31-Jan.-2024 (Evening) : PCM [SOLUTIONS]

MATHEMATICS

Section - A (Single Correct Answer)

1. D

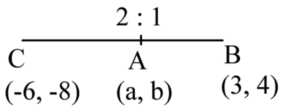
Sol. After giving 2 apples to each child 15 apples left now 15 apples can be distributed in

$^{15+3-1}C_2 = {}^{17}C_2$ ways

$$=\frac{17\times16}{2}=136$$

2. C

Sol. A(a, b), B(3, 4), C(–6, –8)



$\Rightarrow$ a = 0, b = 0 $\quad\Rightarrow$ P(3, 5)

Distance from P measured along x – 2y – 1 = 0

$\Rightarrow$ x = 3 + r cos θ, y = 5 + r sin θ

Where $\tan\theta=\frac{1}{2}$

$$r(2\cos\theta+3\sin\theta)=-17$$

$$\Rightarrow r=\left|\frac{-17\sqrt{5}}{7}\right|=\frac{17\sqrt{5}}{7}$$

3. B

Sol. $z_1 + z_2 = 5$

$$z_1^3+z_2^3=20+15i$$

$$z_1^3+z_2^3=(z_1+z_2)^3-3z_1z_2(z_1+z_2)$$

$$z_1^3+z_2^3=125-3z_1\cdot z_2(5)$$

$\Rightarrow 20 + 15i = 125 - 15z_1z_2$

$\Rightarrow 3z_1z_2 = 25 - 4 - 3i$

$\Rightarrow 3z_1z_2 = 21 - 3i$

$\Rightarrow z_1 \cdot z_2 = 7 - i$

$\Rightarrow (z_1 + z_2)^2 = 25$

$$\Rightarrow z_1^2+z_2^2=25-2(7-i)$$

$$\Rightarrow 11+2i$$

$$(z_1^2+z_2^2)^2=121-444i$$

$$\Rightarrow z_1^4+z_2^4+2(7-i)^2=117+44i$$

$$\Rightarrow z_1^4+z_2^4=117+44i-2(49-1-14i)$$

$$\Rightarrow |z_1^4+z_2^4|=75$$

4. B

Sol. (y – 2) = m(x – 8)

$\Rightarrow$ x-intercept

$$\Rightarrow\left(\frac{-2}{m}+8\right)$$

$\Rightarrow$ y-intercept

$\Rightarrow$ (–8m + 2)

$$\Rightarrow OA+OB=\frac{-2}{m}+8-8m+2$$

$$f'(m)=\frac{2}{m^2}-8=0$$

$$\Rightarrow m^2=\frac{1}{4}$$

$$\Rightarrow m=\frac{-1}{2}$$

$$\Rightarrow f\left(\frac{-1}{2}\right)=18$$

$\Rightarrow$ Minimum = 18

5. C

Sol. $f(x)=\int_{-x}^{x}(|t|-t^2)e^{-t^2}dt$

$$\Rightarrow f'(x)=2,\ (|x|-x^2)e^{-x^2}\ ....(1)$$

$$g(x)=\int_0^{x^2}t^{\frac{1}{2}}e^{-t}dt$$

$$g'(x)=xe^{-x^2}(2x)-0$$

$$f'(x)+g'(x)=2xe^{-x^2}-2x^2e^{-x^2}+2x^2e^{-x^2}$$

Integrating both sides w.r.t. x

$$f(x)=+g(x)=\int_0^{\alpha} 2xe^{-x^2}dx$$

$x^2 = t$

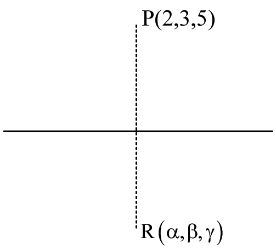
$$\Rightarrow \int_0^{\sqrt{\alpha}} e^{-t}dt=[-e^{-t}]_0^{\sqrt{\alpha}}$$

$$=-e^{(\log_e(9)^{-1})+1}$$

$$\Rightarrow 9(f(x)g(x))=\left(1-\frac{1}{9}\right)9=8$$

6. B

Sol.



$\because \overrightarrow{PR} \perp (2,\ 3,\ 4)$

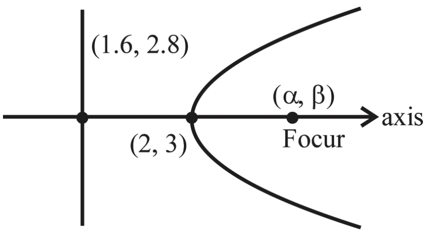
$\therefore \overrightarrow{PR}\cdot(2,\ 3,\ 4)=0$

$(\alpha-2,\ \beta-3,\ \gamma-5)\cdot(2,\ 3,\ 4)=0$

$\Rightarrow\ 2\alpha+3\beta+4\gamma=4+9+20=33$

7. D

Sol.



Slope of axis $=\frac{1}{2}$

$$y-3=\frac{1}{2}(x-2)$$

$$\Rightarrow 2y-6=x-2$$

$$\Rightarrow 2y-x-4=0$$

$2x+y-6=0$

$4x+2y-12=0$

$\alpha+1.6=4\Rightarrow\alpha=2.4$

$\beta+2.8=6\Rightarrow\beta=3.2$

Ellipse passes through (2.4, 3.2)

$$\Rightarrow \frac{\left(\frac{24}{10}\right)^2}{a^2}+\frac{\left(\frac{32}{10}\right)^2}{b^2}=1 \quad ....(1)$$

Also $1-\frac{b^2}{a^2}=\frac{1}{2}=\frac{b^2}{a^2}=\frac{1}{2}$

$\Rightarrow a^2=2b^2$

Put in (1) $\Rightarrow b^2=\frac{328}{25}$

$$\Rightarrow \left(\frac{2b^2}{a}\right)^2=\frac{4b^2}{a^2}\times b^2=4\times\frac{1}{2}\times\frac{328}{25}=\frac{656}{25}$$

