MATHEMATICS

1. A

Sol. $\lim_{x\to 0}\dfrac{e-e^{\frac{1}{2x}\ln(1+2x)}}{x}$

$=\lim_{x\to 0}(-e)\dfrac{\left(e^{\frac{\ln(1+2x)}{2x}}-1\right)}{x}$

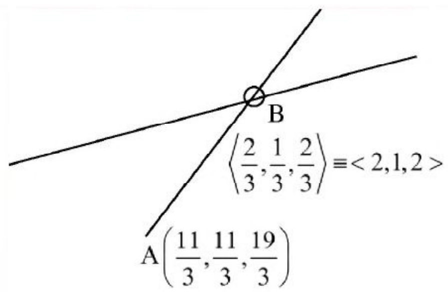
$=\lim_{x\to 0}(-e)\dfrac{\ln(1+2x)-2x}{2x^2}$

$=(-e)\times(-1)\dfrac{4}{2\times 2}=e$

2. A

Sol. $\dfrac{x-1}{2-1}=\dfrac{y-2}{3-2}=\dfrac{z-3}{5-3}$

$\Rightarrow \dfrac{x-1}{1}=\dfrac{y-2}{1}=\dfrac{z-3}{2}=\lambda$



$B(1+\lambda,\ 2\lambda,\ 3+2\lambda)$

D.R. of $AB =<\dfrac{3\lambda-8}{3},\ \dfrac{3\lambda-5}{3},\ \dfrac{6\lambda-10}{3}>$

$B\left(\dfrac{5}{3},\ \dfrac{8}{3},\ \dfrac{13}{3}\right)\dfrac{3\lambda-8}{3\lambda-5}=\dfrac{2}{1}\Rightarrow 3\lambda-8=6\lambda-10$

$3\lambda=2$

$\lambda=\dfrac{2}{3}$

$AB=\dfrac{\sqrt{36+9+36}}{3}=\dfrac{9}{3}=3$

3. A

Sol. $\sqrt{1-(y'(x))^2}=y(x)$

$1-\left(\dfrac{dy}{dx}\right)^2=y^2$

$\left(\dfrac{dy}{dx}\right)^2=1-y^2$

$\dfrac{dy}{\sqrt{1-y^2}}=dx$ OR $\dfrac{dy}{\sqrt{1-y^2}}=-dx$

$\Rightarrow \sin^{-1}y = x + c,\ \sin^{-1}y = -x + c$

$x = 0,\ y = 0 \Rightarrow c = 0$

$\sin^{-1} y = x$, as $y\ge 0$

$\sin x = y$

$\Rightarrow \dfrac{dy}{dx}=\cos x$

$\dfrac{d^2y}{dx^2}=-\sin x$

$\Rightarrow -\sin x + \sin x + 1 = 1$

4. A

Sol. $\dfrac{z-2i}{2+2i}+\dfrac{\bar{z}+2i}{\bar{z}-2i}=0$

$z\bar{z}-2i\bar{z}-2iz+4(-1)$

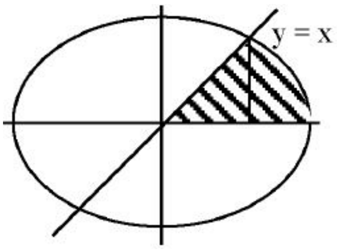
$+z\bar{z}+2zi+2\bar{z}i+4(-1)=0$

$\Rightarrow 2|z|^2=8\Rightarrow |z|=2$

$|z-(6+81)|_{maximum}=10+2=12$

5. B

Sol. $\dfrac{x^2}{18}+\dfrac{y^2}{6}=1$



$$\frac{x^2}{18}+\frac{3x^2}{18}=1\Rightarrow 4x^2=18\Rightarrow x^2=\frac{9}{2}$$

$$\int_{\sqrt{2}}^{3\sqrt{2}}\frac{\sqrt{18-x^2}}{\sqrt{3}}dx$$

$$=\frac{1}{\sqrt{3}}\left(\frac{x\sqrt{18-x^2}}{2}+\frac{18}{2}\sin^{-1}\frac{x}{3\sqrt{2}}\right)_{\frac{3}{\sqrt{2}}}^{3\sqrt{2}}$$

$$=\frac{1}{\sqrt{3}}\left(9\times\frac{\pi}{2}-\frac{3}{2\sqrt{2}}\times\frac{3\sqrt{3}}{\sqrt{2}}-9\times\frac{\pi}{6}\right)$$

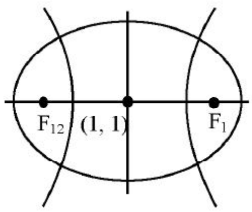
Required Area

$$=\frac{1}{2}\times\frac{9}{2}+\left(\frac{18\pi}{6}-\frac{9\sqrt{3}}{4}\right)\frac{1}{\sqrt{3}}$$

$$=\sqrt{3}\pi$$

6. B

Sol.



$$e_1=\sqrt{1-\frac{75}{100}}=\frac{5}{10}=\frac{1}{2}$$

$e_2 = 2$

$F_1(6, 1)$, $F_2(-4, 1)$

$$2ae_2=10\Rightarrow a=\frac{5}{2}\Rightarrow 2a=5$$

$$\Rightarrow \alpha=5$$

$$4=1+\frac{b^2}{a^2}\Rightarrow b^2=3a^2$$

$$b=\sqrt{3}\times\frac{5}{2}$$

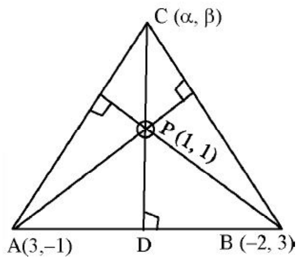
$$\beta=5\sqrt{3}$$

$$3\alpha^2+2\beta^2=3\times25+2\times25\times3$$

$$=225$$

7. C

Sol.



$$M_{AB}=\frac{4}{-5}\Rightarrow M_{DP}=\frac{5}{4}$$

Equation of PC is $y-1=\dfrac{5}{4}(x-1)$ ....(1)

$$M_{AP}=\frac{2}{-2}=-1\Rightarrow M_{BC}=+1$$

Equation of BC is $y - 3 = (x + 2)$ .....(2)

On solving (1) and (2)

$$x+4=\frac{5}{4}(x-1)\Rightarrow 4x+16=5x-5\Rightarrow \alpha=21$$

$$\Rightarrow \beta=y=x+5=26$$

$$\alpha+\beta=47$$

Equation of $\perp$ bisector of AP

$y - 0 = (x - 2)$ ....(3)

Equation of $\perp$ bisector of AB

$y-1=\dfrac{5}{4}\left(x-\dfrac{1}{2}\right)$ .....(4)

On solving (3) & (4)

$$(x-3)4=5x-\frac{5}{2}$$

$$x=\frac{-19}{2}=h$$

$$y=\frac{-23}{2}=k$$

$$\Rightarrow 2(h+k)=-42$$

8. C

Sol.

