MATHEMATICS

Section - A (Single Correct Answer)

1. A

Sol. $f(x) = (a + b - 2c)\, x^2 + (b + c - 2a)\, x + (c + a - 2b)$

$f(x) = a + b - 2c + b + c - 2a + c + a - 2b = 0$

$f(1) = 0$

$\therefore \alpha \cdot 1 = \dfrac{c+a-2b}{a+b-2c}$

$\alpha = \dfrac{c+a-2b}{a+b-2c}$

If $-1 < \alpha < 0$

$-1 < \dfrac{c+a-2b}{a+b-2c} < 0$

$b + c < 2a$ and $b > \dfrac{a+c}{2}$

therefore, b cannot be G.M. between a and c.

If, $0 < \alpha < 1$

$0 < \dfrac{c+a-2b}{a+b-2c} < 1$

$b > c$ and $b < \dfrac{a+c}{2}$

Therefore, b may be the G.M. between a and c.

2. D

Sol. Put x = 1

$\therefore a = 1$

$$b = \lim_{x\to 0} \frac{\int_0^x \frac{\ln(1+t)}{1+t^{2024}}\,dt}{x^2}$$

Using L'HSOPITAL Rule

$$b = \lim_{x\to 0} \frac{\ln(1+x)}{(1+x^{2024})} \times \frac{1}{2x} = \frac{1}{2}$$

Now, $cx^2 + dx + e = 0$, $x^2 + x + 4 = 0$ $(D < 0)$

$\therefore \dfrac{c}{1} = \dfrac{d}{1} = \dfrac{c}{4}$

3. A

Sol. $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$

$a = 3, b = 5$

$e = \sqrt{1 - \dfrac{9}{25}} = \dfrac{4}{5}$ $\therefore$ foci $= (0, \pm be) = (0, \pm 4)$

$\therefore e_H = \dfrac{4}{5} \times \dfrac{15}{8} = \dfrac{3}{2}$

Let equation hyperbola

$\dfrac{x^2}{A^2} - \dfrac{y^2}{B^2} = -1$

$\therefore B \cdot e_H = 4$ $\therefore B = \dfrac{8}{3}$

$\therefore A^2 = B^2(e_H^2 - 1) = \dfrac{64}{9}\left(\dfrac{9}{4} - 1\right)$ $\therefore A^2 = \dfrac{80}{9}$

$\therefore \dfrac{x^2}{\frac{80}{9}} - \dfrac{y^2}{\frac{64}{9}} = -1$

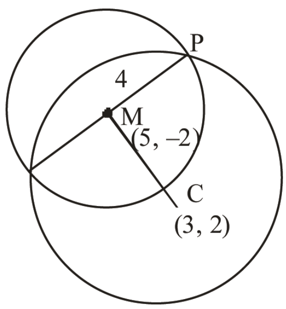
Directrix : $y = \pm \dfrac{B}{e_H} = \pm \dfrac{16}{9}$

$PS = e \cdot PM = \dfrac{3}{2}\left|\dfrac{14}{3} \cdot \sqrt{\dfrac{2}{5}} - \dfrac{16}{9}\right|$

$= 7\sqrt{\dfrac{2}{5}} - \dfrac{8}{3}$

4. C

Sol.



$2x + 3y = 12$

$3x - 2y = 5$

$13x = 39$

$x = 3, y = 2$

Center of given circle is (5, –2)

Radius $\sqrt{25+4-13} = 4$

$\therefore CM = \sqrt{4+16} = 5\sqrt{2}$

$\therefore CP = \sqrt{16+20} = 6$

5. D

Sol. $y^2 \le 4x,\ x < 4$

$$\frac{xy(x-1)(x-2)}{(x-3)(x-4)} > 0$$

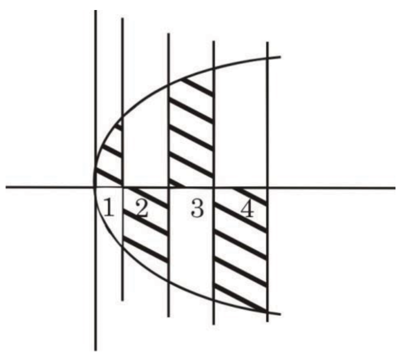
Case - I : $y > 0$

$$\frac{x(x-1)(x-2)}{(x-3)(x-4)} > 0$$

$x \in (0,\ 1) \cup (2,\ 3)$

Case II : $y < 0$

$$\frac{x(x-1)(x-2)}{(x-3)(x-4)} < 0,\ x \in (1,\ 2) \cup (3,\ 4)$$



$$\text{Area} = 2\int_0^4 \sqrt{x}\,dx \quad = 2 \cdot \frac{2}{3}[x^{3/2}]_0^4 = \frac{32}{2}$$

6. D

Sol. $f(x) = \dfrac{4x+3}{6x-4}$

$$g(x) = \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4} = \frac{34x}{34} = x$$

$g(x) = x \quad \therefore g(g(g(4))) = 4$

7. D

Sol. $\lim\limits_{x\to 0} \dfrac{e^{2|\sin x|} - 2|\sin x| - 1}{x^2}$

$$\lim_{x\to 0} \frac{e^{2|\sin x|} - 2|\sin x| - 1}{|\sin x|^2} \times \frac{\sin^2 x}{x^2}$$

Let |sin x| = t

$$\lim_{x\to 0} \frac{e^{2t} - 2t - 1}{t^2} \times \lim_{x\to 0} \frac{\sin^2 A}{x^2}$$

$$= \lim_{x\to 0} \frac{2e^{2t} - 2}{2t} \times 1 = 2 \times 1 = 2$$

8. D

Sol. $D = \begin{vmatrix} 1 & -2 & 1 \\ 2 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix}$

$= 1(\alpha\beta + 3) + 2(2\beta - 9) + 1(-2 - 3\alpha)$

$= a\beta + 3 + 4\beta - 18 - 2 - 3\alpha$

For infinite solutions $D = 0$, $D_1 = 0$, $D_2 = 0$ and $D_3 = 0$

$D = 0$

$\alpha\beta - 3\alpha + 4\beta = 17$ ....(1)

$$D_1 = \begin{vmatrix} -4 & -2 & 1 \\ 5 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix}$$

$$D_2 = \begin{vmatrix} 1 & -4 & 1 \\ 2 & 5 & 3 \\ 3 & 3 & \beta \end{vmatrix} = 0$$

