MATHEMATICS

1. B

Sol. $f(0^-) = \lim_{x\to 0^-} = \dfrac{2\sin^2 x}{x^2} = 2 = \alpha$

$f(0^+) = \lim_{x\to 0^+} \beta \times \sqrt{2}\dfrac{\sin\frac{x}{2}}{2\frac{x}{2}} = \dfrac{\beta}{\sqrt{2}} = 2$

$\Rightarrow \beta = 2\sqrt{2}$

$\alpha^2 + \beta^2 = 4 + 8 = 12$

2. B

Sol. A B C

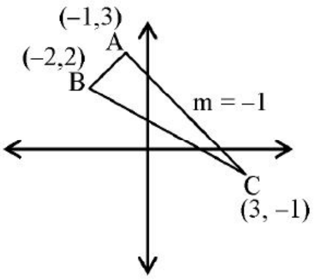
7R, 5B 5R, 7B 6R, 6B

$P(B) = \dfrac{1}{3}\cdot\dfrac{5}{12} + \dfrac{1}{3}\cdot\dfrac{7}{12} + \dfrac{1}{3}\cdot\dfrac{6}{12}$

required probability $= \dfrac{\frac{1}{3}\cdot\frac{5}{12}}{\frac{1}{3}\cdot\left[\frac{5}{12}+\frac{7}{12}+\frac{6}{12}\right]} = \dfrac{5}{18}$

3. C

Sol.



equation of AC $\rightarrow$ $x + y = 2$

equation of line parallel to AC $x + y = d$

$\left|\dfrac{d-2}{\sqrt{2}}\right| = 1$

equation of new required line

$\boxed{x + y = 2 - \sqrt{2}}$

4. C

Sol. $\int dy = \int \dfrac{(2x^2+2x+3)}{x^4+2x^3+3x^2+2x+2}dx$

$y = \int \dfrac{(2x^2+2x+3)}{(x^2+1)(x^2+2x+2)}dx$

$y = \int \dfrac{dx}{x^2+2x+2} + \int \dfrac{dx}{x^2+1}$

$y = \tan^{-1}(x+1) + \tan^{-1}x + C$

$y(-1) = \dfrac{-\pi}{4}$

$\dfrac{-\pi}{4} = 0 - \dfrac{\pi}{4} + C \Rightarrow C = 0$

$\Rightarrow y = \tan^{-1}(x+1) + \tan^{-1}x$

$y(0) = \tan^{-1}1 = \dfrac{\pi}{4}$

5. D

Sol. $y = \dfrac{2x^2-3x+8}{2x^2+3x+8}$

$x^2(2y-2) + x(3y+3) + 8y - 8 = 0$

use $D \geq 0$

$(3y+3)^2 - 4(2y-2)(8y-8) \geq 0$

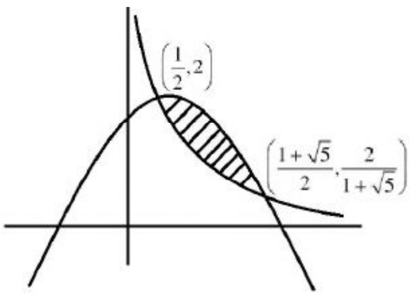
$(11y-5)(5y-11) \leq 0$

$\Rightarrow y \in \left[\dfrac{5}{11}, \dfrac{11}{5}\right]$

$y = 1$ is also included

6. B

Sol.



$$A=\int_{\frac{1}{2}}^{\frac{1+\sqrt{5}}{2}}\left(1+3x-2x^2-\frac{1}{x}\right)dx$$

$$A=\left[x+\frac{3x^2}{2}-\frac{2x^3}{3}-\ell nx\right]_{\frac{1}{2}}^{\frac{1+\sqrt{5}}{2}}$$

$$A=\frac{1+\sqrt{5}}{2}+\frac{3}{2}\left(\frac{1+\sqrt{5}}{2}\right)^2-\frac{2}{3}\left(\frac{1+\sqrt{5}}{2}\right)^3-\ell n\left(\frac{1+\sqrt{5}}{2}\right)$$

$$-\frac{1}{2}-\frac{3}{2}\left(\frac{1}{4}\right)+\frac{2}{3}\left(\frac{1}{8}\right)+\ell n\left(\frac{1}{2}\right)$$

$$A=\frac{1}{2}+\frac{\sqrt{5}}{2}+\frac{3}{8}+\frac{3}{4}\sqrt{5}+\frac{15}{8}-\frac{4}{3}-\frac{2}{3}\sqrt{5}$$

$$-\frac{1}{2}-\frac{3}{8}+\frac{1}{12}-\ell n(1+\sqrt{5})$$

$$=\sqrt{5}\left(\frac{1}{2}+\frac{3}{4}-\frac{2}{3}\right)+\frac{15}{8}-\frac{4}{3}+\frac{1}{12}-\ell n(1+\sqrt{5})$$

$$=\frac{14}{24}\sqrt{5}+\frac{15}{24}-\ell n(1+\sqrt{5})$$

7. C

Sol. $\begin{vmatrix}1 & \sqrt{2}\sin\alpha & \sqrt{2}\cos\alpha \\ 1 & \sin\alpha & -\cos\alpha \\ 1 & \cos\alpha & \sin\alpha\end{vmatrix}=0$

$$\Rightarrow 1-\sqrt{2}\sin\alpha(\sin\alpha+\cos\alpha)+\sqrt{2}\cos\alpha(\cos\alpha-\sin\alpha)=0$$

$$\Rightarrow 1+\sqrt{2}\cos 2\alpha-\sqrt{2}\sin 2\alpha=0$$

$$\cos 2\alpha-\sin 2\alpha=-\frac{1}{\sqrt{2}}$$

$$\cos\left(2\alpha+\frac{\pi}{4}\right)=-\frac{1}{2}$$

$$2\alpha+\frac{\pi}{4}=2n\pi\pm\frac{2\pi}{3}$$

$$\alpha+\frac{\pi}{8}=n\pi\pm\frac{\pi}{3}$$

$n=0,$

$$x=\frac{\pi}{3}-\frac{\pi}{8}=\frac{5\pi}{24}$$

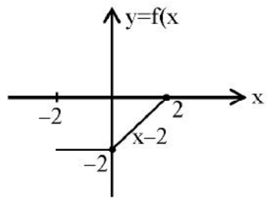
8. B

Sol. ${}^{18}C_3-{}^{5}C_3-{}^{6}C_3-{}^{7}C_3$

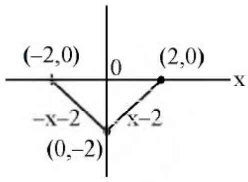
$=751$

9. A

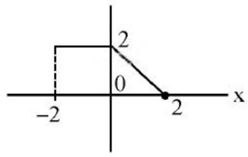
Sol.