x	C	2C	3C	4C	5C	6C
f	2	1	1	1	1	1

$$\bar{x}=\frac{(2+2+3+4+5+6)C}{7}=\frac{22C}{7}$$

$$\text{Var}(x)=\frac{c^2(2+2^2+3^2+4^2+5^2+6^2)}{7}$$

$$-\left(\frac{22c}{7}\right)^2$$

$$=\frac{92c^2}{7}-c^2\times\frac{484}{49}$$

$$=\frac{(644-484)c^2}{49}=\frac{160c^2}{49}$$

$$160=\frac{160\times c^2}{49}\Rightarrow c=7$$

9. A

Sol. $f(x)=\dfrac{1}{2+\sin 3x+\cos 3x}$

$$\left[\frac{1}{2+\sqrt{2}},\ \frac{1}{2-\sqrt{2}}\right]$$

$$\frac{\alpha}{\beta}=\frac{a+b}{2\sqrt{ab}}=\frac{1}{2}\left(\sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}\right)$$

$$=\frac{1}{2}\left(\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}}+\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}\right)$$

$$=\frac{(2-\sqrt{2})+(2+\sqrt{2})}{2\times\sqrt{2}}=\sqrt{2}$$

10. B

Sol. $\bar{a}=\hat{i}-2\hat{j}-3\hat{k}$

$$\bar{a}=2\hat{i}+\hat{j}-\hat{k}$$

$$\bar{a}\times\bar{r}=\bar{a}\times\bar{b};\ \bar{a}\cdot\bar{r}=0$$

$$\Rightarrow \bar{a}\times(\bar{r}-\bar{b})=\bar{0}$$

$$\Rightarrow \bar{a}=\lambda(\bar{r}-\bar{b})$$

$$\bar{a}\cdot\bar{a}=\lambda(\bar{a}\cdot\bar{r}-\bar{a}\cdot\bar{b})$$

$$14=-7\lambda\Rightarrow\lambda=-2$$

$$\frac{-\bar{a}}{2}=\bar{r}-\bar{b}\Rightarrow\bar{r}=\bar{b}-\frac{\bar{a}}{2}$$

$$=\frac{2\bar{b}-\bar{a}}{2}=\frac{3\hat{i}+\hat{k}}{2}$$

Statement (I) is incorrect

$$\cos 2A+\cos 2B+\cos 2c\ge-\frac{3}{2}$$

$$2A+2B+2C=2\pi$$

cos 2A + cos 2B + cos 2C

= –1 – 4 cos A · cos B · cos C

$$\ge -1-4\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$$

$$=-\frac{3}{2}$$

Statement (II) is correct.

11. A

Sol. $\displaystyle\lim_{x\to\frac{\pi}{2}}\frac{0-\{\sin(2x)+\cos(x)\}\cdot 3x}{2\left(x-\frac{\pi}{2}\right)}$

$$=\lim_{x\to\frac{\pi}{2}}\frac{-\{2\sin x\cos x+\cos x\}3x^2}{2\left(x-\frac{\pi}{2}\right)}$$

$$=\lim_{x\to\frac{\pi}{2}}\left\{\frac{2\sin x\sin\left(\frac{\pi}{2}-x\right)}{2\left(x-\frac{\pi}{2}\right)}+\frac{\sin\left(\frac{\pi}{2}-x\right)}{2\left(\frac{\pi}{2}-x\right)}\right\}3x^2$$

$$=\left(1(1)+\frac{1}{2}\right)3\left(\frac{\pi}{2}\right)^2$$

$$=\frac{9\pi^2}{8}$$

12. A

Sol. $T_{r+1} = {}^9C_r (x^{2/3})^{9-r}\left(\frac{x^{-2/5}}{2}\right)^r$

$= {}^9C_r\left(\frac{1}{2}\right)^r (r)^{\left(6-\frac{2r}{3}-\frac{2r}{3}\right)}$

for coefficient of $x^{2/3}$, put $6-\frac{2r}{3}-\frac{2r}{5}=\frac{2}{3}$

$\Rightarrow r=5$

$\therefore$ Coefficient of $x^{2/3}$ is $={}^9C_5\left(\frac{1}{5}\right)^5$

For coefficient of $x^{-2/5}$, put $6-\frac{2r}{3}-\frac{2r}{5}=-\frac{2}{5}$

$\Rightarrow r=6$

Coefficient of $x^{-2/5}$ is ${}^9C_6\left(\frac{1}{2}\right)^6$

$\text{Sum} = {}^9C_5\left(\frac{1}{2}\right)^5 + {}^9C_6\left(\frac{1}{2}\right)^6 = \frac{21}{4}$

13. D

Sol. $BCB^{-1} = A$

$\Rightarrow (BCB^{-1})\,(BCB^{-1}) = A.A$

$\Rightarrow BCI\,CB^{-1} = A^2$

$\Rightarrow BC^2B^{-1} = A^2$

$\Rightarrow B^{-1}(BC^2B^{-1})B = B^{-1}(A.A)B$

From equation (1)

$C^2 = A^{-1}.A.B$

$C^2 = B$

Also $AB^{-1} = A^{-1}$

$\Rightarrow AB^{-1}\,A = A^{-1}\,A = I$

$\Rightarrow A^{-1}(AB^{-1}A) = A^{-1}I$

$B^{-1}A = A^{-1}$

Now characteristics equation of C^2 is

$|C_2-\lambda I|=0$

$|B-\lambda I|=0$

$\Rightarrow \begin{vmatrix} 1-\lambda & 3 \\ 1 & 5-\lambda \end{vmatrix}=0$

$\Rightarrow (1-\lambda)(5-1)-3=0 \Rightarrow (\lambda^2-6\lambda+5)-3=0$

$\Rightarrow \lambda^2-6\lambda+2=0$

$\Rightarrow \beta^2-6B+2I=0$

$\Rightarrow C^4-6C^2+2I=0$

$\alpha=-6$

$\beta=2$

$\therefore 2\beta-\alpha=4+6=10$

14. D

Sol. $\ln(y) = 3\sin^{-1}x$

$\frac{1}{y}\cdot y' = 3\left(\frac{1}{\sqrt{1-x^2}}\right)$

$\Rightarrow y' = \frac{3y}{\sqrt{1-x^2}}$ at $x=\frac{1}{2}$

$\Rightarrow y' = \frac{3e^{3\left(\frac{\pi}{6}\right)}}{\frac{\sqrt{3}}{2}} = 2\sqrt{3}e^{\frac{\pi}{2}}$

$\Rightarrow y'' = 3\left(\frac{\sqrt{1-x^2}\,y' - y\frac{1}{2\sqrt{1-x^2}}(-2x)}{(1-x^2)}\right)$

$\Rightarrow (1-x^2)y'' = 3\left(3y+\frac{xy}{\sqrt{1-x^2}}\right)$

$\downarrow$ at $x=\frac{1}{2}$, $y=e^{3\sin^{-1}\left(\frac{1}{2}\right)}=e^{3\left(\frac{\pi}{6}\right)}=e^{\frac{\pi}{2}}$

$(1-x^2)y''\big|_{\text{at } x=\frac{1}{2}} = 3\left(3e^{\frac{\pi}{2}}+\frac{\frac{1}{2}\left(e^{\frac{\pi}{2}}\right)}{\frac{\sqrt{3}}{2}}\right)$