$\Rightarrow 1(5\beta - 9) + 4(2\beta - 9) + 1(6 - 15) = 0$

$13\beta - 9 - 36 - 9 = 0$

$13\beta = 54,\ \beta = \dfrac{54}{13}$ put in (1)

$$\frac{54}{13}\alpha - 3\alpha + 4\left(\frac{54}{13}\right) = 17$$

$54\alpha - 39\alpha + 216 = 221$

$15\alpha = 5 \qquad \alpha = \dfrac{1}{3}$

Now, $12\alpha + 13\beta = 12 \cdot \dfrac{1}{3} + 13 \cdot \dfrac{54}{13}$

$= 4 + 54 = 58$

9. C

Sol. $\frac{dx}{dy}=\frac{x}{y}\left(\ln\left(\frac{x}{y}\right)+1\right)$

Let $\frac{x}{y}=t \Rightarrow x=ty$

$\frac{dx}{dy}=t+y\frac{dt}{dy}$

$y\frac{dt}{dy}=t\ln(t) \Rightarrow \frac{dt}{t\ln(t)}=\frac{dy}{y}$

$\Rightarrow \int\frac{dt}{t\cdot\ln(t)}=\int\frac{dy}{y}$

$\Rightarrow \int\frac{dp}{p}=\int\frac{dy}{y}$ let $\ln t = p$

$\frac{1}{t}dt=dp$

$\Rightarrow \ln p=\ln y+c$

$\ln(\ln t)=\ln y+c$

$\ln\left(\ln\left(\frac{x}{y}\right)\right)=\ln y+c$

at $x = e$, $y = 1$

$\ln\left(\ln\left(\frac{e}{1}\right)\right)=\ln(t)+c \Rightarrow c=0$

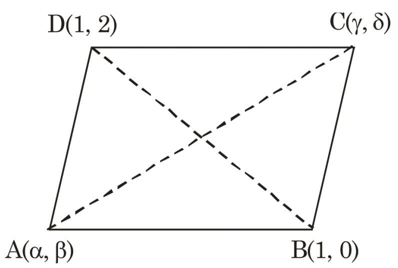
$\ln\left|\ln\left(\frac{x}{y}\right)\right|=\ln y$

$\left|\ln\left(\frac{x}{y}\right)\right|=e^{\ln y}$

$\left|\ln\left(\frac{x}{y}\right)\right|=y$

10. D

Sol.



Let E is mid point of diagonals

$\frac{\alpha+\gamma}{2}=\frac{1+1}{2}$ & $\frac{\beta+\delta}{2}=\frac{2+0}{2}$

$\alpha+\gamma=2$ $\quad\quad$ $\beta+\delta=2$

$2(\alpha+\beta+\gamma+\delta)=2(2+2)=8$

11. A

Sol. $\frac{dy}{dx}=\frac{\sin x+y\cos x}{\sin x\cdot\cos x\left(\frac{1}{\cos x}-\sin x\cdot\frac{\sin x}{\cos x}\right)}$

$=\frac{\sin x+y\cos x}{\sin x(1-\sin^2 x)}$

$\frac{dy}{dx}=\sec^2 x+y\cdot 2(\operatorname{cosec}2x)$

$\frac{dy}{dx}-2\operatorname{cosec}(2x)\cdot y=\sec^2 x$

$\frac{dy}{dx}+p\cdot y=Q$

$I.F.=e^{\int pdx}=e^{\int -2\operatorname{cosec}(2x)dx}$

Let $2x = t$

$2\frac{dx}{dt}=1$

$dx=\frac{dt}{2}$

$=e^{-\int \operatorname{cosec}(t)dt}$

$=e^{-\ln\left|\tan\frac{t}{2}\right|}$

$=e^{-\ln|\tan x|}=\frac{1}{|\tan x|}$

$y(IF)=\int Q(IF)dx+c$

$\Rightarrow y\frac{1}{|\tan x|}=\int\sec^2 x\cdot\frac{1}{|\tan x|}+c$

$y\cdot\frac{1}{|\tan x|}=\int\frac{dt}{|t|}+c$ $\quad\quad$ for $\tan x = t$

$y\cdot\frac{1}{|\tan x|}=\ln|t|+c$

y = |tan x| (ln |tan x| + c)

Put $x=\frac{\pi}{4},\ y=2$

$2=\ln 1+c\Rightarrow c=2$

y = |tan x| (ln |tan x| + 2)

$y\left(\frac{\pi}{3}\right)=\sqrt{3}(\ln\sqrt{3}+2)$

12. D

Sol. $\vec{p}\times\vec{b}-\vec{c}\times\vec{b}=\vec{0}$

$(\vec{p}-\vec{c})\times\vec{b}=\vec{0}$

$\vec{p}-\vec{c}=\lambda\vec{b}\Rightarrow\vec{p}+\vec{c}+\lambda\vec{b}$

Now, $\vec{p}\cdot\vec{a}=0$ (given)

So, $\vec{c}\cdot\vec{a}+\lambda\vec{a}\cdot\vec{b}=0$

$(3-3-8)+\lambda(12+1-14)=0$

$\lambda=-8$

$\vec{p}=\vec{c}-8\vec{b}$

$\vec{p}=-31\hat{i}-11\hat{j}-52\hat{k}$

So, $\vec{p}\cdot(\hat{i}-\hat{j}-\hat{k})$

= –31 + 11 + 52 = 32

13. D

Sol. General term of the sequence,

$$T_r=\frac{r}{1-3r^2+r^4}$$

$$T_r=\frac{r}{r^4-2r^2+1-r^2}$$

$$T_r=\frac{r}{(r^2-1)^2-r^2}$$

$$T_r=\frac{r}{(r^2-r-1)(r^2+r-1)}$$

$$T_r=\frac{\frac{1}{2}[(r^2+r-1)-(r^2-r-1)]}{(r^2-r-1)(r^2+r-1)}$$

$$=\frac{1}{2}\left[\frac{1}{r^2-r-1}-\frac{1}{r^2+r-1}\right]$$

Sum of 10 terms,

$$\sum_{r=1}^{10}T_r=\frac{1}{2}\left[\frac{1}{-1}-\frac{1}{109}\right]=\frac{-55}{109}$$

