## 31-Jan-2024 (Evening) : PCM

## MATHEMATICS

## Section - A (Single Correct Answer)

1. The number of ways in which 21 identical apples can be distributed among three children such that each child gets at least 2 apples, is
(A) 406
(B) 130
(C) 142
(D) 136
2. Let $\mathrm{A}(\mathrm{a}, \mathrm{b}), \mathrm{B}(3,4)$ and $(-6,-8)$ respectively denote the centroid, circumcentre and orthocentre of a triangle. Then, the distance of the point $\mathrm{P}(2 \mathrm{a}+3,7 \mathrm{~b}+5)$ from the line $2 \mathrm{x}+3 \mathrm{y}-4=0$ measured parallel to the line $x-2 y-1=0$ is
(A) $\frac{15 \sqrt{5}}{7}$
(B) $\frac{17 \sqrt{5}}{6}$
(C) $\frac{17 \sqrt{5}}{7}$
(D) $\frac{\sqrt{5}}{17}$
3. Let $z_{1}$ and $z_{2}$ be two complex number such that $z_{1}+z_{2}=5$ and $z_{1}^{3}+z_{2}^{3}=20+15 i$. Then $\left|z_{1}^{4}+z_{2}^{4}\right|$ equals
(A) $30 \sqrt{3}$
(B) 75
(C) $15 \sqrt{15}$
(D) $25 \sqrt{3}$
4. Let a variable line passing through the centre of the circle $x^{2}+y^{2}-16 x-4 y=0$, meet the positive coordinate axes at the point A and B . Then the minimum value of $\mathrm{OA}+\mathrm{OB}$, where O is the origin, is equal to
(A) 12
(B) 18
(C) 20
(D) 24
5. Let $\mathrm{f}, \mathrm{g}:(0, \infty) \rightarrow \mathrm{R}$ be two functions defined by $\mathrm{f}(\mathrm{x})=\int_{-\mathrm{x}}^{\mathrm{x}}\left(|\mathrm{x}|-\mathrm{t}^{2}\right) \mathrm{e}^{-t^{2}} d t$ and $\mathrm{g}(\mathrm{x})=\int_{0}^{2} \mathrm{t}^{1 / 2} \mathrm{e}^{-t} d t$. Then the value of $\left(\mathrm{f}(\sqrt{\log 9})+\mathrm{g}\left(\sqrt{\log _{\mathrm{e}} 9}\right)\right)$ is equal to
(A) 6
(B) 9
(C) 8
(D) 10
6. Let $(\alpha, \beta, \gamma)$ be mirror image of the point $(2,3,5)$ in the line $\frac{x-1}{2}-\frac{y-2}{3}-\frac{z-3}{4}$. Then $2 \alpha+3 \beta+4 \gamma$ is equal to
(A) 32
(B) 33
(C) 31
(D) 34
7. Let $P$ be a parabola with vertex $(2,3)$ and directrix $2 x+y=6$. Let an ellipse $E: \frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$, $a>b$ of eccentricity $\frac{1}{\sqrt{2}}$ pass through the focus of the parabola. P. Then the square of the length of the latus rectum of E , is
(A) $\frac{385}{8}$
(B) $\frac{347}{8}$
(C) $\frac{512}{25}$
(D) $\frac{656}{25}$
8. The temperature $\mathrm{T}(\mathrm{t})$ of a body at time $\mathrm{t}=0$ is $160^{\circ} \mathrm{F}$ and it decreases continuously as per the differential equation $\frac{\mathrm{dT}}{\mathrm{dt}}=-\mathrm{K}(\mathrm{T}-80)$, where K is positive constant. If $\mathrm{T}(15)=120 \mathrm{oF}$, then $\mathrm{T}(45)$ is equal to
(A) $85^{\circ} \mathrm{F}$
(B) $95^{\circ} \mathrm{F}$
(C) $90^{\circ} \mathrm{F}$
(D) $80^{\circ} \mathrm{F}$
9. Let $2^{\text {nd }}, 8^{\text {th }}$ and $44^{\text {th }}$, terms of a non-constant A.P. be respectively the $1^{\text {st }}, 2^{\text {nd }}$ and $3^{\text {rd }}$ terms of G.P. If the first term of A.P. is 1 then the sum of first 20 terms is equal to -
(A) 980
(B) 960
(C) 990
(D) 970
10. Let $f: \rightarrow R \rightarrow(0, \infty)$ be strictly increasing function such that $\lim _{x \rightarrow \infty} \frac{f(7 x)}{f(x)}=1$. Then, the value of $\lim _{x \rightarrow \infty}\left[\frac{f(5 x)}{f(x)}-1\right]$ is equal to
(A) 4
(B) 0
(C) $7 / 5$
(D) 1
11. The area of the region enclosed by the parabola $y=4 x-x^{2}$ and $3 y=(x-4)^{2}$ is equal to
(A) $\frac{32}{9}$
(B) 4
(C) 6
(D) $\frac{14}{3}$
12. Let the mean and the variance of 6 observation $a, b, 68,44,48,60$ be 55 and 194 , respectively if $a>b$, then $a+3 b$ is
(A) 200
(B) 190
(C) 180
(D) 210
13. If the function $f:(-\infty,-1] \rightarrow(a, b]$ defined by $f(x)=e^{x^{3}-3 x+1}$ is one-one and onto, then the distance of the point $P(2 b+a+2)$ from the line $x+e^{-3} y=4$ is :
(A) $2 \sqrt{1+\mathrm{e}^{6}}$
(B) $4 \sqrt{1+\mathrm{e}^{6}}$
(C) $3 \sqrt{1+\mathrm{e}^{6}}$
(D) $\sqrt{1+\mathrm{e}^{6}}$
14. Consider the function $f:(0, \infty) \rightarrow R$ defined by $f(x)=e^{-\left|\log _{c} x\right|}$. If $m$ and $n$ be respectively the number of points at which $f$ is not continuous and $f$ is not differentiable, then $m+n$ is
(A) 0
(B) 3
(C) 1
(D) 2
15. The number of solutions, of the equation $e^{\sin x}-2 e^{-\sin x}=2$ is
(A) 2
(B) more than 2
(C) 1
(D) 0
16. If $a=\sin ^{-1}(\sin (5))$ and $b=\cos ^{-1}(\cos (5))$, then $a^{2}+b^{2}$ is equal to
(A) $4 \pi^{2}+25$
(B) $8 \pi^{2}-40 \pi+50$
(C) $4 \pi^{2}-20 \pi+50$
(D) 25
17. If for some $\mathrm{m}, \mathrm{n} ;{ }^{6} \mathrm{C}_{\mathrm{m}}+2\left({ }^{6} \mathrm{C}_{\mathrm{m}+1}\right)+{ }^{6} \mathrm{C}_{\mathrm{m}+2}>{ }^{8} \mathrm{C}_{3}$ and ${ }^{\mathrm{n}-1} \mathrm{P}_{3}:{ }^{\mathrm{n}} \mathrm{P}_{4}=1: 8$, then $\mathrm{n} \mathrm{P}_{\mathrm{m}+1}+{ }^{\mathrm{n}+1} \mathrm{C}_{\mathrm{m}}$ is equal to
(A) 380
(B) 376
(C) 384
(D) 372
18. A coin is based so that a head is twice as likely to occur as a tail. If the coin is tossed 3 times, then the probability of getting two tails and one head is -
(A) $\frac{2}{9}$
(B) $\frac{1}{9}$
(C) $\frac{2}{27}$
(D) $\frac{1}{27}$
19. Let A be a $3 \times 3$ real matrix such that $\mathrm{A}\left(\begin{array}{l}1 \\ 0 \\ 1\end{array}\right)=2\left(\begin{array}{l}1 \\ 0 \\ 1\end{array}\right), \mathrm{A}\left(\begin{array}{l}-1 \\ 0 \\ 1\end{array}\right)=4\left(\begin{array}{l}-1 \\ 0 \\ 1\end{array}\right), \mathrm{A}\left(\begin{array}{l}0 \\ 1 \\ 0\end{array}\right)=2\left(\begin{array}{l}0 \\ 1 \\ 0\end{array}\right)$. Then, the system $(A-31)\left(\begin{array}{l}x \\ y \\ z\end{array}\right)=\left(\begin{array}{l}1 \\ 2 \\ 3\end{array}\right)$ has
(A) unique solution
(B) exactly two solutions
(C) no solution
(D) infinitely many solutions
20. The shortest distance between lines $L_{1}$ and $L_{2}$, where $L_{1}: \frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+4}{2}$ and $L_{2}$ is the line passing through the points $\mathrm{A}(-4,4,3) \cdot \mathrm{B}(-1,6,3)$ and perpendicular to the line $\frac{\mathrm{x}-3}{-2}=\frac{\mathrm{y}}{3}=\frac{\mathrm{z}-1}{1}$, is
(A) $\frac{121}{\sqrt{221}}$
(B) $\frac{24}{\sqrt{117}}$
(C) $\frac{141}{\sqrt{121}}$
(D) $\frac{42}{\sqrt{117}}$

