MATHEMATICS

Section - A (Single Correct Answer)

1. D

Sol. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \lambda^2 \\ 1 & 3 & \lambda \end{vmatrix} = 0$

$\Rightarrow 2\lambda^2 - \lambda - 1 = 0$

$\lambda = 1, \ -\frac{1}{2}$

$\begin{vmatrix} 1 & 1 & 5 \\ 2 & \lambda^2 & 9 \\ 3 & \lambda & \mu \end{vmatrix} = 0 \Rightarrow \mu = 13$

Infinite solution $\lambda = 1$ & $\mu = 13$

For unique solution $\lambda \neq 1$

For solution $\lambda = 1$ & $\mu \neq 13$

If $\lambda \neq 1$ & $\mu \neq 13$

Considering the case when $\lambda = -\frac{1}{2}$ and $\mu \neq 13$

this will generate no solution case

2. B

Sol. $3\sin\alpha\cos\beta + 3\sin\beta\cos\alpha$

$= 2\sin\alpha\cos\beta - 2\sin\beta\cos\alpha$

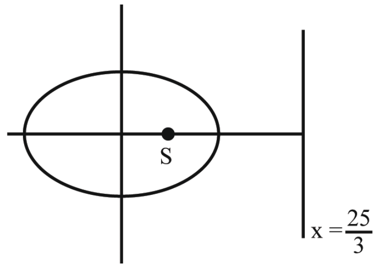
$5\sin\beta\cos\alpha = -\sin\alpha\cos\beta$

$\tan\beta = -\frac{1}{5}\tan\alpha$

$\tan\alpha = -5\tan\beta$

3. D

Sol. $\left.\begin{matrix} A = (10,\ 0) \\ B\left(0,\ \frac{50}{7}\right) \end{matrix}\right\} P = (3,\ 5)$



ae = 3

$\frac{a}{e} = \frac{25}{3}$

a = 5

b = 4

Length of LR $= \frac{2b^2}{a} = \frac{32}{5}$

4. A

Sol. $|\vec{b}|^2 = 6$; $|\vec{a}||\vec{b}|\cos\theta = 3\sqrt{2}$

$|\vec{a}|^2|\vec{b}|^2\cos^2\theta = 18$

$|\vec{a}|^2 = 6$

Also $1 + \alpha^2 + \beta^2 = 6$

$\alpha^2 + \beta^2 = 5$

to find

$(\alpha^2 + \beta^2)|\vec{a}|^2|\vec{b}|^2\sin^2\theta$

$= (5)(6)(6)\left(\frac{1}{2}\right)$

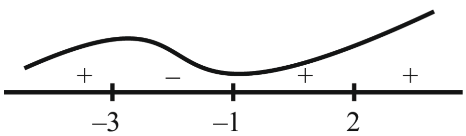
$= 90$

5. C

Sol. $f'(x) = (x + 3)^2 \cdot 3(x - 2)^2 + (x - 2)^3\, 2(x + 3)$

$= 5(x + 3)(x - 2)^2(x + 1)$

$f'(x) = 0,\ x = -3, -1, 2$



$f(-4) = -216$

$f(-3) = 0,\ f(4) = 49 \times 8 = 392$
$M = 392,\ m = -216$
$M - m = 392 + 216 = 608$
Ans = '3'

6. C

Sol. $1^{st}\text{ GP} \Rightarrow t_1 = a,\ t_3 = b = ar^2 \Rightarrow r^2 = \frac{b}{a}$

$$t_{11} = ar^{10} = a(r^2)^5 = a\cdot\left(\frac{b}{a}\right)^5$$

$$2^{nd}\text{ G.P.} \Rightarrow T_1 = a,\ T_5 = ar^4 = b$$

$$\Rightarrow r^4 = \left(\frac{b}{a}\right) \Rightarrow r = \left(\frac{b}{a}\right)^{1/4}$$

$$T_p = ar^{p-1} = a\left(\frac{b}{a}\right)^{\frac{p-1}{4}}$$

$$t_{11} = T_p \Rightarrow a\left(\frac{b}{a}\right)^5 = a\left(\frac{b}{a}\right)^{\frac{p-1}{4}}$$

$$\Rightarrow 5 = \frac{p-1}{4} \Rightarrow p = 21$$

7. A

Sol. Cocus of point P(x, y) whose distance from Gives
$X + 2y + 7 = 0$ & $2x - y + 8 = 0$ are equal is

$$\frac{x+2y+7}{\sqrt{5}} = \pm\frac{2x-y+8}{\sqrt{5}}$$

$(x + 2y + 7)^2 - (2x - y + 8)^2 = 0$
Combined equation of lines
$(x - 3y + 1)(3x + y + 15) = 0$
$3x^2 - 3y^2 - 8xy + 18x - 44y + 15 = 0$

$$x^2 - y^2 - \frac{8}{3}xy + 6x - \frac{44}{3}y + 5 = 0$$

$x^2 - y^2 + 2h\,xy + 2gx\ 2 + 2fy + c = 0$

$$h = \frac{4}{3},\ g = 3,\ f = -\frac{22}{3},\ c = 5$$

$$g + c + h - f = 3 + 5 - \frac{4}{3} + \frac{22}{3} = 8 + 6 = 14$$

8. B

Sol. $|\vec{b}| = 1$ & $|\vec{b}\times\vec{a}| = 2$

$$(\vec{b}\times\vec{a})\cdot\vec{b} = \vec{b}\cdot(\vec{b}\times\vec{a}) = 0$$

$$|(\vec{b}\times\vec{a}) - \vec{b}|^2 = |\vec{b}\times\vec{a}|^2 + |\vec{b}|^2$$
$$= 4 + 1 = 5$$