8. C

Sol. $\frac{dT}{dt}=-k(T-80)$

$$\int_{160}^{T}\frac{dT}{(T-80)}=\int_0^1 -Kdt$$

$[\ln|T-80|]_{160}^{T}=-kt$

$\ln|T-80|-\ln 80=-kt$

$$\ln\left|\frac{T-80}{80}\right|=-kt$$

$T=80+80e^{-kt}$

$120=80+80e^{-k.45}$

$$\frac{40}{80}=e^{-k15}=\frac{1}{2}$$

$\therefore T(45)=80+80e^{-k.45}$

$=80+80(e^{-k.15})^3$

$$=80+80\times\frac{1}{8}$$

$=90$

9. D

Sol. $1+d,\ 1+7d,\ 1+43d$ are in GP

$(1+7d)^2=(1+d)(1+43d)$

$1+49d^2+14d=1+44d+43d^2$

$6d^2-30d=0$

$d=5$

$$S_{20}=\frac{20}{2}[2\times1+(20-1)\times5]$$

$= 10\ [2 + 95]$

$= 970$

10. B

Sol. $f:R\to(0,\ \infty)$

$$\lim_{x\to\infty}\frac{f(7x)}{f(x)}=1$$

$\because$ f is increasing

$\therefore f(x)<f(5x)<f(7x)$

$$\because \frac{f(x)}{f(x)}<\frac{f(5x)}{f(x)}<\frac{f(7x)}{f(x)}$$

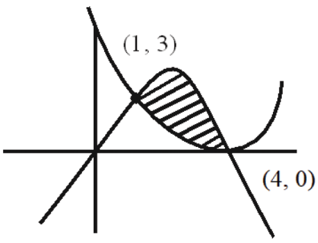
$$1<\lim_{x\to0}\frac{f(5x)}{f(x)}<1$$

$$\therefore \left[\frac{f(5x)}{f(x)}-1\right]$$

$\Rightarrow 1-1=0$

11. C

Sol.



$$\text{Area}=\left|\int_1^4\left[(4x-x^2)-\frac{(x-4)^3}{3}\right]\right|dx$$

$$\text{Area}=\left|\frac{4x^2}{2}-\frac{x^3}{3}-\frac{(x-4)^3}{9}\right|_1^4$$

$$=\left|\left(\frac{64}{2}-\frac{64}{3}-\frac{4}{2}+\frac{1}{3}-\frac{27}{9}\right)\right|$$

$\Rightarrow (27-21)=6$

12. C

Sol. a, b, 68, 44, 48, 60

Mean = 55 $\qquad$ a > b

Variance = 194 $\qquad$ a + 3b

$$\frac{a+b+68+44+48+60}{6}=55$$

$\Rightarrow$ 220 + a + b = 330

$\therefore$ a + b = 110 $\qquad$ ......(1)

Also,

$$\sum\frac{(x_i-\overline{x})^2}{n}=194$$

$\Rightarrow (a-55)^2+(b-55)^2+(68-55)^2$
$+ (44-55)^2+(48-55)^2+(60-55)^2=194\times6$

$\Rightarrow (a-55)^2+(b-55)^2+169+121+49+25=1164$

$\Rightarrow (a-55)^2+(b-55)^2=1164-364=800$

$a^2+3025-110a+b^2+3025-110b=800$

$\Rightarrow a^2+b^2=800-6050+12100$

$a^2+b^2=6850$ $\qquad$ ......(2)

Solve (1) & (2);

a = 75, b = 35

$\therefore$ a + 3b = 75 + 3 (35) = 75 +105 = 180

13. A

Sol. $f(x)=e^{x^3-3x+1}$

$$f'(x)=e^{x^3-3x+1}\cdot(3x^2-3)$$

$$=e^{x^3-3x+1}\cdot3(x-1)(x+1)$$

For $f'(x)\ge0$

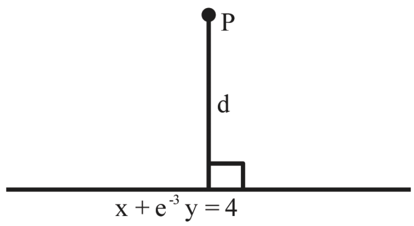
$\therefore f(x)$ is increasing function

$\therefore a={}^{-\infty}=f(-\infty)$

$b=e^{-1+31}=e^3=f(-1)$

P(2b + 4, a + 2)

$\therefore P(2e^3+4,\ 2)$



$$d=\frac{(2e^3+4)+2e^{-3}-4}{\sqrt{1+e^{-6}}}=2\sqrt{1+e^6}$$

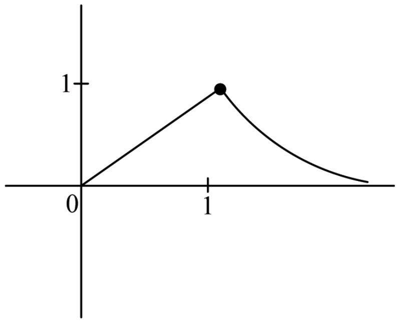
14. C

$f:(0,\infty)\to R$

$f(x)=e^{-|\log_e x|}$

$$f(x)=\frac{1}{e^{|\ln x|}}=\begin{cases}\frac{1}{e^{-\ln x}}; & 0<x<1\\ \frac{1}{e^{\ln x}}; & x\ge 1\end{cases}$$

$$\begin{cases}\frac{1}{\frac{1}{x}}=x; & 0<x<1\\ \frac{1}{x}, & x\ge 1\end{cases}$$



m = 0 (No point at which function is not continuous) n = 1 (Not differentiable)

$\therefore\ m+n=1$

15. D

Sol. Take $e^{\sin x}=t(t>0)$

$\Rightarrow t-\frac{2}{t}=2$

$\Rightarrow \frac{t^2-2}{t}=2$

$\Rightarrow t^2-2t-2=0$

$\Rightarrow t^2-2t+1=3$

$\Rightarrow (t-1)^2=3$

$\Rightarrow t=1\pm\sqrt{3}$

$\Rightarrow t=1\pm1.73$

$\Rightarrow$ t = 2.73 or –0.73 (rejected as t > 0)

$\Rightarrow e^{\sin x}=2.73$

$\Rightarrow \log_e e^{\sin x}=\log_e 2.73$

$\Rightarrow \sin x=\log_e 2.73>1$

So no solution.

