04-April-2024 (Evening Batch) : JEE Main Paper

MATHEMATICS

1. B

Sol. $\lim_{x\to 0} f(x) = a\ell n2\ell n3$

$$\lim_{n\to 0}\frac{72^x - 9^x - 8^x + 1}{\sqrt{2}-\sqrt{1+\cos x}} = \lim_{x\to 0}\frac{(8^x-1)(9^x-1)}{\sqrt{2}-\sqrt{1+\cos x}}$$

$$\lim_{n\to 0}\left(\frac{8^x-1}{x}\right)\left(\frac{9^x-1}{x}\right)\left(\frac{x^2}{1-\cos x}\right)\left(\sqrt{2}+\sqrt{1+\cos x}\right)$$

$\therefore \ell n8 \times \ell n9 \times 2 \times 2\sqrt{2} = 24\sqrt{2}\ell n2\ell n3$

$\therefore a = 24\sqrt{2},\ a^2 = 576\times 2 = 1152$

2. A

Sol. $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 0,\ \lambda > 0$

$|\vec{a}|^2 - |\vec{b}|^2 = 0 \rightarrow 1+\lambda^2+9 = 9+1+4$

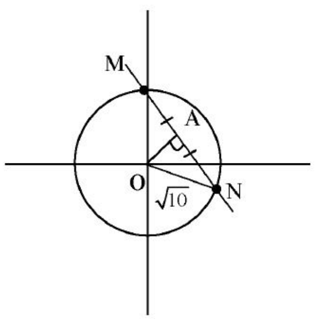
$\therefore \lambda = 2,\ \cos\theta = \dfrac{\vec{a}-\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \dfrac{3-\lambda-6}{\sqrt{14}\cdot\sqrt{14}}$

$14\cos\theta = 3-8 = -5$

$\therefore (14\cos\theta)^2 = 25$

3. B

Sol.



$C : x^2 + y^2 = 10$

$AN = \dfrac{MN}{2} = 1$

$\therefore$ In $\Delta OAN \rightarrow (ON)^2 = (OA)^2 + (AN)^2$

$10 = (OA)^2 + 1 \rightarrow OA = 3$

Perpendicular distance of center from

$PQ = \dfrac{|0+0-2|}{\sqrt{2}} = \sqrt{2}$

$= 3+\sqrt{2}$ or $3-\sqrt{2}$

4. B

Sol. All $((x_1y_1), (x_1, y_1))$ are in R where

$x_1, y_1 \in N\ \therefore$ R is reflexive

$((1, 1), (2, 3)) \in R$ but $((2, 3), (1, 1)) \notin R$

$\therefore$ R is not symmetric

$((2, 4), (3, 3)) \in R$ and $((3, 3), (1, 3)) \in R$ but $((2, 4), (1, 3)) \notin R$

$\therefore$ R is not transitive

5. $2b = a + c,\ b^2 = (a + 1)(c + 3)$,

$\dfrac{a+b+c}{3} = 8 \rightarrow b = 8,\ a+c = 16$

$64 = (a + 1)(19 - a) = 19 + 18a - a^2$

$a^2 - 18a - 45 = 0 \rightarrow (a-15)(a+3) = 0, (a > 10)$

$a = 15, c = 1, b = 8$

$((abc)^{1/3})^3 = abc = 120$

6. C

Sol. $Adj(A) = \begin{bmatrix}1 & -2\\ 0 & 1\end{bmatrix}$

$(AdjA)^2 = \begin{bmatrix}1 & -4\\ 0 & 1\end{bmatrix}$

|
|

$(AdjA)^{10} = \begin{bmatrix}1 & -20\\ 0 & 1\end{bmatrix}$

$B = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + \begin{bmatrix}1 & -2\\ 0 & 1\end{bmatrix} + \begin{bmatrix}1 & -4\\ 0 & 1\end{bmatrix} + \ldots + \begin{bmatrix}1 & -20\\ 0 & 1\end{bmatrix}$

$B = \begin{bmatrix}11 & -110\\ 0 & 11\end{bmatrix} \Rightarrow$ sum of elements of B

$= -88$

7. B

Sol. $\dfrac{1\times2^2+2\times3^2+.....+100\times(101)^2}{1^2\times2+2^2\times3+.....+100^2\times101}=\dfrac{\sum_{r=1}^{100}r(r+1)^2}{\sum_{r=1}^{100}r^2(r+1)}$

$$=\frac{\sum_{r=1}^{100}(r^3+2r^2+r)}{\sum_{r=1}^{100}(r^3+r^2)}=\frac{\left(\frac{n(n+1)}{2}\right)^2+\frac{2\cdot n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}}{\left(\frac{n(n+1)}{2}\right)^2+\frac{n(n+1)(2n+1)}{6}}$$

$$=\frac{\frac{n(n+1)}{2}\left[\frac{n(n+1)}{2}+\frac{2}{3}\cdot(2n+1)+1\right]}{\frac{n(n+1)}{2}\left[\frac{n(n+1)}{2}+\frac{(2n+1)}{3}\right]};\text{Put } n=100$$

$$=\frac{\frac{100(101)}{2}+\frac{2}{3}(201)+1}{\frac{100\times101}{2}+\frac{201}{3}}=\frac{5185}{5117}=\frac{305}{301}$$

8. B

$$\lim_{x\to0}\frac{f(x)}{x^3}$$

Using L. Hopital Rule.

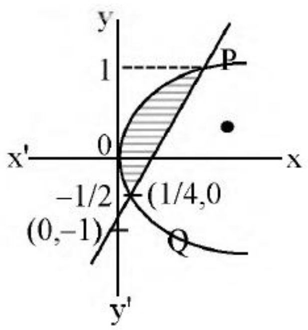
$$\lim_{x\to0}\frac{f'(x)}{3x^2}=\lim_{x\to0}\frac{x+\sin(1-e^x)}{3x^2}\ \text{(Again L Hopital)}$$

Using L.H. Rule

$$=\lim_{x\to0}\frac{-[\sin(1-e^x)(-e^x)\cdot e^x+\cos(1-e^x)\cdot e^x]}{6}$$

$$=-\frac{1}{6}$$

9. D



Shaded area $=\int_{-\frac{1}{2}}^{1}(x_{Right}-x_{Lift})dy$

$y^2=2x$

$y=4x-1$ Solve

$y=1,\ y=-\frac{1}{2}$

Shaded area $=\int_{-\frac{1}{2}}^{1}\left(\frac{y+1}{4}-\frac{y^2}{2}\right)dy$

$$=\left(\frac{1}{4}\left(\frac{y^2}{2}+y\right)-\frac{y^3}{6}\right)_{-\frac{1}{2}}^{1}=\frac{9}{32}$$

