MATHEMATICS

Section - A (Single Correct Answer)

1. A

Sol. P(4W4B/2W2B) =

$$\frac{P(4W\ 4B) \times P(2W\ 2B\ /\ 4W\ 4B)}{P(2W\ 6B) \times P(2W\ 2B\ /\ 2W\ 6B) + P(3W\ 5B) \times P(2W\ 2B\ /\ 3W\ 5B) + \ldots\ldots\ldots\ldots + P(6W2B) \times P(2W2B\ /\ 6W2B)}$$

$$= \frac{\frac{1}{5}\times\frac{{}^4C_2\times{}^4C_2}{{}^8C_4}}{\frac{1}{5}\times\frac{{}^2C_2\times{}^6C_2}{{}^8C_4}+\frac{1}{5}\times\frac{{}^3C_2\times{}^5C_2}{{}^8C_4}+\ldots\ldots+\frac{1}{5}\times\frac{{}^6C_2\times{}^2C_2}{{}^8C_4}}$$

$$=\frac{2}{7}$$

2. C

Sol. $\int_0^{\frac{\pi}{4}}\frac{x\,dx}{\sin^4(2x)+\cos^4(2x)}$

Let 2x = t then $dx=\frac{1}{2}dt$

$$I=\frac{1}{4}\int_0^{\frac{\pi}{2}}\frac{t\,dt}{\sin^4 t+\cos^4 t}$$

$$I=\frac{1}{4}\int_0^{\frac{\pi}{2}}\frac{\left(\frac{\pi}{2}-t\right)dt}{\sin^4\left(\frac{\pi}{2}-t\right)+\cos^4\left(\frac{\pi}{2}-t\right)}$$

$$I=\frac{1}{4}\int_0^{\frac{\pi}{2}}\frac{\frac{\pi}{2}dt}{\sin^4 t+\cos^4 t}-I$$

$$2I=\frac{\pi}{8}\int_0^{\frac{\pi}{2}}\frac{dt}{\sin^4 t+\cos^4 t}$$

$$2I=\frac{\pi}{8}\int_0^{\frac{\pi}{2}}\frac{\sec^4 t\,dt}{\tan^4 t+1}$$

Let tan t = y then $\sec^2 t$ dt = dy

$$2I=\frac{\pi}{8}\int_0^{\infty}\frac{(1+y)^2dy}{1+y^4}$$

$$=\frac{\pi}{16}\int_0^{\infty}\frac{1+\frac{1}{y^2}}{y^2+\frac{1}{y^2}}dy$$

Put $y-\frac{1}{y}=p$

$$I=\frac{\pi}{16}\int_{-\infty}^{\infty}\frac{dp}{p^2+(\sqrt{2})^2}$$

$$=\frac{\pi}{16\sqrt{2}}\left[\tan^{-1}\left(\frac{p}{\sqrt{2}}\right)\right]_{-\infty}^{\infty}$$

$$I=\frac{\pi^2}{16\sqrt{2}}$$

3. B

Sol. $A=\begin{bmatrix}\sqrt{2} & 1\\ -1 & \sqrt{2}\end{bmatrix}\Rightarrow \det(A)=3$

$B=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\Rightarrow \det(B)=1$

Now $C=ABA^T\Rightarrow \det(C)=(\det(A))^2 \times \det(B)$

|C| = 9

Now $|X| = |A^TC^2A|$

$= |A^T|\ |C|^2\ |A|$

$= |A|^2\ |C|^2$

$= 9\times 81$

$= 729$

4. A

Sol. Finding tan (A + B) we get

$\Rightarrow \tan(A+B)=$

$$\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{\frac{1}{\sqrt{x(x^2+x+1)}}+\frac{\sqrt{x}}{\sqrt{x^2+x+1}}}{1-\frac{1}{x^2+x+1}}$$

$$\Rightarrow \tan(A+B)=\frac{(1+x)(\sqrt{x^2+x+1})}{(x2)+x(\sqrt{x})}$$

$$\frac{(1+x)(\sqrt{x^2+x+1})}{(x^2+x)(\sqrt{x})}$$

$$\tan(A+B)=\frac{\sqrt{x^2+x+1}}{x\sqrt{x}}=\tan C$$

$A + B = C$

5. C

Sol. Total ways to partition 5 into 4 parts are :

5, 0, 0, 0 $\Rightarrow$ 1 ways

4, 1, 0, 0 $\Rightarrow \frac{5!}{4!}=5$ ways

3, 2, 0, 0 $\Rightarrow \frac{5!}{3!2!}=10$ ways

2, 2, 0, 1 $\Rightarrow \frac{5!}{2!2!2!}=15$ ways

2, 1, 1, 1 $\Rightarrow \frac{5!}{2!(1!)^3 3!}=10$ ways

3, 1, 1, 0 $\Rightarrow \frac{5!}{3!2!}=10$ ways

Total $\Rightarrow$ 1 + 5 + 10 15 + 10 + 10 = 51 ways

6. D

Sol. Let Z = x + iy

Then $(x - 1)^2 + y^2 = 1 \rightarrow (1)$

& $(\sqrt{2}-1)(2x)-i(2iy)=2\sqrt{2}$

$\Rightarrow (\sqrt{2}-1)x+y=\sqrt{2}\rightarrow(2)$

Solving (1) & (2) we get

Either x = 1 or $x=\frac{1}{2-\sqrt{2}}\rightarrow(3)$

On solving (3) with (2) with get

For x = 1 $\Rightarrow$ y = 1 $\Rightarrow$ $Z_2 = 1 + i$

& for

$$x=\frac{1}{2-\sqrt{2}}\Rightarrow y=\sqrt{2}-\frac{1}{\sqrt{2}}\Rightarrow Z_1=\left(1+\frac{1}{\sqrt{2}}\right)+\frac{i}{\sqrt{2}}$$

Now

$$\left|\sqrt{2}z_1-z_2\right|^2$$

$$=\left|\left(\frac{1}{\sqrt{2}}+1\right)\sqrt{2}i-(1+i)\right|^2$$

$=(\sqrt{2})^2$

$=2$

7. B

Sol. Median = 170 $\Rightarrow$ 125, a, b, 170, 190, 210, 230

Median deviation about

Median =

$$\frac{0+45+60+20+40+170-a+170-b}{7}=\frac{205}{7}$$

$\Rightarrow a+b=300$

$$\text{Mean}=\frac{170+125+230+190++210+a+b}{7}=175$$

Mean deviation

About mean =

$$\frac{50+175-a+175-b+5+15+35+55}{7}=30$$

8. A

Sol. $\vec{a}=-5\hat{i}+j-3\hat{k}$

$\vec{b}=\hat{i}+2\hat{j}-4\hat{k}$

$(\vec{a}\times\vec{b})\times\hat{i}=(\vec{a}\cdot\hat{i})\vec{b}-(\vec{b}\cdot\hat{i})\vec{a}$

