10

2 × (2 × 5 × 4 × 3) = 240

8. D

Sol. $f(x)=\frac{\sin 3x+\alpha\sin x-\beta\cos 3x}{x^3}$

is continuous at x = 0

$$\lim_{x\to0}\frac{3x-\frac{(3x)^3}{\underline{|3}}+....+\alpha\left(x-\frac{x^3}{\underline{|3}}....\right)-\beta\left(1-\frac{(3x)^2}{\underline{|2}}...\right)}{x^3}=f(0)$$

$$\lim_{x\to0}\frac{-\beta+x(3+\alpha)+\frac{9\beta x^2}{\underline{|2}}+\left(\frac{-27}{\underline{|3}}-\frac{\alpha}{\underline{|3}}\right)x^3.....}{x^3}=f(0)$$

for exist

$$\beta=0,\ 3+\alpha=0,\ -\frac{27}{\underline{|3}}-\frac{\alpha}{\underline{|3}}=f(0)$$

$$\alpha=-3,\ -\frac{27}{6}-\frac{(-3)}{6}=f(0)$$

$$f(0)=\frac{-27+3}{6}=-4$$

9. A

Sol. $I=\int_0^{\pi/4}\frac{136\sin x}{3\sin x+5\cos x}dx$

136 sin x = A(3 sin x + 5 cos x) + B(3 cos x – 5 sin x)

136 = 3A – 5B .....(1)

0 = 5A + 3B .....(2)

$$3B=-5A\Rightarrow B=-\frac{5}{3}A$$

$$136=3A-5\left(-\frac{5}{3}A\right)$$

$$136=3A+\frac{25}{3}A$$

$$136=\frac{34A}{3}$$

$$\Rightarrow A=\frac{136\times3}{34}=12$$

$$B=\frac{-5}{3}(12)=-20$$

$$I=\int_0^{\pi/4}\frac{A(3\sin x+5\cos x)}{3\sin x+5\cos x}+\int_0^{\pi/4}\frac{B(3\cos x-5\sin x)}{3\sin x+5\cos x}$$

$$=A(x)_0^{\pi/4}+B[\ell n(3\sin x+5\cos x)]_0^{\pi/4}$$

$$=12\left(\frac{\pi}{4}\right)-20\ell n\left(\frac{3}{\sqrt2}+\frac{5}{\sqrt2}\right)-\ell n(0+5)$$

$$=3\pi-20\ell n4\sqrt2+20\ell n5$$

$$=3\pi-20\times\frac{5}{2}\ell n2+20\ell n5$$

$$=3\pi-50\ell n2+20\ell n5$$

10. C

$ax^2 + bx + c = 0$

a, b, c ∈ {1, 2, 3, 4, 5,6 ,7, 8}

Repeated roots D = 0

$\Rightarrow b^2 - 4ac = 0 \Rightarrow b^2 = 4ac$

$$\text{Pro}=\frac{8}{8\times8\times8}=\frac{1}{64}$$

⇒ (a, b, c)

(1, 2, 1) ; (2, 4, 2) ; (1, 4, 4) ; (4, 4, 1) ; (3, 6, 3) ; (2, 8, 8) ; (8, 8, 2) ; (4, 8, 4)

8 case

11. A

Sol. |A| = 3, |B| = 2

$|A^TA(\text{adj}(2A))^{-1}\ (\text{adj}(4B))\ (\text{adj}(AB))^{-1}AA^T|$

$= 3\times3\times|(\text{adj}(2A))^{-1}|\times|\text{adj}(4B)|\times|(\text{adj}(AB))^{-1}|\times3\times3$

↓ ↓ ↓

$$\frac{1}{|\text{adj}(2A)|}\qquad 2^{12}\times2^2\qquad \frac{1}{|\text{adj}(AB)|}$$

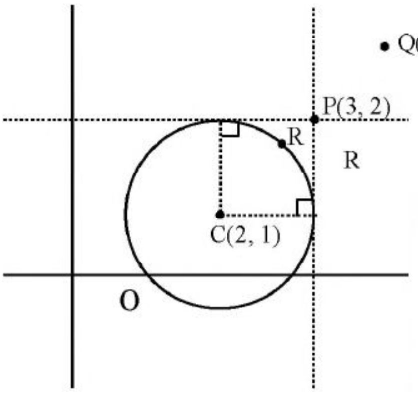
$= \dfrac{1}{2^6 |adjA|}$ $\quad$ $= \dfrac{1}{|adjB \cdot adjA|}$

$= \dfrac{1}{2^6 \cdot 3^2}$ $\quad$ $= \dfrac{1}{2^2 \cdot 3^2}$

$= 3^4 \cdot \dfrac{1}{2^6 \cdot 3^2} \cdot 2^{12} \cdot 2^2 \cdot \dfrac{1}{2^2 \cdot 3^2} = 64$

12. D

Sol.