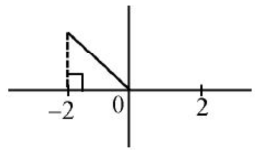
f(|x|) →



|f(x)|



$$h(x)=\begin{cases}x-2+2-x=0, & 0\le x\le 2\\ -x-2+2=-x, & -2\le x<0\end{cases}$$



$$\Rightarrow\int_0^2 h(x)dx=0 \text{ and } \int_{-2}^{0}h(x)dx=2$$

10. A

Sol. $T_{r+1}={}^{15}C_r\left(5^{\frac{1}{3}}\right)^r\left(2^{\frac{1}{5}}\right)^{15-r}$

$$={}^{15}C_r 5^{\frac{r}{3}}\cdot 2^{\frac{15-r}{5}}$$

$R=3\lambda,\ 15\mu$

$\Rightarrow r=0,\ 15$

2 rational terms

$\Rightarrow\ {}^{15}C_0\ 2^3+{}^{15}C_{15}\ (5)^5$

$=8+3125=3133$

11. D

Sol. $\vec{C}=C_1\hat{i}+C_2\hat{j}+C_3\hat{k}$

$C_1^2+C_2^2+C_3^2=1$

$\vec{C}\cdot(2\hat{i}+2\hat{j}-2\hat{k})=|C|\sqrt{9}\cos 60^{o}$

$$2C_1 + 2C_2 - C_3 = \frac{3}{2}$$

$C_1 - C_3 = 1$

$$C_1 + 2C_2 = \frac{1}{2}$$

$$C_1 = \frac{\sqrt{2}}{3} + \frac{1}{2}$$

$$C_2 = \frac{-1}{3\sqrt{2}}$$

$$C_3 = \frac{\sqrt{2}}{3} - \frac{1}{2}$$

12. D

Sol. $p^2 = 2q$

$2 = a + 6d$ ...(i)

$p = a + 7d$ ...(ii)

$q = a + 12d$ ...(iii)

$p - 2 = d$ ((ii) – (i))

$q - p = 5d$ ((iii) – (ii))

$q - p = 5(p - 2)$

$q = 6p - 10$

$p^2 = 2(6p - 10)$

$p^2 - 12p + 20 = 0$

$p = 10, 2$

$p = 10$; $q = 50$

$d = 8$

$a = -46$

2, 10, 50, 250, 1250

$ar^4 = a + (n - 1)d$

$1250 = -46 + (n - 1)8$

$n = 163$

13. A

Sol. $\frac{\Sigma x_i}{6} = 2$ and $\frac{\Sigma x_i^2}{N} - \mu^2 = 23$

$\alpha + \beta = 10$

$\alpha^2 + \beta^2 = 52$

solving we get $\alpha = 4,\ \beta = 6$

$$\frac{\Sigma |x_i - \bar{x}|}{6} = \frac{5+2+5+8+2+4}{6} = \frac{13}{3}$$

14. D

Sol. $\text{Sum} = 8 = -\frac{b}{a}$

$\text{Product} = 12 = \frac{1}{a} \quad \Rightarrow a = \frac{1}{12}$

$b = -\frac{2}{3}$

$$2a + b = \frac{2}{12} - \frac{2}{3} = -\frac{1}{2}$$

$$6a + b = \frac{6}{12} - \frac{2}{3} = -\frac{1}{6}$$

sum = –8

$x^2 + 8x + 12 = 0$

15. B

Sol. $z = x + iy$

$\bar{z} = x - iy$

$\bar{z}^2 = x^2 - y^2 - 2ixy$

$\Rightarrow x^2 - y^2 - 2ixy + \sqrt{x^2 + y^2} = 0$

$\Rightarrow x = 0$ or $y = 0$

$-y^2 + |y| = 0 \qquad x^2 + |x| = 0$

$|y| = |y|^2 \qquad \Rightarrow x = 0$

$y = 0, \ \pm 1$

$\Rightarrow i, \ -i \qquad \Rightarrow \alpha = i - i = 0$

are roots $\qquad \beta = i(-i) = 1$

$4(0 + 1) = 4$

16. A

Sol. PQ line

$$\frac{x-1}{4} = \frac{y+2}{-2} = \frac{z-3}{4}$$

pt $(4t + 1, -2t - 2, 4t + 3)$

$\text{distance}^2 = 16t^2 + 4t^2 + 16t^2 = 81$

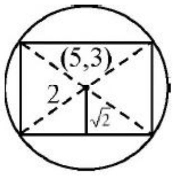
$$t = \pm\frac{3}{2}$$

pt(7, –5, 9)

$\alpha^2 + \beta^2 + \gamma^2 = 155$

option (A)

17. B

Sol. 

$y = x + c$ & $x + y + d = 0$

$\left|\dfrac{5-3+c}{\sqrt{2}}\right| = \sqrt{2}$ $\left|\dfrac{8+d}{\sqrt{2}}\right| = \sqrt{2}$

$|c + 2| = 2$ $8 + d = \pm 2$

$c = 0, -4$ $d = -10, -6$

pts (5, 5), (3, 3), (7, 3), (5, 1)

$\sum (x_i^2 + y_1^2) = 25 + 25 + 9 + 9 + 49 + 9 + 25 + 1$

$= 152$

Option (D)

18. A

Sol. $-1 \le \dfrac{3x - 22}{2x - 19} \le 1$ $\dfrac{3x^2 - 8x + 5}{x^2 - 3x - 10} > 0$

$x \in \left(5, \dfrac{41}{5}\right]$

$3\alpha + 10\beta = 97$

Option (A)

19. A

Sol. $f(x) = 2$

when $x = 0$

$\because g'(f(x))f'(x) = 1$

$g'(2) = \dfrac{1}{f'(0)}$

$\because f'(x) = 5x^4 + \dfrac{2}{4}e^{x/4}$

$g'(2) = 2$

Ans = 16 Option (A)