16. B

Sol. $a=\sin^{-1}(\sin 5)=5-2\pi$

and $b=\cos^{-1}(\cos 5)=2\pi-5$

$\therefore a^2+b^2=(5-2\pi)^2+(2\pi-5)^2$

$=8\pi^2-40\pi+50$

17. D

Sol. ${}^6C_m+2({}^6C_{m+1})+{}^6C_{m+2}>{}^8C_3$

${}^7C_{m+1}+{}^7C_{m+2}>{}^8C_3$

${}^8C_{m+2}>{}^8C_3$

$\therefore m=2$

And ${}^{n-1}P_3:{}^nP_4=1:8$

$$\frac{(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)}=\frac{1}{8}$$

$\therefore n=8$

$\therefore {}^nP_{m+1}+{}^{n+1}C_m={}^8P_3+{}^9C_2$

$=8\times7\times6+\frac{9\times8}{2}$

$=372$

18. A

Sol. Let probability of tail is $\frac{1}{3}$

$\Rightarrow$ Probability of getting head $=\frac{2}{3}$

$\therefore$ Probability of getting 2 tails and 1 head

$=\left(\frac{1}{3}\times\frac{2}{3}\times\frac{1}{3}\right)\times3$

$=\frac{2}{27}\times3$

$=\frac{2}{9}$

19. A

Sol. Let $A=\begin{bmatrix}x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{bmatrix}$

Given $A\begin{bmatrix}1\\0\\1\end{bmatrix}=\begin{bmatrix}2\\0\\2\end{bmatrix}$ .....(1)

$$\therefore \begin{bmatrix} x_1+z_1 \\ x_2+z_2 \\ x_3+z_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

$\therefore x_1 + z_1 = 2$ .....(2)

$x_2 + z_2 = 0$ ....(3)

$x_3 + z_3 = 0$ ....(4)

$$\text{Given } A\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \\ 4 \end{bmatrix}$$

$$\therefore \begin{bmatrix} -x_1+z_1 \\ -x_2+z_2 \\ -x_3+x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$$

$\Rightarrow -x_1 + z_1 = -4$ ....(5)

$-x_2 + x_2 = 0$ .....(6)

$-x_3 + z_3 = 4$

$$\text{Given } A\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$\therefore\ y_1 = y_2 = 2,\ y_3 = 0$

$\therefore$ from (2), (3), (4), (5), (6) and (7)

$x_1 = 3x,\ x_2 = 0,\ x_3 = -1$

$y_1 = 0,\ y_2 = 2,\ y_3 = 0$

$z_1 = -1,\ z_2 = 0,\ z_3 = 3$

$$\therefore A = \begin{bmatrix} 3 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$

$$\therefore \text{Now } (A-3)\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} -z \\ -y \\ -x \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$[z = -1], [y = -2], [x = -3]$

20. C

Sol. $L_2 = \dfrac{x+4}{3} = \dfrac{y-4}{2} = \dfrac{z-3}{0}$

$$\therefore S.D = \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ 2 & -3 & 2 \\ 3 & 2 & 0 \end{vmatrix}}{|\vec{n}_1 \times \vec{n}_2|}$$

$$= \frac{\begin{vmatrix} 5 & -5 & -7 \\ 2 & -3 & 2 \\ 3 & 2 & 0 \end{vmatrix}}{|\vec{n}_1 \times \vec{n}_2|}$$

$$= \frac{141}{|-4\hat{i} + 6\hat{j} + 13\hat{k}|}$$

$$= \frac{141}{\sqrt{16+36+169}}$$

$$= \frac{141}{\sqrt{221}}$$

Section - B (Numerical Value Type)

21. 15

Sol. $\displaystyle\int_0^{\pi} \frac{x^2 \sin x \cdot \cos x}{\sin^4 x + \cos^4 x} dx$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x \cdot \cos x}{\sin^4 x + \cos^4 x}(x^2 - (\pi - x)^2)dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x \cdot \cos x(2\pi x - \pi^2)}{\sin^4 x + \cos^4 x}$$

$$= 2\pi\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx - \pi^2 \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx$$

$$= 2\pi \cdot \frac{\pi}{4}\int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x}dx - \pi^2 \int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x}dx$$

$$= -\frac{\pi^2}{2}\int_0^{\frac{\pi}{2}} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x}dx$$

$$= -\frac{\pi^2}{2}\int_0^{\frac{\pi}{2}} \frac{\sin x \cos x\, dx}{1 - \sin^2 x \times \cos^2 x}$$

$$= -\frac{\pi^2}{2}\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{2 - \sin^2 2x}dx$$

$$= -\frac{\pi^2}{2}\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{1 + \cos^2 2x}dx$$

Let $\cos 2x = t$

22. 36

Sol. $(a^2 + b^2) x^2 - 2b (a + c) x + b^2 + c^2 = 0$

$\Rightarrow a^2x^2 - 2abx + b^2 + b^2x^2 - 2bcx + c^2 = 0$

$\Rightarrow (ax - b)^2 + (bx - c)^2 = 0$

$\Rightarrow ax - b = 0, bx - c = 0$

$\Rightarrow a + b > c \quad b + c > a \quad c + a > b$

$a + ax > bx$	$ax + bx > a$	$ax^2 + a > ax$
$a + ax > ax^2$	$ax + ax^2 > a$	$x^2 - x + 1 > 0$
$x^2 - x - 1 < 0$	$x^2 + x - 1 > 0$	always true

$$\frac{1-\sqrt{5}}{2} < x < \frac{1+\sqrt{5}}{2}$$

$$x < \frac{-1-\sqrt{5}}{2}, \text{ or } x > \frac{-1+\sqrt{5}}{2}$$

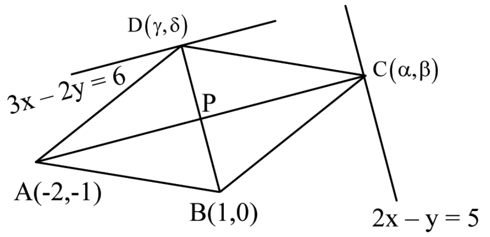
$$\Rightarrow \frac{\sqrt{5}-1}{2} < x < \frac{\sqrt{5}+1}{2}$$

$$\Rightarrow \alpha = \frac{\sqrt{5}-1}{2},\ \beta = \frac{\sqrt{5}+1}{2}$$

$$12(\alpha^2 + \beta^2) = 12\left(\frac{(\sqrt{5}-1)^2 + (\sqrt{5}+1)^2}{4}\right) = 36$$

23. 32

Sol.



$$P \equiv \left(\frac{\alpha-2}{2}, \frac{\beta-1}{2}\right) \equiv \left(\frac{\gamma+1}{2}, \frac{\delta}{2}\right)$$

$$\frac{\alpha-2}{2} = \frac{\gamma+1}{2} \text{ and } \frac{\beta-1}{2} = \frac{\delta}{2}$$

$\Rightarrow \alpha - \gamma = 3$ ......(1), $\beta - \delta = 1$ ....(2)

Also, (γ, δ) lies on $3x - 2y = 6$

$3\gamma - 2\delta = 6$ .....(3)

and (α, β) lies on $2x - y = 5$

$\Rightarrow 2\alpha - \beta = 5$ ....(4)

Solving (1), (2), (3), (4)