Coordinates of the centre will be (2, 1)

Equation of circle will be

$(x - 2)^2 + (y - 1)^2 = 1$

$QC = \sqrt{(5-2)^2 + (5-1)^2}$

QC = 5

shortest distance

= RQ = CQ – CR

= 5 – 1 = 4

13. B

Sol. Centre of the circle $= \left(\dfrac{3}{2}, 1\right)$

Equation of diameter = 2x + 3y – k = 0

$2\left(\dfrac{3}{2}\right) + 3(1) - k = 0$

$\Rightarrow k = 6$

Now, Equation of ellipse becomes

$x^2 + 9y^2 = 36$

$\dfrac{x^2}{6^2} + \dfrac{y^2}{2^2} = 1$

length of LR $= \dfrac{2b^2}{a} = \dfrac{2 \cdot 2^2}{6} = \dfrac{8}{6} = \dfrac{4}{3} = \dfrac{m}{n}$

$\therefore 2m + n = 2(4) + 3 = 11$

14. C

Sol. Statement I :

$(|z_1| + |z_2|)\left|\dfrac{z_1}{|z_1|} + \dfrac{z_2}{|z_2|}\right|$

Since $\left|\dfrac{z_1}{|z_1|} + \dfrac{z_2}{|z_2|}\right| \le \left|\dfrac{z_1}{|z_1|}\right| + \left|\dfrac{z_2}{|z_2|}\right|$

$\left|\dfrac{z_1}{|z_1|} + \dfrac{z_2}{|z_2|}\right| \le \left|\dfrac{z_1}{z_1}\right| + \left|\dfrac{z_2}{z_2}\right|$

$\left|\dfrac{z_1}{|z_1|} + \dfrac{z_2}{|z_2|}\right| \le 2$

$(|z_1| + |z_2|)\left(\left|\dfrac{z_1}{|z_1|} + \dfrac{z_2}{|z_2|}\right|\right) \le 2(|z_1| + |z_2|)$

$\therefore$ statement I is correct

For **Statement II :**

$\dfrac{a}{|y-z|} = \dfrac{b}{|z-x|} = \dfrac{c}{|x-y|}$

$\dfrac{a^2}{|y-z|^2} = \dfrac{b^2}{|z-x|} = \dfrac{c^2}{|x-y|^2} = \lambda$

$a^2 = \lambda(|y-z|^2) = \lambda(y-z)(\overline{y}-\overline{z})$

$b^2 = \lambda(z-x)(\overline{z}-\overline{x})$ and $c^2 = \lambda(x-y)(\overline{x}-\overline{y})$

$\dfrac{a^2}{y-z} + \dfrac{b^2}{z-x} + \dfrac{c^2}{x-y} = \lambda(\overline{y}-\overline{z}+\overline{z}-\overline{x}+\overline{x}-\overline{y}) = 0$

Statement II is false

15. A

Sol. $4\left(\dfrac{1-\tan^2\theta/2}{1+\tan^2\theta/2}\right) - 3\left(\dfrac{2\tan\dfrac{\theta}{2}}{1+\tan^2\dfrac{\theta}{2}}\right) = 1$

let $\tan\dfrac{\theta}{2} = t$

$\dfrac{4-4t^2-6t}{1+t^2} = 1$

$4 - 4t^2 - 6t = 1 + t^2$

$\Rightarrow 5t^2 + 6t - 3 = 0$

$$\Rightarrow t=\frac{-6\pm\sqrt{36-4(5)(-3)}}{2(5)}$$

$$=\frac{-6\pm\sqrt{96}}{10}$$

$$=\frac{-6\pm4\sqrt{6}}{10}$$

$$t=\frac{-3+2\sqrt{6}}{5}$$

$$\cos\theta=\frac{1-t^2}{1+t^2}=\frac{1-\left(\frac{2\sqrt{6}-3}{5}\right)^2}{1+\left(\frac{2\sqrt{6}-3}{5}\right)^2}=\frac{1-\left(\frac{24+9-12\sqrt{6}}{25}\right)}{1+\left(\frac{24+9-12\sqrt{6}}{25}\right)}$$

$$=\frac{25-33+12\sqrt{6}}{25+33-12\sqrt{6}}=\frac{12\sqrt{6}-8}{58-12\sqrt{6}}=\frac{6\sqrt{6}-4}{29-6\sqrt{6}}\times\frac{29+6\sqrt{6}}{29+6\sqrt{6}}$$

$$=\frac{100+150\sqrt{6}}{625}=\frac{4+6\sqrt{6}}{25}\times\frac{4-6\sqrt{6}}{4-6\sqrt{6}}$$

$$=\frac{-200}{25(4-6\sqrt{6})}=\frac{-8}{4-6\sqrt{6}}=\frac{4}{3\sqrt{6}-2}$$

16. D

Sol. $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+....+\frac{1}{\sqrt{99}+\sqrt{100}}=m$

$$\frac{\sqrt{1}-\sqrt{2}}{1}+\frac{\sqrt{2}-\sqrt{3}}{-1}.....\frac{\sqrt{99}-\sqrt{100}}{-1}=m$$

$$\sqrt{100}-1=m\Rightarrow m=9$$

$$\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+....+\frac{1}{99\cdot100}=n$$

$$\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}....\frac{1}{99}-\frac{1}{100}=n$$