$=3e^{\frac{\pi}{2}}\left(3+\frac{1}{\sqrt{3}}\right)$

$(1-x^2)y''-xy\big|_{\text{at } x=\frac{1}{2}}$

$$=3e^{\frac{\pi}{2}}\left(3+\frac{1}{\sqrt{3}}\right)-\frac{1}{2}\left(2\sqrt{3}e^{\frac{\pi}{2}}\right)=9e^{\frac{\pi}{2}}$$

15. D

Sol. $I=\int_{1/4}^{3/4}\cos\left(2\cot^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)dx\right)$

$$\int_{1/4}^{3/4}\cos\left(2\left(\tan^{-1}\sqrt{\frac{1+x}{1+x}}\right)\right)dx$$

$$\int_{1/4}^{3/4}\frac{1-\tan^2\left(\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right)}{1+\tan^2\left(\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right)}dx$$

$$=\int_{1/4}^{3/4}\frac{1-\left(\frac{1+x}{1-x}\right)}{1+\left(\frac{1+x}{1-x}\right)}dx=\int_{1/4}^{3/4}\frac{-2x}{2}dx$$

$$=\int_{1/4}^{3/4}(-x)dx=-\left(\frac{x^2}{2}\right)_{1/4}^{3/4}$$

$$=-\frac{1}{2}\left[\frac{9}{16}-\frac{1}{16}\right]$$

$$=-\frac{1}{4}$$

16. D

Sol. $\sum_{n=0}^{\infty}ar^n=57$

$$a+ar+ar^2+\infty=57$$

$$\frac{a}{1-r}=57 \qquad .....(I)$$

$$\sum_{n=0}^{\infty}a^3r^{3n}=9747$$

$$a^3+a^2\cdot r^3+a^3\cdot r^6+.....\infty=9746$$

$$\frac{a^3}{1-r^3}=9746 \qquad ......(II)$$

$$\frac{(I)^3}{(II)}\Rightarrow\frac{\frac{a^3}{(1-r)^3}}{\frac{a^2}{1-r^3}}=\frac{57^3}{9717}=19$$

On solving, $r=\frac{2}{3}$ and $r=\frac{3}{2}$ (rejected)

a = 19

$$\therefore a+18r=19+18\times\frac{2}{3}=31$$

17. C

Sol. Favourable cases = $^6C_3$

Total out comes = 6^3

Probability of getting greater number than previous one $=\frac{^6C_3}{r^3}=\frac{20}{216}=\frac{5}{54}$

18. B

Sol. $I=\int_{-1}^{2}1.\log_e\left(x+\sqrt{x^2+1}\right)dx$

$$=x\log_e\left(x+\sqrt{x^2+1}\right)-\int_{-1}^{2}\left(\frac{1+\frac{x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}}\right)dx$$

$$=x\log_e\left(x+\sqrt{x^2+1}\right)-\int_{-1}^{2}\frac{x}{\sqrt{x^2+1}}dx$$

$$=x\log_e\left(x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}\Big|_{-1}^{2}$$

$$=\left(2\log_e\left(2+\sqrt{5}\right)-\sqrt{5}\right)$$

$$-\left(\log_e\left(-1+\sqrt{2}\right)-\sqrt{2}\right)$$

$$=\log_e\left(2+\sqrt{5}\right)^2-\sqrt{5}+\log_e\left(\sqrt{2}-1\right)+\sqrt{2}$$

$$=\sqrt{2}-\sqrt{5}+\log_e\left(\frac{\left(2+\sqrt{5}\right)^2}{\sqrt{2}+1}\right)$$

$$=\sqrt{2}-\sqrt{5}+\log_e\left(\frac{9+4\sqrt{5}}{\sqrt{2}+1}\right)$$

19. B

Sol. $x^2 - \sqrt{2x} - \sqrt{3}$ $\langle^{\alpha}_{\beta}$

$\alpha^{n+2} - \sqrt{2}\alpha^{n+1} - \sqrt{3}\alpha^n = 0$

and $\beta^{n+2} - \sqrt{2}\beta^{n+1} - \sqrt{3}\beta^n = 0$

Substracting

$(\alpha^{n+2} - \beta^{n+2}) - \sqrt{2}(\alpha^{n+1} - \beta^{n+1}) - \sqrt{3}(\alpha^n - \beta^n) = 0$

$\Rightarrow P_{n+2} - \sqrt{2}P_{n+1} - \sqrt{3}P_n = 0$

Put n = 10

$P_{12} - \sqrt{2}P_{11} - \sqrt{3}P_{10} = 0$

$n = 9$

$P_{11} - \sqrt{2}P_{10} - \sqrt{3}P_9 = 0$

$11\left(\sqrt{3}\cdot P_{10} + \sqrt{2}P_{11}\right) - 10\left(\sqrt{2}P_{10} - P_{11}\right)$

$= 0 - 10\left(-\sqrt{3}P_9\right) = 10\sqrt{3}P_9$

20. D

Sol. Area of parallelogram $= \frac{1}{2}|\vec{d}_1 \times \vec{d}_2$

$A = \frac{1}{2}|(\vec{a}+\vec{b})\times(\vec{b}+\vec{c})| = \frac{\sqrt{21}}{2}$

so, $\vec{a}+\vec{b} = \hat{i} + \alpha\hat{j} + 2\hat{k}$

$\vec{b}+\vec{c} = -\hat{i} + \beta\hat{j}$

$(\vec{a}+\vec{b})\times(\vec{b}+\vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \alpha & 2 \\ -1 & \beta & 0 \end{vmatrix}$

$= \hat{i}(-2\beta) - \hat{j}(2) + \hat{k}(\beta+\alpha)$

$|(\vec{a}+\vec{b})\times(\vec{b}+\vec{c})| = \sqrt{4\beta^2 + 4 + (\alpha+\beta)^2} = \sqrt{21}$

$4\beta^2 + 4 + \alpha^2 + \beta^2 + 2\alpha\beta = 21$

$\alpha^2 + 5\beta^2 - 12 = 17$

$\alpha^2 + 5\beta^2 = 29$

and $\alpha\beta = -6$

and given α, β are integers

so,

$\alpha = -3,\ \beta = 2$

or

$\alpha = 3,\ \beta = -2$

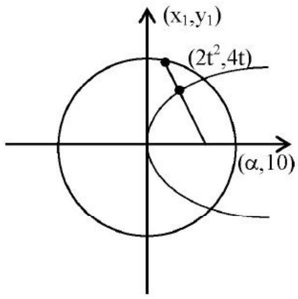
$(\alpha_1, \beta_1) = (-3, 2)$

$(\alpha_2, \beta_2) = (3, -2)$

$\alpha_1^2 + \beta_1^2 - \alpha_2\beta_2 = 9 + 4 + 6 = 19$

21. 80

Sol.