$\alpha = -3,\ \beta = -11,\ \gamma = -6,\ \delta = -12$

$|\alpha + \beta + \gamma + \delta| = 32$

24. 25

Sol. $(x + 3)^{n-1} + (x + 3)^{n-2} (x + 2) + (x + 3)^{n-3} (x + 2)^2 + \ldots\ldots + (x + 2)^{n-1}$

$$\sum \alpha_r = 4^{n-1} + 4^{n-2} \times 3 + 4^{n-1} \times 3^2 \ldots\ldots + 3^{n-1}$$

$$= 4^{n-1}\left[1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 \ldots. + \left(\frac{3}{4}\right)^{n-1}\right]$$

$$= 4^{n-1} \times \frac{1 - \left(\frac{3}{4}\right)^n}{1 - \frac{3}{4}}$$

$$= 4^n - 3^n = \beta^n - \gamma^n$$

$\beta = 4,\ \gamma = 3$

$\beta^2 + \gamma^2 = 16 + 9 = 25$

25. 7

Sol. $|A| = 2$

$$\underbrace{\text{adj}(\text{adj}(\text{adj}\ldots(a)))}_{2024 \text{ times}} = |A|^{(n-1)^{2024}}$$

$$= |A|^{2^{2024}}$$

$= 2^{2^{2024}}$

$2^{2024} = (2^2)\ 2^{2022} = 4(8)^{674} = 4(9-1)^{674}$

$\Rightarrow 2^{2024} \equiv 4 (\text{mod } 9)$

$\Rightarrow 2^{2024} \equiv 9m + 4,\ m \leftarrow \text{even}$

$2^{9m+4} \equiv 16 \cdot (2^3)^{3m} = 16(\text{mol } 9)$

$\equiv 7$

26. 38

Sol. $(\vec{a} \times \vec{b}) \times \vec{c} = 2(\vec{a} \times \vec{b}) + 24\hat{j} - 6\hat{k}$

$(5\hat{i} + \hat{j} + 4\hat{k}) \times \vec{c} = 2(7\hat{i} - 7\hat{j} - 7\hat{k}) + 24\hat{j} - 6\hat{k}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 1 & 4 \\ x & y & z \end{vmatrix} = 14\hat{i} + 10\hat{j} - 20\hat{k}$$

$\Rightarrow \hat{i}(z - 4y) - \hat{j}(5z - 4x) + \hat{k}(5y - x) = 14\hat{i} + 10\hat{j} - 20\hat{k}$

$z - 4y = 14,\ 4x - 5z = 10,\ 5y - x = -20$

$(a - b + i) \cdot \vec{c} = -3$

$(2\hat{i} + 3\hat{j} - 2\hat{k}) \cdot \vec{c} = -3$

$2x + 3y - 2z = -3$

$\therefore\ x = 5,\ y = -3,\ z = 2$

$|\vec{c}|^2 = 25 + 9 + 4 = 38$

27. 81

Sol. $$\lim_{x \to 0} \frac{ax^2\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...\right) - b\left(x - \frac{x^2}{2} + \frac{x^3}{3} - ....\right) + cx\left(1 - x + \frac{x^2}{x!} - \frac{x^3}{3!} + ...\right)}{x^3 \cdot \frac{\sin x}{x}}$$

$$= \lim_{x \to 0} \frac{(c - b)x + \left(\frac{b}{2} - c + a\right)x^2 + \left(a - \frac{b}{3} + \frac{c}{2}\right)x^3 + ...}{x^3} = 1$$

$c - b = 0,\ \frac{b}{2} - c + a = 0$

$a - \frac{b}{3} + \frac{c}{2} = 1 \qquad a = \frac{3}{4} \qquad b = c = \frac{3}{2}$

$a^2 + b^2 + c^2 = \frac{9}{16} + \frac{9}{4} + \frac{9}{4}$

$16(a^2 + b^2 + c^2) = 81$

28. 22

Sol. $\frac{x - 4}{12} = \frac{x + 6}{4} = \frac{z + 2}{6}$

$$\frac{x - 4}{\frac{6}{7}} = \frac{y + 6}{\frac{2}{7}} = \frac{z + 2}{\frac{3}{7}} = 21$$

$$\left(21 \times \frac{6}{7} + 4,\ \frac{2}{7} \times 21 - 6,\ \frac{3}{7} \times 21 - 2\right)$$

$= (22, 0, 7) = (a, b, c)$

$\therefore \sqrt{324 + 144 + 16} = 22$

29. 9

Sol. $\sec^2 x \frac{dx}{dy} + e^{2y} \tan^2 x + \tan x = 0$

$$\left(\text{Put } \tan x = t \Rightarrow \sec^2 x \frac{dx}{dy} = \frac{dt}{dy}\right)$$

$\frac{dt}{dy} + e^{2y} \times t^2 + t = 0$

$\frac{dt}{dy} + t = -t^2 \cdot e^{2y}$

$\frac{1}{t^2} \frac{dt}{dy} + \frac{1}{t} = -e^{2y}$

$$\left(\text{Put } \frac{1}{t} = u \quad \frac{-1}{t^2} \frac{dt}{dy} = \frac{du}{dy}\right)$$

$\frac{-d}{dy} + u = -e^{2y}$

$\frac{du}{dy} - u = e^{2y}$

$\text{I.F.} = e^{-\int dy} = e^{-y}$

$ue^{-y} = \int e^{-y} \times e^{2y} dy$

$\frac{1}{\tan x} \times e^{-y} = e^y + c$

$x = \frac{\pi}{4},\ y = 0,\ c = 0$

$x = \frac{\pi}{6},\ y = \alpha$

$\sqrt{3}e^{-\alpha} = e^{\alpha} + 0$

$e^{2\alpha} = \sqrt{3}$ $\qquad$ $e^{8\alpha} = 9$

30. 66

Sol. R = {(3, 2), (6, 4), (9, 6), (12, 8),..........(99, 66)}

n(R) = 33

∴ 66

PHYSICS

Section - A (Single Correct Answer)

31. A

Sol. $a = \frac{(m_1 - m_2)g}{(m_1 + m_2)} = \frac{g}{8}$

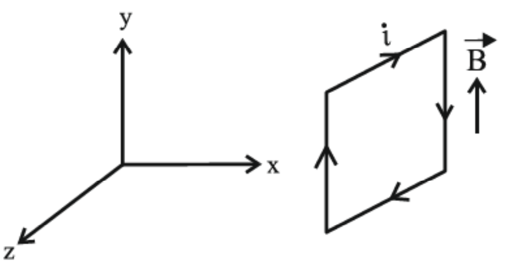
$8m_1 - 8m_2 = m_1 + m_2$

$7m_1 = 9m_2$

$\frac{m_1}{m_2} = \frac{9}{7}$

32. B

Sol.