## Section - B (Numerical Value Type)

21. $\left|\frac{120}{\pi^{3}} \int_{0}^{\pi} \frac{x^{2} \sin x \cos x}{\sin ^{4} x+\cos ^{4} x} d x\right|$ is equal to $\qquad$
22. Let $\mathrm{a}, \mathrm{b}, \mathrm{c}$ be the length of three sides of a triangle satisfying the condition
$\left(a^{2}+b^{2}\right) x^{2}-2 b(a+c) \cdot x+\left(b^{2}+c^{2}\right)=0$.
If the set of all possible values of $x$ is the interval $(\alpha, \beta)$, then $12\left(\alpha^{2}+\beta^{2}\right)$ is equal to $\qquad$ .
23. Let $\mathrm{A}(-2,-1), \mathrm{B}(1,0), \mathrm{C}(\alpha, \beta)$ and $\mathrm{D}(\gamma, \delta)$ be the vertices of a parallelogram ABCD . If the point C lies on $2 x-y=5$ and the point $D$ lies on $3 x-2 y=6$, then the value of $|\alpha+\beta+\gamma+\delta|$ is equal to $\qquad$ .
24. Let the coefficient of $x r$ in the expansion of
$(x+3)^{\mathrm{n}-1}+(\mathrm{x}+3)^{\mathrm{n}-2}(\mathrm{x}+2)+(\mathrm{x}+3)^{\mathrm{n}-3}(\mathrm{x}+2)^{2}+\ldots \ldots .+(\mathrm{x}+2)^{\mathrm{n}-1}$ be $\alpha_{\mathrm{r}}$.
If $\sum_{\mathrm{r}=0}^{\mathrm{n}} \alpha_{\mathrm{r}}=\beta^{\mathrm{n}}-\gamma^{\mathrm{n}}, \beta-\gamma \in \mathrm{N}$, then the value of $\beta^{2}+\gamma^{2}$ equals $\qquad$ .
25. Let A be a $3 \times 3$ matrix and $\operatorname{det}(\mathrm{A})=2$. If $\mathrm{n}=\operatorname{det}(\underbrace{(\operatorname{adj}(\operatorname{adj}(\ldots .(\operatorname{adj}))))}_{2024-\mathrm{times}}$