$T = S_1$

$xx_1 + yy_1 = x_1^2 + y_1^2$

$\alpha x_1 = x_1^2 + y_1^2$

$\alpha(2t^2) = 4t^4 + 16t^2$

$\alpha = 2t^2 + 8$

$\frac{\alpha - 8}{2} = t^2$

Also, $4t^2 + 16t^2 - 4 < 0$

$t^2 = -2 + \sqrt{5}$

$\alpha = 4 + 2\sqrt{5}$

$\therefore \alpha \in (8,\ 4 + 2\sqrt{5})$

$\therefore (2q - p)^2 = 80$

22. 252

Sol. $f(x) = -(p^2 - 6p + 8)\cos 4n + 2(2 - p)n + 7$

$f^1(x) = +4(p^2 - 6p + 8)\sin 4x + (4 - 2p) \neq 0$

$\sin 4x \neq \frac{2p - 4}{4(p-4)(p-2)}$

$$\sin 4x \neq \frac{2(p-2)}{4(p-4)(p-2)}$$

$p \neq 2$

$$\sin 4x \neq \frac{1}{2(p-4)}$$

$$\Rightarrow \left|\frac{1}{2(p-4)}\right| > 1$$

on solving we get

$$\therefore p \in \left(\frac{7}{2}, \frac{9}{2}\right)$$

Hence $a = \frac{7}{2}$, $b = \frac{9}{2}$

$\therefore 16ab = 252$

23. 61

Sol. $\frac{dy}{dx} - 3y = \alpha$

$$\text{If} = e^{\int -3dx} = e^{-3x}$$

$$\therefore y - e^{-3x} = \int e^{-3x} \cdot \alpha dx$$

$$ye^{-3x} = \frac{\alpha e^{-3x}}{-3} + c$$

$(*e^{3x})$

$$y = \frac{\alpha}{-3} + C \cdot e^{3x}$$

on substituting $x = 0, y = 1$

$x \to -\infty$, $y = 7$

we get $y = 7 - 6e^{3x}$

$9f(-\log_e 3) = 61$

24. 70

Sol. N = a b c

(i) All distinct digits

$a + b + c = 14$

$a \geq 1$

$b, c \in \{0 \text{ to } 9\}$

by hit & trial : 8 cases

(6, 5, 3)	(8, 6, 0)	(9, 4, 1)
(7, 6, 1)	(8, 5, 1)	(9, 3, 2)
(7, 5, 2)	(8, 4, 2)	
(7, 4, 3)		(9, 5, 0)

(ii) 2 same, 1 diff $\qquad a = b \,;\, c$

$2a + c = 14$

by values :

$$\left.\begin{matrix}(3, 8)\\(4, 6)\\(5, 4)\\(6, 2)\\(7, 0)\end{matrix}\right\} \text{Total } \frac{3!}{2!} \times 5 - 1$$

= 14 cases

(iii) all same :

$3a = 14$

$a = \frac{14}{3} \times$ rejected

0 cases

Hence, Total cases :

$8 \times 3! + 2 \times (4) + 14$

$= 48 + 22$

$= 70$

25. 24

Sol. $2x + 3y = 23$

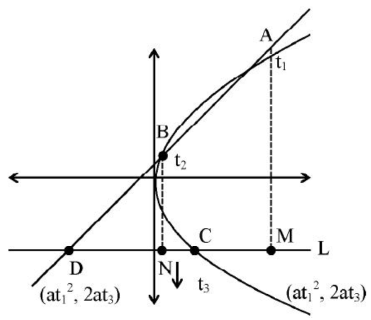
$x = 1$	$y = 7$
$x = 4$	$y = 5$
$x = 7$	$y = 3$
$x = 10$	$y = 1$

A	B
(1, 7)	1
(4, 5)	4
(7, 3)	7
(10, 1)	10

The number of one-one functions from A to B is equal to 4!

26. 36

Sol.



$m_{AB} = m_{AD}$

$$\Rightarrow \frac{2}{t_1+t_2} = \frac{2a(t_1-t_3)}{at_1^2-\alpha}$$

$$\Rightarrow at_1^2 - \alpha = a\{t_1^2 - t_1t_3 + t_1t_2 - t_2t_3\}$$

$$\Rightarrow \alpha = a(t_1t_3 + t_2t_3 - t_1t_2)$$

AM = $|2a(t_1 - t_3)|$, BN = $|2a(t_2 - t_3)|$,

$$CD = |at_3^2 - \alpha|$$

$$CD = |at_3^2 - a(t_1t_3 + t_2t_3 - t_1t_2)|$$

$$= a|t_3^2 - t_1t_3 - t_2t_3 + t_1t_2|$$

$$= a|t_3(t_3 - t_1) - t_2(t_3 - t_1)|$$

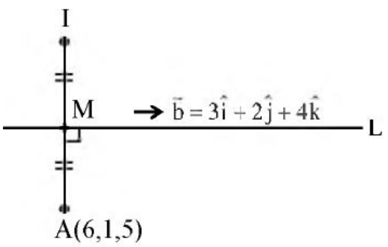
$$CD = a|(t_3 - t_2)(t_3 - t_1)|$$

$$\left(\frac{AM \cdot BN}{CD}\right)^2 = \left\{\frac{2a(t_1-t_3)\cdot 2a(t_2-t_3)}{a(t_3-t_2)(t_3-t_1)}\right\}^2$$

$$16a^2 = 16 \times \frac{9}{4} = 36$$

27. 62

Sol.



Let $M(3\lambda+1,\ 2\lambda,\ 4\lambda+2)$

$$\overline{AM} \cdot \vec{b} = 0$$

$$\Rightarrow 9\lambda - 15 + 4\lambda - 2 + 16\lambda - 12 = 0$$

$$\Rightarrow 29\lambda = 29$$

$$\Rightarrow \lambda = 1$$

M(4, 2, 6), I = (2, 3, 7)

Required Distance $= \sqrt{4+9+49} = \sqrt{62}$

Ans. 62

28. 1011

Sol. $$\left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \ldots + \frac{1}{\alpha+2012}\right)$$

$$-\left\{\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots+\left(\frac{1}{2023}-\frac{1}{2024}\right)\right\} = \frac{1}{2024}$$

$$\Rightarrow \left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \ldots + \frac{1}{\alpha+2012}\right)$$

$$-\left\{\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)+\ldots+\frac{1}{2023}\right.$$

$$\left.-\frac{1}{2024} - 2\left(\frac{1}{2}+\frac{1}{4}+\ldots+\frac{1}{2022}\right)\right\} = \frac{1}{2024}$$

$$\Rightarrow \left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \ldots + \frac{1}{\alpha+2012}\right)$$

$$-\left(\frac{1}{1}+\frac{1}{2}+\ldots+\frac{1}{2023}\right)$$

$$+\frac{1}{2024} + \left(\frac{1}{1}+\frac{1}{2}+\ldots+\frac{1}{1011}\right) = \frac{1}{2024}$$

$$\Rightarrow \frac{1}{\alpha+1} + \frac{1}{\alpha+2} + \ldots + \frac{1}{\alpha+2012}$$

$$= \frac{1}{1012} + \frac{1}{1013} + \ldots + \frac{1}{2023}$$

$$\Rightarrow \alpha = 1011$$

29. 0

Sol. $$2\sin^{-1}x + 3\cos^{-1}x = \frac{2\pi}{5}$$

$$\Rightarrow \pi + \cos^{-1}x = \frac{2\pi}{5}$$

$$\Rightarrow \cos^{-1}x = \frac{-3\pi}{5}$$

Not possible

Ans. 0

30. 450

Sol. $$A = \begin{pmatrix} 2 & -5 \\ 3 & m \end{pmatrix},\ B = \begin{pmatrix} 20 \\ m \end{pmatrix}$$

$$x = \begin{pmatrix} x \\ y \end{pmatrix}$$

$2x - 5y = 20$ ....(1)