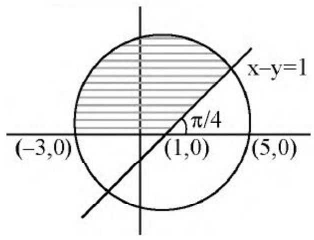
10. B

Sol. $|z-1|\le2\Rightarrow(x-1)^2+y^2\le4$ ....(1)

$(z+\bar{z})+i(z-\bar{z})\le2\Rightarrow2x+i(2iy)\le2$

$\Rightarrow x-y\le1$ ....(2)

$\text{Im}(z)\ge0\Rightarrow y\ge0$ ....(3)



Required area

= Area of semi-circle – area of sector A

$\frac{1}{2}\pi(2)^2-\frac{\pi}{2}$

$=\frac{3\pi}{2}$

11. Let $I=\int_{-1}^{+1}\frac{\cos\alpha x}{1+3^x}dx$ ....(I)

$$I=\int_{-1}^{+1}\frac{\cos\alpha}{1+3^{-x}}dx$$

$\left(\text{using }\int_a^b f(x)dx=\int_a^b f(a+b-x)dx\right)$ ....(II)

Add (I) and (II)

$$2I=\int_{-1}^{+1}\cos(\alpha x)dx=2\int_{0}^{1}\cos(\alpha x)dx$$

$$I=\frac{\sin\alpha}{\alpha}=\frac{2}{\pi}(\text{given})$$

$$\therefore \alpha=\frac{\pi}{2}$$

12. B

Sol. $f(x)=3\sqrt{x-2}+\sqrt{4-x}$

$x-2\geq 0$ & $4-x\geq 0$

$\therefore x\in[2,\ 4]$

Let $x=2\sin^2\theta+4\cos^2\theta$

$\therefore f(x)=3\sqrt{2}\,|\cos\theta|+\sqrt{2}\,|\sin\theta|$

$\therefore \sqrt{2}\leq\sqrt{2}\,|\cos\theta|+\sqrt{2}\,|\sin\theta|\leq\sqrt{9\times 2+2}$

$\sqrt{2}\leq 3\sqrt{2}\,|\cos\theta|+\sqrt{2}\,|\sin\theta|\leq\sqrt{20}$

$\therefore \alpha=\sqrt{2}\quad \beta=\sqrt{20}$

$\alpha^2+2\beta^2=2+40=42$

13. A

Sol. Coeff of $x^4={}^nC_4$

Coeff. of $x^5={}^nC_5$

Coeff. of $x^6={}^nC_6$

${}^nC_4, {}^nC_5, {}^nC_6$ .... AP

$2\cdot{}^nC_5={}^nC_4+{}^nC_6$

$$2=\frac{{}^nC_4}{{}^nC_5}+\frac{{}^nC_6}{{}^nC_5}\quad\left\{\frac{{}^nC_r}{{}^nC_{r-1}}=\frac{n-r+1}{r}\right\}$$

$$2=\frac{5}{n-4}+\frac{n-5}{6}$$

$12(n-4)=30+n^2-9n+20$

$n^2-21n+98=0$

$(n-14)(n-7)=0$

n_{max} 14 n_{min} 7

14. A

Sol. Let $H:\frac{x^2}{a^2}-\frac{y^2}{b^2}=1\qquad (b^2=a^2(e^2-1))$

$\therefore$ equation of $C_1=x^2+y^2=a^2$

$Ar.=36\pi$

$\pi a^2=36\pi$

$a=6$

Now radius of C_2 can be $a(e-1)$ or $a(e+1)$

for $r=a(e-1)$	for $r=a(e+1)$
$Ar.=4\pi$	$\pi r^2=4\pi$
$\pi a^2(e-1)^2=4\pi$	$a^2(e+1)^2=4$
$36\pi(e-1)^2=4\pi$	$36(e+1)^2=4$
$e-1=\frac{1}{3}$	$e+1=\frac{1}{3}$
$e=\frac{4}{3}$	$-\frac{2}{3}$ Not possible

$$\therefore b^2=36\left(\frac{16}{9}-1\right)=28$$

$$\therefore LR=\frac{2b^2}{a}=\frac{2\times 28}{6}=\frac{28}{3}$$

15. B

Sol. $\sum P_i=1$

$a+2a+a+b+2b+3b=1$

$4a+6b=1$ ....(I)

$$E(x)=\text{ mean }=\frac{46}{9}$$

$$\sum P_iX_i=\frac{46}{9}\Rightarrow 4a+4a+4b+12b+24b=\frac{46}{9}$$

$$8a+40b=\frac{46}{9}$$

$$4a+20b=\frac{23}{9}\qquad ....(II)$$

Subtract (I) from (II) we get

$$b=\frac{1}{9}\ \&\ a=\frac{1}{12}$$

Variance $=E(x_i^2)-E(x_i)^2$

$E(x_i^2)=0^2\times 9^2+2^2\times 2a+4^2(a+b)+6^2(2b)+8^2(3b)$

$=24a+280b$

Put $a=\frac{1}{12}\quad b=\frac{1}{9}$

$$E(x_i^2) = 2 + \frac{280}{9} = \frac{298}{9}$$

$$\therefore \sigma^2 = E(x_i^2) - E(x_i)^2$$

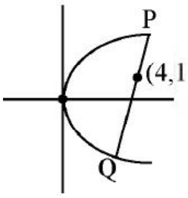
$$= \frac{298}{9} - \left(\frac{46}{9}\right)^2$$

$$\sigma^2 = \frac{298}{9} - \frac{2116}{81}$$

$$= \frac{566}{81}$$

16. D

Sol.



$T = S_1$

$y - 6(x + 4)$

$= 1 - 48$

$6x - y = 23$

Option D $\left(\frac{1}{2}, \ -20\right)$ will satisfy

17. B

Sol. $\cos^{-1} x - \left(\frac{\pi}{2} - \cos^{-1} y\right) = \alpha$

$$\cos^{-1} x + \cos^{-1} y = \frac{\pi}{2} + \alpha$$

$$\alpha \in \left[-\frac{\pi}{2}, \ \pi\right], \ \frac{\pi}{2} + \alpha \in \left[0, \ \frac{3\pi}{2}\right]$$

$$\cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right) = \frac{\pi}{2} + \alpha$$

$$xy - \sqrt{1-x^2}\sqrt{1-y^2} = \frac{\pi}{2} + \alpha$$

$$(xy + \sin\alpha) = (1-x^2)(1-y^2)$$

$x^2y^2 + 2\ xy \sin a + \sin^2 a = 1 - x^2 - y^2 + x^2y^2$

$$x^2 + y^2 + 2xy\sin\alpha = 1 - \sin^2\alpha$$

$$x^2 + y^2 + 2xy\sin\alpha = \cos^2\alpha$$

Min. value of $\cos^2\alpha = 0$

At $\boxed{\alpha = \frac{\pi}{2}}$

Option (B) is correct

18. B

Sol. $\frac{dy}{dx} + y\left(\frac{2x^3 + 8x}{(x^2+4)^2}\right) = \frac{2}{(x^2+4)^2}$