Then the remainder when n is divided by 9 is equal to $\qquad$ -.
26. Let $\vec{a}=3 \hat{i}+2 \hat{j}+\hat{k}, \vec{b}=2 \hat{i}-\hat{j}+3 \hat{k}$ and $\vec{c}$ be a vector such that

$$
(\vec{a}+\vec{b}) \times \vec{c}=2(\vec{a} \times \vec{b})+24 \hat{j}-6 \hat{k} \text { and }(\vec{a}-\vec{b}+\hat{i}) \cdot \vec{c}=-3
$$

Then $|\overrightarrow{\mathrm{c}}|^{2}$ is equal to $\qquad$ .
27. If $\lim _{x \rightarrow 0} \frac{\operatorname{ax}^{2} e^{x}-b \log _{e}(1+x)+c x e^{-x}}{x^{2} \sin x}=1$, then $16\left(a^{2}+b^{2}+c^{2}\right)$ is equal to $\qquad$ .
28. A line passes through $\mathrm{A}(4,-6,-2)$ and $\mathrm{B}(16,-2,4)$. The point $\mathrm{P}(\mathrm{a}, \mathrm{b}, \mathrm{c})$ where $\mathrm{a}, \mathrm{b}, \mathrm{c}$ are non-negative integers, on the line $A B$ lies at a distance of 21 units, from the point $A$. The distance between the points $P(a, b, c)$ and $Q(4,-12,3)$ is equal to $\qquad$ .
29. Let $\mathrm{y}=\mathrm{y}(\mathrm{x})$ be the solution of the differential equation, $\sec ^{2} x d x+\left(e^{2 y} \tan ^{2} x+\tan x\right) d y=0,0<x<\frac{\pi}{2}, y\left(\frac{\pi}{4}\right)=0$. If $y\left(\frac{\pi}{6}\right)=\alpha$. Then $e^{8 \alpha}$ is equal to $\qquad$ .
30. Let $\mathrm{A}=\{1,2,3, \ldots \ldots \ldots \ldots .100\}$. Let R be a relation on A defined by $(\mathrm{x}, \mathrm{y}) \in \mathrm{R}$ if and only if $2 \mathrm{x}=3 \mathrm{y}$.

Let $R_{1}$ be a symmetric relation on $A$ such that $R \subset R_{1}$ and the number of elements in $R_{1}$ is $n$. Then, the minimum value of $n$ is $\qquad$ .
31. A light string passing over a smooth light fixed pulley connects two blocks of masses $m_{1}$ and $m_{2}$. If the acceleration of the system is $\mathrm{g} / 8$, then the ratio of masses is