$3x + my = m$ .....(2)

$$\Rightarrow y = \frac{2m-60}{2m+15}$$

$$y < 0 \Rightarrow m \in \left(\frac{-15}{2}, 30\right)$$

$$x = \frac{25m}{2m+15}$$

$$x < 0 \Rightarrow m \in \left(\frac{-15}{2}, 0\right)$$

$$\Rightarrow m \in \left(\frac{-15}{2}, 0\right)$$

$|A| = 2m + 15$

Now,

$$8\int_{\frac{-15}{2}}^{0} (2m+15)dm = 8\{m^2 + 15m\}_{\frac{-15}{2}}^{0}$$

$$\Rightarrow 8\left\{-\left(\frac{225}{4} - \frac{225}{2}\right)\right\}$$

$$= 8 \times \frac{225}{4} = 450$$

PHYSICS

Section - A (Single Correct Answer)

31. A

Sol. By conservation of momentum

$p_i = p_f$

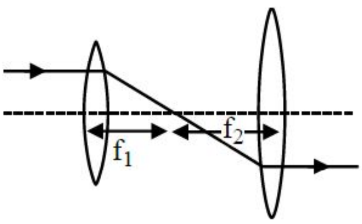
$O = m_1 u_1 + m_2 u_2$

$$\frac{u_1}{u_2} = -\left[\frac{1}{2}\right] \text{ as } \frac{m_1}{m_2} = \frac{2}{1}$$

move in opposite direction with speed ratio 1 : 2

32. C

Sol.



$D = f_1 + f_2 = 25$ cm

Paraxial parallel rays pass through focus and ray from focus of convex lens will become parallel

33. B

Sol. $KE = \frac{nf_1 RT}{2}$

$T_i = -78°C \rightarrow 273 + [-78°C] = 195$ K

KE α T

To double the K.E energy temp also become double

$T_f = 390$ K

$T_f = 117°C$

34. D

Sol. Hydrogen will be in first excited state therefore it will emit one spectral line corresponding to transition b/w energy level 2 to 1

35. B

Sol. $E_0 = B_0C$

$E_0 = 3 \times 10^8 \times (3.5 \times 10^{-7}) \sin(1.5 \times 10^3 x + 0.5 \times 10^{11} t)$

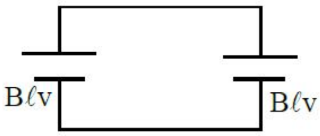
$E_0 = 105 \sin(1.5 \times 10^3 x + 0.5 \times 10^{11} t)$ Vm^{-1}

Data inconsistent while calculating speed of wave.

You can challenge for data.

36. C

Sol. At t = 10 sec complete loop is in magnetic field therefore no change in flux

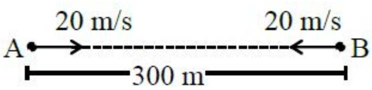


$$e = \frac{d\phi}{dt} = 0$$

e = 0 for complete loop

37. C

Sol.



$|\vec{u}_{BA}| = 40 m/s$

$|\vec{a}_{BA}| = 4 m/s$

Apply $(v^2 = u^2 + 2as)_{relative}$

$O = (40)^2 + 2(-4)(S)$

S = 200 m

Remaining distance = 300 – 200 = 100 m

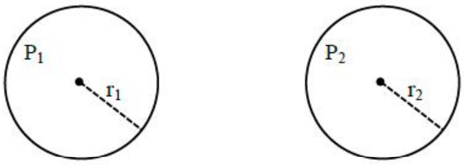
38. C

Sol. Theory

Zener diode used as voltage regulator

39. D

Sol.



$P_1 - P_0 = \frac{4T}{r_1}$ $\quad$ $P_2 - P_0 = \frac{4T}{r_2}$

$P_1 - P_0 = 3\ (P_2 - P_0)$

$\frac{4T}{r_1} = 3\frac{4T}{r_2}$

$r_2 = 3r_1$

$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{1}{27}$

40. B

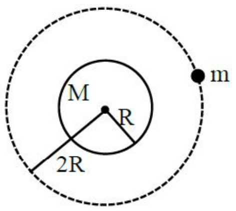
Sol. $\lambda = \frac{h}{\sqrt{2mE}}$ or $E = h\nu$

$[ML^2 T^{-2}] = h\ [T^{-1}]$

$h = [ML^2T^{-1}]$

41. D

Sol.



Total energy $= \frac{-GMm}{2(2R)}$

if energy $= \frac{10^4 R}{6}$ is added then

$\frac{-GMm}{4R} + \frac{10^4 R}{6} = \frac{-GMm}{2r}$

where r is new radius of revolving and $g = \frac{GM}{R^2}$

$-\frac{mgR}{4} + \frac{10^4 R}{6} = -\frac{mgR^2}{2r} \left(m = 10^3 kg\right)$

$-\frac{10^3 \times 10 \times R}{4} + \frac{10^4 R}{6} = -\frac{10^3 \times 10 \times R^2}{2r}$

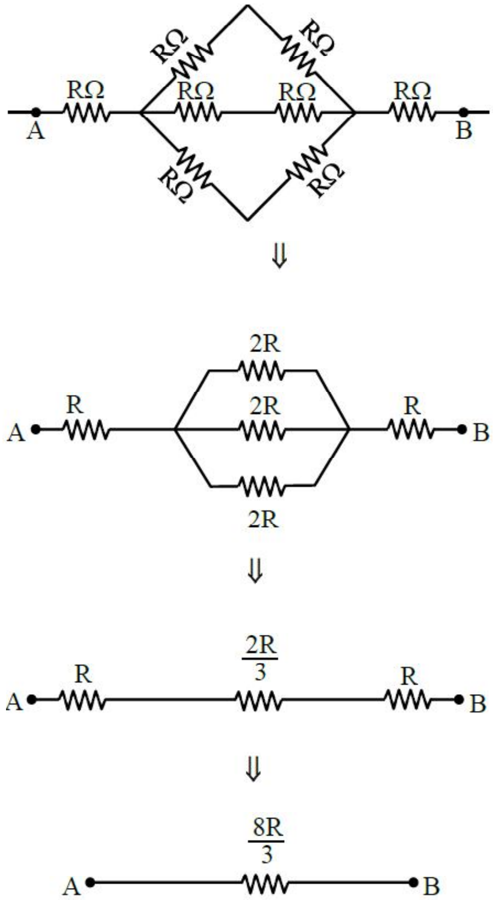
$-\frac{1}{4} + \frac{1}{6} = -\frac{R}{2r}$

$r = 6R$

42. B

Sol. From symmetry we can remove two middle resistance.