20. C

Sol. $|adj(A - 2A^T)\,(2A - A^T)| = 28$

$|(A - 2A^T)\,(2A - A^T)| = 24$

$|A - 2A^T|\,|2A - A^T| = \pm 16$

$(A - 2A^T)^T = A^T - 2A$

$|A - 2A^T| = |A^T - 2A|$

$\Rightarrow |A - 2A^T|^2 = 16$

$|A - 2A^T| = \pm 4$

$$\begin{bmatrix} 1 & 2 & \alpha \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 2 & 0 \\ 4 & 0 & 2 \\ 2\alpha & 2 & 4 \end{bmatrix}$$

$$\begin{vmatrix} -1 & 0 & \alpha \\ -3 & 0 & -1 \\ -2\alpha & -1 & -2 \end{vmatrix}$$

$1 + 3\alpha = 4$

$3\alpha = 3$

$\alpha = 1$

$$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{vmatrix} = -1 - 3 = -4$$

$|A|^2 = 16$

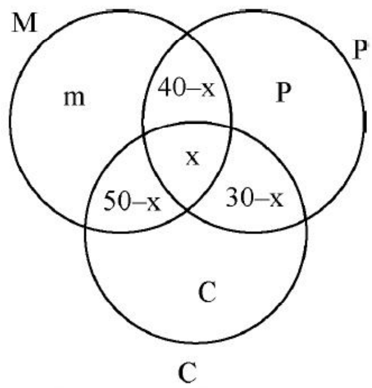
21. 100

Sol. $$\lim_{x \to 1} \frac{\frac{1}{3}(5x+1)^{-2/3}5 - \frac{1}{3}(x+5)^{-2/3}}{\frac{1}{2}(2x+3)^{-1/2} \cdot 2 - \frac{1}{2}(x+4)^{-1/2}}$$

$= \dfrac{8}{3}\dfrac{\sqrt{5}}{6^{2/3}}$ $m = 8$, $n = 3$

$8m = 12n = 100$

22. 45

Sol. 

$125 \le m + 90 - x \le 130$

$85 \le P + 70 - x \le 95$

$75 \le c + 80 - x \le 90$

$m + P + C + 120 - 2x = 210$

$\Rightarrow 15 \le x \le 45$ & $30 - x \ge 0$

$\Rightarrow 15 \le x \le 30$

$30 + 15 = 45$

23. 7

Sol. $ye^{-x} = \int (e^{-x} + 4e^{-x}\sin x)dx$

$ye^{-x} = -e\text{-}x - 2(e^{-x}\sin x\ e^{-x}\cos x) + C$

$y = -1 - 2(\sin x + \cos x) + ce^{x}$

$\because y(\pi) = 1 \Rightarrow c = 0$

$y(\pi/2) = -1 - 2 = -3$

Ans = 10 – 3 = 7

24. 48

Sol. $\frac{38}{3\sqrt{5}}\hat{k} = \frac{(5\hat{i} + 5\hat{j} - 9\hat{k})}{\sqrt{5}} \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1 & -3 & 2 \end{vmatrix}$

$\frac{38}{3\sqrt{5}}\hat{k} = \frac{19}{\sqrt{5}}$

$k = \frac{19}{\sqrt{5}}$

$k = \frac{3}{2}$

$\int_0^{3/2} [x^2] = \int_0^1 0 + \int_1^{\sqrt{2}} 1 + \int_{\sqrt{2}}^{3/2} 2$

$= \sqrt{2} - 1 + 2\left(\frac{3}{2} - \sqrt{2}\right)$

$= 2 - \sqrt{2}$

$\alpha = 2$

$\Rightarrow 6\alpha^3 = 48$

25. 25

Sol. Given $|A| = 2$

trace A = –3

and $A^2 + xA + yI = 0$

$\Rightarrow x = 3, y = 2$

so, information is incomplete to determine eccentricity of hyperbola (e) and length of latus rectum of hyperbola (ℓ)

26. 8

Sol. $f(x) = 1 + \frac{(1+x)}{1!} + \frac{(1+x)^2}{2!} + \frac{(1+x)^3}{3!} + ...$

$\frac{e^{(1+x)}}{1+x} = \frac{1}{x+1} + 1 + \frac{(1+x)}{2!} + \frac{(1+x)^2}{3!} + \frac{(1+x)^2}{4!}$

coeff. x^2 in RHS : $1 + \frac{{}^2C_2}{3} + \frac{{}^3C_2}{4} + ... = a$

coeff. x^2 in L.H.S.

$e\left(1 + x + \frac{x^2}{2!}\right).....\left(1 - x + \frac{x^2}{2!}......\right)$

is $e - e + \frac{e}{2!} = a$

$b = 1 + \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + ... = e^2$

$\frac{2b}{a^2} = 8$

27. 27

Sol. Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$A\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\Rightarrow a_1 + a_2 + a_3$ ....(1)

$\Rightarrow b_1 + b_2 + b_3$ ....(2)

$\Rightarrow c_1 + ca_2 + c_3$ ....(3)

Now,

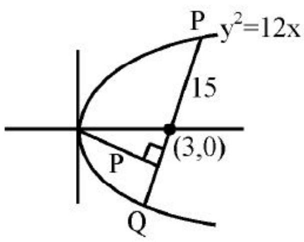
$|A| = (a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2)$

$- (a_3b_2c_1 + a_2bc_3 + a_1b_3c_2)$

$\therefore$ From above in formation, clearly $|A|_{max} = 27$, when $a_1 = 3, b_2 = 3, c_3 = 3$

28. 72

Sol.



length of focal chord $= 4a\operatorname{cosec}^2\theta = 15$

$12\operatorname{cosec}^2\theta = 15$

$\sin^2\theta = \frac{4}{5}$

$\tan^2\theta = 4$

$\tan\theta = 2$

equation $\frac{y-0}{x-3} = 2$

$y = 2x - 6$

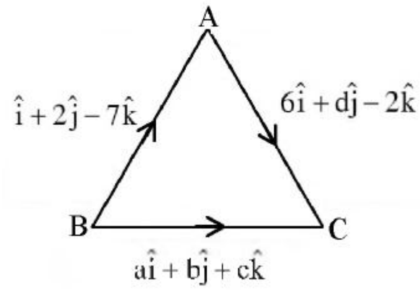
$2x - y - 6 = 0$

$P = \frac{6}{\sqrt{5}}$

$10p^2 = 10 \cdot \frac{36}{5} = 72$

29. 54

Sol.