$$\frac{dy}{dx} + y\left(\frac{2x}{x^2+4}\right) = \frac{2}{(x^2+4)^2}$$

$$IF = e^{\int \frac{2x}{x^2+4}dx}$$

$IF = x^2 + 4$

$$y^{\times}(x^2+4) = \int \frac{2}{(x^2+4)^2} \times (x^2+4)$$

$$y(x^2+4) = 2\int \frac{dx}{x^2+2^2}$$

$$y(x^2+4) = \frac{2}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$$

$0 = 0 + c = c = 0$

$$y(x^2+4) = \tan\left(\frac{x}{2}\right)$$

y at $x = 2$

$y(4 + 4) = \tan^{-1}(1)$

$$\boxed{y(2) = \frac{\pi}{32}}$$

Option (D) is correct

19. D

Sol. $\vec{d} = \lambda(\vec{b} + \vec{c})$

$$\vec{a} \cdot \vec{d} = \lambda(\vec{b} \cdot \vec{a} + \vec{c} \cdot \vec{a})$$

$$1 = \lambda(1 + x + 5)$$

$$1 = \lambda(x + 6) \qquad ....(1)$$

$|\vec{d}| = 1$ $\qquad \boxed{\frac{1}{\lambda} = x + 6}$

$$|\lambda(\vec{b} + \vec{c})| = 1$$

$$|\lambda((x+2)\hat{i}+6\hat{j}-2\hat{k})|=1$$

$$\lambda^2((x+2)^2+6^2+2^2)=1$$

$x^2 + 4x + 4 + 36 + 4 = (x + 6)^2$

$x^2 + 4x + 44 = x^2 + 12x + 36$

$8x = 8, x = 1$

$$\begin{vmatrix}1 & 1 & 1\\ 2 & 4 & -5\\ x & 2 & 3\end{vmatrix}=(\vec{a}\times\vec{b})\cdot\vec{c}$$

$$\begin{vmatrix}0 & 0 & 1\\ -2 & 9 & -4\\ x-2 & -1 & 3\end{vmatrix}=2-9(x-2)$$

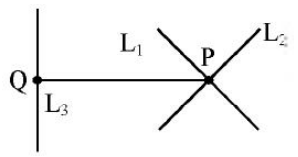
$= 20 - 9x$

at $x = 1$

$20 - 9 = 11$

Option (D) is correct

20. C

Sol. 

$$L_1 \equiv \frac{x^2-1}{1}=\frac{y-4}{5}=\frac{z-2}{1}=\lambda$$

$P(\lambda+2,\ 5\lambda+4,\ \lambda+2)$

$$L_2 \equiv \frac{x-3}{2}=\frac{y-2}{3}=\frac{z-3}{2}$$

$P(2\mu+3,\ 3\mu+2,\ 2\mu+3)$

$\lambda+2=2\mu+3 \qquad 3\mu+2=5\lambda+4$

$\lambda=2\mu+1 \qquad 3\mu=5\lambda+2$

$3\mu=5(2\mu+1)+2$

$3\mu=10\mu+7$

$\mu=-1 \quad \lambda=-1$

Both satisfies (P)

$P(1, -1, 1)$

$$L_3 \equiv \frac{x}{1/4}=\frac{y}{1/2}=\frac{z}{1}$$

$$L_3=\frac{x}{1}=\frac{y}{2}=\frac{z}{4}=k$$

Coordinates of $Q(k, 2k, 4k)$

DR's of PQ $= < k - 1, 2k + 1, 4k - 1 >$ PQ $\perp$ to L_3

$(k - 1) + 2(2k + 1) + 4(4k - 1) = 0$

$k - 1 + 4k + 2 + 16k - 4 = 0$

$$k=\frac{1}{7}$$

$$Q\left(\frac{1}{7},\ \frac{2}{7},\ \frac{4}{7}\right)$$

$$PQ=\sqrt{\left(1-\frac{1}{7}\right)^2+\left(-1-\frac{2}{7}\right)^2+\left(1-\frac{4}{7}\right)^2}$$

$$=\sqrt{\frac{36}{49}+\frac{81}{49}+\frac{9}{49}}=\frac{\sqrt{126}}{7}$$

$$PQ=\frac{3\sqrt{14}}{7}$$

Option (C) will satisfy

21. 4

Sol. $$D=(\sin 2\theta)^2-4\left(1-\frac{\sin^2 2\theta}{2}\right)\left(1-\frac{3}{4}\sin^2 2\theta\right)$$

$$=(\sin 2\theta)^2-4\left(1-\frac{5}{4}\sin^2 2\theta+\frac{3}{8}\sin^4 2\theta\right)$$

$$D=-\frac{3}{2}\sin^4 2\theta+6\sin^2 2\theta-4>0$$

$$3\sin^4 2\theta-12\sin^2 2\theta+8<0$$

$$\sin^2 2\theta=\frac{12\pm\sqrt{12^2-12.8}}{6}=\frac{12\pm 4\sqrt{3}}{6}=\frac{6\pm 2\sqrt{3}}{3}$$

$$\sin^2 2\theta=2\pm\frac{2}{\sqrt{3}},\ \text{but } \sin^2 2\theta\in[0,\ 1]$$

$$\therefore \alpha=2-\frac{2}{\sqrt{3}},\ \beta=1\to(\alpha-2)^2=\frac{4}{3},\ (\beta-1)^2=0$$

$$\boxed{3(\alpha-2)^2+(\beta-1)^2=4}$$

22. 1

Sol. $\int \text{cosec}^3 x \cdot \text{cosec}^2 x dx = 1$

By applying integration by parts

$I = -\cot x \,\text{cosec}^3 x + \int \cot x(-3\,\text{cosec}^2 x \cot x \,\text{cosec}\, x)dx$

$I = -\cot \text{cosec}^3 x - 3\int \text{cosec}^3 x(\text{cosec}^2 x - 1)dx$

$I = -\cot x\,\text{cosec}^3 x - 3I + 3\int \text{cosec}^3 x dx$

let

$I_1 = \int \text{cosec}^3 x dx = -\text{cosec}\, x \cot x - \int \cot^2 x\,\text{cosec}\, x dx$

$I_1 = -\text{cosec}\, x \cot x - \int (\text{cosec}^2 x - 1)\text{cosec}\, x dx$

$2I_1 = -\text{cosec}\, x \cot x + \ell n\left|\tan \frac{x}{2}\right|$

$I_1 = -\frac{1}{2}\text{cosec}\, x \cot x + \frac{1}{2}\ell n\left|\tan \frac{x}{2}\right|$