$$1-\frac{1}{100}=n$$

$$\frac{99}{100}=n$$

$$(m,\ n)=\left(9,\ \frac{99}{100}\right)$$

$= 99 - 99 = 0$

Ans. Option (D) $11x - 100y = 0$

17. D

$f(x) = x^5 + 2x^3 + 3x + 1$

$f'(x) = 5x^4 + 6x^2 + 3$

$f'(1) = 5 + 6 + 3 = 14$

$g(f(x))$ — x

$g'(f(x))\, f'(x) = 1$

for $f(x) = 7$

$\Rightarrow x^5 + 2x^3 + 3x + 1 = 7$

$\Rightarrow x = 1$

$$g'(7)f'(1)=1\Rightarrow g'(7)=\frac{1}{f'(1)}=\frac{1}{14}$$

$x = 1,\ f(x) = 7 \Rightarrow g(7) = 1$

$$\frac{g(7)}{g'(7)}=\frac{1}{1/14}=14$$

18. A

Sol. $A(1, -1, 2)$

$B(5, 7, -6)$

$C(3, 4, -10)$

$D(-1, -4, -2)$

$$\text{Area}=\frac{1}{2}\left|\overline{AC}\times\overline{BD}\right|=\frac{1}{2}|(2\hat{i}+5\hat{j}-12\hat{k})\times(6\hat{i}+11\hat{j}-4\hat{k})|$$

$$=\frac{1}{2}|112\hat{i}-64\hat{j}-8\hat{k}|$$

$$=4|14\hat{i}-5\hat{j}-\hat{k}|$$

$$=4\sqrt{196+64+1}$$

$$=4\sqrt{261}$$

$$=12\sqrt{29}$$

19. A

Sol. $\int_{-\pi}^{\pi}\frac{2y(1+\sin y)}{1+\cos^2 y}dy$

$$=\int_{-\pi}^{\pi}\frac{2y}{1+\cos^2 y}dy+\int_{-\pi}^{\pi}\frac{2y\sin y}{1+\cos^2 y}dy$$

(Odd) (Evev)

$$=0+2.2\int_0^{\pi}y\left(\frac{\sin y}{1+\cos^2 y}\right)dy$$

$$I = 4\int_0^{\pi} \frac{y \sin y}{1+\cos^2 y} dy$$

$$I = 4\int_0^{\pi} \frac{(\pi - y)\sin y}{1+\cos^2 y} dy$$

$$2I = 4\int_0^{\pi} \frac{\pi \sin y}{1+\cos^2 y} dy$$

$$I = 2\pi\int_0^{\pi} \frac{\sin y}{1+\cos^2 y} dy$$

$$= 2\pi(-\tan^{-1}(\cos y))_0^{\pi}$$

$$= -2\pi\left[\left(-\frac{\pi}{4}\right) - \left(\frac{\pi}{4}\right)\right]$$

$$= -2\pi\left[-\frac{2\pi}{4}\right] = \pi^2$$

20. D

Sol. $\frac{2-x}{3} = \frac{3y-2}{4\lambda+1} = 4-z$ .....(1)

$$\frac{x-2}{(-3)} = \frac{y-\frac{2}{3}}{\left(\frac{4\lambda+1}{3}\right)} = \frac{z-4}{(-1)}$$

$\frac{x+3}{3\mu} = \frac{1-2y}{6} = \frac{5-z}{7}$ .....(2)

$$\frac{x+3}{3\mu} = \frac{y-\frac{1}{2}}{(-3)} = \frac{z-5}{(-7)}$$

Right angle

$$\Rightarrow (-3)(3\mu) + \left(\frac{4\lambda+1}{3}\right)(-3) + (-1)(-7) = 0$$

$$-9\mu - 4\lambda - 1 + 7 = 0$$

$$4\lambda + 9\mu = 6$$

21. 56

Sol. X = denotes number of defective

x	0	1	2	3
P(x)	$\frac{7}{15}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$
x_1^2	0	1	4	9
$P_i x_1^2$	0	$\frac{5}{12}$	$\frac{20}{12}$	$\frac{9}{12}$
$p_i x_i$	0	$\frac{5}{12}$	$\frac{10}{12}$	$\frac{3}{12}$

$$\mu = \Sigma p_i x_i = \frac{18}{12}$$

$$\Sigma p_i x_1^2 = \frac{34}{12}$$

$$\sigma^2 = \Sigma p_i x_1^2 - (\mu)^2$$

$$= \frac{34}{12} - \left(\frac{18}{12}\right)^2 = \frac{17}{6} - \frac{9}{4}$$

$$\frac{34-24}{12} = \frac{7}{12}$$

$$96\sigma^2 = 96 \times \frac{7}{12} = 56$$

22. 54

Sol. $(1+2x-3x^3)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$

General term $m\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$

$$= {}^9C_r \cdot \frac{3^{9-2r}}{2^{9-r}}(-1)^r \cdot x^{18-3r}$$

Pur r = 6 to get coeff. of $x^0 = {}^9C_6 \cdot \frac{1}{6^3} \cdot x^0 = \frac{7}{18}x^0$