$\text{Area} = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -7 \\ 6 & d & -2 \end{vmatrix} = 15\sqrt{2}$

$(-47d)\hat{i} - \hat{j}(-2+42) + \hat{k}(d-12)$

$(7d-4)^2 + (40)^2 + (d-12)^2 = 1800$

$50d^2 - 80d - 40 = 0$

$5d^2 - 8d - 4 = 0$

$5d^2 - 10d - 2d - 4$

$5d(d-2) + 2(d-2) = 0$

$d = 2 \text{ or } d = -\frac{2}{5}$

$\because d > 0,\ d = 2$

$(a+1)\hat{i} + (b+2)\hat{j} + (c-7)\hat{k} = 6\hat{i} + 2\hat{j} - 2\hat{k}$

$a + 1 = 6\ \&\ b + 2 = 2,\ c - 7 = -2$

$a = 5 \quad b = 0 \quad c = 5$

$|AB| = \sqrt{1+4+49} = \sqrt{54}$

$|BC| = \sqrt{25+25} = \sqrt{50}$

$|AC| = \sqrt{86+4+4} = \sqrt{44}$

Ans. 54

30. 8

Sol. $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1+\frac{1}{2}\sin 2x}dx = \int_0^{\frac{\pi}{4}} \frac{-\cos 2x}{2+\sin 2x}dx$

$\int \frac{1}{2+\sin 2x} - \int \frac{\cos 2x}{2+\sin 2x}$

$(I_1) \quad - \quad (I_2)$

$(I_1) = \int \frac{dx}{2+\frac{2\tan x}{1+\tan^2 x}}$

$\int_0^{\frac{\pi}{4}} \frac{\sec^2 x dx}{2\tan^2 x + 2\tan x + 2}$

$\tan x = t$

$\frac{1}{2}\int_0^1 \frac{dt}{\left(t+\frac{1}{2}\right)^2 + \frac{3}{4}} = \frac{\pi}{6\sqrt{3}}$

$I_2 = \int_0^{\pi/4} \frac{\cos 2x}{2+\sin 2x}dx = \frac{1}{2}\left(\ell n\frac{3}{2}\right)$

$I_1 - I_2 = \frac{1}{\sqrt{3}}\frac{\pi}{6} + \frac{1}{2}\ell n\frac{2}{3}$

$\Rightarrow a = 2, b = 6$

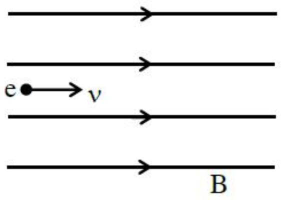
Ans. 6

PHYSICS

Section - A (Single Correct Answer)

31. B

Sol.



Since $\vec{v} \parallel \vec{B}$ so force on electron due to magnetic field is zero. So it will move along axis with uniform velocity.

32. D

Sol. $\vec{E} = \hat{i}40\cos\omega\left(t - \frac{z}{c}\right)$

$\vec{E}$ is along +x direction

$\vec{v}$ is along +z direction

So direction of $\vec{B}$ will be along +y and magnitude of B will be E/c

So answer is $\frac{40}{c}\cos\omega\left(t - \frac{z}{c}\right)\hat{j}$

33. D

Sol. Balancing mass number and atomic number

$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{144}_{56}Ba + ^{89}_{36}Kr + 3^{1}_{0}n$

34. D

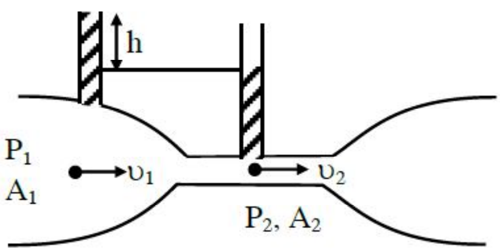
Sol. $f^{-1} = v^{-1} - u^{-1}$

$-f^{-2}\,df = -v^{-2}dv - u^{-2}du$

$$\frac{df}{f^2} = \frac{dv}{v^2} + \frac{du}{u^2}$$

35. D

Sol.



Applying Bernoulli's equation

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

[h_1 & h_2 are height of point from any reference level]

Given $V_1 = V_2 = 0$ (for statement-1)

$\therefore$ $P_1 - P_2 = \rho g(h_2 - h_2)$

For statement-2

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$P_1 - P_2 = \rho g h$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

$$\rho g h = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

$2gh = v_2^2 - v_1^2$

Hence answer (D)

36. B

Sol. Given $R_0 = 8\Omega$, $R_{100} = 10\Omega$

$\therefore$ $R_{100} = R_0(1 + \alpha\Delta T)$

Also, $R_{400} = R_0(1 + \alpha\Delta T^1)$

$\therefore$ $10 = 8(1 + \alpha \times 100)$

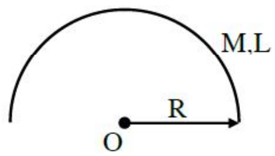
$\Rightarrow$ $100\alpha = 1/4$

$\therefore$ $R_{400} = 8(1 + 400\alpha) = 8(1 + 1) = 16\Omega$

Hence option (B)

37. D

Sol.



We have $R = L/\pi$

$$g_0 = \frac{2G\frac{M}{L}}{R} = \frac{2GM\pi}{L^2}$$

$$\therefore F_m = mg_0 = \frac{2GM\pi m}{L^2}$$

Hence option (D)

38. C

Sol. We know that per °C change is equivalent to 1.8° change in °F.

$\therefore$ 40° change on celcius scale will corresponds to 72° change on Fahrenheit scale.

Hence option (C)

39. A

Sol. We know that $P_{eq} = \Sigma P_i$

$\therefore$ given all lenses are identical

$\therefore$ $5P = 25D$

$\therefore$ $P = 5D$

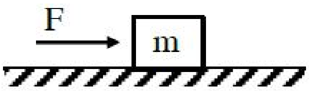
$$\therefore \frac{1}{f} = 5 \Rightarrow f = \frac{1}{5}m = 20cm$$

Hence option (A)

40. C

Sol. Given lights are of same wavelength.
Hence stopping potential will remain same.
Since $I_2 > I_1$, hence saturation current corresponding to I_2 will be greater than that corresponding to I_1.
Hence option (C)

41. B

Sol. 

$$F = ma \Rightarrow a = \frac{F}{m} = \frac{kt}{m}$$

a vs t will be straight line passing through origin.
Since option (B).