$4I = -\frac{1}{4}\text{cosec} \cot x\left(\text{cosec}^2 x + \frac{3}{2}\right) + \frac{3}{8}\ell n\left|\tan \frac{x}{2}\right| + c$

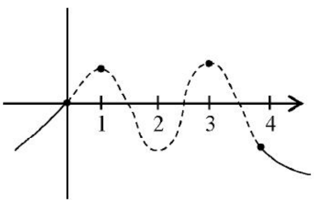
$\therefore \alpha = \frac{-1}{4},\ \beta = \frac{3}{8} \rightarrow \boxed{8(\alpha+\beta) = 1}$

23. 5

Sol. $(3f'f'' + ff''')(x) = ((ff'' + (f')^2)(x))'$

$(ff'' + (f')^2)(x) = ((ff')(x))'$

$\therefore (3f'f'' + f''')(x) = (f(x) \cdot f'(x))''$



min. roots of (x) $\rightarrow$ 4

$\therefore$ min. roots of f '(x) $\rightarrow$ 3

$\therefore$ min. roots of (f(x) · f '(x)) $\rightarrow$ 7

$\boxed{\therefore \text{ min. roots of } (f(x)\cdot f'(x))'' \rightarrow 5}$

24. 1024

Sol. $f(f(x)) = \frac{2f(x)}{\sqrt{1+9f^2(x)}} = \frac{4x}{\sqrt{1+9x^2+9.2^2x^2}}$

$f(f(f(x))) = \frac{2^3x/\sqrt{1+9x^2}}{\sqrt{1+9(1+2^2)\frac{2^2x^2}{1+9x^2}}}$

$= \frac{2^3x}{\sqrt{1+9x^2(1+2^2+2^4)}}$

$\therefore$ By observation

$\alpha = 1+2^2+2^4+....+2^{18} = 1\left(\frac{(2^2)^{10}-1}{2^2-1}\right) = \frac{2^{20}-1}{3}$

$3\alpha + 1 = 2^{20} \rightarrow \sqrt{3\alpha+1} = 2^{10} = \boxed{1024}$

25. 5

Sol. Let $A = \begin{bmatrix} a & b \\ b & d \end{bmatrix}$

$\begin{bmatrix} a & b \\ b & d \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$, $ad - b^2 = 1$

a + b = 3, b + d = 7, (3 – b) (7 – b) – b^2 = 1

21 – 10b = 1 $\rightarrow$ b = 2, a = 1, d = 5

$A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$

$A^{-1} = \alpha A + \beta I$

$\begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \alpha+\beta & 2\alpha \\ 2\alpha & 5\alpha+\beta \end{bmatrix}$

26. 5626

Sol.

From Group A	From Group B	Ways of Selection
4M	4W	${}^4C_4\,{}^4C_4 = 1$
3M 1W	1M 3W	${}^4C_3\,{}^5C_1\,{}^5C_1\,{}^4C_3 = 400$
2M 2W	2M 2W	${}^4C_2\,{}^5C_2\,{}^5C_2\,{}^4C_2 = 3600$
1M 3W	3M 1W	${}^4C_1\,{}^5C_3\,{}^5C_3\,{}^4C_1 = 1600$
4W	4M	${}^5C_4\,{}^5C_4 = 25$
	Total	**5626**

Ans. 5626

27. 8288

$P(W) = \frac{1}{3}$ $\qquad$ $P(L) = \frac{2}{3}$

x = number of matches that team wins

y = number of matches that team loses

$|x-y| \le 2$ and $x + y = 10$

$|x-y| = 0, 1, 2 \qquad x, y \in N$

Case-I : $|x-y| = 0 \Rightarrow x = y$

$\because x + y = 10 \Rightarrow x = 5 = y$

$$P(|x-y|=0) = {}^{10}C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^5$$

Case-II: $|x-y| = 1 \Rightarrow x - y = \pm 1$

$x = y+1$	$x = y-1$
$\because x+y=10$	$\because x+y=10$
$2y = 9$	$2y = 11$
Not possible	Not possible

Case-III: $|x-y| = 2 \Rightarrow x - y = \pm 2$

$x - y = 2$ OR $x - y = -2$

$\because x + y = 10 \quad \because x + y = 10$

$x = 6, y = 4 \quad x = 4, y = 6$

$$P(|x-y|=2) = {}^{10}C_6\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^4 + {}^{10}C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^6$$

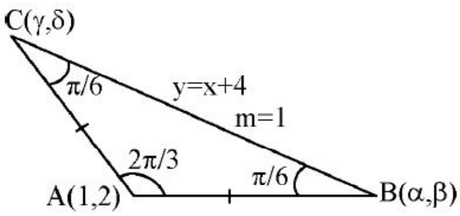
$$p = {}^{10}C_5\frac{2^5}{3^{10}} + {}^{10}C_6\frac{2^4}{3^{10}} + {}^{10}C_4\frac{2^6}{3^{10}}$$

$$3^9 p = \frac{1}{3}({}^{10}C_5 2^5 + {}^{10}C_6 2^4 + {}^{10}C_4 2^6)$$

$= 8288$

28. 14

Sol.



Equation of line passes through point A(1, 2) which makes angle $\frac{\pi}{6}$ from y = x + 4 is

$$y - 2 = \frac{1 \pm \tan\frac{\pi}{6}}{1 \mp \tan\frac{\pi}{6}}(x-1)$$

$$y - 2 = \frac{\sqrt{3} \pm 1}{\sqrt{3} \mp 1}(x-1)$$

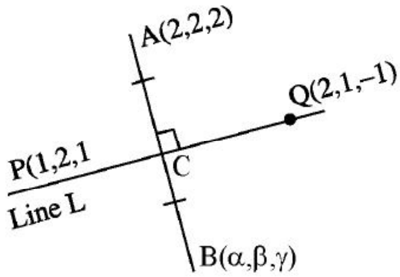
$\oplus$	$\ominus$
$y - 2 = (2+\sqrt{3})(x-1)$	$y - 2 = (2-\sqrt{3})(x-1)$
solve with $y = x + 4$	solve with $y = x + 4$
$x + 2 = (2+\sqrt{3})x - 2 - \sqrt{3}$	$x + 2 = (2-\sqrt{3}x - 2 + \sqrt{3})$
$x = \frac{4+\sqrt{3}}{1+\sqrt{3}}$	$x = \frac{4-\sqrt{3}}{1-\sqrt{3}}$

$$\alpha^2 + \gamma^2 = \left(\frac{4+\sqrt{3}}{1+\sqrt{3}}\right)^2 + \left(\frac{4-\sqrt{3}}{1-\sqrt{3}}\right)^2$$

$\alpha^2 + \gamma^2 = 14$

29. 6

Sol.