(A) $9 / 7$
(B) $8 / 1$
(C) $4 / 3$
(D) $5 / 3$
32. A uniform magnetic field of $2 \times 10^{-3} \mathrm{~T}$ acts along positive Y -direction. A rectangular loop of sides 20 cm and 10 cm with current of 5 A is Y-Z plane. The current is in anticlockwise sense with reference to negative X axis. Magnitude and direction of the torque is :
(A) $2 \times 10^{-4} \mathrm{~N}-\mathrm{m}-$ along positive Z -direction
(B) $2 \times 10^{-4} \mathrm{~N}-\mathrm{m}-$ along negative Z -direction
(C) $2 \times 10^{-4} \mathrm{~N}$-m - along positive X -direction
(D) $2 \times 10^{-4} \mathrm{~N}-\mathrm{m}-$ along positive Y -direction
33. The measured value of the length of a simple pendulum is 20 cm with 2 mm accuracy. The time for 50 oscillations was measured to be 40 seconds with 1 second resolution. From these measurements, the accuracy in the measurement of acceleration due to gravity is $\mathrm{N} \%$. The value of N is :
(A) 4
(B) 8
(C) 6
(D) 5
34. Force between two point charges $q_{1}$ and $q_{2}$ placed in vacuum at ' $r$ ' cm apart is $F$. Force between them when placed in a medium having dielectric $K=5 \mathrm{at}{ }^{\prime} \mathrm{r} / 5^{\prime} \mathrm{cm}$ apart will be:
(A) $\mathrm{F} / 25$
(B) 5 F
(C) $\mathrm{F} / 5$
(D) 25 F
35. An AC voltage $\mathrm{V}=20 \sin 200 \pi \mathrm{t}$ is applied to a series LCR circuit which drives a current $I=10 \sin \left(200 \pi t+\frac{\pi}{3}\right)$. The average power dissipated is:
(A) 21.6 W
(B) 200 W
(C) 173.2 W
(D) 50 W
36. When unpolarized light is incident at an angle of $60^{\circ}$ on a transparent medium from air. The reflected ray is completely polarized. The angle of refraction in the medium is
(A) $30^{\circ}$
(B) $60^{\circ}$
(C) $90^{\circ}$
(D) $45^{\circ}$
37. The speed of sound in oxygen at S.T.P. will be approximately:
(Given, $\mathrm{R}=8.3 \mathrm{JK}^{-1}, \gamma=1.4$ )
(A) $310 \mathrm{~m} / \mathrm{s}$
(B) $333 \mathrm{~m} / \mathrm{s}$
(C) $341 \mathrm{~m} / \mathrm{s}$
(D) $325 \mathrm{~m} / \mathrm{s}$
38. A gas mixture consists of 8 moles of argon and 6 moles of oxygen at temperature T. Neglecting all vibrational modes, the total internal energy of the system is
(A) 29 RT
(B) 20 RT
(C) 27 RT
(D) 21 RT
39. The resistance per centimeter of a meter bridge wire is $r$, with $X \Omega$ resistance in left gap. Balancing length from left end is at 40 cm with $25 \Omega$ resistance in right gap. Now the wire is replaced by another wire of 2 r resistance per centimeter. The new balancing length for same settings will be at
(A) 20 cm
(B) 10 cm
(C) 80 cm
(D) 40 cm
40. Given below are two statements:

Statement I: Electromagnetic waves carry energy as they travel through space and this energy is equally shared by the electric and magnetic fields.
Statement II: When electromagnetic waves strike a surface, a pressure is exerted on the surface.
In the light of the above statements, choose the most appropriate answer from the options given below:
(A) Statement I is incorrect but Statement II is correct
(B) Both Statement I and Statement II are correct.
(C) Both Statement I and Statement II are incorrect.
(D) Statement I is correct but Statement II is incorrect.
41. In a photoelectric effect experiment a light of frequency 1.5 times the threshold frequency is made to fall on the surface of photosensitive material. Now if the frequency is halved and intensity is doubled, the number of photo electrons emitted will be:
(A) Doubled
(B) Quadrupled
(C) Zero
(D) Halved
42. A block of mass 5 kg is placed on a rough inclined surface as shown in the figure.


If $\vec{F}_{1}$ is the force required to just move the block up the inclined plane and $\overrightarrow{\mathrm{F}}_{2}$ is the force required to just prevent the block from sliding down, then the value of $\left|\overrightarrow{\mathrm{F}}_{1}\right|-\left|\overrightarrow{\mathrm{F}}_{2}\right|$ is :
[Use : $\mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2}$ ]
(A $\quad 20 \sqrt{3} \mathrm{~N}$
(B) $\quad 50 \sqrt{3} \mathrm{~N}$
(C) $\frac{5 \sqrt{3}}{2} \mathrm{~N}$
(D) 10 N
43. By what percentage will the illumination of the lamp decrease if the current drops by $20 \%$ ?
(A) $46 \%$
(B) $26 \%$
(C) $36 \%$
(D) $56 \%$
44. If two vectors $\vec{A}$ and $\vec{B}$ having equal magnitude $R$ are inclined at an angle $\theta$, then
(A) $|\overrightarrow{\mathrm{A}}-\overrightarrow{\mathrm{B}}|=\sqrt{2} \mathrm{R} \sin \left(\frac{\theta}{2}\right)$
(B) $|\overrightarrow{\mathrm{A}}+\overrightarrow{\mathrm{B}}|=2 \mathrm{R} \sin \left(\frac{\theta}{2}\right)$
(C) $|\overrightarrow{\mathrm{A}}+\overrightarrow{\mathrm{B}}|=2 \mathrm{R} \cos \left(\frac{\theta}{2}\right)$
(D) $|\overrightarrow{\mathrm{A}}-\overrightarrow{\mathrm{B}}|=2 \mathrm{R} \cos \left(\frac{\theta}{2}\right)$
45. The mass number of nucleus having radius equal to half of the radius of nucleus with mass number 192 is
(A) 24
(B) 32
(C) 40
(D) 20
46. The mass of the moon is $1 / 144$ times the mass of a planet and its diameter $1 / 16$ times the diameter of a planet. If the escape velocity on the planet is $v$, the escape velocity on the moon will be
(A) $\mathrm{v} / 3$
(B) $\quad \mathrm{v} / 4$
(C) $\mathrm{v} / 12$
(D) $\mathrm{v} / 6$
47. A small spherical ball of radius $r$, falling through a viscous medium of negligible density has terminal velocity ' $v$ '. Another ball of the same mass but of radius $2 r$, falling through the same viscous medium will have terminal velocity:
(A) $\mathrm{v} / 2$
(B) $\mathrm{v} / 4$
(C) $4 v$
(D) 2 v
48. A body of mass 2 kg begins to move under the action of a time dependent force given by $\overrightarrow{\mathrm{F}}=\left(6 t \hat{\mathrm{i}}+6 \mathrm{t}^{2} \hat{j}\right) \mathrm{N}$. The power developed by the force at the time $t$ is given by:
(A) $\left(6 t^{4}+9 t^{5}\right) \mathrm{W}$
(B) $\left(3 t^{3}+6 t^{5}\right) \mathrm{W}$
(C) $\left(9 t^{5}+6 t^{3}\right) W$
(D) $\left(9 t^{3}+6 t^{5}\right) \mathrm{W}$
49.