$=-5\vec{b}-\vec{a}$

$=(((-5\vec{b}-\vec{a})\times\hat{i})\times\hat{i})$

$=((-11\hat{j}+23\hat{k})\times\hat{i})\times\hat{i}$

$\Rightarrow(11\hat{k}+23\hat{j})\times\hat{i}$

$\Rightarrow(11\hat{j}-23\hat{k})$

$\vec{c}\cdot(-\hat{i}+\hat{j}+\hat{k})=11-23=-12$

9. C

Sol. $(\sqrt{3}+\sqrt{2})^x+(\sqrt{3}-\sqrt{2})^x=10$

Let $(\sqrt{3}+\sqrt{2})^x=t$

$t+\frac{1}{t}=10$

$t^2 - 10t + 1 = 0$

$$t=\frac{10\pm\sqrt{100-4}}{2}=5\pm2\sqrt{6}$$

$(\sqrt{3}+\sqrt{2})^x=(\sqrt{3}\pm\sqrt{2})^2$

x = 2 or x = –2

Number of solutions = 2

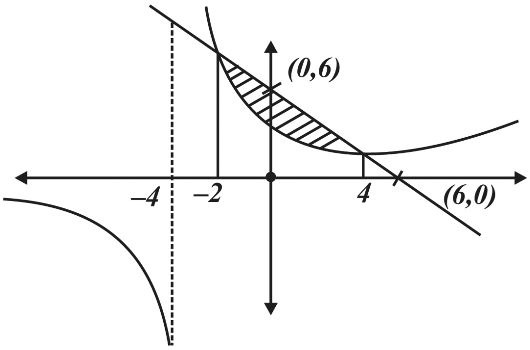
10. C

Sol. $xy + 4y = 16,\qquad x + y = 6$

$y(x + 4) = 16$ _____(1) , $x + y = 6$ _____(2)

on solving, (1) & (2)

we get $x = 4,\ x = -2$



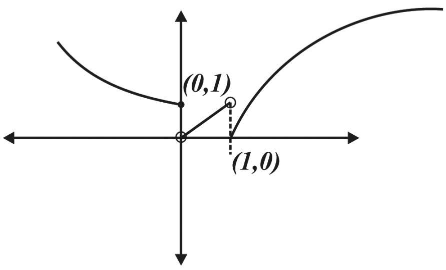
$$\text{Area} = \int_{-2}^{4}\left((6-x)-\left(\frac{16}{x+4}\right)\right)dx$$

$= 30 - 32\ln 2$

11. B

Sol. $$g(f(x)) = \begin{cases} f(x), & f(x) \ge 0 \\ e^{f(x)}, & f(x) < 0 \end{cases}$$

$$g(f(x)) = \begin{cases} e^{-x}, & (-\infty, 0] \\ e^{\ln x}, & (0, 1) \\ \ln x, & [1, \infty) \end{cases}$$



Graph of g(f(x))

$g(f(x)) \Rightarrow$ Many one into

12. B

Sol. Using family planes

$2x + 3y - z - 5 = k_1(x + \alpha y + 3z + 4) + k_2(3x - y + \beta z - 7)$

$2 = k_1 + 3k_2,\ 3 = k_1\alpha - k_2,\ -1 = 3k_1 + \beta k_2,\ -5 = 4k_1 - 7k_2$

On solving we get

$$k_2 = \frac{13}{19},\ k_1 = \frac{-1}{19},\ \alpha = 70,\ \beta = \frac{-16}{13}$$

$$13\alpha\beta = 13(-70)\left(\frac{-16}{13}\right)$$

$= 1120$

13. C

Sol. $e_h = \sqrt{1+\sin^2\theta}$

$e_c = \sqrt{1-\sin^2\theta}$

$e_h = \sqrt{7}e_c$

$1+\sin^2\theta = 7(1-\sin^2\theta)$

$\sin^2\theta = \frac{6}{8} = \frac{3}{4}$

$\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{3}$

14. D

Sol. $\frac{dy}{dx} = 2x(x+y)^3 - x(x+y) - 1$

$x + y = t$

$\frac{dt}{dx} - 1 = 2xt^3 - xy - 1$

$\frac{dt}{2t^3 - t} = xdx$

$\frac{tdt}{2t^4 - t^2} = xdx$

Let $t^2 = z$

$$\int \frac{dz}{2(2z^2 - z)} = \int xdx$$

$$\int \frac{dz}{4z\left(z-\frac{1}{2}\right)} = \int xdx$$

$$\ln\left|\frac{z-\frac{1}{2}}{z}\right| = x^2 + k$$

$$z = \frac{1}{2-\sqrt{e}}$$

15. D

Sol. At x = 1, f(x) is continuous therefore,

$f(1^-) = f(1) = f(1^+)$

$f(1) = 3 + c$ ....(1)

$$f(1^+) = \lim_{h\to 0} 2(1+h)+1$$

$$f(1^+) = \lim_{h\to 0} 3+2h = 3 \quad .....(2)$$

from (1) & (2)

$c = 0$

at x = 0, f(x) is continuous therefore,

$f(0^-) = f(0) = f(0^+)$ .....(3)

$f(0) = f(0^+) = 2$ .....(4)