42. D

Sol. Velocity just before collision $= \sqrt{2gh}$

Velocity just after collision $= \sqrt{2g\left(\frac{h}{2}\right)}$

$$\therefore \Delta KE = \frac{1}{2}m(2gh) - \frac{1}{2}mgh$$

$$= \frac{1}{2}mgh$$

$\therefore$ % loss in energy

$$= \frac{\Delta KE}{KE_i} \times 100 = \frac{\frac{1}{2}mgh}{\frac{1}{2}mg2h} \times 100 = 50\%$$

Hence option (D)

43. C

Sol. Comparing the given equation with standard equation of standing

$$\left[\frac{n}{\lambda}\right] = [\omega] = T^{-1}$$

$[nt] = [\lambda] = L$

$[n] = [\lambda\omega] = LT^{-1}$

$[x] = [\lambda] = L$

Hence option (C)

44. B

Sol. Given, $102.5 = u + \frac{a}{2}(2n-1)$ &

$$115 = u + \frac{a}{2}(2n+3)$$

$$\Rightarrow 102.5 = u + an - \frac{a}{n} \ \&$$

$$115 = u + an + \frac{3a}{2}$$

$12.5 = 2a \Rightarrow a = 6.25 \text{ m/s}^2$

Hence option (B)

45. D

Sol. Let potential gradient be λ.

$\therefore \quad i \times 10 = \lambda \times 500 = \varepsilon - ir_s$

$\Rightarrow \quad 500\lambda = \varepsilon - 50\lambda r_s$

Also,

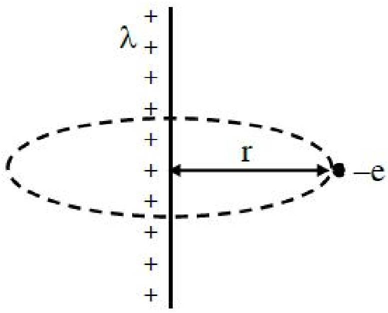
$i' \times 1 = \lambda \times 400 = \varepsilon - i'r_s$

$\Rightarrow \quad 400\lambda = \varepsilon - 400\lambda r_s$

$\therefore \quad 100\lambda = 350\lambda r_s \Rightarrow rs = 10/35 \approx 0.3\Omega$

Hence option (D)

46. B

Sol. 

Electric field E at a distance r due to infinite long wire is $E = \frac{2k\lambda}{r}$

Force of electron $\Rightarrow F = eE$

$$F = e\left(\frac{2k\lambda}{r}\right)$$

$$F = \frac{2k\lambda e}{r}$$

This force will provide required centripetal force

$$F = \frac{mv^2}{r} = \frac{2k\lambda e}{r}$$

$$v = \sqrt{\frac{2k\lambda e}{m}}$$

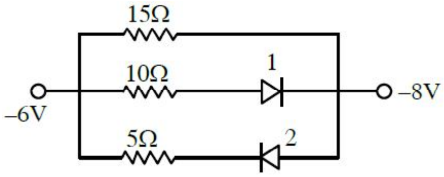
$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{2k\lambda e}{m}\right)$$

$= k\lambda e$

This is constant so option (B) is correct.

47. C

Sol.

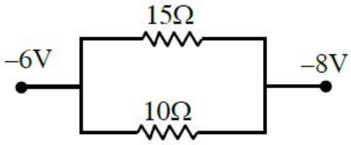


Diode 2 is in reverse bias

So current will not flow in branch of 2nd diode, So we can assume it to be broken wire.

Diode 1 is in forward bias

So it will behave like conducting wire. So new circuit will be



Correct answer (C)

48. B

Sol. For ideal gas

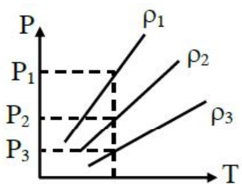
PV = nRT

$$PV = \frac{m}{M}RT$$

$$P = \left(\frac{M}{V}\right)\frac{RT}{M}$$

$$P = \frac{\rho RT}{M}$$

(Where m is mass of gas and M is molecular mass of gas)



for same temperature $P_1 > P_2 > P_3$

So $\rho_1 > \rho_2 > \rho_3$

So correct answer is (B)

49. D

Sol. x = 2 + 4t

$$\frac{dx}{dt} = v_x = 4$$

$$\frac{dv_x}{dt} = a_x = 0$$

$y = 3t + 8t^2$

$$\frac{dy}{dt} = v_y = 3 + 16t$$

$$\frac{dv_y}{dt} = a_y = 16$$

the motion will be uniformly accelerated motion.

For path

x = 2 + 4t

$$\frac{(x-2)}{4} = t$$

Put this value of t is equation of y

$$y = 3\left(\frac{x-2}{4}\right) + 8\left(\frac{x-2}{4}\right)^2$$

this is a quadratic equation so path will be parabola.

Correct answer (D)

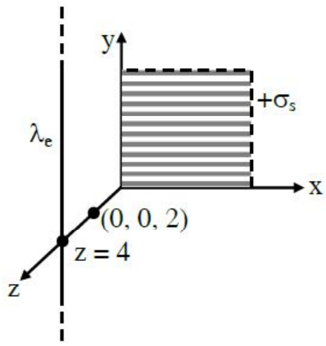
50. D

Sol. This is possible when phase difference is π/2 between current and voltage so correct answer will be (D)

Section - B (Numerical Value)

51. 16

Sol.