14. D

Sol. A vector in the direction of the required line can be obtained by cross product of

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 5\\ -1 & 3 & 2\end{vmatrix}$$

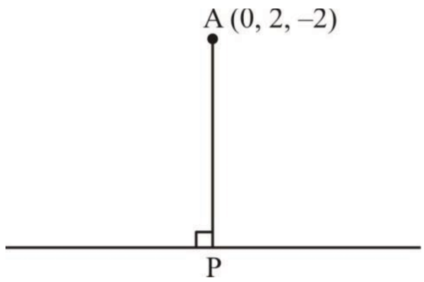
$=-9\hat{i}-9\hat{j}+9\hat{k}$

Required line,

$\vec{r}=(5\hat{i}-4\hat{j}+3\hat{k})+\lambda'(-9\hat{i}-9\hat{j}+9\hat{k})$

$\vec{r}=(5\hat{i}-4\hat{j}+3\hat{k})+\lambda(\hat{i}+\hat{j}-\hat{k})$

Now distance of (0, 2, –2)



P.V. of $P\equiv(5+\lambda)\hat{i}+(\lambda-4)\hat{j}+(3-\lambda)\hat{k}$

$\overrightarrow{AP}=(5+\lambda)\hat{i}+(\lambda-6)\hat{j}+(5-\lambda)\hat{k}$

$5+\lambda+\lambda-6-5+\lambda=0$

$\lambda=2$

$|\overrightarrow{AP}|=\sqrt{49+16+9}$

$|\overrightarrow{AP}|=\sqrt{74}$

15. A

Sol. Let $\sin^{-1}\alpha=A,\ \sin^{-1}\beta=B,\ \sin^{-1}\gamma=C$

$A+B+C=\pi$

$(\alpha+\beta)^2-\gamma^2=3\alpha\beta$

$\alpha^2+\beta^2-\gamma^2=\alpha\beta$

$$\frac{\alpha^2+\beta^2-\gamma^2}{2\alpha\beta}=\frac{1}{2}$$

$$\Rightarrow\cos C=\frac{1}{2}$$

$\sin C=\gamma$

$$\cos C=\sqrt{1-\gamma^2}=\frac{1}{2}$$

$$\gamma=\frac{\sqrt{3}}{2}$$

16. D

Sol. Probability of drawing first red and then white

$$=\frac{10}{75}\times\frac{30}{75}=\frac{4}{75}$$

17. D

Sol. Let g(x) = ax + b

Now function f(x) in continuous at x = 0

$$\therefore \lim_{x\to 0^+} f(x)=f(0)$$

$$\Rightarrow \lim_{x\to 0}\left(\frac{1+x}{2+x}\right)^{\frac{1}{x}}=b$$

$$\Rightarrow 0=b$$

$$\therefore g(x)=ax$$

Now, for x > 0

$$f'(x)=\frac{1}{x}\cdot\left(\frac{1+x}{2+x}\right)^{\frac{1}{x}-1}\cdot\frac{1}{(2+x)^2}$$

$$+\left(\frac{1+x}{2+x}\right)^{\frac{1}{x}}\cdot\ln\left(\frac{1+x}{2+x}\right)\cdot\left(-\frac{1}{x^2}\right)$$

$$\therefore f'(1)=\frac{1}{9}-\frac{2}{3}\cdot\ln\left(\frac{2}{3}\right)$$

And f(–1) = g(–1) = –a

$$\therefore a=\frac{2}{3}\ln\left(\frac{2}{3}\right)-\frac{1}{9}$$

$$\therefore g(3)=2\ln\left(\frac{2}{3}\right)-\frac{1}{3}$$

$$=\ln\left(\frac{4}{9\cdot e^{1/3}}\right)$$

18. C

Sol. $f(0)=\begin{vmatrix}0&1&1\\2&0&6\\0&4&-2\end{vmatrix}=12$

$$f'(x)=\begin{vmatrix}3x^3&4x&3\\3x^2+2&2x&x^3+6\\x^3-x&4&x^2-2\end{vmatrix}+$$

$$\begin{vmatrix}x^3&2x^2+1&1+3x\\6x&2&3x^2\\x^3-x&4&x^2-2\end{vmatrix}+$$

$$\begin{vmatrix}x^3&2x^2+1&1+3x\\3x^2+2&2x&x^3+6\\3x^2-1&0&2x\end{vmatrix}$$

$$\therefore f'(0)=\begin{vmatrix}0&0&3\\2&0&6\\0&4&-2\end{vmatrix}+\begin{vmatrix}0&1&1\\0&2&0\\0&4&-2\end{vmatrix}+\begin{vmatrix}0&1&1\\2&0&6\\-1&0&0\end{vmatrix}$$

$$=24-6=18$$

$$\therefore 2f(0)+f'(0)=42$$

19. D

Sol. 3 bad apples, 15 good apples.

Let X be no of bad apples

Then $P(X=0)=\frac{{}^{15}C_2}{{}^{18}C_2}=\frac{105}{153}$

$$P(X=1)=\frac{{}^{3}C_1\times{}^{13}C_1}{{}^{18}C_2}=\frac{45}{153}$$

$$P(X=2)=\frac{{}^{3}C_2}{{}^{18}C_2}=\frac{3}{153}$$

$$E(X)=0\times\frac{105}{153}+1\times\frac{45}{153}+2\times\frac{3}{153}=\frac{51}{153}$$

$$=\frac{1}{3}$$

Var (X) = E(X^2) – (E(X))^2

$$=0\times\frac{105}{153}+1\times\frac{45}{151}+4\times\frac{3}{153}-\left(\frac{1}{3}\right)^2$$

$$=\frac{57}{153}-\frac{1}{9}=\frac{40}{153}$$

20. B

Sol. $ax^2 + 2(a+1)x + 9a + 4 < 0 \ \forall x \in R$

$\therefore a < 0$

Section - B (Numerical Value Type)

21. 176

Sol. $I = \int_0^{\frac{\pi}{2}} \sin 2x \cdot (\cos x)^{\frac{11}{2}} \left(1 + (\cos x)^{\frac{5}{2}}\right)^{\frac{1}{2}} dx$