$f(0^-)$ has to be equal to 2

$$\lim_{h\to 0}\frac{a-b\cos(2h)}{h^2}$$

$$\lim_{h\to 0}\frac{a-b\left\{1-\frac{4h^2}{2!}+\frac{16h^4}{4!}+....\right\}}{h^2}$$

$$\lim_{h\to 0}\frac{a-b+b\left\{2h^2-\frac{2}{3}h^4......\right\}}{h^2}$$

for limit to exist a – b = 0 and limit is 2b ....(5)

from (3), (4) & (5)

a = b = 1

checking differentiability at x = 0

$$\text{LHD}: \lim_{h\to 0}\frac{\frac{1-\cos 2h}{h^2}-2}{-h}$$

$$\lim_{h\to 0}\frac{1-\left(1-\frac{4h^2}{2!}+\frac{16h^4}{4!}.....\right)-2h^2}{-h^3} = 0$$

$$\text{RHD}: \lim_{h\to 0}\frac{(0+h)^2+2-2}{h} = 0$$

Function is differentiable at every point in its domain

$\therefore$ m = 0

m + a + b + c = 0 + 1 + 1 + 0 = 2

16. C

Sol. $e = \frac{1}{\sqrt{2}} = \sqrt{1-\frac{b^2}{a^2}} \Rightarrow \frac{1}{2} = 1-\frac{b^2}{a^2}$

$$\frac{2b^2}{a} = 14$$

$$e_H = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{1}{2}} = \sqrt{\frac{3}{2}}$$

$$(e_H)^2 = \frac{3}{2}$$

17. D

Sol. 3, a, b, c $\rightarrow$ A.P

$\Rightarrow$ 3, 3 + d, 3 + 2d, 3 + 3d

3, a – 1, b + 1, c + 9

$\rightarrow$ G.P $\Rightarrow$ 3, 2 + d, 4 + 2d, 12 + 3d

a = 3 + d $(2 + d)^2 = 3(4 + 2d)$

b = 3 + 2d d = 4, –2

c = 3 + 3d

If d = 4

G.P $\Rightarrow$ 3, 6, 12, 24

a = 7

b = 11

c = 15

$$\frac{a+b+c}{3} = 11$$

18. D

Sol. $x^2 + y^2 = 4$

C(0, 0) $r_1 = 2$

$C'(2\lambda,\ 0)$ $r_2 = \sqrt{4\lambda^2-9}$

$|r_1 - r_2| < CC' < |r_1 + r_2|$

$$\left|2-\sqrt{4\lambda^2-9}\right| < |2\lambda| < 2+\sqrt{4\lambda^2-9}$$

$$4+\lambda^2-9-4\sqrt{4\lambda^2-9} < 4\lambda^2$$

True $\lambda \in R$ .....(1)

$$4\lambda^2 < 4+4\lambda^2-9+4\sqrt{4\lambda^2-9}$$

$$5 < 4\sqrt{4\lambda^2-9} \text{ and } \lambda^2 \geq \frac{9}{4}$$

$$\frac{25}{16} < 4\lambda^2 - 9 \qquad \lambda \in \left(-\infty,\ -\frac{3}{2}\right] \cup \left[\frac{3}{2},\ \infty\right)$$

$$\frac{169}{64} < \lambda^2$$

$$\lambda \in \left(-\infty, -\frac{13}{8}\right) \cup \left(\frac{13}{8}, \infty\right) \Rightarrow R - \left[-\frac{13}{8}, \frac{13}{8}\right]$$

as per question $a = -\frac{13}{8}$ and $b = \frac{13}{8}$

$\therefore$ required point is (–1, 6) with satisfies option (4)

19. B

Sol. $5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 2, \forall x \neq 0$ .....(1)

Substitute $x \to \frac{1}{x}$

$$5f\left(\frac{1}{x}\right) + 4f(x) = \frac{1}{x^2} - 2 \text{ .....(2)}$$

On solving (1) and (2)

$$f(x) = \frac{5x^2 - 2x^2 - 4}{9x^2}$$

$y = 9x^2 f(x)$

$y = 5x^4 - 2x^2 - 4$ ....(3)

$$\frac{dy}{dx} = 20x^3 - 4x$$

for strictly increasing

$$\frac{dy}{dx} > 0$$

$4x(5x^2 - 1) > 0$

$$x \in \left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$$

20. B

Sol. Passing points of lines L_1 & L_2 are

$(\lambda, 2, 1)$ & $(\sqrt{3}, 1, 2)$

$$S.D = \frac{\begin{vmatrix} \sqrt{3}-\lambda & -1 & 1 \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix}}$$

$$1 = \left|\frac{\sqrt{3}-\lambda}{\sqrt{3}}\right|$$

$$\lambda = 0, \lambda = 2\sqrt{3}$$

<u>Section - B (Numerical Value Type)</u>

21. 14

Sol. $(t + 1)dx = (2x + (t + 1)^4)dt$

$$\frac{dx}{dt} = \frac{2x + (t+1)^4}{t+1}$$

$$\frac{dx}{dt} - \frac{2x}{t+1} = (t+1)^3$$

$$I.F. = e^{-\int \frac{2}{t+1}dx} = e^{-2\ln(t+1)} = \frac{1}{(t+1)^2}$$

$$\frac{x}{(t+1)^2} = \int \frac{1}{(t+1)^2}(t+1)^3 dt + c$$

$$\frac{x}{(t+1)^2} = \frac{(t+1)^2}{2} + c$$

$$\Rightarrow c = \frac{3}{2}$$

$$x = \frac{(t+1)^4}{2} + \frac{3}{2}(t+1)^2$$

put, $t = 1$

$x = 2^3 + 6 = 14$

22. 169

Sol. $x + 2y + 3z = 42, \quad x, y, z \geq 0$

$z = 0$ $\quad x + 2y = 42 \Rightarrow 22$

$z = 1$ $\quad x + 2y = 39 \Rightarrow 20$

$z = 2$ $\quad x + 2y = 36 \Rightarrow 19$

$z = 3$ $\quad x + 2y = 33 \Rightarrow 17$

$z = 4$ $\quad x + 2y = 30 \Rightarrow 16$

$z = 5$ $\quad x + 2y = 27 \Rightarrow 14$

$z = 6$ $\quad x + 2y = 24 \Rightarrow 13$

$z = 7$ $\quad x + 2y = 21 \Rightarrow 11$

$z = 8$ $\quad x + 2y = 18 \Rightarrow 10$

$z = 9$ $\quad x + 2y = 15 \Rightarrow 8$

$z = 10$ $\quad x + 2y = 12 \Rightarrow 7$

z = 11 $\quad x + 2y = 9 \Rightarrow 5$
z = 12 $\quad x + 2y = 6 \Rightarrow 4$
z = 13 $\quad x + 2y = 3 \Rightarrow 2$
z = 14 $\quad x + 2y = 0 \Rightarrow 1$
Total : 169