$\vec{M} = i\vec{A}$

$= 5 \times (0.2) \times (0.1)\left(-\hat{i}\right)$

$\vec{\tau} = \vec{M} \times \vec{B} = 0.1\left(-\hat{i}\right) \times \left(2 \times 10^{-3}\right)\left(\hat{j}\right)$

$= 2 \times 10^{-4}\left(-\hat{k}\right)$ N-m

33. C

Sol. $T = 2\pi\sqrt{\frac{l}{g}}$

$g = \frac{4\pi^2 l}{T^2}$

$\frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T}$

$= \frac{0.2}{20} + 2\left(\frac{1}{40}\right) = \frac{0.3}{20}$

Percentage change $= \frac{0.3}{20} \times 100 = 6\%$

34. B

Sol. In air $F = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r_2}$

In medium

$F' = \frac{1}{4\pi(K\epsilon_0)} \frac{q_1 q_2}{(r')^2} = \frac{25}{4\pi(5\epsilon_0)} \frac{q_1 q_2}{(r)^2} = 5F$

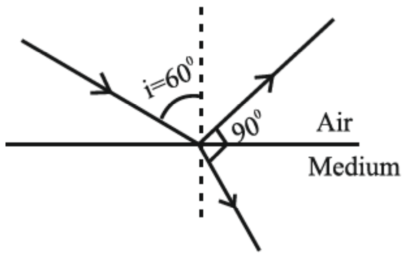
35. D

Sol. < P > = IV cosϕ

$= \frac{20}{\sqrt{2}} \times \frac{10}{\sqrt{2}} \times \cos 60° = 50W$

36. A

Sol. By Brewster's law



At complete reflection refracted ray and reflected ray are perpendicular.

37. A

Sol. $v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{1.4 \times 8.3 \times 273}{32 \times 10^{-3}}}$

$= 314.8541 \simeq 315 m/s$

38. C

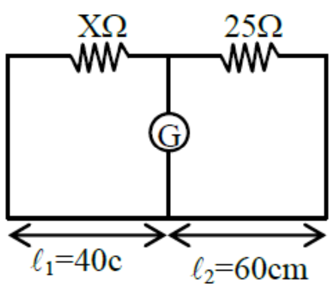
Sol. $U = nC_V T$

$\Rightarrow U = n_1 C_{V_1} T + n_2 C_{V_2} T$

$\Rightarrow 8 \times \frac{3R}{2} \times T + 6 \times \frac{5R}{2} \times T = 27RT$

39. D

Sol.



$$\frac{25}{rl_1} = \frac{X}{rl_2} \quad .....(i)$$

$$\frac{25}{2rl_1'} = \frac{X}{2rl_2'} \quad ..... (ii)$$

From (i) and (ii)

$l'_2 = l_2 = 40$ cm

40. B

Sol. $\frac{1}{2}\varepsilon_0 E^2 = \frac{B^2}{2\mu_0}$

$\because$ E = CB and $C = \frac{1}{\mu_0 \varepsilon_0}$

41. C

Sol. Since $\frac{f}{2} < f_0$ i.e. the incident frequency is less than threshold frequency. Hence there will be no emission of photoelectrons.

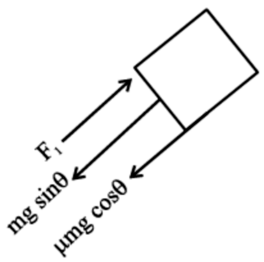
$\Rightarrow$ current = 0

42. B

Sol. $f_K = \mu mg \cos \theta$

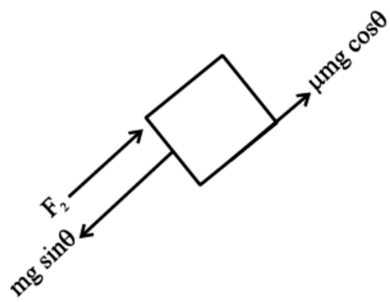
$= 0.1 \times \frac{50 \times \sqrt{3}}{2}$

$= 2.5\sqrt{3}N$



$F_1 = mg \sin \theta + f_K$

$= 25 + 2.5\sqrt{3}$



$F_2 = mg \sin \theta - f_K$

$= 25 - 2.5\sqrt{3}$

$\therefore F_1 - F_2 = 5\sqrt{3}N$

43. C

Sol. $P = i^2R$

$P_{int} = I^2_{int}\ R$

$P_{final} = (0.8\ I_{int})^2\ R$

% change in power =

$$\frac{P_{final} - P_{int}}{P_{int}} \times 100 = (0.64 - 1) \times 100 = -36\%$$

44. C

Sol. The magnitude of resultant vector

$R' = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

Here a = b = R

Then $R' = \sqrt{R^2 + R^2 + 2R^2\cos\theta}$

$= R\sqrt{2}\sqrt{1+\cos\theta}$

$= \sqrt{2}R\sqrt{2\cos^2\frac{\theta}{2}}$

$= 2R\cos\frac{\theta}{2}$

45. A

Sol. $R_1 = \frac{R_2}{2}$

$R_0 (A_1)^{1/3} = \frac{R_0}{2}(A_2)^{1/3}$

$A_1 = \frac{1}{8}A_2$

$A_1 = \frac{192}{8} = 24$

46. A

Sol. $V_{escape} = \sqrt{\frac{2GM}{R}}$

$V_{planet} = \sqrt{\frac{2GM}{R}} = V$

$V_{Moon} = \sqrt{\frac{2GM \times 16}{144R}} = \frac{1}{3}\sqrt{\frac{2GM}{R}}$

$V_{Moon} = \frac{V_{Planet}}{3} = \frac{V}{3}$

47. A

Sol. Since density is negligible hence Buoyancy force will be negligible

At terminal velocity.

$Mg = 6\pi\eta rv$

$V \propto \frac{1}{r}$ (as mass is constant)

Now, $\frac{v}{v'} = \frac{r'}{r}$

$r' = 2r$

So, $v' = \frac{v}{2}$

48. D

Sol. $\vec{F} = \left(6t\hat{i} + 6t^2\hat{j}\right)N$

$$\vec{F} = m\vec{a} = \left(6t\hat{i} + 6t^2\hat{j}\right)$$

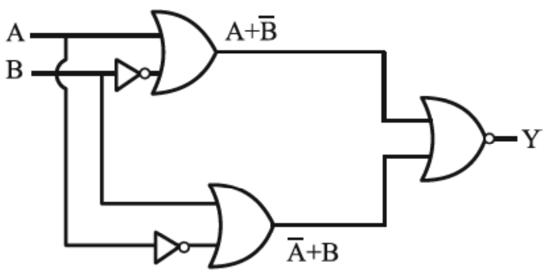
$$\vec{a} = \frac{\vec{F}}{m} = \left(3t\hat{i} + 3t^2\hat{j}\right)$$

$$\vec{v} = \int_0^t \vec{a}dt = \frac{3t^2}{2}\hat{i} + t^3\hat{j}$$

$$P = \vec{F}\cdot\vec{v} = \left(9t^3 + 6t^5\right)W$$

49. C

Sol.