The output of the given circuit diagram is

(A) | A | B | Y |  |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 |  |
|  | 1 | 0 | 0 |
|  | 0 | 1 | 0 |
|  | 1 | 1 | 1 |

(B) | A | B | Y |
| :--- | :--- | :--- |
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 1 | 0 |

(C) | A | B | Y |
| :--- | :--- | :--- |
| 0 | 0 | 0 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |

(D)

| A | B | Y |
| :--- | :--- | :--- |
| 0 | 0 | 0 |
| 1 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 1 | 0 |

50. Consider two physical quantities $A$ and $B$ related to each other as $E=\frac{B-x^{2}}{A t}$ where $E, x$ and $t$ have dimensions of energy, length and time respectively. The dimension of $A B$ is
(A) $\mathrm{L}^{-2} \mathrm{M}^{1} \mathrm{~T}^{0}$
(B) $\quad \mathrm{L}^{2} \mathrm{M}^{-1} \mathrm{~T}^{1}$
(C) $\quad \mathrm{L}^{-2} \mathrm{M}^{-1} \mathrm{~T}^{1}$
(D) $\quad \mathrm{L}^{0} \mathrm{M}^{-1} \mathrm{~T}^{1}$

## Section - B (Numerical Value Type)

51. In the following circuit, the battery has an emf of 2 V and an internal resistance of $\frac{2}{3} \Omega$. The power consumption in the entire circuit is $\qquad$ W.

52. Light from a point source in air falls on a convex curved surface of radius 20 cm and refractive index 1.5 . If the source is located at 100 cm from the convex surface, the image will be formed at $\qquad$ cm from the object.
53. The magnetic flux $\phi$ (in weber) linked with a closed circuit of resistance $8 \Omega$ varies with time (in seconds) as $\phi=5 \mathrm{t}^{2}-36 \mathrm{t}+1$. The induced current in the circuit at $\mathrm{t}=2 \mathrm{~s}$ is $\qquad$ A.
54. Two blocks of mass 2 kg and 4 kg are connected by metal wire going over a smooth pulley as shown in figure. The radius of wire is $4.0 \times 10^{-5} \mathrm{~m}$ and Young's modulus of the metal is $2.0 \times 10^{11} \mathrm{~N} / \mathrm{m}^{2}$. The longitudinal strain developed in the wire is $\frac{1}{\alpha \pi}$. The value of $\alpha$ is $\qquad$ .
[Use : $\mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2}$ )

55. A body of mass ' $m$ ' is projected with a speed ' $u$ ' making an angle of $45^{\circ}$ with the ground. The angular momentum of the body about the point of projection, at the highest point is expressed as $\frac{\sqrt{2} \mathrm{mu}^{3}}{\mathrm{Xg}}$. The value of ' X ' is $\qquad$ .
56. Two circular coils P and Q of 100 turns each have same radius of $\pi \mathrm{cm}$. The currents in P and R are 1 A and 2 A respectively. P and Q are placed with their planes mutually perpendicular with their centers coincide. The resultant magnetic field induction at the center of the coils is $\sqrt{\mathrm{x}} \mathrm{mT}$, where $\mathrm{x}=$ $\qquad$ . [Use $\mu_{0}=4 \pi \times 10^{-7} \mathrm{TmA}^{-1}$ ]
57. The distance between charges +q and -q is $2 l$ and between +2 q and -2 q is $4 l$. The electrostatic potential at point P at a distance r from centre O is $-\alpha\left[\frac{\mathrm{q} l}{\mathrm{r}^{2}}\right] \times 10^{9} \mathrm{~V}$, where the value of $\alpha$ is $\qquad$ -

$$
\left(\text { Use } \frac{1}{4 \pi \varepsilon_{0}}=9 \times 10^{9} \mathrm{Nm}^{2} \mathrm{C}^{-2}\right)
$$


58. Two identical spheres each of mass 2 kg and radius 50 cm are fixed at the ends of a light rod so that the separation between the centers is 150 cm . Then, moment of inertia of the system about an axis perpendicular to the rod and passing through its middle point is $\frac{x}{20} \mathrm{~kg} \mathrm{~m}^{2}$, where the value of x is $\qquad$ -
59. The time period of simple harmonic motion of mass M in the given figure is $\pi \sqrt{\frac{\alpha \mathrm{M}}{5 \mathrm{~K}}}$, where the value of $\alpha$ is $\qquad$ -

60. A nucleus has mass number $\mathrm{A}_{1}$ and volume $\mathrm{V}_{1}$. Another nucleus has mass number $\mathrm{A}_{2}$ and volume $\mathrm{V}_{2}$. If relation between mass number is $A_{2}=4 A_{1}$, then $\frac{V_{2}}{V_{1}}=$ $\qquad$ -.
CHEMISTRY $\quad$ Section - A (Single Correct Answer)
61. Match List I with List II :