23. 678

Sol. coeff. of x^{30} in $\dfrac{(x+1)^6(1+x^2)^7(1-x^3)^8}{x^6}$

coeff. of x^{36} in $(1 + x)^6 (1 + x^2)^7 (1 - x^3)^8$

General term

$^6C_{r_1}\,{}^7C_{r_2}\,{}^8C_{r_3}(-1)^{r_3}x^{r_1+2r_2+3r_3}$

$r_1 + 2r_2 + 3r_3 = 36$

Case I: $r_1 + 2r_2 = 12$ (Taking $r_3 = 8$)

r_1	r_2	r_3
0	6	8
2	5	8
4	4	8
6	3	8

Case II: $r_1 + 2r_2 = 15$ (Taking $r_3 = 7$)

r_1	r_2	r_3
1	7	7
3	6	7
5	5	7

Case III: $r_1 + 2r_2 = 18$ (Taking $r_3 = 6$)

r_1	r_2	r_3
4	7	6
6	6	6

Coeff. = 7 + (15 x 21) + (15 x 35) + (35)
$-(6 \times 8) - (20 \times 7 \times 8) - (6 \times 21 \times 8) + (15 \times 28)$
$+ (7 \times 28) = -678 = \alpha$

$|\alpha| = 678$

24. 6699

Sol. 3, 7, 11, 15,...., 403
2, 5, 8, 11, ..., 404
LCM (4, 3) = 12
11, 23, 35,....let (403)
403 = 11 + (n – 1) x 12

$\dfrac{392}{12} = n - 1$

$33{\cdot}66 = n$

n = 33

Sum $\dfrac{33}{2}(22 + 32 \times 12)$

$= 6699$

25. 18

Sol. Finding right hand limit

$\lim\limits_{x\to0^+} f(x) = \lim\limits_{h\to0} f(0+h)$

$= \lim\limits_{h\to0} f(h)$

$= \lim\limits_{h\to0} \dfrac{\cos^{-1}(1-h^2)\sin^{-1}(1-h)}{h(1-h^2)}$

$= \lim\limits_{h\to0} \dfrac{\cos^{-1}(1-h^2)}{h}\left(\dfrac{\sin^{-1}1}{1}\right)$

Let $\cos^{-1}(1-h^2) = \theta \Rightarrow \cos\theta = 1 - h^2$

$= \dfrac{\pi}{2}\lim\limits_{\theta\to0} \dfrac{\theta}{\sqrt{1-\cos\theta}}$

$= \dfrac{\pi}{2}\lim\limits_{\theta\to0} \dfrac{1}{\sqrt{\dfrac{1-\cos\theta}{\theta^2}}}$

$= \dfrac{\pi}{2}\dfrac{1}{\sqrt{1/2}}$

$R = \dfrac{\pi}{\sqrt{2}}$

Now finding left hand limit

$L = \lim\limits_{x\to0^-} f(x)$

$= \lim\limits_{h\to0} f(-h)$

$= \lim\limits_{h\to0} \dfrac{\cos^{-1}(1-\{-h\}^2)\sin^{-1}(1-\{-h\})}{\{-h\}-\{-h\}^3}$

$= \lim\limits_{h\to0} \dfrac{\cos^{-1}(1-(-h+1)^2)\sin^{-1}(1-(-h+1))}{(-h+1)-(-h+1)^3}$

$= \lim\limits_{h\to0} \dfrac{\cos^{-1}(-h^2+2h)\sin^{-1}h}{(1-h)(1-(1-h)^2)}$

$= \lim\limits_{h\to0} \left(\dfrac{\pi}{2}\right)\dfrac{\sin^{-1}h}{(1-(1-h)^2)}$

$$=\frac{\pi}{2}\lim_{h\to 0}\left(\frac{\sin^{-1}h}{-h^2+2h}\right)$$

$$=\frac{\pi}{2}\lim_{h\to 0}\left(\frac{\sin^{-1}h}{h}\right)\left(\frac{1}{-h+2}\right)$$

$$L=\frac{\pi}{4}$$

$$\frac{32}{\pi^2}(L^2+R^2)=\frac{32}{\pi^2}\left(\frac{\pi^2}{2}+\frac{\pi^2}{16}\right)$$

$= 16 + 2$

$= 18$

26. 72

Sol. $x^2 + y^2 = 3$ and $x^2 = 2y$

$y^2 + 2y - 3 = 0 \Rightarrow (y + 3)(y - 1) = 0$

$y = -3$ or $y = 1$

$y=1\ x=\sqrt{2}\Rightarrow P(\sqrt{2},\ 1)$

p lies on the line

$\sqrt{2}x+y=\alpha$

$\sqrt{2}(\sqrt{2})+1=\alpha$

$\alpha=3$

For circle C_1

Q_1 lies on y aixs

Let $Q_1(0,\ \alpha)$ coordinates

$R_1=2\sqrt{3}$ (Given)

Line L act as tangnet

Apply P = r (condition of tangency)

$$\Rightarrow\left|\frac{\alpha-3}{\sqrt{3}}\right|=2\sqrt{3}$$

$\Rightarrow|\alpha-3|=6$

$\alpha-3=6$ or $\alpha-3=6$

$\Rightarrow\alpha=9$ $\qquad\qquad \alpha=-3$

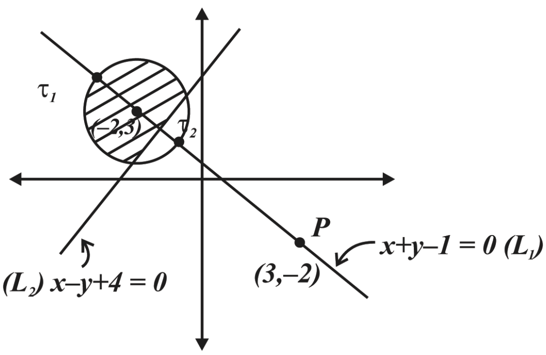
$$\Delta PQ_1Q_2=\frac{1}{2}\begin{vmatrix}\sqrt{2} & 1 & 1\\ 0 & 9 & 1\\ 0 & -3 & 1\end{vmatrix}$$

$$=\frac{1}{2}(\sqrt{2}(12))=6\sqrt{2}$$

$(\Delta PQ_1Q_2)^2=72$

27. 36

Sol.