Put r = 7 to get coeff. of $x^{-3} = {}^9C_r \cdot \frac{3^{-5}}{2^2}(-1)^7 \cdot x^{-3}$

$$= -{}^9C_7 \cdot \frac{1}{3^5 \cdot 2^2} \cdot x^{-3} = \frac{-1}{27}x^{-3}$$

$$(1+2x-3x^3)\left(\frac{7}{18}x^0 - \frac{1}{27}x^{-3}\right)$$

$$\frac{7}{18}+\frac{3}{27}=\frac{7}{18}+\frac{1}{9}=\frac{7+2}{18}=\frac{9}{18}=\frac{1}{2}$$

$$\therefore 108\cdot\frac{1}{2}=54$$

23. 72

Sol. $y = x^2 - 5x$ and $y = 7x - x^2$

$$\int_0^6 (g(x)-f(x))dx$$

$$\int_0^6 (7x-x^2)-(x^2-5x)dx$$

$$\int_0^6 (12x-2x^2)dx=\left[12\frac{x^2}{2}-\frac{2x^3}{3}\right]_0^6$$

$$\Rightarrow 6(6)^2-\frac{2}{3}(6)^3$$

$= 216 - 144 = 72$ unit^2

24. 125

Sol. $(x^1 + x^2 ....+ x^6)^4$

$$x^4\cdot\left(\frac{1-x^6}{1-x}\right)^4$$

$x^4\cdot(1 - x^6)^4 \cdot(1 - x)^{-4}$

$x^4[1 - 4x^6 + 6x^{12}....] [(1 - x)^{-4}]$

$(x^4 - 4x^{10} + 6x^{16}....) (1 - x)^{-4}$

$(x^4 - 4x^{10} + 6x^{16}) (1 + {}^{15}C_{12}\, x^{12} + {}^{9}C_6 x^6 ....)$

$({}^{15}C_{12} - 4 \cdot {}^{9}C_6 + 6)x^{16}$

$({}^{15}C_3 - 4 \cdot {}^{9}C_6 + 6)$

$= 35 \times 13 - 6 \times 8 \times 7 + 6$

$= 455 - 336 + 6$

$= 125$

25. 18

Sol. $|2a - 1| = 3[a] + 2\{a\}$

$|2a - 1| = [a] + 2a$

Case -1 : $a>\frac{1}{2}$

$2a - 1 = [a] + 2a$

$[a]=-1\ \therefore a\in[-1,\ 0)$ Reject

Case -2 : $a<\frac{1}{2}$

$-2a + 1 = [a] + 2a$

$a = I + f$

$-2(I + f) + 1 = I + 2I + 2f$

$$I=0,\ f=\frac{1}{4}\ \ \therefore a=\frac{1}{4}$$

Hence $a=\frac{1}{4}$

$$72\sum_{a\in S} a=72\times\frac{1}{4}=18$$

26. 24

Sol. $$\lim_{t\to x}\frac{t^2f(x)-x^2f(t)}{t-x}=1$$

$$\lim_{t\to x}\frac{2t\cdot f(f)-x^2f'(x)}{1}=1$$

$2x \cdot f(x) - 2f'(x) = 1$

$$\frac{dy}{dx}-\frac{2}{x}\cdot y=\frac{-1}{x^2}$$

$$\text{I.f.}=e^{\int -\frac{2}{x}dx}=\frac{1}{x^2}$$

$$\therefore \frac{y}{x^2}=\int -\frac{1}{x^4}dx+C$$

$$\frac{y}{x^2}=\frac{1}{3x^3}+C$$

Put $f(1) = 1$

$$C=\frac{2}{3}$$

$$y=\frac{1}{3x}+\frac{2x^2}{3}$$

$$y=\frac{2x^3+1}{3x}$$

$$f(2)=\frac{17}{6}$$

$$f(3)=\frac{55}{9}$$

$$2f(2)+3f(3)=\frac{17}{3}+\frac{55}{3}=\frac{72}{3}=24$$

27. 910

d $\rightarrow$ common diff.

$A_k = -kd[2a + (2k - 1)d]$

$A_3 = -153$

$\Rightarrow$ 153 — 13d[2a + 5d]

51 — d[2a + 5d] ....(1)

$A_5 = -435$

435 = 5d[2a + 9d]

87 = d[2a + 9d]

(2) – (1)

$36 = 4d^2$

d = 3, a = 1

$a_{17} - A_7 = 49 - [-7.3[2 + 39]] = 910$

28. 30

Sol. $(\vec{a} + 2\vec{b}) \times \vec{c} = 3(\vec{c} \times \vec{a})$

$(2\vec{b} + 4\vec{a}) \times \vec{c} = 0$

$\vec{c} = \lambda(4\vec{a} + 2\vec{b}) = \lambda(8\hat{i} - 14\hat{j} + 30\hat{k})$

$\vec{a} \cdot \vec{c} = 130$

$8\lambda + 42\lambda + 210\lambda = 130$

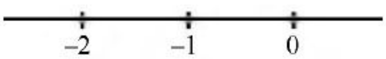
$\lambda = \frac{1}{2}$

$\vec{c} = 4\hat{i} - 7\hat{j} + 15\hat{k}$

$\vec{b} \cdot \vec{c} = 8 + 7 + 15 = 30$

29. C

Sol.



Case-1

$x \geq 0$

$x^2 + 2x - 5x - 5 - 1 = 0$

$x^2 - 3x - 6 = 0$

$x = \frac{3 \pm \sqrt{9 + 24}}{2} = \frac{3 \pm \sqrt{33}}{2}$

One positive root

Case-2

$-1 \leq x < 0$

$-x^2 - 2x - 5x - 5 - 1 = 0$

$x^2 + 7x + 6 = 0$

$(x + 6)(x + 1) = 0$

$x = -1$

one root in range

Case-3

$-2 \leq x < -1$

$x^2 - 2x + 5x + 5 - 1 = 0$

$x^2 - 3x - 4 = 0$

$(x - 4)(x + 1) = 0$

No root in range

Case-4

$x < -2$

$x^2 + 7x + 4 = 0$

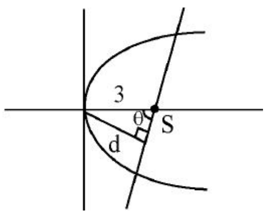
$x = \frac{-7 \pm \sqrt{49 - 16}}{2} = \frac{7 \pm \sqrt{33}}{2}$

one root in range

Total number of distinct roots are 3

30. 108

Sol.