Put $\cos x = t^2 \Rightarrow \sin x\, dx = -2t\, dt$

$\therefore I = 4\int_0^1 t^2 \cdot t^{11} \sqrt{(1+t^5)}(t)dt$

$I = 4\int_0^1 t^{14}\sqrt{1+t^5}dt$

Put $1 + t^5 = k^2$

$\Rightarrow 5t^4 dt = 2k\, dk$

$\therefore I = 4 \cdot \int_1^{\sqrt{2}} (k^2-1)^2 \cdot k \frac{2k}{5} dk$

$I = \frac{8}{5}\int_1^{\sqrt{2}} k^6 - 2k^4 + k^2 dk$

$I = \frac{8}{5}\left[\frac{k^7}{7} - \frac{2k^5}{5} + \frac{k^3}{3}\right]_1^{\sqrt{2}}$

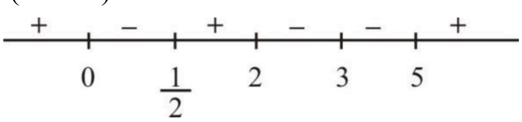
$I = \frac{8}{5}\left[\frac{8\sqrt{2}}{7} - \frac{8\sqrt{2}}{5} + \frac{2\sqrt{2}}{3} - \frac{1}{7} + \frac{2}{5} - \frac{1}{3}\right]$

$I = \frac{8}{5}\left[\frac{22\sqrt{2}}{105} - \frac{8}{105}\right]$

$\therefore 525 \cdot I = 176\sqrt{2} - 64$

22. 27

Sol. $f'(x) = (e^x - 1)(2x - 1)^5 (x - 2)^7 (x - 3)^{12} (2x - 10)^6$



Local minima at $x = \frac{1}{2},\ x = 5$

Local maxima at $x = 0$, $x = 2$

$\therefore p = 0 + 4 = 4,\ q = \frac{1}{2} + 5 = \frac{11}{2}$

Then $p^2 + 2q = 16 + 11 = 27$

23. 3734

Sol. We have III, TT, D, S, R, B, U, O, N

Number of words with selection (a, a, a, b)

$= {}^8C_1 \times \frac{4!}{3!} = 32$

Number of words with selection (a, a, b, b)

$= \frac{4!}{2!2!} = 6$

Number of words with selection (a, a, b, c)

$= {}^2C_1 \times {}^8C_2 \times \frac{4!}{2!} = 672$

Number of words with selection (a, b, c, d)

$= {}^9C_4 \times 4! = 3024$

$\therefore$ total $= 3024 + 672 + 6 + 32$

$= 3734$

24. 12

Sol. $\frac{x}{1} = \frac{y}{1} = \frac{z-1}{0} = r \to Q(r,\ r,\ 1)$

$\frac{x}{1} = \frac{y}{-1} = \frac{z+1}{0} = k \to R(k,\ -k,\ -1)$

$\overline{PQ} = (a-r)\hat{i} + (a-r)\hat{j} + (a-1)\hat{k}$

$a = r + a - r = 0.$

$2a = 2r \to a = r$

$\overline{PR} = (a-k)i + (a+k)\hat{j} + (a+1)\hat{k}$

$a - k - a - k = 0 \Rightarrow k = 0$

As, $PQ \perp PR$

$(a - r)(a - k) + (a - r)(a + k) + (a - 1)(a + 1) = 0$

$a = 1$ or -1

$12a^2 = 12$

25. 118

Sol. $(1+x)(1-x^2)\left(1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3}\right)^5$

$= (1+x)(1-x^2)\left(\left(1+\frac{1}{x}\right)^2\right)^5$

$= \frac{(1+x)^2(1-x)(1+x)^{15}}{x^{15}}$

$= \frac{(1+x)^{17} - x(1+x)^{17}}{x^{15}}$

= coeff (x^3) in the expansion ≈ coeff (x^{18}) in $(1 + x)^{17} - x(1 + x)^{17}$

$= 0 - 1$

$= -1$

coeff (x^{-13}) in the expansion ≈ coeff (x^2) in $(1 + x)^{17} - x(1 + x)^{17}$

$= \binom{17}{2} - \binom{17}{1}$

$= 17 \times 8 - 17$

$= 17 \times 7$

$= 119$

Hence Answer = 119 – 1 = 118

26. 3

Sol. $(\sqrt{2})^x = 2^x \Rightarrow x = 0 \Rightarrow \alpha = 1$

$$z = \frac{\pi}{4}(1+i)^4\left[\frac{\sqrt{\pi} - \pi i - i - \sqrt{\pi}}{\pi + 1} + \frac{\sqrt{\pi} - i - \pi i - \sqrt{\pi}}{1 + \pi}\right]$$