New circuit is



43. B

Sol. As per gauss theorem,

$\phi = \frac{q_{in}}{\epsilon_0} = \frac{q + (-2q) + 5q}{\epsilon_0}$

$\frac{4q}{\epsilon_0}$

44. C

Sol. In uniform magnetic field,

$R = \frac{mv}{qB} = \frac{\sqrt{2m(KE)}}{qB}$

Since same K.E

$R \propto \frac{\sqrt{m}}{q}$

$$\therefore \frac{R_{deutron}}{R_{proton}} = \sqrt{\frac{m_d}{m_p}} \times \frac{q_p}{q_d}$$

$$= \sqrt{2} \times 1$$

$$\therefore \gamma_d : \gamma_p = \sqrt{2} : 1$$

45. B

Sol. E_{photon} = (work function) + $K.E_{max}$

$\therefore$ $4.13 = 3.13 + KE_{max}$

$\therefore$ $K.E_{max} = 1$ eV

46. C

Sol. In each fusion reaction, $4\,{}^1_1H$ nucleus are used.

Energy released per Nuclei of ${}^1_1H = \frac{26.7}{4}$MeV

$\therefore$ Energy released by 2 kg hydrogen (E_H)

$$= \frac{2000}{1} \times N_A \times \frac{26.7}{4}\text{MeV}$$

&

$\therefore$ Energy released by 2 kg Vranium (E_V)

$$= \frac{2000}{235} \times N_A \times 200\text{MeV}$$

So,

$$\frac{E_H}{E_V} = 235 \times \frac{26.7}{4 \times 200} = 7.84$$

$\therefore$ Approximately close to 7.62

47. B

Sol. $W_{AB} = \int PdV$ (Assuming T to be constant)

$$= \int \frac{RTdV}{V^3}$$

$$= RT\int_2^4 V^{-3}dV$$

$$= 8 \times 300 \times \left(-\frac{1}{2}\left[\frac{1}{4^2} - \frac{1}{2^2}\right]\right) = 225 \text{ J}$$

$$W_{BC} = P\int_4^2 dV = 10(2-4) = -20\text{J}$$

$W_{CA} = 0$

$\therefore$ $W_{cycle} = 205$ J

Note : Data is inconsistent in process AB.

So needs to be challenged.

48. D

Sol. $T_1 \sin45° = F$

$T_1 \cos45° = T_2 = 1 \times g$

$\therefore$ $\tan 45° = F/g$

$\therefore$ F = 10N

49. C

Sol. $V_T = \frac{2g}{9}\frac{R^2[\rho_B - \rho_L]}{\eta}$

$$\Rightarrow V_T = \frac{2}{9} \times \frac{10 \times (10^{-4})^2}{9.8 \times 10^{-6}}\left[10^5 - 10^3\right]$$

$\Rightarrow V_T = 224.5$

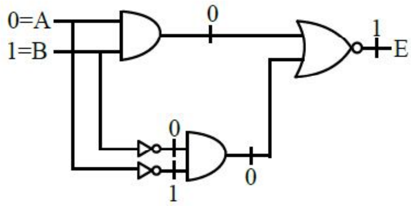
when ball fall from height (h)

$$V = \sqrt{2gh}$$

$$h = \left(\frac{V^2}{2g}\right) = 2518\text{m}$$

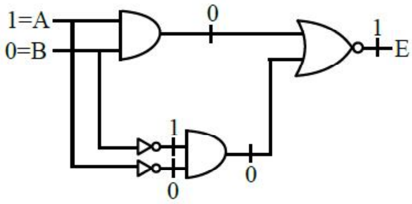
50. A

Sol. **For x**



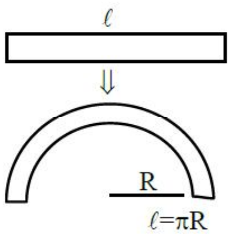
x = 1

For y



51. 28

Sol. Magnetic moment of straight wire = m x l = 44



Magnetic moment of arc

= m × 2 r

$$= m \times \frac{2l}{\pi}$$

$$= \frac{44 \times 2}{\pi} = \frac{88}{\pi} = 28$$

52. 22

Sol. m = 0.5 kg

F = – 50 (x)

ma = (– 50x)

0.5 a = – 50x

a = (– 100x)

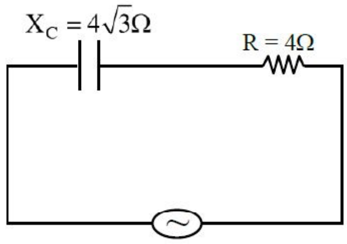
$W^2 = 100 \Rightarrow (w = 10)$

$$T = \frac{2\pi}{10} = \left(\frac{\pi}{5}\right) = \frac{22}{7\times 15} = \left(\frac{22}{35}\right)$$

$$\frac{\pi}{35} = \frac{22}{35} \Rightarrow \boxed{x = 22}$$

53. 4

Sol.



$$Z = \sqrt{R^2 + X^2L}$$

$$Z = \sqrt{4^2 + \left(4\sqrt{3}\right)^2} = 8\Omega$$

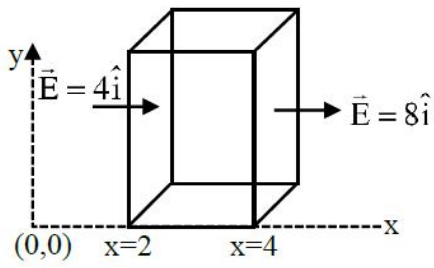
$$V_{rms} = \frac{V}{\sqrt{2}} = \frac{8\sqrt{2}}{\sqrt{2}} = (8V)$$

$$I_{rms} = \frac{V_{rms}}{z} = \frac{8}{8} = 1A$$

Power dissipated = $I^2_{rms} \times R = 1 \times 4 = (4W)$

54. 16

Sol.



$\vec{E} = 2x\hat{i}$

$\phi = \vec{E}\cdot\vec{A}$

$\phi_{in} = -4 \times 4 = -16 Nm^2/c$

$\phi_{out} = 8 \times 4 = 32$ Nm/c

$d_{net} = \phi_{in} + \phi_{out} = -16 + 32 = 16$ Nm

55. 3

Sol. For slipping

a = g sin θ

$$l = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2l}{g\sin\theta}}$$

For rolling

$$a' = \frac{g\sin\theta}{1+\frac{k^2}{R^2}}\left[k = \frac{R}{\sqrt{2}}\right]$$

$$\Rightarrow a' = \frac{2g\sin\theta}{3}$$

$$l = \frac{1}{2}a'(t')^2$$

$$\Rightarrow t' = \sqrt{\frac{6l}{2g\sin\theta}} = \sqrt{\frac{\alpha}{2}}\sqrt{\frac{2l}{g\sin\theta}}$$

$$\Rightarrow \boxed{\alpha = 3}$$

56. 2500

Sol. $Req = \frac{10^4 R}{10^4 + R}$

E = 4V, I = 2mA

$$I = \frac{E}{Req} \Rightarrow 2\times 10^{-3} = \frac{4\left(10^4 + R\right)}{10^4 R}$$

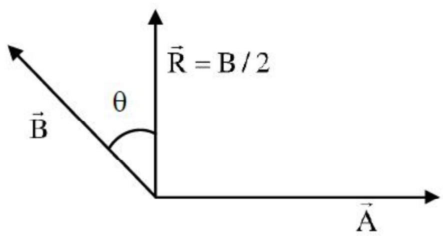
$\Rightarrow 20R = 40000 + 4R$

$16R = 40000$

$R = 2500\Omega$

57. 150

Sol.