If $A = 0; \bar{A} = 1$

$A = 1; \bar{A} = 0$

$B = 0; \bar{B} = 1$

$B = 1; \bar{B} = 0$

$$Y = \overline{\left(A + \bar{B}\right) + \left(\bar{A} + B\right)} = \overline{(1+1)} = 0$$

50. B

Sol. $[B] = L^2$

$$A = \frac{x^2}{tE} = \frac{L^2}{TML^2T^{-2}} = \frac{1}{MT^{-1}}$$

$$[A] = M^{-1}T$$

$$[AB] = \left[L^2M^{-1}T^1\right]$$

Section - B (Numerical Value Type)

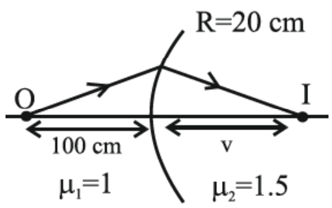
51. 3

Sol. $R_{eq} = \frac{4}{3}\Omega$

$$\therefore P = \frac{V^2}{R_{eq}} = \frac{4}{4/3} = 3W$$

52. 200

Sol.



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.5}{v} - \frac{1}{-100} = \frac{1.5 - 1}{20}$$

$v = 100$ cm

Distance from object

$= 100 + 100 = 200$ cm

53. 2

Sol. $\varepsilon = -\left(\frac{d\phi}{dt}\right) = 10t - 36$

at $t = 2$, $\varepsilon = 16$ V

$$i = \frac{\varepsilon}{R} = \frac{16}{8} = 2A$$

54. 12

Sol. $T = \left(\frac{2m_1m_2}{m_1 + m_2}\right)g = \frac{80}{3}N$

$$A = \pi r^2 = 16\pi \times 10^{-10} m^2$$

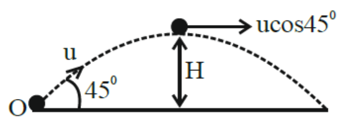
$$\text{Strain} = \frac{\Delta l}{l} = \frac{F}{AY} = \frac{T}{AY}$$

$$= \frac{80/3}{16\pi \times 10^{-10} \times 2 \times 10^{11}} = \frac{1}{12\pi}$$

$\alpha = 12$

55. 8

Sol.

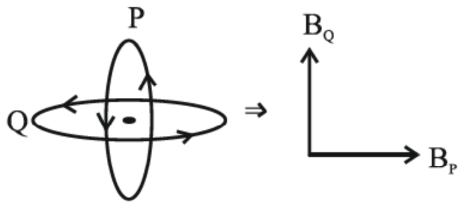


$$L = mu\cos\theta \frac{u^2\sin^2\theta}{2g}$$

$$= mu^3 \frac{1}{4\sqrt{2}} \Rightarrow x = 8$$

56. 20

Sol.



$$B_P = \frac{\mu_0 N i_1}{2r} = \frac{\mu_0 \times 1 \times 100}{2\pi} = 2 \times 10^{-3}\,T$$

$$B_Q = \frac{\mu_0 N i_2}{2r} = \frac{\mu_0 \times 2 \times 100}{2\pi} = 4 \times 10^{-3}\,T$$

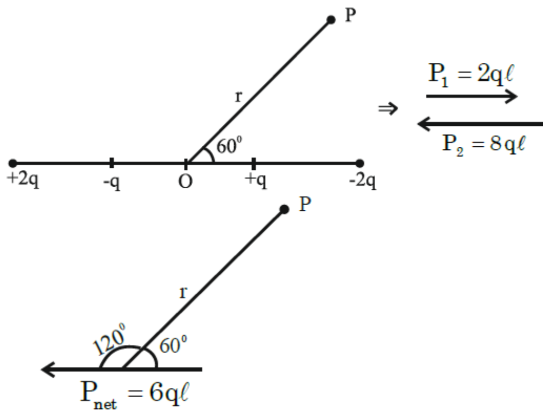
$$B_{net} = \sqrt{B_P^2 + B_Q^2}$$

$$= \sqrt{20}\,mT$$

$x = 20$

57. 27

Sol.



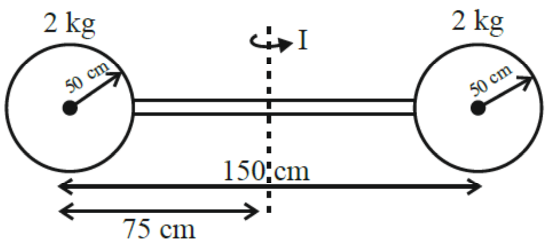
$$V = \frac{K\vec{p}\cdot\vec{r}}{r^3} = \frac{9\times10^9(6ql)}{r^2}\cos(120°)$$

$$= -(27)\left(\frac{ql}{r^2}\right)\times 10^9\ Nm^2c^{-2}$$

$\Rightarrow \quad \alpha = 27$

58. 53

Sol.



$$I = \left(\frac{2}{5}mR^2 + md^2\right)\times 2$$

$$I = 2\left(\frac{2}{5}\times 2\times\left(\frac{1}{2}\right)^2 + 2\times\left(\frac{3}{4}\right)^2\right) = \frac{53}{20}\,kg - m^2$$

$X = 53$

59. 12

Sol. $k_{eq} = \frac{2k\cdot k}{3k} + k = \frac{5k}{3}$

Angular frequency of oscillation $(\omega) = \sqrt{\frac{k_{eq}}{m}}$

$$(\omega) = \sqrt{\frac{5k}{3m}}$$

Period of oscillation $(\tau) = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3m}{5k}}$

$$= \pi\sqrt{\frac{12m}{5k}}$$

60. 4

Sol. For a nucleus

Volume: $V = \frac{4}{3}\pi R^3$

$$R = R_0(A)^{1/3}$$

$$V = \frac{4}{3}\pi R_0^3 A$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{A_2}{A_1} = 4$$

CHEMISTRY

Section - A (Single Correct Answer)