$$= -\frac{\pi i}{2}(1 + 4i + 6i^2 + 4i^3 + 1)$$

$$= 2\pi i$$

$$\beta = \frac{2\pi}{\frac{\pi}{2}} = 4$$

Distance from (1, 4) to $4x - 3y = 7$

Will be $\frac{15}{5} = 3$

27. 51

Sol. focii $\equiv (\pm 5,\ 0)$; $\frac{2b^2}{a} = \sqrt{50}$

$ae = 5$ $\qquad b^2 = \frac{5\sqrt{2}a}{2}$

$$b^2 = a^2(1 - e^2) = \frac{5\sqrt{2}a}{2}$$

$$\Rightarrow a(1 - e^2) = \frac{5\sqrt{2}}{2}$$

$$\Rightarrow \frac{5}{e}(1 - e^2) = \frac{5\sqrt{2}}{2}$$

$$\Rightarrow \sqrt{2} - \sqrt{2}e^2 = e$$

$$\Rightarrow \sqrt{2}e^2 + e - \sqrt{2} = 0$$

$$\Rightarrow \sqrt{2}e^2 + 2e - e - \sqrt{2} = 0$$

$$\Rightarrow \sqrt{2}e(e + \sqrt{2}) - 1(1 + \sqrt{2}) = 0$$

$$\Rightarrow (e + \sqrt{2})(\sqrt{2}e - 1) = 0$$

$$\therefore e \neq -\sqrt{2}\ ;\ e = \frac{1}{\sqrt{2}}$$

$\frac{x^2}{b^2} - \frac{y^2}{a^2b^2} = 1$ $\qquad a = 5\sqrt{2}$

$b = 5$

$a^2b^2 = b^2(e_1^2 - 1) \Rightarrow e_1^2 = 51$

28. 48

Sol. $\vec{b} \cdot \vec{c} = (2\vec{a} \times \vec{b}) \cdot \vec{b} - 3|b|^2$

$|b||c| \cos \alpha = -3|b|^2$

$|c| \cos \alpha = -12$, as $|b| = 4$

$\vec{a} \cdot \vec{b} = 2$

$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

$|c|^2 = |(2\vec{a} \times \vec{b}) - 3\vec{b}|^2$

$= 64 \times \frac{3}{4} + 144 = 192$

$|c|^2 \cos^2 \alpha = 144$

$192 \cos^2 \alpha = 144$

$192 \sin^2 \alpha = 48$

29. 16

Sol. All elements are included

Answer is 16

30. 5

Sol. $f(a) + f(1 - a) = 1$

$$M = \int_{f(a)}^{f(1-a)} (1 - x) \cdot \sin^4 x(1 - x)dx$$

$M = N - M$ $\qquad 2M = N$

$\alpha = 2;\ \beta = 1;$

Ans. 5

PHYSICS

Section - A (Single Correct Answer)

31. A

Sol. $KE = \frac{f}{2}kT$

Conceptual

32. C

Sol. $Y = \overline{\overline{A} \cdot \overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$

(De-Morgan's law)

33. A

Sol. $t = \alpha x^2 + \beta x$ (differentiating wrt time)

$$\frac{dt}{dx} = 2\alpha x + \beta$$

$$\frac{1}{v} = 2\alpha x + \beta$$

(differentiating wrt time)

$$-\frac{1}{v^2}\frac{dv}{dt} = 2\alpha\frac{dx}{dt}$$

$$\frac{dv}{dt} = -2\alpha v^3$$

34. D

Sol. $\mu = \dfrac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin\frac{A}{2}}$

$$\frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} = \frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin\frac{A}{2}}$$

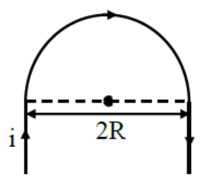
$$\sin\left(\frac{\pi}{2} - \frac{A}{2}\right) = \sin\left(\frac{A+\delta_m}{2}\right)$$

$$\frac{\pi}{2} - \frac{A}{2} = \frac{A}{2} + \frac{\delta m}{2}$$

$$\delta_m = \pi - 2A$$

35. D

Sol.



Note : Direction of magnetic field is in $+\hat{k}$

$\vec{F} = i\vec{l} \times \vec{B}$

$l = 2R$

$\vec{F} = -2iRB\hat{j}$

36. A

Sol. $\frac{1}{\lambda} = \frac{13.6z^2}{hc}\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$ ....(i)

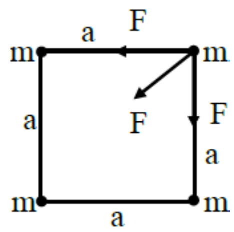
$\frac{1}{\lambda'} = \frac{13.6z^2}{hc}\left[\frac{1}{1^2} - \frac{1}{3^2}\right]$ ....(ii)

On dividing (i) & (ii)

$$\lambda' = \frac{27}{32}\lambda$$

37. B

Sol.



$$F_{net} = \sqrt{2}F + F'$$

$$F = \frac{Gm^2}{a^2} \text{ and } F' = \frac{Gm^2}{(\sqrt{2}a)^2}$$

$$F_{net} = \sqrt{2}\frac{Gm^2}{a^2} + \frac{Gm^2}{2a^2}$$

$$\left(\frac{2\sqrt{2}+1}{32}\right)\frac{Gm^2}{L^2} = \frac{Gm^2}{a^2}\left(\frac{2\sqrt{2}+1}{2}\right)$$

$a = 4L$

38. D

Sol. $PV = nRT$

$$V = \left(\frac{nR}{P}\right)T$$

$$\text{Slope} = \frac{nR}{P}$$

$$\text{Slope} \propto \frac{1}{P}$$

$(\text{Slope})_2 > (\text{Slope})_1$

$P_2 < P_1$

39. B

Sol. $R = \dfrac{\rho L}{\pi\frac{d^2}{4}}$

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} + \frac{2\Delta d}{d}$$

$$\frac{\Delta L}{L} = 0.1\% \text{ and } \frac{\Delta d}{d} = 0.1\%$$

$$\frac{\Delta R}{R} = 0.3\%$$

40. A

Sol. $U_E = \frac{1}{2} \epsilon_0 E^2$

$$U_E = \frac{1}{2} \times 8.85 \times 10^{-12} \times (50)^2$$

$= 1.106 \times 10^{-8}$ J/m^3

41. A

Sol. $F = ax^2 + bt^{1/2}$

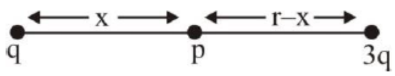
$$[a] = \frac{[F]}{[x^2]} = [M^1L^{-1}T^{-2}]$$

$$[b] = \frac{[F]}{[t^{1/2}]} = [M^1L^1T^{-5/2}]$$

$$\left[\frac{b^2}{a}\right] = \frac{[M^2L^2T^{-5}]}{[M^1L^{-1}T^{-2}]} = [M^1L^3T^{-3}]$$

42. C

Sol.



$$(\vec{E}_{net})_P = 0$$

$$\frac{kq}{x^2} = \frac{k \cdot 3q}{(r-x)^2}$$

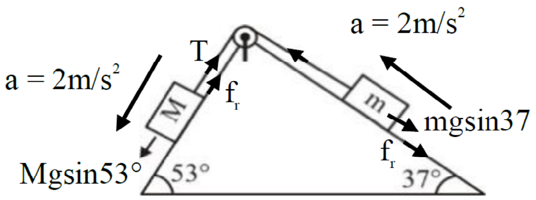
$(r - x)^2 = 3x^2$

$r - x = \sqrt{3}x$

$$x = \frac{r}{\sqrt{3}+1}$$

43. B

Sol.