Clearly for the shaded region z_1 is the intersection of the circle and the line passing through P (L_1) and z_2 is intersection of line L_1 & L_2

Circle : $(x + 2)^2 + (y - 3)^2 = 1$

$L_1 : x + y - 1 = 0$

$L_2 : x - y + 4 = 0$

On solving circle & L_1 we get

$$z_1:\left(-2-\frac{1}{\sqrt{2}},\ 3+\frac{1}{\sqrt{2}}\right)$$

On solving L_1 and z_2 is intersection of line L_1 & L_2 we get $z_2:\left(\frac{-3}{2},\ \frac{5}{2}\right)$

$|z_1|^2+2|z_2|^2=14+5\sqrt{2}+17$

$=31+5\sqrt{2}$

So $\alpha=31$

$\beta=5$

$\alpha+\beta=36$

28. 8

Sol. $$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{8\sqrt{2}\cos x}{(1+e^{\sin x})(1+\sin^4 x)}dx$$

Apply king

$$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{8\sqrt{2}\cos x(e^{\sin x})}{(1+e^{\sin x})(1+\sin^4 x)}dx\ .....(2)$$

adding (1) & (2)

$$2I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{8\sqrt{2}\cos x}{1+\sin^4 x}dx$$

$$I=\int_{0}^{\frac{\pi}{2}}\frac{8\sqrt{2}\cos x}{1+\sin^4 x}dx,$$

$\sin x = t$

$$I=\int_0^1\frac{8\sqrt{2}}{1+t^4}dx$$

$$I=4\sqrt{2}\int_0^1\left(\frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}-\frac{1-\frac{1}{t^2}}{t^2+\frac{1}{t^2}}\right)dt$$

$$I=4\sqrt{2}\int_0^1\frac{\left(1+\frac{1}{t^2}\right)}{\left(t-\frac{1}{t}\right)^2+2}-\frac{\left(1-\frac{1}{t^2}\right)}{\left(t+\frac{1}{t}\right)^2-2}dt$$

Let $t-\frac{1}{t}=z \;\&\; t+\frac{1}{t}=k$

$$=4\sqrt{2}\left[\int_{-\infty}^{0}\frac{dz}{z^2+2}-\int_{\infty}^{2}\frac{dk}{k^2-2}\right]$$

$$=4\sqrt{2}\left[\frac{1}{\sqrt{2}}\tan^{-1}\frac{z}{\sqrt{2}}\right]_{-\infty}^{0}-\left[\frac{1}{2\sqrt{2}}\ln\left(\frac{k-\sqrt{2}}{k+\sqrt{2}}\right)\right]_{\infty}^{2}$$

$$=4\sqrt{2}\left[\frac{\pi}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}\left[\ln\frac{2-\sqrt{2}}{2+\sqrt{2}}\right]\right]$$

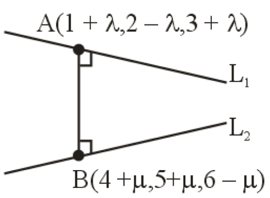
$$=2\pi+2\ln(3+2\sqrt{2})$$

$\alpha = 2$

$\beta = 2$

29. 21

Sol.



$\vec{b}=\hat{i}-\hat{j}+\hat{k}$ (DR's of L_1)

$\vec{d}=\hat{i}+\hat{j}-\hat{k}$ (DR's of L_2)

$$\vec{b}\times\vec{d}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 1\\ 1 & 1 & -1\end{vmatrix}$$

$=0\hat{i}+2\hat{j}+2\hat{k}$ (DR's of Line perpendicular to L_1 and L_2)

DR of AB line

$=(0,\ 2,\ 2)=(3+\mu-\lambda,\ 3+\mu+\lambda,\ 3-\mu-\lambda)$

$$=\frac{3+\mu-\lambda}{0}=\frac{3+\mu+\lambda}{2}=\frac{3-\mu-\lambda}{2}$$

Solving above equation we get

$\mu=-\frac{3}{2}$ and $\lambda=\frac{3}{2}$ point $A=\left(\frac{5}{2},\ \frac{1}{2},\ \frac{9}{2}\right)$

$$B=\left(\frac{5}{2},\ \frac{7}{2},\ \frac{15}{2}\right)$$

Point of AB $=\left(\frac{5}{2},\ 2,\ 6\right)=(\alpha,\ \beta,\ \gamma)$

$2(\alpha+\beta+\gamma)=5+4+12=21$

30. 46

Sol. $n(R_1)$ = 20 + 10 + 6 + 5 + 4 + 3 + 2+ 2 +2

$+2+\underbrace{1+....+1}_{10\text{ times}}$

PHYSICS

Section - A (Single Correct Answer)

31. B

Sol. Conceptual questions

32. B

Sol. $g'=\frac{GMe}{(3R)^2}=\frac{1}{9}g$

$$T=2\pi\sqrt{\frac{l}{g'}}$$

Since the time period of second pendulum is 2 sec.

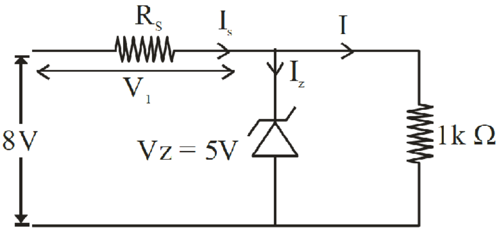
T = 2 sec

$$2 = 2\pi\sqrt{\frac{l}{g}9}$$

$$\boxed{l = \frac{1}{9}\text{m}}$$

33. BONUS

Sol.



Pd across R_s

$V_1 = 8 - 5 = 3V$

Current through the load resistor

$$I = \frac{5}{1\times10^3} = 5\text{mA}$$

Maximum current through Zener diode

$$I_{z\ max.} = \frac{10}{5} = 2\text{mA}$$

And minimum current through Zener diode

$I_{z\ min.} = 0$

$\therefore\ I_{s\ max.} = 5 + 2 = 7\text{mA}$

And $R_{s\ min} = \frac{V_1}{I_{s\ max}} = \frac{3}{7}\text{k}\Omega$

Similarly

$I_{s\ min.} = 5\text{mA}$

And $R_{s\ max} = \frac{V_1}{I_{s\ max}} = \frac{3}{5}\text{k}\Omega$

$$\therefore \frac{3}{7}\text{k}\Omega < R_s < \frac{3}{5}\text{k}\Omega$$

34. C

Sol. $i = \frac{E_{eq}}{r_{eq}} = \frac{8\times5}{8\times0.2}$

$I = 25A$

$V = E - ir = 5 - 0.2 \times 25 = 0$

35. A

Sol. $V_C = \frac{q_{net}}{C_{net}} = \frac{CV + 2CV}{2C}$

$$V_C = \frac{3V}{2}$$

Loss of energy

$$= \frac{1}{2}CV^2 + \frac{1}{2}C(2V)^2 - \frac{1}{2}2C\left(\frac{3V}{2}\right)^2$$

$$= \left(\frac{CV^2}{4}\right)$$

36. A

Sol. $C_V = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2} = \frac{2\times\frac{3}{2}R + 6\times\frac{5}{2}R}{2+6} = \frac{9}{4}R$

37. B

Sol. $T = m\omega^2 l$

$400 = 0.5\omega^2 \times 0.5$

$\omega = 40$ rad/s.