$$B\cos\theta = \frac{B}{2}$$

$\Rightarrow \quad \theta = 60°$

So, angle between A &B is 90° + 60° = 150°

58. 4

Sol. (μ – 1) t = nλ

(1.5 – 1) t = 4 × 500 × 10

$t = 4000 \times 10^{-9}$ m

t = 4 μm

59. 58

Sol. $W = \int_{x_1}^{x_2} F dx$

$W = \int_{2}^{4} (3x^2 + 2x - 5) dx$

$W = \left[x^3 + x^2 - 5x\right]_2^4$

W = [60 – 2] J = 58 J

60. 1027

Sol. $R = R_0(1 + \alpha\Delta T)$

$62 = 50\,[1 + 2.4 \times 10^{-4}\,\Delta T]$

$\Delta T = 1000°C$

$\Rightarrow T - 27° = 1000°C$

$T = 1027°C$

CHEMISTRY

Section - A (Single Correct Answer)

61. (B)

Sol.: The candela is the luminous intensity of a source that emits monochromatic radiation of frequency radiation of frequency 540×10^{12} Hz and has a radiant intensity in that direction of $\frac{1}{683}$ w/sr. It is unit of Candela.

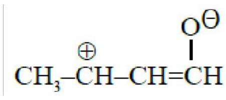
62. (B)

Sol.: $CH_3 - CH = CH - \overset{\overset{\displaystyle O}{||}}{C}H$ (III)

↓

Non Polar R.S.

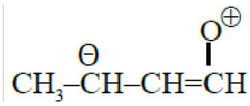
More No of covalent bond



(II)

↓

Having –ve charge on more electronegative atom



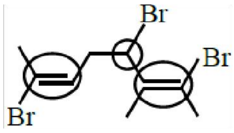
(I)

↓

Having –ve charge on less electronegative atom

Stability order III > II > I

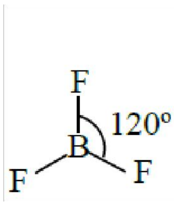
63. (A)

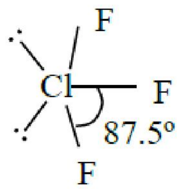
Sol.: 

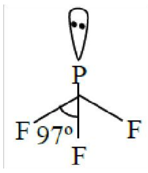
There are three stereo center

So No of stereoisomer = 2^3 = 8

64. (C)







Order of bond angle is

$ClF_3 < PF_3 < BF_3$

65. (B)

Sol.: (A) Br_2 water test is test of unsaturation in which reddish orange colour of bromine water disappears.

(B) Alcohols given Red colour with ceric ammonium nitrate.

(C) Phenol gives Violet colour with natural ferric chloride.

(D) Aldehyde & Ketone give Yellow/Orange/Red Colour compounds with 2, 4-DNP i.e., 2, 4-Dinitrophenyl hydrazine.

66. (C)

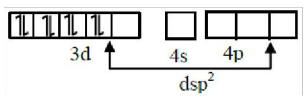
Sol.: A-IV, B-III, C-I, D-II

67. (D)

Sol.: (A) $K_2[\overset{+2}{Ni}(CN)_4]$

Ni^{2+}:[Ar]$3d^8 4s^o$, (CN^- is S.F.L)

Pre hybridization state of Ni^{+2}

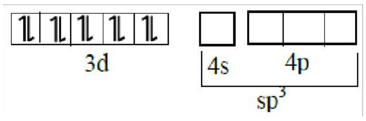


(B) $[Ni(CO)_4]$

Ni : [Ar] $3d^8\ 4s^2$

CO is S.F.L, so pairing occur

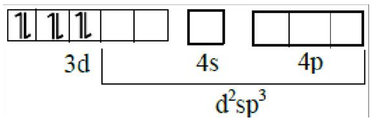
Pre hybridization state of Ni



(C) $[Co(NH_3)_6]Cl_3$

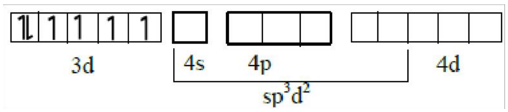
Co^{+3}: $[Ar]3d^6 4s^0$

With Co^{3+}, NH_3 act as S.F.L.



(D) $Na_3[CoF_6]$

Co^{3+} : $[Ar]3d^6$ ($F^{\ominus}$: W.F.L)



68. (B)

Sol.: $EDTA^{4-} \rightarrow$ Hexadentate ligand

$[Ca(EDTA)]^{2-}$

So Coordination environment is octahedral

69. (A)

So.: Glucose is soluble in water due to presence of alcohol functional group and extensive hydrogen bonding.

Glucose exist is open chain as well as cyclic forms in its aqueous solution.

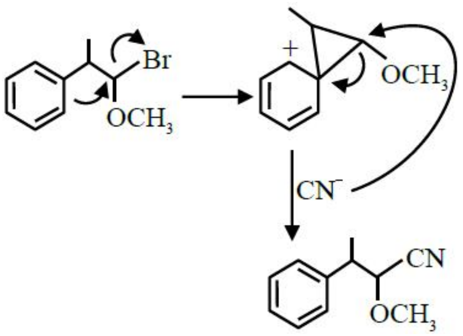
Glucose having 6C atoms so it is hexose and having aldehyde functional group so it is aldose.

Thus, aldohexose.

Glucose is monomer unit in sucrose with fructose

70. (A)

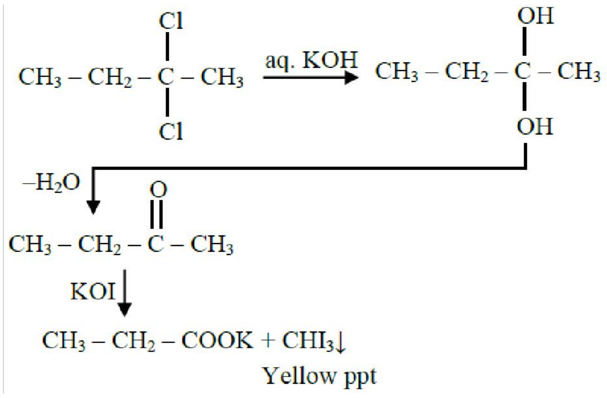
Sol.:



Due to NGP effect of phenyl ring Nucleophilic substitution of Br will occurs.

71. (B)

Sol.:



72. (B)

Sol.: $AB_{2(s)} \rightleftharpoons A^{+2}_{(aq)} + 2B^{-}_{(aq)}$

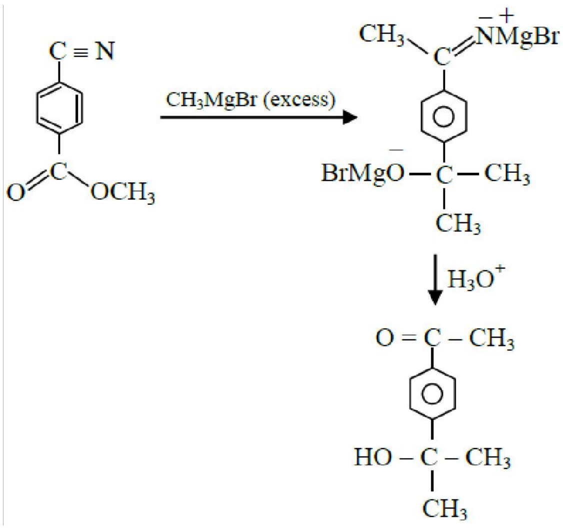
$K_{sp} = [A^{+2}][B^{-}]^2$

$= 1.2\times10^{-4}\times(2.4\times10^{-4})^2$

$=6.91\times10^{-12}M^3$

73. (B)

Sol.:



74. (C)

Sol.: On moving down the group in transition elements, stability of higher oxidation state increases, due to increase in effective nuclear charge.