DR's of Line $L \equiv -1:1:2$

Dr's of $AB \equiv \alpha - 2 : \beta - 2 : \gamma - 2$

$AB \perp_{ar} L \Rightarrow 2 - \alpha + \beta - 2 + 2\gamma - 4 = 0$

$2\gamma + \beta - \alpha = 4 \qquad$ ....(1)

Let C is mid-point of AB

$$C\left(\frac{\alpha+2}{2}, \frac{\beta+2}{2}, \frac{\gamma+2}{2}\right)$$

$$\text{Dr's of } PC = \frac{\alpha}{2} : \frac{\beta-2}{2} : \frac{\gamma}{2}$$

$$\text{line } L \parallel PC \Rightarrow \frac{-\alpha}{2} = \frac{\beta-2}{2} = \frac{\gamma}{4} = K \text{ (let)}$$

$\alpha = -2K$

$\beta = 2k + 2$

$\gamma = 4K$

use in (1) $\Rightarrow K = \frac{1}{6}$

value of $\alpha + \beta + 6\gamma = 24K + 2 = 6$

30. 31

Sol. $\frac{dy}{dx} = (x+y+2)^2$ ....(1), $y(0) = -2$

Let $x + y + 2 = v$

$1 + \frac{dy}{dx} = \frac{dv}{dx}$

from (1) $\frac{dv}{dx} = 1 + v^2$

$\int \frac{dv}{1+v^2} = \int dx$

$\tan^{-1}(v) = x + C$

$\tan^{-1}(x + y + 2) = x + C$

at $x = 0$ $y = -2 \Rightarrow C = 0$

$\Rightarrow \tan^{-1}(x + y + 2) = x$

$y = \tan x - x - 2$

$f(x) = \tan x - 2,\ x \in \left[0, \frac{\pi}{3}\right]$

$f'(x) = \sec^2 x - 1 > 0 \Rightarrow f(x) \uparrow$

$f_{min} = f(0) = -2 = \beta$

$f_{max} = f\left(\frac{\pi}{3}\right) = \sqrt{3} - \frac{\pi}{3} - 2 = \alpha$

now $(3\alpha + \pi)^2 + \beta^2 = \gamma + \delta\sqrt{3}$

$\Rightarrow (3\alpha + \pi)^2 + \beta^2 = (3\sqrt{3} - 6)^2 + 4$

$\gamma + \delta\sqrt{3} = 67 - 36\sqrt{3}$

$\Rightarrow \gamma = 67$ and $\delta = -36 \Rightarrow \gamma + \delta = 31$

PHYSICS

Section - A (Single Correct Answer)

31. B

Sol. Since CH_4 is polyatomic Non-Linear

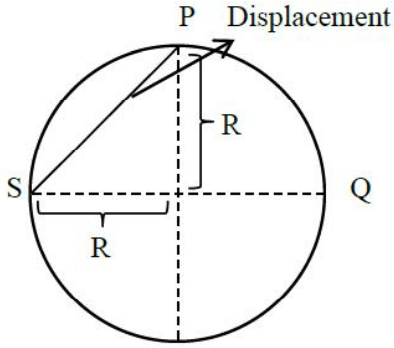
D.O.F of CH_4

T. DOF = 3

R. DOF = 3

32. B

Sol.



$\therefore$ Displacement $= R\sqrt{2} = 2\sqrt{2} = \sqrt{8}$ km

33. B

Sol. At stable equilibrium

$U = -\ mB \cos 0° = -\ mB$

At unstable equilibrium

$U = -\ mB \cos 180° = +\ mB$

$W = \Delta U$

W.D. = 2 mB

$= 2\ (0.5)\ 8 \times 10^{-2} = 8 \times 10^{-2}$ J

34. B

Sol. Diode should be in forward biased to calculate dynamic resistance

Hence correct answer would be 2.

35. C

Sol. $\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4$

36. B

Sol. $Y = \overline{\overline{A} \cdot \overline{B}}$

By De-Morgan Law

$Y = \overline{\overline{A + B}}$

$Y = A + B$

Hence OR gate

37. A

Sol. Since, Intensity $\propto$ width of slit (ω)

so, $I_1 = I$, $I_2 = 4I$

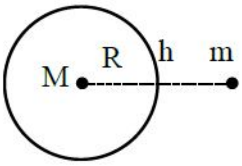
$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = I$

$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = 9I$

$\frac{I_{max}}{I_{min}} = \frac{9I}{I} = \frac{9}{I}$

38. B

Sol.



$$\Rightarrow \frac{GMm}{(R+h)^2} = \frac{mv^2}{(R+h)}$$

$$\Rightarrow \frac{GM}{(R+h)} = v^2 \quad ....(1)$$

$\Rightarrow v = (R + h)\omega$

$$\Rightarrow v = (R+h)\frac{2\pi}{T} \quad ....(2)$$

$\Rightarrow GM = gR^2 \quad ....(3)$

Put value from (2) & (3) in eq. (1)

$$\Rightarrow \frac{gR^2}{(R+h)} = (R+h)^2\left(\frac{2\pi}{T}\right)^2$$

$$\Rightarrow \frac{T^2R^2g}{(2\pi)^2} = (R+h)^3$$

$$\Rightarrow \left[\frac{T^2R^2g}{(2\pi)^2}\right]^{1/3} - R = h$$

39. D

Sol. **A** – V lags by 90° from I hence option (I) is correct.

B – V lead by 90° from I hence option (IV) is correct

C – In LCR resonance XL = XC. Hence circuit is purely resistive so option (II) is correct

D – In LCR series V is at some angle from I hence (III) is correct

Hence option (D) is correct.

40. C

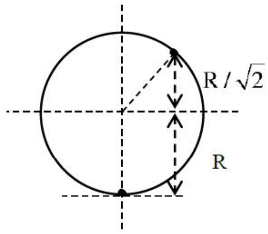
Sol. Statement I is correct as we know contact angle depends on cohesine and adhesive forces.

Statement II is incorrect because height of liquid is given by $h = \frac{2T\cos\theta_C}{\rho gr}$ where r is radius of Tube (assuming length of capillary is sufficient)

Hence option (C) is correct.

41. B

Sol.



Apply W.E.T. from A to B

$\Rightarrow \quad W_{mg} = K_B - K_A$

$$\Rightarrow \quad mg \times \left(\frac{R}{\sqrt{2}} + R\right) = \frac{1}{2}mv_B^2 - 0\{v_A = 0 \text{ rest}\}$$

$$\Rightarrow \quad mgR\frac{(\sqrt{2}+1)}{\sqrt{2}} = \frac{1}{2}mv_B^2$$

$$\Rightarrow \quad \sqrt{gR\frac{2(\sqrt{2}+1)}{\sqrt{2}}} = v_B$$

$$\Rightarrow \quad \sqrt{\frac{10\times14\times2(2.4)}{1.4}} = v_B \Rightarrow 21.9 = v_B$$

Hence option (B) is correct

42. A

Sol. Rated power & voltage gives resistance

$$R = \frac{V^2}{P} = \frac{(200)^2}{50} = \frac{40000}{50}$$

$R = 800$

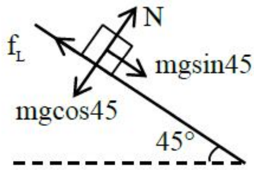
$$P = \frac{(V_{applied})^2}{R} = \frac{(100)^2}{800}$$

P = 12.5 watt

Hence option (A) is correct.