61. **(D)**

Sol. $[Cr(H_2O)_6]^{3+}$ Contains Cr^{3+} : [Ar] $3d^3$: $t_{2g}^3 e_g^0$

$[Fe(H_2O)_6]^{3+}$ Contains Fe^{3+} : [Ar] $3d^5$: $t_{2g}^3 e_g^2$

$[Ni(H_2O)_6]^{2+}$ Contains Ni^{2+} : [Ar] $3d^8$: $t_{2g}^6 e_g^2$

$[V(H_2O)_6]^{3+}$ Contains V^{3+} : [Ar] $3d^2$: $t_{2g}^2 e_g^0$

62. **(A)**

Sol. $CaCO_3(s) \xrightarrow{\Delta} CaO(s) + CO_2(g)$

$MgCO_3(s) \xrightarrow{\Delta} MgO(s) + CO_2(g)$

Let the weight of $CaCO_3$ be x gm

$\therefore$ weight of $MgCO_3 = (2.21 - x)$gm

Moles of $CaCO_3$ decomposed = moles of CaO formed

$$\frac{x}{100} = \text{moles of CaO formed}$$

$$\therefore \text{ weight of CaO formed} = \frac{x}{100} \times 56$$

Moles of $MgCO_3$ decomposed = moles of MgO formed

$$\frac{(2.21 - x)}{84} = \text{moles of MgO formed}$$

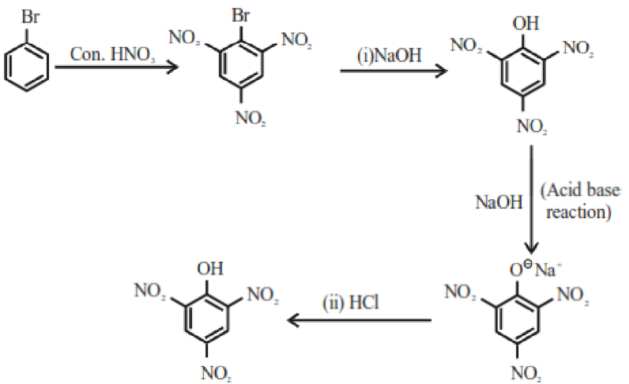
$$\therefore \text{ weight of MgO formed} = \frac{2.21 - x}{84} \times 40$$

$$\Rightarrow \frac{2.21 - x}{84} \times 40 + \frac{x}{100} \times 56 = 1.152$$

$\therefore$ x = 1.1886 g = weight of $CaCO_3$ & weight of $MgCO_3$ = 1.0214 g

63. **(A)**

Sol.



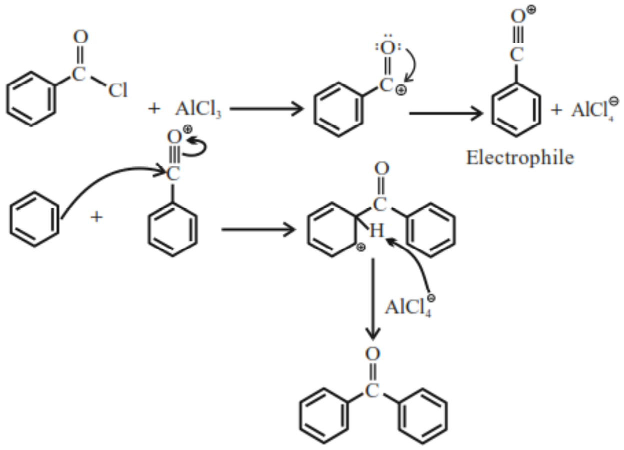
64. **(A)**

Sol. $S_1 : S_8 + 12OH^- \rightarrow 4S^{2-} + 2S_2O_3^{2-} + 6H_2O$

S_2 : ClO_4^- cannot undergo disproportionation reaction as chlorine is present in it's highest oxidation state.

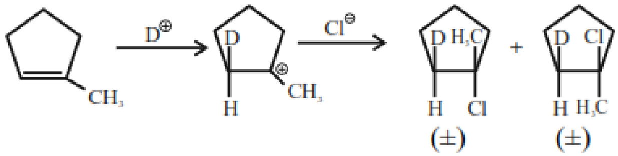
65. **(D)**

Sol.



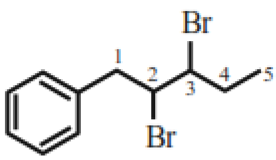
66. **(C) or (D)**

Sol.



67. **(C)**

Sol.



2, 3-dibromo-1-phenylpentane

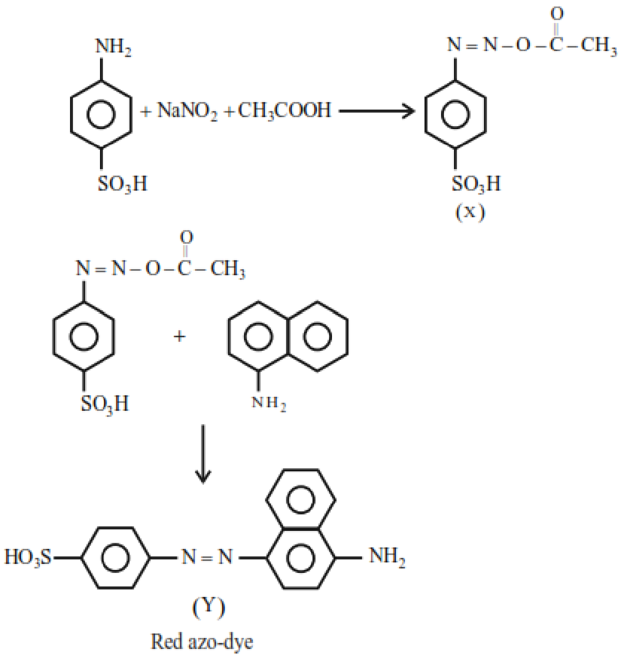
68. **(D)**

Sol. $[Ni(CO)_4] \rightarrow$ diamagnetic, sp^3 hybridisation, number of unpaired electrons = 0

$[NiCl_4]^{2-} \rightarrow$ paramagnetic, sp^3 hybridisation, number of unpaired electrons = 2

69. **(D)**

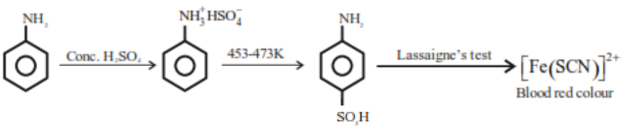
Sol.



This is known as Griess-Ilosvay test.

70. **(D)**

Sol.