For M block

$10g \sin 53° - \mu (10g) \cos 53° - T = 10 \times 2$

$T = 80 - 15 - 20$

$T = 45$ N

For m block

$T - mg \sin 37° - \mu mg \cos 37° = m \times 2$

$45 = 10$ m

$m = 4.5$ kg

44. B

Sol. $\varepsilon = N\left(\frac{\Delta\phi}{t}\right)$

$\Delta\phi = (\Delta B)A$

$B_i = 5000$ T,

$B_f = 3000$ T

$d = 0.02$ m

$r = 0.01$ m

$\Delta\phi = (\Delta B)A$

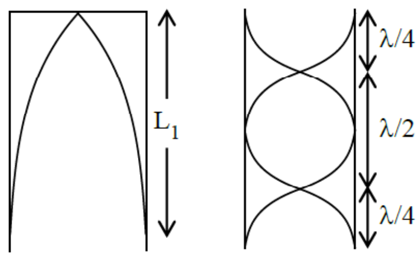
$= (2000)\pi(0.01)^2 = 0.2\pi$

$$\varepsilon = N\left(\frac{\Delta\phi}{t}\right) \Rightarrow 22 = N\left(\frac{0.2\pi}{2}\right)$$

$N = 70$

45. D

Sol.



$\frac{\lambda}{4} = L_1$

$2\left(\frac{\lambda}{2}\right) = \lambda$

$v = f\lambda$

$f_2 = \frac{2v}{2L_2}$

$v = f_1(4L_1)$

$f_2 = \frac{v}{L_2}$

$f_1 = \frac{v}{4L_1}$

$f_1 = f_2$

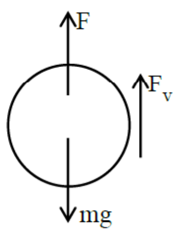
$\Rightarrow L_2 = 4L_1$

$60 = 4 \times L_1$

$L_1 = 15$ cm

46. B

Sol.



$$mg - F_B - F_v = ma$$

$$\left(\rho\frac{4}{3}\pi r^3\right)g - \left(\rho L\frac{4}{3}\pi r^3\right)g - 6\pi\eta r v = m\frac{dv}{dt}$$

Let $\frac{4}{3m}\pi R^3 g(\rho - \rho_L) = K_1$ and $\frac{6\pi\eta r}{m} = K_2$

$$\frac{dv}{dt} = K_1 - K_2 v$$

$$\int_0^v \frac{dv}{K_1 - K_2 v} = \int_0^t dt$$

$$-\frac{1}{K_2} ln[K_1 - K_2 v]_0^v = t$$

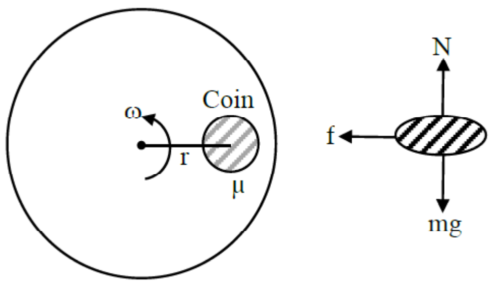
$$ln\left(\frac{K_1 - K_2 v}{K_1}\right) = -K_2 t$$

$$K_1 - K_2 v = K_1 e^{-K_2 t}$$

$$v = \frac{K_1}{K_2}\left[1 - e^{-K_2 t}\right]$$

47. C

Sol.



$N = mg$
$f = m\omega^2 r$
$f = \mu N$
$\mu mg = mr\omega^2$

$$\omega = \sqrt{\frac{\mu g}{r}}$$

48. B

Sol. Series :

$R_{eq} = R_1 + R_2$

$$2R(1 + \alpha_{eq}\Delta\theta) = R(1 + \alpha_1\Delta\theta) + R(1 + \alpha_2\Delta\theta)$$

$$2R(1 + \alpha_{eq}\Delta\theta) = 2R(\alpha_1 + \alpha_2)R\Delta\theta$$

$$\alpha_{eq} = \frac{\alpha_1 + \alpha_2}{2}$$

Parallel :

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{\frac{R}{2}(1 + \alpha_{eq}\Delta\theta)} = \frac{1}{R(1 + \alpha_1\Delta\theta)} + \frac{1}{R(1 + \alpha_2\Delta\theta)}$$

$$\frac{2}{1 + \alpha_{eq}\Delta\theta} = \frac{1}{1 + \alpha_1\Delta\theta} + \frac{1}{1 + \alpha_2\Delta\theta}$$

$$\frac{2}{1 + \alpha_{eq}\Delta\theta} = \frac{1 + \alpha_2\Delta\theta + 1 + \alpha_1\Delta\theta}{(1 + \alpha_1\Delta\theta)(1 + \alpha_2\Delta\theta)}$$

$$2[(1 + \alpha_1\Delta\theta)(1 + \alpha_2\Delta\theta)]$$

$$= [2 + (\alpha_1 + \alpha_2)\Delta\theta][1 + \alpha_{eq}\Delta\theta]$$

$$2[1 + \alpha_1\Delta\theta + \alpha_2\Delta\theta + \alpha_1\alpha_2\Delta\theta]$$

$$= 2 + 2\alpha_{eq}\Delta\theta + (\alpha_1 + \alpha_2)\Delta\theta + \alpha_{eq}(\alpha_1 + \alpha_2)\Delta\theta^2$$

Neglecting small terms

$2 + 2(\alpha_1 + \alpha_2)\Delta\theta = 2 + 2\alpha_{eq}\Delta\theta + (\alpha_1 + \alpha_2)\Delta\theta$

$(\alpha_1 + \alpha_2)\Delta\theta = 2\alpha_{eq}\Delta\theta$

$$\alpha_{eq} = \frac{\alpha_1 + \alpha_2}{2}$$

49. B

Sol. $|\vec{p}_1| = |\vec{p}_2|$

$$KE = \frac{p^2}{2M}; \text{ p same}$$

$$KE \propto \frac{1}{m}$$

$$\frac{KE_1}{KE_2} = \frac{p^2/2M_1}{p^2/2M_2} = \frac{M_2}{M_1}$$

50. C

Sol. $E = \phi + K_{max}$

$\phi = hc/\lambda_0$

$K_{max} = eV_0$

$$8e = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} \qquad ......(i)$$

$$2e = \frac{hc}{3\lambda} - \frac{hc}{\lambda_0} \qquad ....(ii)$$

on solving (i) & (ii)

$\lambda_0 = 9\lambda$

Section - B (Numerical Value Type)