$$\frac{E_S}{E_l} = \frac{\sigma}{2\epsilon_0} \times \frac{2\pi\epsilon_0 r}{\lambda} = \frac{\pi \times \sigma r}{\lambda}$$

$$= \frac{\pi \times 2\lambda \times 2}{\lambda} = \frac{4\pi}{1}$$

∴ n = 16

52. 7

Sol. $$\Delta E = 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.9eV$$

$$\Delta E = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{\Delta E}$$

$P_i = P_f$

$0 = -mv + \frac{h}{\lambda'}$

$\Rightarrow v = \frac{h}{m\lambda'}$

$\Delta E = \frac{1}{2}mv^2 + \frac{hc}{\lambda'}$

$= \frac{1}{2}m\left(\frac{h}{m\lambda'}\right)^2 + \frac{hc}{\lambda'}$

Now $\Delta E = \frac{h^2}{2m\lambda'^2} + \frac{hc}{\lambda'}$

$\lambda'^2\Delta E - hc\lambda' - \frac{h^2}{2m} = 0$

$$\lambda' = \frac{hc \pm \sqrt{h^2c^2 + \frac{4\Delta E h^2}{2m}}}{2\Delta E}$$

$$\lambda' = \frac{hc \pm hc\sqrt{1 + \frac{2\Delta E}{mc^2}}}{2\Delta E}$$

$$\frac{\lambda'}{\lambda} = \frac{1 + \left(1 + \frac{2\Delta E}{mc^2}\right)^{\frac{1}{2}}}{2} = \frac{1 + 1 + \frac{\Delta E}{mc^2}}{2}$$

$\frac{\lambda'}{\lambda} = 1 + \frac{\Delta E}{2mc^2}$

$\frac{\lambda' - \lambda}{\lambda} = \frac{\Delta E}{2mc^2} = \frac{1.9 \times 1.6 \times 10^{-19}}{2 \times 1.67 \times 10^{-27} \times 9 \times 10^{16}} = 10^{-9}$

$\therefore$ % change $\approx 10^{-7}$

Correct answer 7

53. 50

Sol. Force on segment parallel to x-axis will cancel each other. Hence Fnet will be due to portion parallel to y-axis.

$F = 0.5 \times 0.5 \times 6 \times 0.2 - 0.5 \times 0.5 \times 0.2 \times 5$

$= 0.5 \times 0.5 \times 0.2$

$= 0.25 \times 0.2$

$= 50 \times 10^{-3}$ N

$= 50$ mN

54. 8

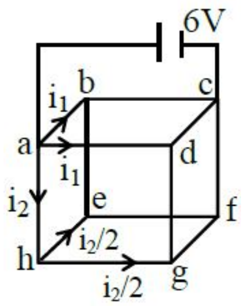
Sol. $I_{rms} = \sqrt{\frac{\int i^2 dt}{\int dt}}$

$I_{rms} = \sqrt{(6)^2 + \frac{(\sqrt{56})^2}{2}}$

$= \sqrt{36 + 28} = \sqrt{64} = 8A$

55. 1

Sol.



From symmetry, current through e-b & g-d = 0

$\therefore R_{eq} = \frac{3}{4} \times R = \frac{3}{2}\Omega$

$\therefore$ Current through battery $= \frac{6 \times 2}{3} = 4A$

$i_2 = \frac{4}{8} \times 2 = 1A$

$\therefore$ ΔV across e-f $= \frac{i_2}{2} \times R = \frac{1}{2} \times 2 = 1V$

56. 7

Sol. $\omega = \Delta U = S\Delta A$

$36960 \text{ erg} = \frac{40\text{dyne}}{\text{cm}} 8\pi\left[(r)^2 - \left(\frac{7}{2}\right)^2\right]\text{cm}^2$

$r = 7$ cm

57. 9

Sol. $n_2\lambda_2 = n_1\lambda_1$

$\frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2} = \frac{450}{650} = \frac{9}{13}$

$n_2 = 9$

58. 5

Sol. $3 = K(a - l)$

$2 = K(b - l)$

$T = K(3a - 2b - l)$

$T = K(3(a - l) - 2(b - l))$

$= K\left[3\left(\frac{3}{K}\right) - 2\left(\frac{2}{K}\right)\right]$

$= 9 - 4 = 5$ N

59. 6

Sol. $\left|\vec{F}_1\right| = F$

$\left|\vec{F}_R\right| = \left|\vec{F}_2\right| = 3F$

$F_R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$

$9F^2 = F^2 + 9F^2 + 6F^2\cos\theta$

$\cos\theta = -\frac{1}{6}$

$\theta = \cos^{-1}\left(\frac{1}{-6}\right)$

n = – 6

|n| = 6

60. 7

Sol. Gain in P.E. = Loss in K.E.

$mgh = \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$

$h \propto 1 + \frac{K^2}{R^2}$

$\frac{h_1}{h_2} = \frac{1+\frac{2}{5}}{1+1} = \frac{7}{5\times 2} = \frac{7}{10}$

n = 7

CHEMISTRY

Section - A (Single Correct Answer)

61. (C)

Sol. $2e^- + 2H^+(aq) \rightarrow H_2(g)$

$E = E^O - \frac{0.059}{n}\log\frac{P_{H_2}}{[H^+]^2}$

$0 = 0 - \frac{0.059}{2}\log\frac{P_{H_2}}{(10^{-7})^2}$

$\log\frac{P_{H_2}}{(10^{-7})^2} = 0$

$\frac{P_{H_2}}{10^{-14}} = 1$

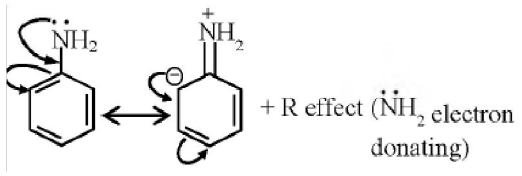
$P_{H_2} = 10^{-14}$ bar

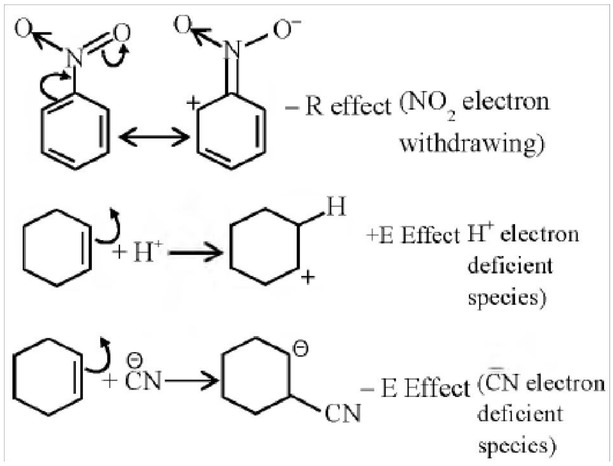
62. (A)