## LIST-I (Complex ion) LIST-II (Electronic configuration)

A. $\left[\mathrm{Cr}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}\right]^{3+}$
I. $\quad \mathrm{t}_{2 \mathrm{~g}}^{2} \mathrm{e}_{\mathrm{g}}^{0}$
B. $\left[\mathrm{Fe}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}\right]^{3+}$
II. $\quad t_{2 g}^{3} e_{g}^{0}$
C. $\left[\mathrm{Ni}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}\right]^{2+}$
III. $t_{2 g}^{3} e_{g}^{2}$
D. $\left[\mathrm{V}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}\right]^{3+}$
IV. $\quad t^{6}{ }_{2 g} e_{g}^{2}$

Choose the correct answer from the options given below :
(A) A-III, B-II, C-IV, D-I
(B) A-IV, B-I, C-II, D-III
(C) A-IV, B-III, C-I, D-II
(D) A-II, B-III, C-IV, D-I
62. A sample of $\mathrm{CaCO}_{3}$ and $\mathrm{MgCO}_{3}$ weighed 2.21 g is ignited to constant weight of 1.152 g . The composition of mixture is :
[Given molar mass in $\mathrm{g} \mathrm{mol}^{-1} \mathrm{CaCO}_{3}: 100, \mathrm{MgCO}_{3}: 84$ ]
(A) $1.187 \mathrm{~g} \mathrm{CaCO}_{3}+1.023 \mathrm{~g} \mathrm{MgCO}_{3}$
(B) $1.023 \mathrm{~g} \mathrm{CaCO}_{3}+1.023 \mathrm{~g} \mathrm{MgCO}_{3}$
(C) $1.187 \mathrm{~g} \mathrm{CaCO}_{3}+1.187 \mathrm{~g} \mathrm{MgCO}_{3}$
(D) $1.023 \mathrm{~g} \mathrm{CaCO}_{3}+1.187 \mathrm{~g} \mathrm{MgCO}_{3}$
63. Identify A and B in the following reaction sequence.

(A)


(B)

$B=$

(C)
$\mathrm{A}=$

$\mathrm{B}=$

(D) $\mathrm{A}=$

,
$B=$

64. Given below are two statements :

Statement I : $\mathrm{S}_{8}$ solid undergoes disproportionation reaction under alkaline conditions to form $\mathrm{S}^{2-}$ and $\mathrm{S}_{2} \mathrm{O}_{3}{ }^{2-}$.
Statement II : $\mathrm{ClO}_{4}^{-}$can undergo disproportionation reaction under acidic condition.
In the light of the above statements, choose the most appropriate answer from the options given below.
(A) Statement I is correct but statement II is incorrect
(B) Statement I is incorrect but statement II is correct
(C) Both statement I and statement II are incorrect
(D) Both statement I and statement II are correct
65. Identify major product ' P ' formed in the following reaction.

(A)

(B)

(C)

(D)

66. Major product of the following reaction is -

(A)

(B)

(C)

(D)

67. Identify structure of 2,3-dibromo-1-phenylpentane.
(A)

(B)

(C)

(D)

68. Select the option with correct property -
(A) $\left[\mathrm{Ni}(\mathrm{CO})_{4}\right]$ and $\left[\mathrm{NiCl}_{4}\right]^{2-}$ both diamagnetic
(B) $\quad\left[\mathrm{Ni}(\mathrm{CO})_{4}\right]$ and $\left[\mathrm{NiCl}_{4}\right]^{2-}$ both paramagnetic
(C) $\quad\left[\mathrm{NiCl}_{4}\right]^{2-}$ diamagnetic, $\left[\mathrm{Ni}(\mathrm{CO})_{4}\right]$ paramagnetic
(D) $\left[\mathrm{Ni}(\mathrm{CO})_{4}\right]$ diamagnetic, $\left[\mathrm{NiCl}_{4}\right]^{2-}$ paramagnetic
69. The azo-dye $(\mathrm{Y})$ formed in the following reactions is Sulphanilic acid $+\mathrm{NaNO}_{2}+\mathrm{CH}_{3} \mathrm{COOH} \rightarrow \mathrm{X}$.

(A)

(B)

(C)

(D)

70. Given below are two statements :

Statement I : Aniline reacts with conc. $\mathrm{H}_{2} \mathrm{SO}_{4}$ followed by heating at $453-473 \mathrm{~K}$ gives p-aminobenzene sulphonic acid, which gives blood red colour in the 'Lassaigne's test'.
Statement II : In Friedel-Craft's alkylation and acylation reactions, aniline forms salt with the $\mathrm{AlCl}_{3}$ catalyst. Due to this, nitrogen of aniline aquires a positive charge and acts as deactivating group.
In the light of the above statements, choose the correct answer from the options given below.
(A) Statement I is false but statement II is true.
(B) Both statement I and statement II are false.
(C) Statement I is true but statement II is false.
(D) Both statement I and statement II are true.
71. $\quad \mathrm{A}_{(\mathrm{g})} \rightleftharpoons \mathrm{B}_{(\mathrm{g})}+\frac{\mathrm{C}}{2}{ }_{(\mathrm{g})}$