51. 5

Sol. $\vec{F} = q(\vec{v} \times \vec{B})$

$$5e\hat{k} = e(3\hat{i} + 5\hat{j}) \times (B_0\hat{i} + 2B_0\hat{j})$$

$$5e\hat{k} = e(6B_0\hat{k} - 5B_0\hat{k})$$

$\Rightarrow \quad B_0 = 5T$

52. 2

Sol. Without dielectric

$$Q = \frac{A \in_0}{d} V$$

with dielectric

$$Q = \frac{A \in_0 V}{d - t + \frac{t}{K}}$$

given

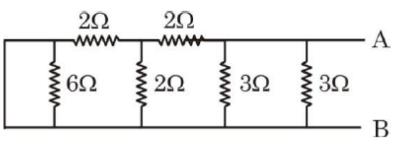
$$\frac{A \in_0 V}{d - t + \frac{t}{K}} = (1.25)\frac{A \in_0 V}{d}$$

$$\Rightarrow 1.25\left(3 + \frac{2}{K}\right) = 5$$

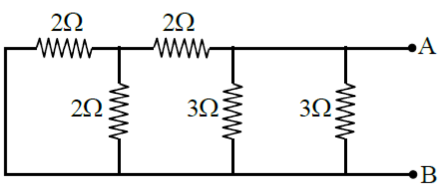
$\Rightarrow K = 2$

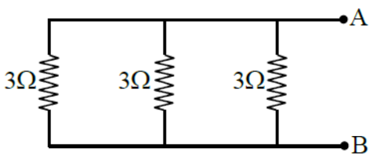
53. 1

Sol.



6Ω is short circuit





$$R_{eq} = 3 \times \frac{1}{3} = 1\Omega$$

54. 6

Sol. Using work energy theorem

$$W = \Delta KE = 0 - \left(\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\right)$$

$$W = 0 - \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$$

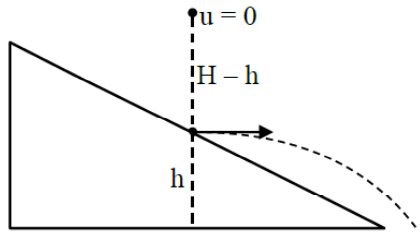
$$= -\frac{1}{2} \times 50 \times 0.4^2\left(1 + \frac{1}{2}\right) = -6J$$

Absolute work = +6J

W = – 6J W = 6J

55. 2

Sol.



Total time of flight = T

$$T = \sqrt{\frac{2h}{g}} + \sqrt{\frac{2(H-h)}{g}}$$

For max. time $= \frac{dT}{dh} = 0$

$$\sqrt{\frac{2}{g}}\left(\frac{-1}{2\sqrt{H-h}} + \frac{1}{2\sqrt{h}}\right) = 0$$

$$\sqrt{H-h} = \sqrt{h}$$

$$h = \frac{H}{2} \Rightarrow \frac{H}{h} = 2$$

56. 13

Sol. For incoherent wave $I_1 = I_A + I_B \Rightarrow I_1 = I_0 + 9I_0$

$I_1 = 10\ I_0$

For coherent wave

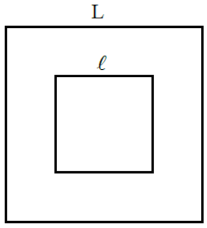
$$I_2 = I_A + I_B + 2\sqrt{I_A I_B}\cos 60°$$

$$I_2 = I_0 + 9I_0 + 2\sqrt{9I_0^2} \cdot \frac{1}{2} = 13I_0$$

$$\frac{I_1}{I_2} = \frac{10}{13}$$

57. 128

Sol.



Flux linkage for inner loop.

$\phi = B_{center} \cdot l^2$

$$= 4 \times \frac{\mu_0 i}{4\pi \frac{L}{2}}(\sin 45 + \sin 45) l^2$$

$$\phi = 2\sqrt{2}\frac{\mu_0 i}{\pi L} l^2$$

$$M = \frac{\phi}{i} = \frac{2\sqrt{2}\mu_0 l^2}{\pi L} = 2\sqrt{2}\frac{\mu_0}{\pi}$$

$$= 2\sqrt{2}\frac{4\pi}{\pi} \times 10^{-7}$$

$$= 8\sqrt{2} \times 10^{-7}\,H$$

$$= \sqrt{128} \times 10^{-7}\,H$$

$x = 128$

58. 18

Sol. $\beta = \frac{-\Delta P}{\frac{\Delta V}{V}}$

$$\Delta P = -\beta \frac{\Delta V}{V}$$

$$\rho g h = -\beta \frac{\Delta V}{V}$$

$$10^3 \times 10 \times h = -9 \times 10^8 \times \left(-\frac{0.02}{100}\right)$$

$\Rightarrow h = 18$ m

59. 7

Sol. $v = \omega\sqrt{A^2 - x^2}$

at $x = \frac{2A}{3}$

$$v = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2} = \frac{\sqrt{5}A\omega}{3}$$

New amplitude = A'

$$v' = 3v = \sqrt{5}A\omega = \omega\sqrt{(A')^2 - \left(\frac{2A}{3}\right)^2}$$

$$A' = \frac{7A}{3}$$

60. 1

Sol. $E = \Delta mc^2$

$= 0.4 \times 10^{-3} \times (3 \times 10^8)^2$

$= 3600 \times 10^7$ kWs

$$= \frac{3600 \times 10^7}{3600}\text{kWh} = 1 \times 10^7\ \text{kWh}$$

CHEMISTRY

Section - A (Single Correct Answer)

61. **(D)**

Sol. Statement I and II are False

Noble gases have low boiling points

Noble gases are held together by weak dispersion forces.

62. **(A)**

Sol. $K_C = \frac{\text{Products ion conc.}}{\text{Reactants ion conc.}}$

$$K_C = \frac{[FeSCN^{2+}]}{[Fe^{3+}][SCN^-]}$$

63. **(D)**

Sol. $(CH_3)_2CO + CS_2$

Exibits positive deviations from Raoult's Law.

64. **(B)**

Sol. Lead sulphate-white

Ammonium sulphide-soluble

Lead iodide-Bright yellow

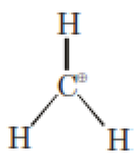
Ammonium arsinomolybdate-yellow

65. **(A)**

Sol. Mn, Ni and Cd metals used in battery industries.

66. **(C)**

Sol.