$\ell = 4a\,\text{cosec}^2\theta$

$\ell = 12 \times \frac{9}{d^2}$

$\ell d^2 = 108$

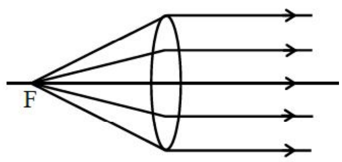
PHYSICS

Section - A (Single Correct Answer)

31. D

Sol. Light emerges parallel

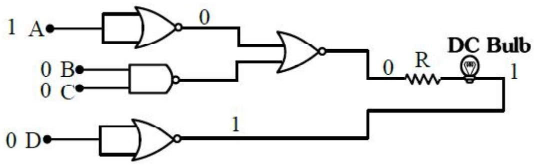
$\therefore$ planor wavefront



32. B

Sol. Bulb will glow if bulb have potential drop on it. One end of bulb must be at high (1) and other must be at low (0).

Option (B) satisfy this condition



33. B

Sol. $[uG] = [(M^1L^{-1}T^{-2})\ (M^{-1}L^3T^{-2})]$

$[uG] = [M^0L^2T^{-4}]$

$[\sqrt{uG}\ ] = [L^1T^{-2}]$

Option (B) is correct

34. A

Sol. Capillary rise

$$h = \frac{2T\cos\theta}{\rho gr}$$

If $\theta = 0°$ then rise is non-zero

$\therefore$ Statement-I is incorrect.

Option(A) is correct

35. A

Sol. $eV_0 = h\nu - \phi$

$$V_0 = \frac{h}{e}\nu - \frac{\phi}{e}$$

M_2 material has higher work function, so statement-(II) is incorrect.

Option (A) is correct.

36. A

Sol. $\vec{R} = \left(2\vec{Q} + 2\vec{P}\right) + \left(2\vec{Q} - 2\vec{P}\right)$

$$\vec{R} = 4\vec{Q}$$

Angle between $\vec{Q}$ and $\vec{R}$ is zero

Option (A) is correct

37. A

Sol.

$$F_e = \frac{kQ_1Q_2}{r^2} = \frac{9\times10^9\times1.6\times10^{-19}\times1.6\times10^{-19}}{r^2}$$

$$F_g = \frac{Gm_1m_2}{r^2}$$

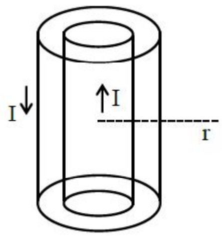
$$= \frac{6.67\times10^{-11}\times9.1\times10^{-31}\times1.6\times10^{-27}}{r^2}$$

$$\frac{F_e}{F_g} \cong 0.23\times10^{40} \cong 2.3\times10^{39}$$

Option (A)

38. C

Sol.



$$\oint \vec{B}.d\vec{l} = \mu_0 i_{enc} = 0$$

$\therefore$ B = 0 outside the cable

39. D

Sol. $F = \frac{k(Ze)(e)}{r^2} = \frac{mv^2}{r}$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}\frac{K(Ze)(e)}{r}$$

$$PE = -\frac{K(Ze)(e)}{r}$$

$$TE = \frac{K(Ze)(e)}{2r} - \frac{K(Ze)(e)}{r} = \frac{-K(Ze)(e)}{2r}$$

2TE = PE

Option (D)

40. C

Sol. Assuming mean free path constant.

$$f \propto v \propto \sqrt{T}$$

$$\frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{300}{400}}$$

$$f_2 = \sqrt{\frac{4}{3}} = f_1 = \frac{2}{\sqrt{3}}Z$$

41. C

Sol. $I_{sphere} = \frac{2}{3}MR^2 = Mk_1^2$

$$I_{cylinder} = \frac{1}{12}M\left(4R^2\right) + \frac{1}{4}MR^2 + M(2R)^2$$

$$= \frac{67}{12}MR^2 = Mk_2^2$$

$$\frac{k_1}{k_2} = \sqrt{\frac{2}{3}\cdot\frac{12}{67}} = \sqrt{\frac{8}{67}}$$

42. A

Sol. ϕ = Mi = BA

$$\Rightarrow Mi = \frac{\mu_0 i}{2b}\pi a^2$$

$\therefore M = \dfrac{\mu_0 \pi a^2}{2b}$

43. A

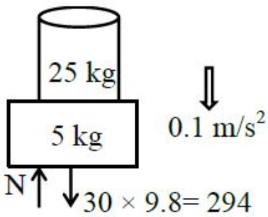
Sol. $KE = \dfrac{1}{2}mv^2 = \dfrac{GMm}{2a}$