The correct relationship between $K_{p}, \alpha$ and equilibrium pressure $P$ is :
(A) $K_{P}=\frac{\alpha^{1 / 2} P^{1 / 2}}{(2+\alpha)^{1 / 2}}$
(B) $\quad K_{P}=\frac{\alpha^{3 / 2} \mathrm{P}^{1 / 2}}{(2+\alpha)^{1 / 2}(1-\alpha)}$
(C) $K_{P}=\frac{\alpha^{1 / 2} P^{3 / 2}}{(2+\alpha)^{3 / 2}}$
(D) $\quad K_{P}=\frac{\alpha^{1 / 2} P^{1 / 2}}{(2+\alpha)^{3 / 2}}$
72. Choose the correct statements from the following.
A. All group 16 elements form oxides of general formula $\mathrm{EO}_{2}$ and $\mathrm{EO}_{3}$ where $\mathrm{E}=\mathrm{S}, \mathrm{Se}, \mathrm{Te}$ and Po. Both the types of oxides are acidic in nature.
B. $\mathrm{TeO}_{2}$ is an oxidising agent while $\mathrm{SO}_{2}$ is reducing in nature.
C. The reducing property decreases from $\mathrm{H}_{2} \mathrm{~S}$ to $\mathrm{H}_{2} \mathrm{Te}$ down the group.
D. The ozone molecule contains five lone pairs of electrons.

Choose the correct answer from the options given below.
(A) A and D only
(B) B and C only
(C) C and D only
(D) A and B only
73. Identify the name reaction.

(A) Stephen reaction
(B) Etard reaction
(C) Gatterman-koch reaction
(D) Rosenmund reduction
74. Which of the following is least ionic ?
(A) $\mathrm{BaCl}_{2}$
(B) AgCl
(C) KCl
(D) $\mathrm{CoCl}_{2}$
75. The fragrance of flowers is due to the presence of some steam volatile organic compounds called essential oils. These are generally insoluble in water at room temperature but are miscible with water vapour in vapour phase. A suitable method for the extraction of these oils from the flowers is -
(A) crystallisation
(B) distillation under reduced pressure
(C) distillation
(D) steam distillation

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76. Given below are two statements :

Statement I : Group 13 trivalent halides get easily hydrolyzed by water due to their covalent nature.
Statement II : $\mathrm{AlCl}_{3}$ upon hydrolysis in acidified aqueous solution forms octahedral $\left[\mathrm{Al}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}\right]^{3+}$ ion. In the light of the above statements, choose the correct answer from the options given below.
(A) Statement I is true but statement II is false
(B) Statement I is false but statement II is true
(C) Both statement I and statement II are false
(D) Both statement I and statement II are true
77. The four quantum numbers for the electron in the outer most orbital of potassium (atomic no. 19) are
(A) $\mathrm{n}=4, \ell=2, \mathrm{~m}=-1, \mathrm{~s}=+\frac{1}{2}$
(B) $\mathrm{n}=4, \ell=0, \mathrm{~m}=0, \mathrm{~s}=+\frac{1}{2}$
(C) $\mathrm{n}=3, \ell=0, \mathrm{~m}=1, \mathrm{~s}=+\frac{1}{2}$
(D) $\mathrm{n}=2, \ell=0, \mathrm{~m}=0, \mathrm{~s}=+\frac{1}{2}$
78. Choose the correct statements from the following.
A. $\mathrm{Mn}_{2} \mathrm{O}_{7}$ is an oil at room temperature
B. $\quad \mathrm{V}_{2} \mathrm{O}_{4}$ reacts with acid to give $\mathrm{VO}_{2}{ }^{2+}$
C. CrO is a basic oxide
D. $\quad \mathrm{V}_{2} \mathrm{O}_{5}$ does not react with acid

Choose the correct answer from the options given below :
(A) A, B and D only
(B) A and C only
(C) A, B and C only
(D) B and C only
79. The correct order of reactivity in electrophilic substitution reaction of the following compounds is :

(A) B $>$ C $>$ A $>$ D
(B) D $>$ C $>$ B $>$ A
(C) A $>$ B $>$ C $>$ D
(D) B $>$ A $>$ C $>$ D
80. Consider the following elements.

Group $\downarrow \begin{aligned} & \mathrm{A}^{\prime} \mathrm{B}^{\prime} \rightarrow \text { Period } \\ & \mathrm{C}^{\prime} \mathrm{D}^{\prime}\end{aligned}$
Which of the following is/are true about $\mathrm{A}^{\prime}, \mathrm{B}^{\prime}, \mathrm{C}^{\prime}$ and $\mathrm{D}^{\prime}$ ?
A. Order of atomic radii : $\mathrm{B}^{\prime}<\mathrm{A}^{\prime}<\mathrm{D}^{\prime}<\mathrm{C}^{\prime}$
B. Order of metallic character : $\mathrm{B}^{\prime}<\mathrm{A}^{\prime}<\mathrm{D}^{\prime}<\mathrm{C}^{\prime}$
C. Size of the element: $\mathrm{D}^{\prime}<\mathrm{C}^{\prime}<\mathrm{B}^{\prime}<\mathrm{A}^{\prime}$
D. Order of ionic radii : $\mathrm{B}^{\prime+}<\mathrm{A}^{\prime+}<\mathrm{D}^{\prime+}<\mathrm{C}^{\prime+}$