43. A

Sol.



mg sin 45 = f_L

mg cos 45 = N

$f_L = \mu_s N$

μ_s = tan 45 = 1

or tan θ = μ_s (θ is angle of repose)

tan 45 = μ_s = 1 correct option (A)

44. B

Sol. $T.E. = \frac{1}{2}kA^2$

since A is same T.E. will be same correct option (B)

45. D

Sol. Intensity of light I = nhν / A

Here n is no. of photons per unit time.

$n = \frac{IA}{h\nu}$ so on increasing frequency ν, n decreases taking intensity constant.

$k_{max} = h\nu - \phi$

So on increasing ν, kinetic energy increases.

46. C

Sol. Moment of momentum is $\vec{r} \times \vec{P}$

$\vec{L} = \vec{r} \times m\vec{v}$

$L = mvr = \frac{nh}{2\pi} = \frac{4h}{2\pi} = \frac{2h}{\pi}$

47. D

Sol. $W = \frac{nR\Delta T}{1-\gamma}$

$TV^{\gamma-1}$ = constant = $T_f (2V)^{\gamma-1}$

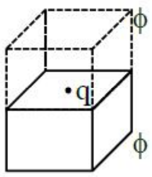
$T_f = T\left(\frac{1}{2}\right)^{\frac{1}{2}} = \frac{T}{\sqrt{2}}$

$W = \frac{R\left(\frac{T}{\sqrt{2}} - T\right)}{1-\frac{3}{2}} = 2RT\frac{\left(\sqrt{2}-1\right)}{\sqrt{2}}$

$= RT\left(2-\sqrt{2}\right)$

48. B

Sol. From



$2\phi = \frac{q}{\epsilon_0}$

$\phi = \frac{q}{2\epsilon_0}$

49. C

Sol. According to principle of homogeneity dimension of LHS should be equal to dimensions of RHS so option (C) is correct.

$T^2 = \frac{4\pi^2 r^3}{GM}$

$\left[T^2\right] = \frac{\left[L^3\right]}{\left[M^{-1}L^3T^{-2}\right]\left[M\right]}$

(Dimension of G is $\left[M^{-1}L^3T^{-2}\right]$)

$\left[T^2\right] = \frac{\left[L^3\right]}{\left[L^3T^{-2}\right]} = \left[T^2\right]$

50. D

Sol. Value of $g = g_s\left(1+\frac{h}{R}\right)^{-2}$

$= g_s\left(1+2\right)^{-2} = \frac{g_s}{9}$

Here g_s = gravitational acceleration at surface

Force = mg = 90 × $\frac{g_s}{9}$ = 100 N

Section - B (Numerical Value)

51. 10

Sol. Given,

$T = 3.14 = \frac{2\pi}{\omega}$

ω = 2 rad/s

$x = 10\sin\left(\omega t + \frac{\pi}{3}\right)$

$v = \frac{dx}{dt} = 10\omega\cos\left(\omega t + \frac{\pi}{3}\right)$

at t = 0

$v = 10\omega\cos\left(\frac{\pi}{3}\right) = 10 \times 2 \times \frac{1}{2}\left[\text{using } \omega = 2\text{rad/s}\right]$

v = 10 m/s

52. 40

Sol. Initial velocity = u = 72 km/h = 20 m/s

v = u + at

⇒ 0 = 20 + a × 4

a = – 5 m/s2

$v_2 - u_2 = 2as \Rightarrow 0_2 - 20^2 = 2(-5).s$

s = 40 m

53. 750

Sol. Before inserting dielectric capacitance is given C_0 = 12.5 pF and charge on the capacitor Q = C_0V

After inserting dielectric capacitance will become $\in_r C_0$.

Change in potential energy of the capacitor

$= E_i - E_f$

$$= \frac{Q^2}{2C_i} - \frac{Q^2}{2C_f} = \frac{Q^2}{2C_0}\left[1 - \frac{1}{\in_r}\right]$$

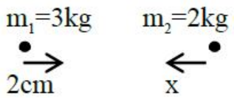
$$= \frac{(C_0V)^2}{2C_0}\left[1 - \frac{1}{\in_r}\right] = \frac{1}{2}C_0V^2\left[1 - \frac{1}{\in_r}\right]$$

Using C_0 = 12.5 pF, V = 12 V, $\in_r = 6$

$$= \frac{1}{2}(12.5)\times 12^2\left[1 - \frac{1}{6}\right] = \frac{1}{2}(12.5)\times 12^2 \times \frac{5}{6}$$

$= 750$ pJ $= 750 \times 10^{-12}$ J

54. 3

Sol. 

$$\Delta X_{C.O.M} = \frac{m_1\Delta x_1 + m_2\Delta x_2}{m_1 + m_2}$$

$$\Rightarrow 0 = \frac{3\times 2 + 2(-x)}{3+2} \Rightarrow x = 3 \text{ cm}$$

55. 208

Sol. $^{235}U \rightarrow {}^{140}Ce + {}^{94}Zr + n$

Disintegration energy

$Q = (m_R - m_P).c^2$

$m_R = 235.0439$ u

$m_P = 139.9054u + 93.9063u + 1.0086$ u

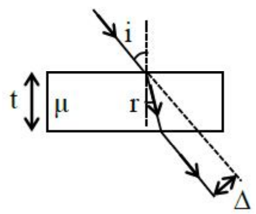
$= 234.8203u$

$Q = (235.0439u - 234.8203u)c^2 = 0.2236\ c^2$

$= 0.2236 \times 931$

$Q = 208.1716$

56. 2

Sol. 