38. D

Sol. $I = \frac{V}{X_C} = 230 \times 300 \times 200 \times 10^{-12} = 13.8\ \mu A$

39. A or B

Sol. For PV^x = constant

If work done by gas is asked then

$$W = \frac{nR\Delta T}{1 - x}$$

Here $x = \frac{3}{2}$

$$\therefore W = \frac{P_2V_2 - P_1V_1}{-\frac{1}{2}}$$

$= 2(P_1V_1 - P_2V_2)$ .... Option (A) is correct

If work done by external is asked then

$W = -2(P_1V_1 - P_2V_2)$ .... Option (B) is correct

40. C

Sol.

$$R = \frac{V}{I_g} - R_g = \frac{100}{5\times10^{-3}} - 50$$

$= 20000 - 50 = 19950\Omega$

41. D

Sol. $\lambda = \frac{h}{p} = \frac{h}{mv} \Rightarrow v = \frac{h}{m\lambda}$

$$\frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} \times \frac{\lambda_\alpha}{\lambda_p} = 4 \times 2 = 8$$

42. D

Sol. 10 MSD = 11 VSD

$$1VSD = \frac{10}{11} MSD$$

LC = 1MSD – 1VSD

$$= 1MSD - \frac{10}{11} MSD$$

$$= \frac{1MSD}{11}$$

$$= \frac{5}{11} \text{units}$$

43. C

Sol. $\omega' = \omega$

$$\frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{LC}}$$

∴ $L'C' = LC$

$L'(4C) = LC$

$$L' = \frac{1}{4}$$

∴ Inductance must be decreased by $\frac{3L}{4}$

44. B

Sol. $\rho = R\frac{\rho}{l}$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta R}{R} + 2\frac{\Delta r}{r} + \frac{\Delta l}{l}$$

$$= \frac{10}{100} + 2 \times \frac{0.05}{0.35} + \frac{0.2}{15}$$

$$= \frac{1}{10} + \frac{2}{7} + \frac{1}{75}$$

$$\frac{\Delta\rho}{\rho} = 39.9\%$$

45. D

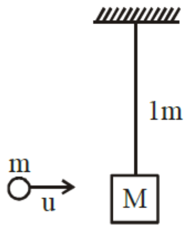
Sol. Angular impulse = change in angular momentum.

[Angular impulse] = [Angular momentum] = [mvr]

= $[M L^2 T^{-1}]$

46. B

Sol.



mu = (M + m)V

$10^{-2} \times 2 \times 10^2 \cong 1 \times V$

$V \cong 2m/s$

$$h = \frac{V^2}{2g} = 0.2 \text{ m}$$

47. A

Sol. $\frac{2\pi R}{T} = V$

Maximum height $H = \frac{v^2 \sin^2\theta}{2g}$

$$4R = \frac{4\pi^2 R^2}{T^2 2g}\sin^2\theta$$

$$\sin\theta = \sqrt{\frac{2gT^2}{\pi^2 R}}$$

$$\theta = \sin^{-1}\left(\frac{2gT^2}{\pi^2 R}\right)^{\frac{1}{2}}$$

48. C

Sol. $f_k = \mu N = 0.04 \times 20g = 8$ Newton

$$a = \frac{60-8}{26} = 2m/s^2$$

49. D

Sol. Transition from n = 1 to n = 3

ΔE = 12.1eV

50. B

Sol. Linear width

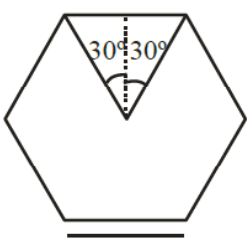
$$W = \frac{2\lambda d}{a} = \frac{2 \times 6 \times 10^{-7} \times 0.2}{1 \times 10^{-5}}$$

$= 2.4 \times 10^{-2} = 24$ mm

Section - B (Numerical Value Type)

51. 72

Sol.



$\frac{4\pi}{6}$m

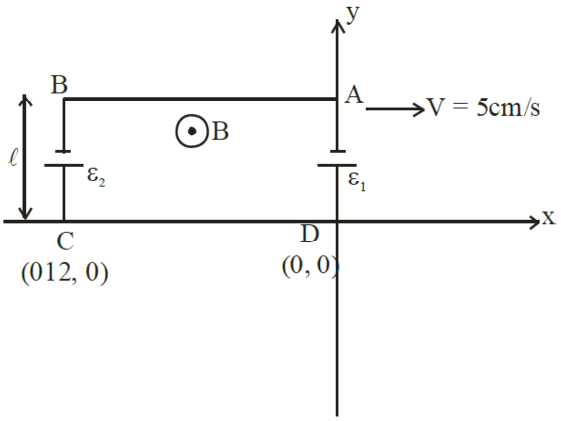
$$B = 6\left(\frac{\mu_0 I}{4\pi r}\right)(\sin 30° + \sin 30°)$$

$$= 6\frac{10^{-7} \times 4\pi\sqrt{3}}{\left(\frac{\sqrt{3}\times 4\pi}{2\times 6}\right)}$$

$= 72 \times 10^{-7}$ T

52. 216

Sol.