Sol. According to spectrochemical series ligand field strength is $CO > H_2O > F^- > S^{2-}$

63. (A)

Sol.:





64. (B)

Sol. Strong acid have weak conjugate base Acidic strength :

H–Cl > CH_3COOH > H_2O > R–OH

Conjugate base strength :

$Cl^- < CH_3COO^- < \bar{O}H < RO^-$

65. (B)

Sol. $NH_4OH \rightleftharpoons NH_4^+ + OH^-$

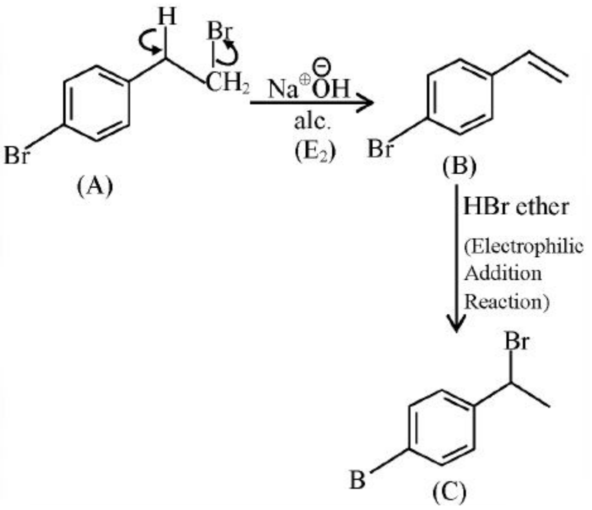
$NH_4Cl \rightarrow NH_4^+ + Cl$

Due to common ion effect of NH_4^+,

$[OH^-]$ decreases in such extent that only group-III cation can be precipitated , due to their very low K_{sp} in the range of 10^{-38}.

66. (C)

Sol.



A and C are position isomer.

67. (A)

Sol. Theory based

68. (A)

Sol. $[V(H_2O)_6]^{3+} \rightarrow d^2sp^3$

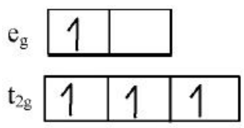
$_{23}V$:- $[Ar]3d^34s^2$

V^{+3} :- $[Ar]3d^2$, n = 2 (even number of unpaired e^-)

$[Cr(H_2O)_6]^{2+} \rightarrow sp^3d^2$

$_{24}Cr$:- $[Ar]3d^54s^1$

Cr^{+2} :- $[Ar]3d^4$, n = 4 (even number of unpaired e^-)



$[Fe(H_2O)_6]^{3+} \rightarrow sp^3d^2$

Fe^{3+} :- $[Ar]3d^54s^0$

n = 5 (odd number of unpaired e^-)

$[Ni(H_2O)_6]^{3+} \rightarrow sp^3d^2$

Ni :- $[Ar]3d^84s^2$

Ni^{+3} :- $[Ar]3d^7$, n = 3 (odd number of unpaired e^-)

$[Cu(H_2O)_6]^{2+} \rightarrow sp^3d^2$

Cu :- $[Ar]3d^94s^0$

n = 1 (odd number of unpaired e^-)

69. (C)

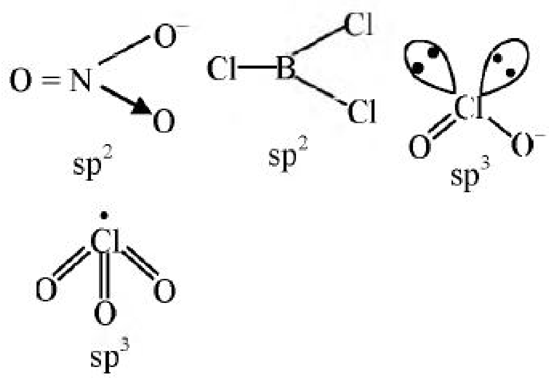
Sol. CH_4 & PF_5 , $\mu_{net} = 0$ (non polar)

μ_{NH_3} > μ_{NF_3}

Vector addition of bond moment and lone pair moment | Vector subtraction of bond moment & lone pair moment

70. (A)

Sol.

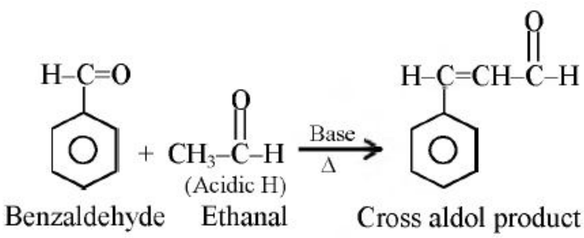


71. (D)

Sol. Hydrogen ion (H^+) shows positive electromeric effect.

72. (D)

Sol. Aldehyde and ketones having acidic α-hydrogen show aldol reaction

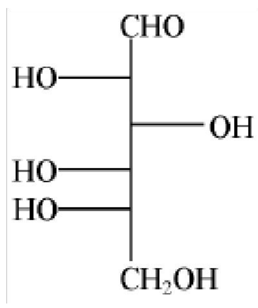


73. (D)

Sol. Hydrazine (NH_2–NH_2) have no carbon so does not show Lassaigne's test.

74. (A)

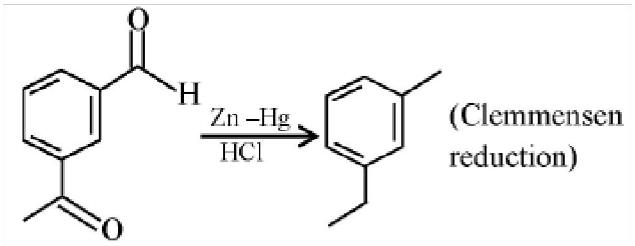
Sol. Structure of L-Glucose is



75. (B)

Sol. Co, Ti, Ni can show +2, +3 and +4 oxidation state, But 'Sc' only shows +3 stable oxidation state.

76. (D)



77. (D)

Sol. N,O, F can't extend their valencies upto their group number due to the non-availability of vacant 2d like orbital.