$PE = -2KE$

$TE = -KE$

44. C

Sol. Taking $g = 9.8\ m/s^2$



$294 - N = 30 \times 0.1$

$N = 291$

45. C

Sol. $T_1 = 2\pi\sqrt{\dfrac{l}{GM}(2R)^2}$

$T_2 = 2\pi\sqrt{\dfrac{l}{GM}(3R)^2}$

$\therefore \dfrac{T_1}{T_2} = \dfrac{2}{3}$

46. A

Sol. Work done by gravity is independent of path. It depends only on vertical displacement so work done in both cases will be same. Option (A) is correct

47. C

Sol. Sum of number by considering significant digit

sum = 4.6 + 4.6 + 4.6 + 4.6 = 18.4

Arithmetic Mean $= \dfrac{\text{sum}}{4} = \dfrac{18.4}{4} = 4.6$

48. A

Sol. $\Delta U = 0$ (Cyclic process)

$\Delta Q = W$ = area of P-V curve.

$= \pi \times (140 \times 10^3\ Pa) \times (140 \times 10^{-6}\ m^3)$

$\Delta Q = 61.6\ J$

49. D

Sol. Here R_2, R_3, R_4 are in parallel

$\dfrac{1}{R_{234}} = \dfrac{1}{R_2} + \dfrac{1}{R_3} + \dfrac{1}{R_4}$

$R_{234} = 2\Omega$

R_{234} is in series with R_1 so

$R_{eq} = R_{234} + R_1 = 2 + 10 = 12\Omega$

$i = \dfrac{12}{12} = 1$ Amp

50. D

Sol. Displacement current is same as conduction current in capacitor.

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$

$= \dfrac{1}{2\pi \times 4 \times 10^3 \times 12 \times 10^{-6}} = 3.317\Omega$

$I = \dfrac{V}{X_C} = \dfrac{40}{3.317} = 12A$

Section - B (Numerical Value)

51. 10

Sol. $\beta = \dfrac{\lambda D}{d} = \dfrac{5 \times 10^{-7} \times 2}{3 \times 10^{-4}} = \dfrac{10 \times 10^{-3}}{3} m$

For 3rd maxima $y_3 = 3\beta = 10 \times 10^{-3}\ m = 10\ mm$

52. 5

Sol. $R = \dfrac{\rho l}{A} \Rightarrow \dfrac{2 \times 10^{-6} \times l}{10^{-5}} = 1 \Rightarrow l = 5$

$mg = Bil$

$B = \dfrac{mg}{il} = \dfrac{5}{2 \times 5} = 0.5 = 5 \times 10^{-1}$ Tesla

53. 4

Sol. $E = \dfrac{E_0}{3} \Rightarrow V = \dfrac{V_0}{3}$

$\dfrac{V_0}{3} = V_0 e^{-\frac{t}{\tau}}$

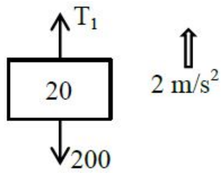
$t = \tau \ln 3$

$6.6 \times 10^{-6} = R\,(1.5 \times 10^{-6})(1.1)$

$R = \dfrac{6}{1.5} = 4\Omega$

54. 240

Sol. FBD of M_1 :

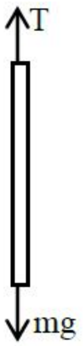


$T_1 - 200 = (4 + 6 + 10) \times 2$

$\therefore \quad T_1 = 240$

55. 600

Sol.



$T = mg$

$\sigma = \frac{T}{A} = \frac{mg}{A}$

$\frac{(\sigma A l)g}{A}$

$\Rightarrow l = \frac{\sigma}{\rho g} = \frac{1.2\times10^8\times3}{6\times10^4\times10} = 600$

56. 19

Sol. $S_n = \frac{1}{2}a(2n-1) = \frac{19a}{2}$

$S_{n-1} = \frac{1}{2}a(2n-3) = \frac{17a}{2}$

$\frac{S_{n-1}}{S_n} = \frac{17}{19} = 1 - \frac{2}{x} \Rightarrow x = 19$

57. 727

Sol. Reaction :

$3\,{}_2^4He \to {}_6^{12}C + \gamma rays$

Mass defect = $\Delta m = (3m_{He} - m_C)$

= (3 × 4.002603 – 12) = 0.007809 u

Energy released

= 931 Δm MeV

= 7.27 MeV = 727 × 10^{-2} MeV

58. 50

Sol. $X_L = \omega L = 100 \times 1 = 100\Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{100\times20\times10^{-6}} = 500\Omega$

$Z = \sqrt{(X_L - X_C)^2 + R^2}$

$\sqrt{(100-500)^2 + 300^2}$

Z = 500 Ω

$i_{rms} = \frac{V_{rms}}{Z} = \frac{50}{500} = 0.1A$

rms voltage across capacitor

$V_{rms} = XC\, i_{rms}$

= 500 × 0.1 = 50V

59. 5

Sol. $i = K\theta$

$\frac{2}{G+R} = K\theta$

$\Rightarrow \frac{1}{\theta} = \frac{(G+R)K}{2} = R\left(\frac{K}{2}\right) + \frac{KG}{2}$

Slope $= \frac{K}{2} = \frac{1}{4} \Rightarrow K = 0.5 = 5\times10^{-1}A$

60. 86

Sol. In parallel combination : Potential difference is same across all

Energy $= \frac{1}{2}(C_1 + C_2 + C_3)V^2$

$= \frac{1}{2}(25+30+45)\times(100)^2\times10^{-6} = 0.5 = E$

In series combination: Charge is same on all.