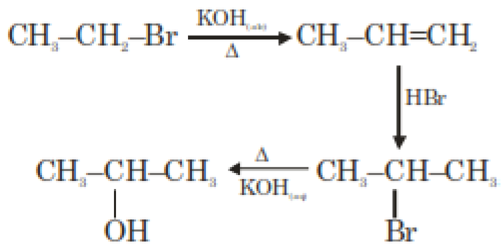
Six electron species

67. **(B)**

Sol. Conductivity of electrolytic cell is affected by concentration of electrolyte, nature of electrolyte and nature of solvent.

68. **(D)**

Sol.



69. **(D)**

Sol. As per NCERT, Assertion (A) and Reason (R) is correct but Reason (R) is not the correct explanation.

70. **(B)**

Sol. Element	**$\Delta_{eg}H$ (kJ/mol)**
F	– 333
S	– 200
Br	– 325
Ar	+ 96

71. **(D)**

Sol. A. CrO_4^{2-} is tetrahedral

B. $2Na_2CrO_4 + 2H^+ \rightarrow Na_2Cr_2O_7 + 2Na^+ + H_2O$

C. As per NCERT, green manganate is paramagnetic with 1 unpaired electron.

D. Statement is correct

E. Statement is correct

72. **(B)**

Sol. Principle used in chromotography is adsorption.

73. **(A)**

Sol.

A	→	B	+	C
P_i		0		0
$P_i - x$		x		x

$$P_t = P_i + x$$

$$P_i - x = P_i - P_t + P_i$$

$$= 2P_i - P_t$$

$$K = \frac{2.303}{t}\log\frac{P_i}{2P_i - P_t}$$

74. **(A)**

Sol. Phenol is more acidic than ethanol because conjugate base of phenoxide is more stable than ethoxide.

75. **(A)**

Sol. 7-Hydroxyheptan-2-one is correct IUPAC name.

76. **(D)**

Sol. B. VBT does not explain stability of complex.

C. Hybridisation of $[Ni(CN)_4]^{-2}$ is dsp^2.

77. **(B)**

Sol. * Molecular orbital should have maximum overlap.

* Symmetry about the molecular axis should be similar.

78. **(B)**

Sol. Glucose $\xrightarrow[\Delta]{NaHCO_3}$ no reaction

Glucose $\xrightarrow[\Delta]{HNO_3}$ saccharic acid

Glucose $\xrightarrow[\Delta]{HI}$ n-hexane

Glucose $\xrightarrow[\Delta]{Br_2}$ Gluconic acid

79. **(C)**

Sol. SnO_2 and PbO_2 are amphoteric.

80. **(B)**

Sol. Fact (NCERT)

<u>Section - B (Numerical Value Type)</u>

81. **(78)**

Sol. CaF_2 does not evolve any gas with concentrated H_2SO_4.

$NaBr \rightarrow$ evolve Br_2

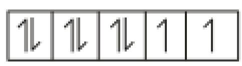
$NaNO_3 \rightarrow$ evolve NO_2

$KI \rightarrow$ evolve I_2

82. **(28)**

Sol. NH_3 act as WFL with Ni^{2+}

$Ni^{2+} = 3d^8$



No. of unpaired electron = 2

$$\mu = \sqrt{n(n+2)} = \sqrt{8} = 2.82 \text{ BM}$$

$$= 28.2 \times 10^{-1} \text{ BM}$$

$$x = 28$$

83. **(50)**

Sol. $CH_{4(g)} + 2O_{2(g)} \rightarrow CO_{2(g)} + 2H_2O_{(\ell)}$

$$n_{CO_2} = \frac{22}{44} = 0.5 \text{ moles}$$

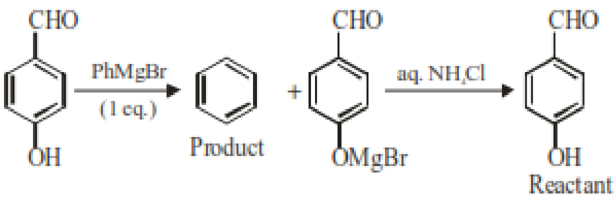
So moles of CH_4 required = 0.5 moles

i.e., 50×10^{-2} mole

x = 50

84. **(0)**

Sol. Product benzene has zero hydroxyl group.



85. **(4)**

Sol. $NH_3 \rightarrow sp^3$

$SO_2 \rightarrow sp^2$

$SiO_2 \rightarrow sp^3$

$BeCl_2 \rightarrow sp$

$CO_2 \rightarrow sp$

$H_2O \rightarrow sp^3$

$CH_4 \rightarrow sp^3$

$BF_3 \rightarrow sp^2$

86. **(10)**

Sol.

$$CH_3CH_2Br + NaOH \begin{cases} \xrightarrow{C_2H_5OH} CH_2{=}CH_2 \\ \xrightarrow{H_2O} CH_3{-}CH_2{-}OH \end{cases}$$

Total number of hydrogen atom in A and B is 10.

87. **(3)**

Sol. $CH_3COONa \rightarrow \dot{C}H_3$

$C_2H_5COONa \rightarrow \dot{C}_2H_5$

$2\dot{C}_2H_5 \rightarrow CH_3-CH_2-CH_2-CH_3$

$2\dot{C}H_3 \rightarrow CH_3-CH_3$

$\dot{C}H_3 + \dot{C}_2H_5 \rightarrow CH_3-CH_2-CH_3$

88. **(163)**

Sol. $\frac{3}{2}O_{2(g)} \rightleftharpoons O_{3(g)}$; $K_p = 2.47 \times 10^{-29}$

$$\Delta_r G^\ominus = -RT\ln K_p$$

$$= -8.314 \times 10^{-3} \times 298 \times \ln(2.47 \times 10^{-29})$$

$$= -8.314 \times 10^{-3} \times 298 \times (-65.87)$$

$$= 163.19 \text{ kJ}$$

89. **(494)**

Sol. $E = \frac{1240}{\lambda(nm)} eV$

$= \frac{1240}{242} eV$

= 5.12 eV

= $5.12 \times 1.6 \times 10^{-19}$

= 8.198×10^{-19} J/atom

= 494 kJ/mol

90. **(5)**

Sol. $Cu^{2+} + 2e^- \rightarrow Cu$

2 Faraday → 1 mol Cu

1 Faraday → 0.5 mol Cu deposit

0.5 mol = 0.5 g atom = 5×10^{-1}

x = 5

• • •