78. (B)

Sol. $M = \frac{n_{NaCl}}{V_{sol}(\text{in L})}$

$$M = \frac{\frac{5.85}{58.5}}{0.5} = 0.2M$$

79. (C)

Sol.: $CH_3 - CH_2 - CH_2 - Br \xrightarrow[80\,^\circ C]{X = \text{conc.alc.NaOH}}$

$CH_3 - CH = CH_2 \xrightarrow{Y = HBr/\text{Acetic acid}} CH_3 - CHBr - CH_3$

80. (B)

Sol.: Correct order of 1st IE

$$\begin{array}{ccccccccccccccc} Li & < & B & < & Be & < & C & < & O & < & N & < & F & < & Ne \\ & & \downarrow & & \downarrow & & & & \downarrow & & \downarrow & & \downarrow & & \\ & & E & < & C & & < & & A & < & B & < & D & & \end{array}$$

Section - B (Numerical Value Type)

81. (125)

Sol. $C_2H_4(g) + H_2(g) \rightarrow C_2H_6(g)$

$\Delta H = BE(C = C) + 4BE\ (C - H) + BE\ (H - H) - BE(C - C) - 6BE\ (C - H)$

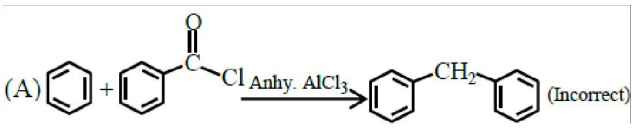
$\Delta H = BE(C = C) + BE(H - H) - BE(C - C) - 2BE(C - H)$

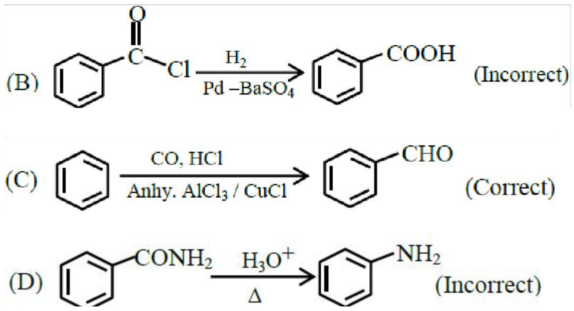
$= 615 + 435 - 347 - 2 \times 414$

$= -125$ kJ

82. (1)

Sol.:





83. (45)

Sol.

$$\underset{\text{Mol.wt.45g}}{CH_3CH_2NH_2} \xrightarrow{NaNO_2 + HCl} \xrightarrow{H_2O} CH_3CH_2{-}OH + \underset{14g}{N_2}$$

given : N_2 evolved is 2.24 L i.e. 0.1 mole.

i.e. $CH_3CH_2NH_2$ (ethyl amine) will be 4.5 g (= 0.1 mole)

Hence the answer = 45×10^{-1} g

84. (8)

Sol: $2\pi r_n = n\lambda_d$

$$2\pi a_0 \frac{n^2}{Z} = n\lambda_d\ ,\ 2\pi a_0 \frac{4^2}{1} = 4\lambda_d$$

$\lambda_d = 8\pi a_0$

85. (50)

Sol. eq.($KMnO_4$) = eq.($H_2C_2O_4$)

$M \times 2 \times 5 = 2 \times 20 \times 2$

$M = 8M$

86. (4)

Sol. $MnO_2 + KOH + O_2 \rightarrow \underset{(A)}{K_2MnO_4} + H_2O$

$K_2MnO_4 \xrightarrow{\text{Neutral/acidic solution}} KMnO_4 + MnO_2$

Mn^{+4} :- $[Ar]3d^3$

$n = 3, \mu\sqrt{3(3+2)} = 3.87\,B.M.$

Nearest integer is (4)

87. (100)

Sol. $K = \frac{K_1K_2}{K_3}$

$$Ae^{\frac{-E_a}{RT}} = \frac{A_1e^{\frac{-E_{a1}}{RT}} A_2e^{\frac{-E_{a2}}{RT}}}{A_3e^{\frac{-E_{a3}}{RT}}}$$

$$Ae^{\frac{-E_a}{RT}} = \frac{A_1A_2}{A_3}e^{\frac{-(E_{a_1}+E_{a_2}-E_{a_3})}{RT}}$$

$$E_a = E_{a_1} + E_{a_2} - E_{a_3}$$

$$400 = 300 + 200 - E_{a_3}$$

$$E_{a_3} = 100 \text{ kJ/mole}$$

88. (707)

Sol. $2 = 0.52 \times m$

$$m = \frac{2}{0.52}$$

According to question, solution is much diluted

$$\text{so} = \frac{\Delta P}{P^o} = \frac{n_{solute}}{n_{solvent}}$$

$$\frac{\Delta P}{P^o} = \frac{m}{1000} \times M_{solvent}$$

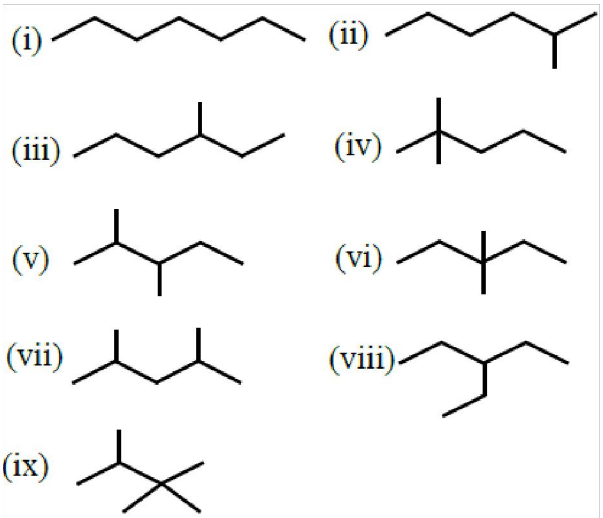
$$\Delta P = P^o \times \frac{m}{1000} \times M_{solvent}$$

$$= 760 \times \frac{\frac{2}{0.52}}{1000} \times 18 = 52.615$$

$P_5 = 760 - 52.615 = 707.385$ mm of Hg

89. (9)

Sol:



90. (2)

Sol. According to M.O.T.

$O_2 \rightarrow$ no. of unpaired electrons = 2

O_2^- $\rightarrow$ no. of unpaired electron = 1

NO $\rightarrow$ no. of unpaired electron = 1

$CN^- \rightarrow$ no. of unpaired electron = 0

$O_2^{2-} \rightarrow$ no. of unpaired electron = 0

☯ ☯ ☯