$i = \theta_C$

$$\Rightarrow \quad i = \sin^{-1}\left(\frac{1}{\mu}\right)$$

$\Rightarrow \quad i = 45°$

and according to snell's law

$1\sin 45° = \sqrt{2}\sin r$

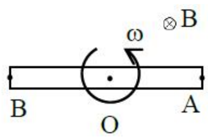
$\Rightarrow \quad r = 30°$

Lateral displacement $\Delta = \frac{t\sin(i-r)}{\cos r}$

$$\Rightarrow \Delta = \frac{4\sqrt{3}\times\sin 15°}{\cos 30°}$$

$\Rightarrow \Delta = 2$cm

57. 0

Sol. 

$$\because V_0 - V_A = \frac{B\omega l^2}{2}$$

$$V_0 - V_B = \frac{B\omega l^2}{2}$$

$\therefore \quad V_A = V_B$

$\therefore \quad V_A - V_B = 0$

58. 32

Sol. $\because R = \frac{\rho l}{A} = \frac{\rho V}{A^2}$

$$\therefore \frac{R_A}{R_B} = \frac{A_B^2}{A_A^2} = \frac{r_B^4}{r_A^4}$$

$$\Rightarrow \frac{R_A}{2} = \left[\frac{4\times 10^{-3}}{2\times 10^{-3}}\right]^4$$

$\Rightarrow R_A = 32\ \Omega$.

59. 5

Sol. $B_A = \frac{\mu_0 i}{2\pi r} + \frac{\mu_0(2i)}{2\pi(3r)} = \frac{5\mu_0 i}{6\pi r}$

$$B_C = \frac{\mu_0(2i)}{2\pi r} + \frac{\mu_0 i}{2\pi(3r)} = \frac{7\mu_0 i}{6\pi r}$$

$$\therefore \frac{B_A}{B_C} = \frac{5}{7}$$

$\therefore$ x = 5

60. 177

Sol. $F = P_0 A + \rho_m ghA$

$$=10^5 \times \frac{22}{7} \times \left(2\times10^{-2}\right)^2$$

$$+1.36\times10^4 \times 10 \times \left(30\times10^{-2}\right)\left(\frac{22}{7}\times\left(2\times10^{-2}\right)^2\right)$$

$F = 51.29 + 125.71 = 177$ N

CHEMISTRY

Section - A (Single Correct Answer)

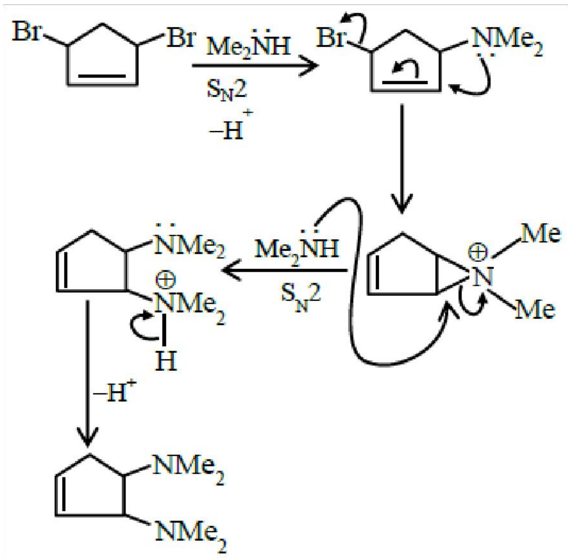
61. (D)

Sol. $K_C^{'} = \left(\frac{1}{K_C}\right)^2 = \left(\frac{1}{4.9\times10^{-2}}\right)^2$

$K'_C = 416.49$

62. (B)

Sol.:



The above mechanism valid for both cis and trans isomers. So the products are same for both cis and trans isomers.

63. (D)

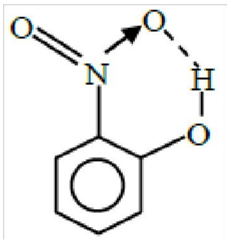
Sol. $2NH_4Cl + MnO_2 + 2H_2SO_4 \xrightarrow{\Delta} MnSO_4$

$+ (NH_4)_2SO_4 + 2H_2O + \underset{\text{greenish yellow solution}}{Cl_2} \uparrow$

64. (B)

Sol. (A) Generally hydrogen bonding exists when H is covalently bonded to the highly electronegative atom like F, O, N.

(B) Intramolecular H bonding is present in



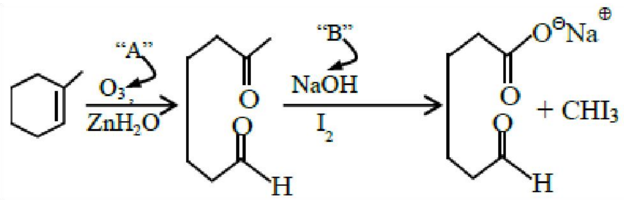
(C) Intermolecular Hydrogen bonding is present in HF

(D) The magnitude has Hydrogen bonding in solid state is greater than liquid state.

(E) Hydrogen bonding has powerfull effect on the structure & properties of compound like melting point, boiling point, density etc.

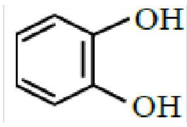
65.(A)

Sol.



66. (C)

Sol.

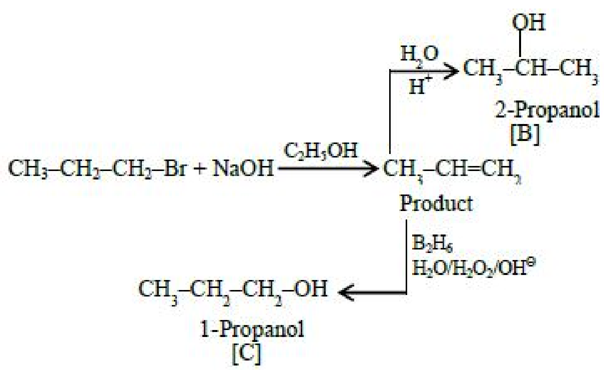


IUPAC name : Benzene-1,2-diol

Common name : catechol

67. (B)

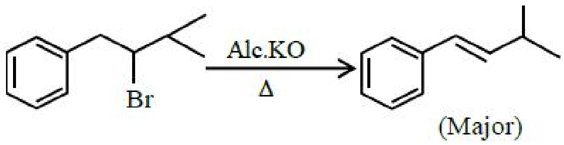
Sol.



68. (C)

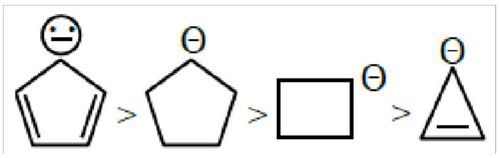
Sol. The most common polar and acidic support used is adsorption chromatography is silica. The surface silanol groups on their supported to adsorb polar compound and work particularly well for basic substances. Alumina is the example of polar and basic adsorbent that is used in adsorption chromatography

69. (B)

Sol.: 

70. (D)

Sol. As we know compound (d) is aromatic and the compound (a) is anti-aromatic. Hence compound (d) is most stable and compound (a) is least stable among these in compound (b) and (c) carbon atom of that positive charge is sp^3 hybridised they on the basis of angle strain theory compound (c) is more stable than compound (b).