B_0 is the magnetic field at origin

$$\frac{dB}{dx} = -\frac{10^{-3}}{10^{-2}}$$

$$\int_{B_0}^{B} dB = -\int_0^x 10^{-1} dx$$

$B - B_0 = -10^{-1}x$

$$B = \left(B_0 - \frac{x}{10}\right)$$

Motional emf in AB = 0

Motional emf in CD = 0

Motional emf in AD = $\varepsilon_1 = B_0 l v$

Magnetic field on rod BC B

$$= \left(B_0 - \frac{(-12\times 10^{-2})}{10}\right)$$

$$\text{Motional emf in BC } = \varepsilon_2 = \left(B_0 + \frac{12\times 10^{-2}}{10}\right) l \times v$$

$\varepsilon_{eq} = \varepsilon_2 - \varepsilon_1 = 300 \times 10^{-7}$ V

For time variation

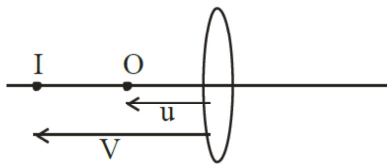
$$(\varepsilon_{eq})' = A\frac{dB}{dt} = 60\times 10^{-7}\text{ V}$$

$(\varepsilon_{eq})net = \varepsilon_{eq} + (\varepsilon_{eq})' = 360 \times 10^{-7}$ V

$$\text{Power } = \frac{(\varepsilon_{eq})^2_{net}}{R} = 216 \times 10^{-9}\text{ W}$$

53. 15

Sol.



v = 3u

v – u = 20 cm

2u = 20 cm

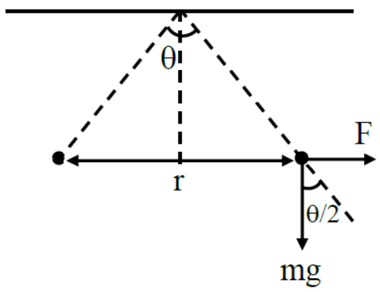
u = 10 cm

$$\frac{1}{(-30)} - \frac{1}{(-10)} = \frac{1}{f}$$

f = 15 cm

54. 3

Sol.



$$\text{In air } \tan\frac{\theta}{2} = \frac{F}{mg} = \frac{q^2}{4\pi\varepsilon_0 r^2 mg}$$

$$\text{In water } \tan\frac{\theta}{2} = \frac{F'}{mg'} = \frac{q^2}{4\pi\varepsilon_0 r^2 mg_{eff}}$$

Equate both equations

$$\varepsilon_0 g = \varepsilon_0\varepsilon_r\, g\left[1 - \frac{1}{1.5}\right]$$

$\varepsilon_r = 3$

55. 27

Sol. $R = R_0A^{1/3}$

$R^3 \propto A$

$$\left(\frac{4.8}{4}\right)^3 = \frac{64}{A} = \frac{64}{A} = (1.2)^3$$

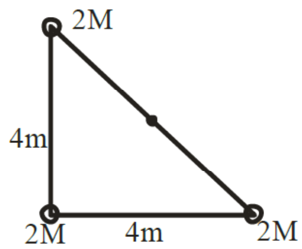
$$\frac{64}{A} = 1.44 \times 1.2$$

$$A = \frac{64}{1.44 \times 1.2} = \frac{1000}{x}$$

$$x = \frac{144 \times 12}{64} = 27$$

56. 3

Sol.



Position vector $\vec{r}_{COM} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3}$

$$\vec{r}_{COM} = \frac{2M \times 0 + 2M \times 4\hat{i} + 2M \times 4\hat{j}}{6M}$$

$$\vec{r} = \frac{4}{3}\hat{i} + \frac{4}{3}\hat{j}$$

$$|\vec{r}| = \frac{4\sqrt{2}}{3}$$

$x = 3$

57. 6

Sol. $f = \dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}$

$f_1 = \dfrac{1}{2}\sqrt{\dfrac{6}{\mu}}$ $\qquad$ $f_2 = \dfrac{1}{2}\sqrt{\dfrac{54}{\mu}}$

$\dfrac{f_1}{f_2} = \dfrac{1}{3}$ $\qquad$ $f_2 - f_1 = 12$

$f_1 = 6$ HZ

58. 9600

Sol. $A = 80\ m^2$

Using Bernonlli equation

$$A(P_2 - P_1) = \frac{1}{2}\rho\left(V_1^2 - V_2^2\right)A$$

$$mg = \frac{1}{2} \times 1\left(70^2 - 50^2\right) \times 80$$

$mg = 40 \times 2400$

$m = 9600$ kg

59. 22

Sol. $q = \int_1^2 i dt = \int_1^2 \left(3t^2 + 4t^3\right)dt$

$$q = \left(t^3 + t^4\right)\Big|_1^2$$

$q = 22C$

60. 52

Sol. $x = 3t^3 + 18t^2 + 16t$

$v = -9t^2 + 36 + 16$

$a = -18t + 36$

$a = 0$ at $t = 2s$

$v = -9(2)^2 + 36 \times 2 + 16$

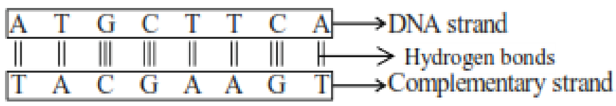
$v = 52$ m/s

CHEMISTRY

Section - A (Single Correct Answer)

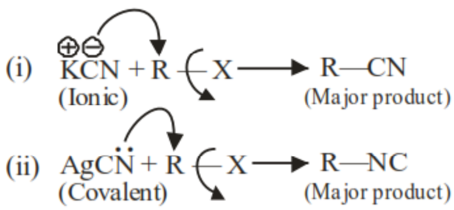
61. **(B)**

Sol. Adenine base pairs with thymine with 2 hydrogen bonds and cytosine base pairs with guanine with 3 hydrogen bonds.



62. **(A)**

Sol.



AgCN is mainly covalent in nature and nitrogen is available for attack, so alkyl isocyanide is formed as main product.

63. **(D)**

Sol. The balanced reaction is,

$Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$

$X = 14$

$Y = 6$

$A = 7$

64. **(A)**

Sol. When a particular oxidation state becomes less stable relative to other oxidation state, one lower, one higher, it is said to undergo disproportionation.

$Cu^+ \rightarrow Cu^{2+} + Cu$

$3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$

65. **(D)**

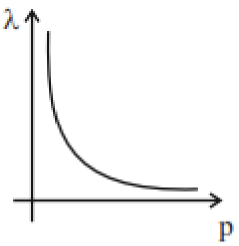
Sol. In F^-, Ne, Na^+ all have $1s^2, 2s^2, 2p^6$ configuration. They have different size due to the difference in nuclear charge.

66. **(A)**

Sol. $\lambda = \frac{h}{p}\left[\lambda \propto \frac{1}{p}\right]$

$\Rightarrow \lambda p = h$ (constant)

So, the plot is a rectangular hyperbola.



67. **(B)**

Sol. $[Ni(H_2O)_6]^{+2} \rightarrow$ Green colour solution due to d-d transition.