$\Rightarrow E^{o}_{Cu^{+2}/Cu} = 0.34V$

$\Rightarrow E^{o}_{H^+/H_2} = 0$

SRP : $Cu^{2+} > H^+$

Cu can't liberate hydrogen gas from weak acid.

75. (D)

Sol.: The carbon-carbon bonds in ethyne is stronger than that in ethene.

(H–C$\equiv$C–H) Ethyne is linear and carbon atoms are SP hybridised

76. (B)

(A) $_{7}N$:[He]$2s^2\ 2p^3$

(B) $_{16}S$:[Ne]$2s^2\ 3p^4$

(C) $_{35}Br$:[Ar]$3d^{10}\ 4s^2\ 4p^5$

(D) $_{36}Kr$:[Ar]$3d^{10}4s^24p^6$

77. (C)

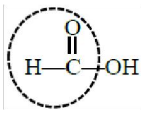
Sol.: Melting point : B > Aℓ > Tℓ > In > Ga

Ionic radius (M^{+3}/pm): Tℓ > In > Ga > Aℓ > B

$(\Delta_{IE}H)_1\left[\frac{kJ}{mol}\right]: B > T\ell > A\ell \approx Ga > In$

Atomic radius (in pm) : Tℓ > In > Aℓ > Ga > B

78. (B)

Sol.: Apart from aldehyde, Formic acid 

also gives silver mirror test with ammonical silver nitrate.

79. (A)

Sol.: $Ha(aq) \rightleftharpoons H^+(aq) + A^-(aq)$

$$K_a = \frac{\alpha^2 C}{1-\alpha}$$

$$\alpha^2 C + K_a\alpha - K_a = 0$$

$$\left(\frac{\lambda_m}{\lambda_m^\infty}\right)^2 + C + K_a\frac{\lambda_m}{\lambda_m^\infty} - K_a = 0$$

$$\lambda_m^2 C + K_a\lambda_m\lambda_m^\infty - K_a(\lambda_m^\infty)^2 = 0$$

80. (B)

Sol.: Einsteinium (atomic No = 99)

[Rn] $5f^{11}\ 6d^0\ 7s^2$

<u>Section - B (Numerical Value Type)</u>

81. (7)

Sol.: Fuming sulphuric acid is a mixture of

conc. H_2SO_4 + SO_3 Or $H_2S_2O_7$

So, Number of Oxygen atoms = 7

82. (6)

Sol.: Among given metals, Cr has maximum IE_2 because Second electron is removed from stable configuration $3d^5$

Cr^+ : [Ar] $3d^5\ 4s^0$

∴ No of unpaired e^- in Cr^+ is 5, n = 5

So, Magnetic moment = $\sqrt{n(n+2)}$ B.M

$= \sqrt{5(5+2)} = 5.92$ BM ≈ 6

83. (23)

Sol.: $X_{methylbenzene} = 0.5$

$$Y_{methylbenzene} = \frac{P_{methylbenzene}}{P_{total}}$$

$$Y_{methylbenzene} = \frac{0.5\times 24}{0.5\times 80 + 0.5\times 24}$$

$$= \frac{12}{40+12} = 0.23 = 23\times 10^{-2}$$

84. (0)

Sol.: $Co^{2+} + H_2S \rightarrow CoS\downarrow$ (Black)

(A)

CoS + Aqua-regia $\rightarrow Co^{2+}(aq) + NOCl + S + H_2O$

(A) (B)

$Co^{2+}(aq) + KNO_2 + CH_3COOH$

↓

$K_3[Co(NO_2)_6]$ + NO + $S + H_2O$

In $K_3[Co(NO_2)_6]$, $Co^{+3}: 3d^6 4s^0$

$Co^{3+}: d^2sp^3$ Hybridisation

Number of unpaired e^- = 0

Magnetic moment $= \sqrt{n(n+2)} = 0$ B.M.

85. (3)

Sol.: $A(g) \rightarrow 2B(g) + C(g)$

$P_{23} = P_0 + 2x = 200$

$P_\infty = 3P_0 = 300$

$P_0 = 100$

$$K = \frac{1}{t}\ln\frac{P_\infty - P_0}{P_\infty - P_t}$$

$$K = \frac{2.3}{23}\log\frac{300-100}{300-200}$$

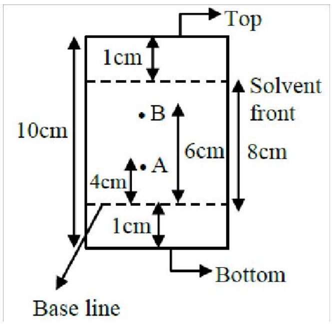
$$= \frac{2.3 \times 0.301}{23} = 0.0301$$

$= 3.01 \times 10^{-2}\ sec^{-1}$

86. (15)

Sol.:

$$R_f = \frac{\text{Distance moved by substance from base line}}{\text{Distance moved by solvent from base line}}$$



$(R_f)_A = \frac{4}{8}$ $\qquad$ $(R_f)_B = \frac{6}{8}$

$$\frac{(R_f)_B}{(R_f)_A} = \frac{6}{8} \times \frac{8}{4}$$

$(R_f)_B = 1.5\ (R_f)_A$

$x = 15$

87. (58)

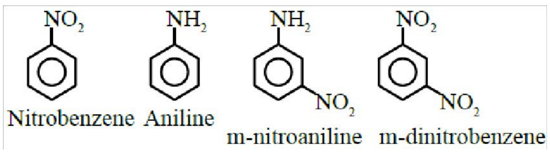
Sol.: $m\Delta V.\Delta x = \frac{h}{4\pi}$

$$\Delta V = \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{-15} \times 4 \times 3.14}$$

$= 57.97 \times 10^{+9}$ m/sec

88. (4)

Sol.: Compounds which can not undergo Friedel Crafts reaction are



89. (6)

Sol.: $O_2(16e) : (\sigma_{1s})^2(\sigma^*_{1s})^2(\sigma_{2s})^2(\sigma^*_{2s})^2$

$$(\sigma_{2p})^2\left[(\pi_{2p})^2 = (\pi_{2p})^2\right],\left[(\pi^*_{2p})^1 = (\pi^*_{2p})^1\right]$$

Number of e^- present in (π^*) of O_2 = 2

Number of e^- present in (π^*) of O_2^+ = 1

Number of e^- present in (π^*) of O_2^- = 3

So total e^- in (π^*) = 2 + 1 + 3 = 6

90. (400)

Sol.: At equilibrium $\Delta G_{PT} = 0$

$\Delta H_{vap} = T\Delta S_{vap}$

$30 \times 1000 = T \times 75$

$T = 400$ K

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