71. (D)

Sol. (i) due to lanthanide contraction $T\ell$ has more I.E. as compared to Ga and $A\ell$

(ii) due to scandide contraction Ga has more I.E. as compared to $A\ell$

72. (B)

Sol. Complex is $\left[\overset{III}{Fe}(NH_3)_2(CN)_4\right]^{\ominus}$

$x = 2$

$y = 4$

so $x + y = 6$

73. (D)

Sol. Fuel cell is used in spaceship and it is type of galvanic cell.

74. (A)

Sol. In compound atoms of different elements combine in fixed ratio by mass.

75. (C)

Sol. Based on biomolecules theory and structure of these named compounds -

(A) α-Glucose and α-Galactose (IV) Epimers.

(B) α-Glucose and β-Glucose (III) Anomers

(C) α-Glucose and α-Fructose (I) Functional isomers

(D) α-Glucose and α-Ribose (II) Homologous

76. (A)

Sol.. (i) $Na < Li < Cl < F$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

LE_1 in kJ/mol 496 520 1256 1681

(ii) $Na < Li < F < Cl$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$\Delta_{eg}H$ in kJ/mol -53 -60 -328 -349

77. A

Sol.: $\wedge_m = \wedge_m^0 - A\sqrt{C}$

Unit of $A\sqrt{C}$ = S cm^2 $mole^{-1}$

Unit of A = S cm^2 $mole^{-3/2}$ $L^{1/2}$

78. (D)

Sol.: ${}_{22}Ti^{+2} \Rightarrow [Ar]3d^2$

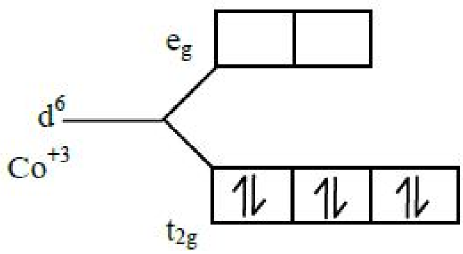
${}_{23}V^{+2} \Rightarrow [Ar]3d^3$

${}_{25}Mn^{+2} \Rightarrow [Ar]3d^5$

${}_{26}Fe^{+2} \Rightarrow [Ar]3d^6$

79. C

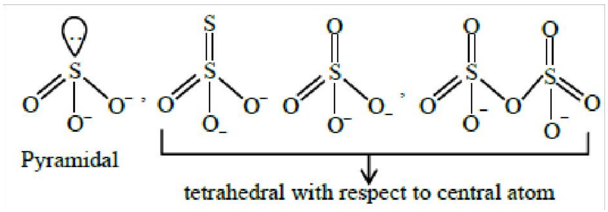
Sol.: $\Rightarrow [Co(H_2O)_6]^{+3}$



No unpaired electrons

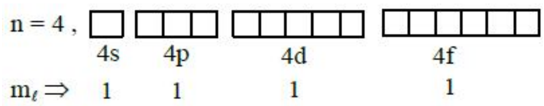
80. (C)

Sol.



Section - B (Numerical Value Type)

81. (4)

Sol. 

So answer is 4.

82. (5)

Sol. Polar molecule $_{(\mu \neq 0)}$: NF_3, H_2O, H_2S, HBr, HCl

Non Polar molecule $_{(\mu \neq 0)}$: $BeCl_2$, BCl_3, XeF_4, CCl_4, CO_2, H_2

So answer is 5.

83. 200

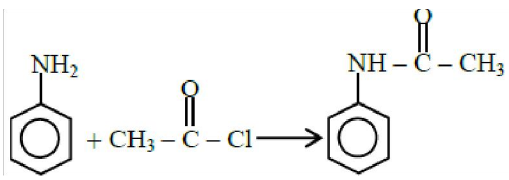
Sol. As isothermal $\Delta U = 0$

and process is irreversible

$Q = -W = -[-P_{ext}(V_2 - V_1)]$

$Q = 5(20 - 60) = -200$ atm-L

84. (95)

Sol.: 

93 g aniline form 135 gm acetanlide

so 6.55 g anilne form $\frac{135}{93} \times 6.55 = 9.5$

95×10^{-1}

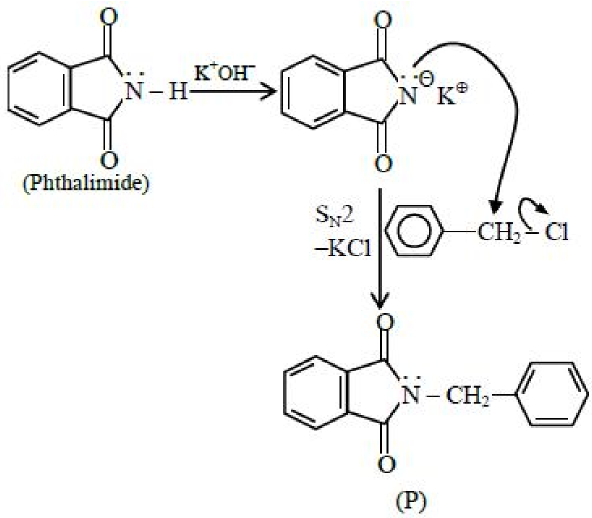
85. (50)

Sol. $K = \frac{2.303}{t} \log \frac{1}{0.1}$

$4.6 \times 10^{-2} = \frac{2.303}{t}$

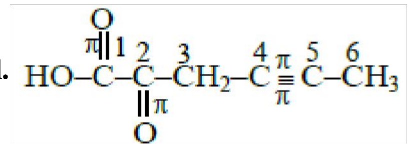
t = 50 sec.

86. (8)



Total number of π-bonds present in product P is 8

87. (18)

Sol. 

2-Oxohex-4-ynoic acid

Number of σ-bonds = 14

Number of π-bonds = 4

= 18

88. (0)

Sol. 'V' has highest enthalpy of atomisation (515 kJ/mol) among first row transition elements.

V_2O_5

Here 'V' is in +5 oxidation state

$V^{+5} \Rightarrow 1s^2\,2s^2\,2p^6\,3s^2\,3p^6$ (no unpaired electrons)

89. (31)

Sol. As moles of water > moles of CH_3COOH water is solvent.

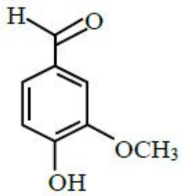
$T°_F - (T_F)_s = K_F \times M$

$0 - (T_F)_S = 1.86 \times \frac{2700/60}{2700/1000}$

$(T_F)_S = -31°C.$

90. (11)

Sol. Vanillin compound is an organic compound molecular formula $C_8H_8O_3$. It is a phenolic aldehyde. Its functional compounds include aldehyde, hydroxyl and ether. It is the primary component of the extract of the vanilla beans.



Total sum of oxygen atoms and π-electrons is 3 + 8 = 11

Total number of oxygen atoms = 3

Total number of π-bonds = 4

$\therefore$ Total number of π-electrons = 8

☯ ☯ ☯