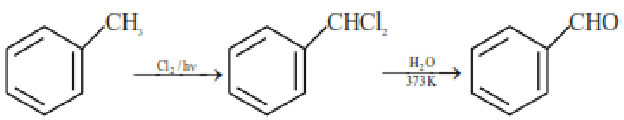
$[Ni(CN)_4]^{-2} \rightarrow$ is diamagnetic and it is colourless.

68. **(D)**

Sol. Unlike NH_3, PH_3 molecules are not associated through hydrogen bonding in liquid state. That is why the boiling point of PH_3 is lower than NH_3.

69. **(B)**

Sol.



70. **(B)**

Sol. Aniline is also known as amino benzene.

71. **(A)**

Sol. In Homoleptic complex all the ligand attached with the central atom should be the same. Hence $[Ni(CN)_4]^{2-}$ is a homoleptic complex.

72. **(D)**

Sol. Higher the electron density in the benzene ring more easily it will be attacked by an electrophile. Phenol has the highest electron density amongst all the given compound.

73. **(C)**

Sol. Heterolytic cleavage of Bond lead to formation of ions.

74. **(C)**

Sol. Increasing order of ionic character

$N_2 < SO_2 < ClF_3 < K_2O < LiF$

Ionic character depends upon difference of electronegativity (bond polarity).

75. **(A)**

Sol.

Salt	Values of i (for different conc. of a Salt)		
	0.1 M	0.01 M	0.001 M
NaCl	1.87	1.94	1.94

i approach 2 as the solution become very dilute.

76. **(B)**

Sol. Kjeldahl's method is used for estimation of Nitrogen where $CuSO_4$ acts as a catalyst.

77. **(A)**

Sol. **Statement (I) :** Potassium hydrogen phthalate is a primary standard for standardisation of sodium hydroxide solution as it is economical and its concentration does not changes with time.

Phenophthalin can acts as indicator in acid base titration as it shows colour in pH range 8.3 to 10.1.

78. **(B)**

Sol. $CH_3(CH_2)_5COOC_2H_5 \xrightarrow{\text{DIBAL-H, } H_2O} CH_3(CH_2)_5CHO$

$C_6H_5COC_6H_5 \xrightarrow{\text{Zn (Hg) \& conc. HCl}} C_6H_5CH_2C_6H_5$

$C_6H_5CHO \xrightarrow[H_2O]{CH_3MgBr} C_6H_5CH(OH)CH_3$

$CH_3COCH_2COOC_2H_5 \xrightarrow{NaBH_4,\ H^+} CH_3CH(OH)CH_2COOC_2H_5$

79. **(D)**

Sol. During free expansion of an ideal gas under adiabatic condition $q = 0, \Delta T = 0, w = 0$.

80. **(A)**

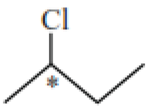
Sol. The NH_2 group in Aniline is *ortho* and *para* directing and a powerful activating group as NH_2 has strong +M effect.

Aniline does not undergo Friedel-Craft's reaction (alkylation and acylation) as Aniline will form complex with $AlCl_3$ which will deactivate the benzene ring.

Section - B (Numerical Value Type)

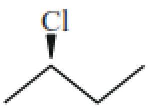
81. **(2)**

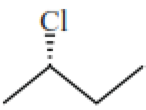
Sol.



There is one chiral centre present in given compound.

So, Total optical isomers = 2



and


82. **(1)**

Sol. $$E = E^{o}_{H^+/H_2} - \frac{0.06}{2}\log\frac{P_{H_2}}{[H^+]^2}$$

$$E = 0.00 - \frac{0.06}{2}\log\frac{2}{[1]^2}$$

$$E = -0.03 \times 0.3 = -0.9 \times 10^{-2}\ V$$

83. **(5)**

Sol.

$SrSO_4$	–	white
$Mg(NH_4)PO_4$	–	white
$BaCrO_4$	–	yellow
$Mn(OH)_2$	–	white
$PbSO_4$	–	white
$PbCrO_4$	–	yellow
AgBr	–	pale yellow
PbI_2	–	yellow
CaC_2O_4	–	white
$[Fe(OH)_2(CH_3COO)]$	–	Brown Red

84. **(17190)**

Sol. $$\lambda t = \ln\frac{(^{14}C/^{12}C)_{atmosphere}}{(^{14}C/^{12}C)_{wood\ sample}}$$

As per the question,

$$\frac{(^{14}C/^{12}C)_{wood}}{(^{14}C/^{12}C)_{atmosphere}} = \frac{1}{8}$$

So, $\lambda t = \ln 8$

$$\frac{\ln 2}{t_{1/2}}t = \ln 8$$

$$t = 3 \times t_{1/2} = 17190 \text{ years}$$

85. **(3)**

Sol. PF_5, PCl_5, $Fe(CO)_5$; Trigonal bipyramidal

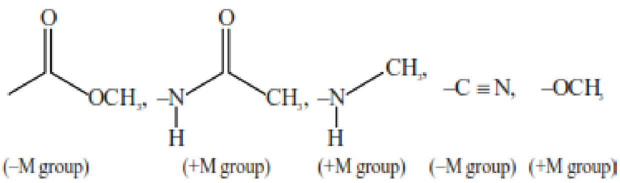
BrF_5 ; square pyramidal

$[PtCl_4]^{-2}$; square planar

BF_3 ; Trigonal planar

86. **(2)**

Sol.



87. **(6)**

Sol. $A_2B \rightarrow 2A^+ + B^{-2}$, B^{-2} has complete octet in its dianionic form, thus in its atomic state it has 6 electrons in its valence shell. As it has negative charge, it has acquired two electrons to complete its octet.

88. **(3)**

Sol. Acidic oxide : Cl_2O_7, SiO_2, N_2O_5

Neutral oxide : CO, NO, N_2O

Amphoteric oxide : Al_2O_3, SnO_2, PbO_2

89. **(24)**

Sol. Limiting Reagent is $PbCl_2$

mmol of $Pb_3(PO_4)_2$ formed

$$= \frac{\text{mmol of } PbCl_2 \text{ reacted}}{3}$$

= 24 mmol

90. **(7)**

Sol. $pH = \frac{pK_w + pK_a - pK_b}{2}$

$pK_a = pK_b$

$\Rightarrow pH = \frac{pK_w}{2} = 7$

● ● ●