$\frac{1}{C_{equ}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{25} + \frac{1}{30} + \frac{1}{45}$

$\frac{1}{C_{equ}} = \frac{(18+15+10)}{450} = \frac{43}{450} \Rightarrow C_{equ} = \frac{450}{43}$

Energy $= \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3}$

$= \frac{Q^2}{2}\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$

$\frac{(V\times C_{equ})^2}{2} \times \frac{1}{C_{equ}} = \frac{V^2 C_{equ}}{2}$

$\frac{(100)^2}{2} \times \frac{450}{43} \times 10^{-6}$

$\Rightarrow \frac{4.5}{86} = \frac{9}{x}E = \frac{9}{x}\times0.5 \Rightarrow x = 86$

CHEMISTRY

Section - A (Single Correct Answer)

61. (D)

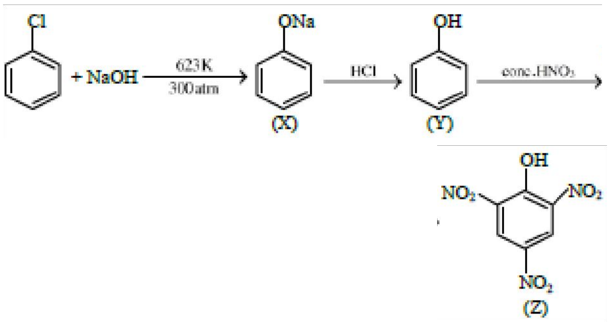
Sol. B, D

62. (A)

Sol. When solid added no effect on equilibrium

63. (C)

Sol.



64. (B)

Sol. Enthalpy of neutralization of SA & SB is always –57 kJ / mol because strong monoacid gives one mole of H^+ and strong mono base gives one mole of OH^- which form one mole of water.

65. (D)

Sol.

	O^{-2}	F^-	Na^+	Mg^{+2}
(No.of e^-)	10	10	10	10

(Ionic radius) $O^{-2} > F^- > Na^+ > Mg^{+2}$

Zeff $O^{-2} < F^- < Na^+ < Mg^{+2}$

66. (B)

Sol. Compounds having same number of carbon atoms follow the boiling point order as:

(Boiling point)$_{\text{Hydrogen bonding}}$ > (Boiling point)$_{\text{high polarity}}$ > (Boiling point)$_{\text{low polarity}}$ > (Boiling point)$_{\text{non polar}}$

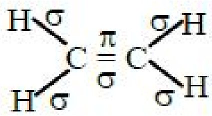
67. (D)

Sol. Statement I : Number of d & f electrons, increases down the group and due to poor shielding of d & f e^-, stability of lower oxidation states increases down the group

Statement II : The atomic size of aluminium is greater than that of gallium.

68. (D)

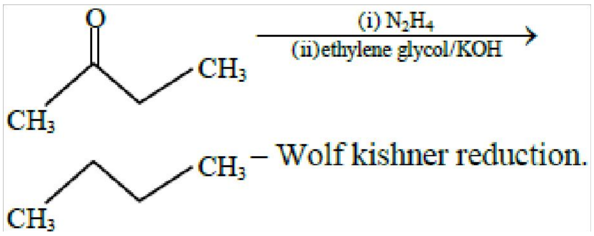
Sol. ethylene is



, it has 5σ bonds and 1π bond.

69. (B)

Sol.:



70. (A)

Sol. In the cathode reaction manganese (Mn) is reduced from the +4 oxidation state to the +3 state.

71. (A)

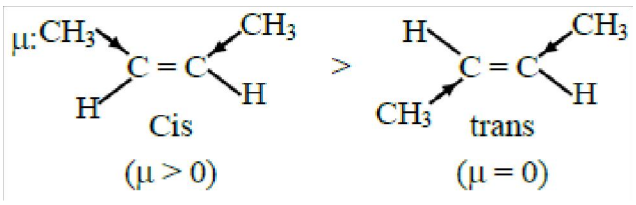
Sol.

	Number of unpaired e^-	$\mu = \sqrt{n(n+2)}$B.M.
$[Co(H_2O_6]^{2+}$	3	3.87
$[Fe(H_2O)_6]^{2+}$	4	4.89
$[Mn(H_2O)_6]^{2+}$	5	5.92
$[Cr(H_2O)_6]^{2+}$	4	4.89

Least paramagnetic behaviour = $[Co(H_2O)_6]^{2+}$

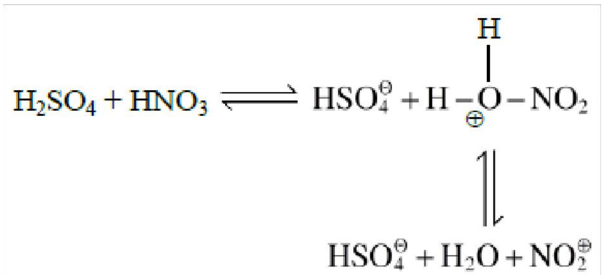
72. (C)

Sol. Dipole moment is a vector quantity and for compound net dipole moment is the vector sum of all dipoles hence dipole moment of cis form is greater than trans form.