71. **(B)**

Sol.

$$A_{(g)} \rightleftharpoons B_{(g)} + \frac{C}{2}_{(g)}$$

$$t = t_{eq} \quad (1-\alpha) \quad \alpha \quad \frac{\alpha}{2}$$

$$P_B = \frac{\alpha}{\left(1+\frac{\alpha}{2}\right)} \cdot P,\ P_A = \frac{(1-\alpha)}{\left(1+\frac{\alpha}{2}\right)} \cdot P,\ P_C = \frac{\frac{\alpha}{2}}{\left(1+\frac{\alpha}{2}\right)} \cdot P$$

$$K_P = \frac{P_B \cdot P_C^{\frac{1}{2}}}{P_A}$$

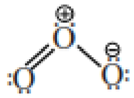
$$= \frac{(\alpha)^{\frac{3}{2}}(P)^{\frac{1}{2}}}{(1-\alpha)(2+\alpha)^{\frac{1}{2}}}$$

72. **(D)**

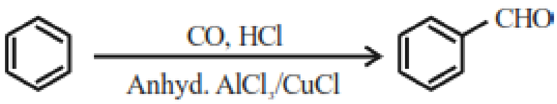
Sol. (A) All group 16 elements form oxides of the EO_2 and EO_3 type where E = S, Se, Te or Po.

(B) SO_2 is reducing while TeO_2 is an oxidising agent.

(C) The reducing property increases from H_2S to H_2Te down the group.

(D)  have six lone pairs.

73. **(C)**

Sol. 

Gatterman-Koch reaction

74. **(B)**

Sol. $AgCl < CoCl_2 < BaCl_2 < KCl$ (ionic character)

Reason : Ag^+ has pseudo inert gas configuration.

75. **(D)**

Sol. Steam distillation technique is applied to separate substances which are steam volatile and are immiscible with water.

76. **(D)**

Sol. In trivalent state most of the compounds being covalent are hydrolysed in water. Trichlorides on hydrolysis in water form tetrahedral $[M(OH)_4]^-$ species, the hybridisation state of element M is sp^3.

In case of aluminium, acidified aqueous solution forms octahedral $[Al(H_2O)_6]^{3+}$ ion.

77. **(B)**

Sol. $_{19}K$ $1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^1$.

Outermost orbital of potassium is 4s-orbital.

$$n = 4,\ \ell = 0,\ m_\ell = 0,\ s = +\frac{1}{2}.$$

78. **(B)**

Sol. (A) Mn_2O_7 is green oil at room temperature.

(B) V_2O_4 dissolve in acids to give VO^{2+} salts.

(C) CrO is basic oxide

(D) V_2O_5 is amphoteric it reacts with acid as well as base.

79. **(D)**

Sol. $-CH_3$ shows +M and +I.

–Cl shows +M and –I but inductive effect dominates.

$-NO_2$ shows –M and –I.

Electrophilic substitution $\alpha \frac{1}{-M \text{ and } -I}$

α + M and +I

Hence, order is B > A > C > D .

80. **(B)**

Sol. In general along the period from left to right, size decreases and metallic character decrease.

In general down the group, size increases and metallic character increases.

B' < A' (size)   C' > A' (size)

D' < C' (size)   D' > B' (size)

B' < A' (metallic character)

D' < C' (metallic character)

$B'^{+} < A'^{+}$ (size)

$D'^{+} < C'^{+}$ (size)

$\therefore$ C statement is incorrect.

Section - B (Numerical Value Type)

81. **(0)**

Sol. $\mu = 1.2 \text{ D} = q \times d$

$\Rightarrow 1.2 \times 10^{-10}$ esu Å = q × 1 Å

$\therefore q = 1.2 \times 10^{-10}$ esu

82. **(399)**

Sol. r = k [A]

So, order of reaction = 1

$t_{1/2}$ = 120 min.

For 90% completion of reaction,

$$\Rightarrow k = \frac{2.303}{t}\log\left(\frac{a}{a-x}\right)$$

$$\Rightarrow \frac{0.693}{t_{1/2}} = \frac{2.303}{t}\log\frac{100}{10} \qquad \therefore t = 399 \text{ min.}$$

83. **(2)**

Sol.

$$R-NH_2 + CH_3-\overset{O}{\overset{\|}{C}}-Cl \longrightarrow R-NH-\overset{O}{\overset{\|}{C}}-CH_3$$

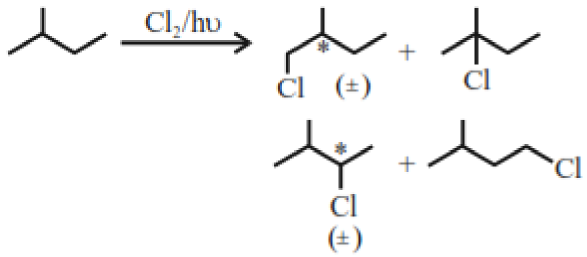
Gain in molecular weight after acylation with one $-NH_2$ group is 42.

Total increase in molecular weight = 84.

$\therefore$ Number of amino group in x = $\frac{84}{42} = 2$

84. **(6)**

Sol.



$\therefore$ Number of isomeric products = 6.

85. **(8)**

Sol. $2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \longrightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$

$\therefore$ Number of moles of H^+ ions required by 1 mole of MnO_4^- to oxidise oxalate ion to CO_2 is 8.

86. **(6)**

Sol. $K_2Cr_2O_7(s) + 4KCl(s) + 6H_2SO_4$ (conc.) $\rightarrow 2CrO_2Cl_2(g) + 6KHSO_4 + 3H_2O$

This reaction is called chromyl chloride test.

Here oxidation state of Cr is +6.

87. **(11)**

Sol. Specific gravity (density) = 1.54 g/cc.

Volume = 1 L = 1000 ml

Mass of solution = 1.54 × 1000

= 1540 g

% purity of H_2SO_4 is 70%

So weight of H_3PO_4 = 0.7 × 1540 = 1078 g

Mole of $H_3PO_4 = \frac{1078}{98} = 11$

Molarity $= \frac{11}{1 \text{ L}} = 11$

88. **(4)**

Sol. **Conductivity (S m⁻¹)**

2.1×10^3, 1.2×10, 3.91, 1×10^3 } conductors at 298.15K

1×10^{-16} Insulator at 298.15 K

1.5×10^{-2}, 1×10^{-7} } Semiconductor at 298.15 K

Therefore number of conductors is 4.

89. **(5)**

Sol. Vitamins A, D, E, K and B_{12} are stored in liver and adipose tissue.

90. **(28721)**

Sol. It is isothermal reversible expansion, so work done negative.

$$W = -2.303\ nRT \log\left(\frac{V_2}{V_1}\right)$$

$$= -2.303 \times 5 \times 8.314 \times 300 \log\left(\frac{100}{10}\right)$$

$$= -28720.713\ J$$

$$= -28721\ J$$

• • •