Choose the correct answer from the options given below :
(A) A only
(B) A, B and D only
(C) A and B only
(D) B, C and D only

## Section - B (Numerical Value Type)

81. A diatomic molecule has a dipole moment of 1.2 D . If the bond distance is $1 \AA$, then fractional charge on each atom is $\qquad$ $\times 10^{-1} \mathrm{esu}$.
[Given : $1 \mathrm{D}=10^{-18}$ esu cm ]
82. $r=k[A]$ for a reaction, $50 \%$ of $A$ is decomposed in 120 minutes. The time taken for $90 \%$ decomposition of $A$ is $\qquad$ minutes.
83. A compound ( x ) with molar mass $108 \mathrm{~g} \mathrm{~mol}^{-1}$ undergoes acetylation to give product with molar mass $192 \mathrm{~g} \mathrm{~mol}^{-1}$. The number of amino groups in the compound ( x ) is $\qquad$ .
84. Number of isomeric products formed by monochlorination of 2-methylbutane in presence of sunlight is
$\qquad$ _.
85. Number of moles of $\mathrm{H}^{+}$ions required by 1 mole of $\mathrm{MnO}_{4}^{-}$to oxidise oxalate ion to $\mathrm{CO}_{2}$ is $\qquad$ .
86. In the reaction of potassium dichromate, potassium chloride and sulfuric acid (conc.), the oxidation state of the chromium in the product is + $\qquad$ -
87. The molarity of 1 L orthophosphoric acid $\left(\mathrm{H}_{3} \mathrm{PO}_{4}\right)$ having $70 \%$ purity by weight (specific gravity 1.54 $\mathrm{g} \mathrm{cm}^{-3}$ ) is $\qquad$ M.
[Molar mass of $\mathrm{H}_{3} \mathrm{PO}_{4}=98 \mathrm{~g} \mathrm{~mol}^{-1}$ ]
88. The values of conductivity of some materials at $298.15 \mathrm{~K} \mathrm{in}_{\mathrm{Sm}}{ }^{-1}$ are $2.1 \times 10^{3}, 1.0 \times 10^{-16}, 1.2 \times 10,3.91$, $1.5 \times 10^{-2}, 1 \times 10^{-7}, 1.0 \times 10^{3}$. The number of conductors among the materials is $\qquad$ .
89. From the vitamins $A, B_{1}, B_{6}, B_{12}, C, D, E$ and $K$, the number vitamins that can be stored in our body is
$\qquad$ _.
90. If 5 moles of an ideal gas expands from 10 L to a volume of 100 L at 300 K under isothermal and reversible condition then work, $w$, is $-x \mathrm{~J}$. The value of x is $\qquad$ .
[Given : $\mathrm{R}=8.314 \mathrm{~J} \mathrm{~K}^{-1} \mathrm{~mol}^{-1}$ ]

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## 31-Jan. 2024 (Evening) : PCM

| MATHEMATICS |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Single Choice Correct |  |  |  |  |  |  |  |  |
| 1. D | 2. | C | 3. | B | 4. | B | 5. | C |
| 6. B | 7. | D | 8. | C | 9. | D | 10. | B |
| 11. C | 12. | C | 13. | A | 14. | C | 15. | D |
| 16. B | 17. | D | 18. | A | 19. | A | 20. | C |
| Numerical Value |  |  |  |  |  |  |  |  |
| 21. 15 | 22. | 36 | 23. | 32 | 24. | 25 | 25. | 7 |
| 26. 38 | 27. | 81 | 28. | 22 | 29. | 9 | 30. | 66 |
| PHYSICS |  |  |  |  |  |  |  |  |
| Single Choice Correct |  |  |  |  |  |  |  |  |
| 31. A | 32. | B | 33. | C | 34. | B | 35. | D |
| 36. A | 37. | A | 38. | C | 39. | D | 40. | B |
| 41. C | 42. | B | 43. | C | 44. | C | 45. | A |
| 46. A | 47. | A | 48. | D | 49. | C | 50. | B |
| Numerical Value |  |  |  |  |  |  |  |  |
| 51. 3 | 52. | 200 | 53. | 2 | 54. | 12 | 55. | 8 |
| 56. 20 | 57. | 27 | 58. | 53 | 59. | 12 | 60. | 4 |
| CHEMISTRY |  |  |  |  |  |  |  |  |
| Single Choice Correct |  |  |  |  |  |  |  |  |
| 61. D | 62. | A | 63. | A | 64. | A | 65. | D |
| 66. C or D | 67. | C | 68. | D | 69. | D | 70. | D |
| 71. B | 72. | D | 73. | C | 74. | B | 75. | D |
| 76. D | 77. | B | 78. | B | 79. | D | 80. | B |
| Numerical Value |  |  |  |  |  |  |  |  |
| 81. 0 | 82. | 399 | 83. | 2 | 84. | 6 | 85. | 8 |
| 86. 6 | 87. | 11 | 88. | 4 | 89. | 5 | 90. | 28721 |