73. (B)

Sol. In nitration of benzene concentrated H_2SO_4 and HNO_3 is used as reagent which generates electrophile $\overset{\oplus}{N}O_2$ in following steps:



Lewis acids can promote the formation of electrophiles not Lewis base

74. (C)

Sol. Experimental order $Br^- < F^- < H_2O < NH_3$

75. (D)

Sol. Ninhydrin test is a test of amino acids. Egg albumin contains protein which is a natural polymer of amino acids which will show positive ninhydrin test

76. (B)

Sol. Mn shows highest oxidation state (Mn^{+7}) in 3d series metals.

77. (D)

Sol. only $C_{12}H_{22}O_{11}$ has 42.1% carbon, 6.4% hydrogen & 51.5 percent oxygen.

78. (D)

Sol.: Phenol is a highly activated compound which can undergo bromination directly with Bromine without any lewis acid.

79. (B)

Sol.: $\wedge_C^{+2} = 57$ Scm^2 mol^{-1}

$\wedge_A^{+2} = 73$ S cm^2 mol^{-1}

$\wedge_{Solution} = \lambda_C^{+2} + \wedge_A^{-2}$

= 57 + 73 = 130

80. (D)

Sol.: More abundant isotope = B^{11}

[Number of neutrons = 6]

x = 6

$B + O_2 \rightarrow B_2O_3$

Oxidation state of B in $B_2O_3 = +3$

So, y = 3

Hence x + y = 9

Section - B (Numerical Value Type)

81. (22)

Sol.: $V = 2.18 \times 10^6 \times \frac{Z}{n}$

$= 21.8 \times 10^5 \times 10^5 \times \frac{1}{1} \approx 22\times10^5$(nearest)

82. (6)

Sol. Fe^{+3} will give green coloured bead when heated at point B.

Number of unpaired e^- in $Fe^{+3} = 5$

$\mu = 5.92$

Nearest integer = 6

83. (150)

Sol. $C_6H_5COOH(S) + \frac{15}{2}O_2(g) \rightarrow 7CO_2(g) + 3H_2O(\ell)$

$\Delta H = \Delta U + \Delta n_gRT$

$= -321.30 - \frac{1}{2}\frac{R}{100}\times 300$

= (–321.30 – 150R) kJ

84. (14)

Sol.: Picric acid is prepared by treating phenol first with concentrated sulphuric acid which converts it to phenol-2,4-disulphonic acid and then with concentrated nitric acid to get 2, 4, 6 trinitrophenol

85. (5)

Sol. Strong oxidising agent = Co^{+3}

No. of unpaired e^- in $Co^{+3}[3d^6] = 4$

Hence $\mu = \sqrt{n(n+2)} = \sqrt{24}$BM

Nearest integer = 5

86. (3)

Sol. $r = K[A]^x[B]^y$

(I) $6 \times 10^{-3} = K[0.1]^x[0.1]^y$

(IV) $2.4 \times 10^{-2} = K[0.4]^x[0.1]^y$

(IV)/(I)

$4 = (4)^x$

x = 1

$r = K[A]^x[B]^y$

(III) $2.88 \times 10^{-1} = K[0.3]^x[0.4]^y$

(II) $7.2 \times 10^{-2} = K[0.3]^x[0.2]^y$

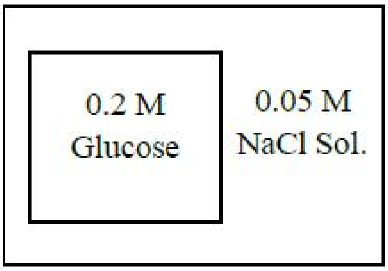
(III)/(II)

$4 = 2^y$

$y = 2$

Overall order = x + y = 1 + 2 = 3

87. (25)

Sol. 

$NaCl \longrightarrow Na^{+} + Cl^{-}$

0.05M 0.05M 0.05M

Total $C_1 = 0.05 + 0.05 = 0.1$ M (NaCl)

$C_2 = 0.2$ M (glucose)

$\pi = (C_2 - C_1)\,RT$

$= (0.2 - 0.1) \times 0.083 \times 300$

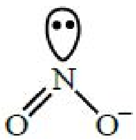
$= 2.49$ bar

$= 24.9 \times 10^{-1}$ bar

88. (2)

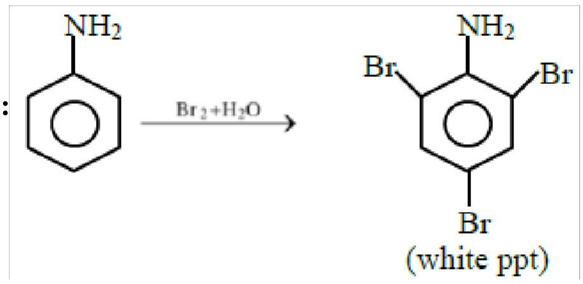
Sol. In Sandmayer reaction only bromobenzene & chlorobenzene are prepared

89. (8)

Sol.: 

Number of valence e^{-} around N-atom = 8

90. (80)

Sol.: 

93 g of aniline produces 330 g of 2, 4, 6–tribromoaniline. Hence 9.3 g of aniline should produce 33g of 2, 4, 6-tribromoaniline. Hence

percentage yield $\dfrac{26.4 \times 100}{33} = 80\%$

☯ ☯ ☯