9. B

Sol. $y = f(x) \Rightarrow \frac{dy}{dx} = f'(x)$

$$\left.\frac{dy}{dx}\right)_{(1,\ f(1))} = f'(1) = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}} \Rightarrow f'(1) = \frac{1}{\sqrt{3}}$$

$$\left.\frac{dy}{dx}\right)_{(3,\ f(3))} = f'(3) = \tan\frac{\pi}{4} = 1 \Rightarrow f'(3) = 1$$

$$27\int_1^3 ((f'(t))^2 + 1)f''(t)dt = \alpha + \beta\sqrt{3}$$

$$I = \int_1^3 ((f'(t))^2 + 1)f''(t)dt$$

$$f'(1) = z \Rightarrow f''(t)dt = dz$$
$$z = f'(3) = 1$$
$$z = f'(1) = \frac{1}{\sqrt{3}}$$

$$I = \int_{1/\sqrt{3}}^{1} (z^2 + 1)dz = \left(\frac{z^3}{3} + z\right)_{1/\sqrt{3}}^{1}$$

$$= \left(\frac{1}{3} + 1\right) - \left(\frac{1}{3}\cdot\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{3}}\right)$$

$$= \frac{4}{3} - \frac{10}{9\sqrt{3}} = \frac{4}{3} - \frac{10}{27}\sqrt{3}$$

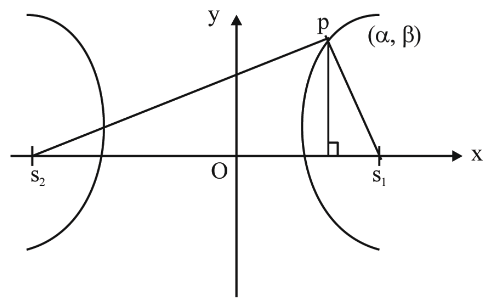
$$\alpha + \beta\sqrt{3} = 27\left(\frac{4}{3} - \frac{10}{27}\sqrt{3}\right) = 36 - 10\sqrt{3}$$

$$\alpha = 36,\ \beta = -10$$

$$\alpha + \beta = 36 - 10 = 26$$

10. C

Sol.



$$\frac{x^2}{9}-\frac{y^2}{4}=1$$

$a^2 = 9,\ b^2 = 4$

$$b^2 = a^2(e^2-1) \Rightarrow e^2 = 1+\frac{b^2}{a^2}$$

$$e^2 = 1+\frac{4}{9}=\frac{13}{9}$$

$$e=\frac{\sqrt{13}}{3} \Rightarrow s_1s_2 = 2ae = 2\times 3\times\sqrt{\frac{13}{3}} = 2\sqrt{13}$$

Area of $\Delta PS_1S_2 = \frac{1}{2}\times\beta\times s_1s_2 = 2\sqrt{13}$

$$\Rightarrow \frac{1}{2}\times\beta\times(2\sqrt{13}) = 2\sqrt{13} \Rightarrow \beta = 2$$

$$\frac{\alpha^2}{9}-\frac{\beta^2}{4}=1 \Rightarrow \frac{\alpha^2}{9}-1=1 \Rightarrow \alpha^2 = 18 \Rightarrow \alpha = 3\sqrt{2}$$

Distance of P from origin $=\sqrt{\alpha^2+\beta^2}$

$$=\sqrt{18+4}=\sqrt{22}$$

11. C

Sol. E_1 : A is selected

A	B
3W, 7R	3W, 2R

E_2 : B is selected

E : white ball is drawn

$P(E_1/E) =$

$$\frac{P(E)\cdot P(E/E_1)}{P(E_1)\cdot P(E/E_1)+P(E_2)\cdot P(E/E_2)} = \frac{\frac{1}{2}\times\frac{3}{10}}{\frac{1}{2}\times\frac{3}{10}\times\frac{1}{2}\times\frac{3}{5}}$$

$$=\frac{3}{3+6}=\frac{1}{3}$$

12. D

Sol. $f(x) = ae^{2x} + be^x + cx$ $\qquad f(0) = -1$

$a + b = -1$

$f'(x) = 2ae^{2x} + be^x + c$ $\qquad f'(\ln 2) = 21$

$8a + 2b + c = 21$

$$\int_0^{\ln 4}(ae^{2x}+be^x)dx = \frac{39}{2}$$

$$\left[\frac{ae^{2x}}{2}+be^x\right]_0^{\ln 4} = \frac{39}{2} \Rightarrow 8a+4b-\frac{a}{2}-b=\frac{39}{2}$$

$15a + 6b = 39$

$15a - 6a - 6 = 39$

$9a = 45 \Rightarrow a = 5$

$b = -6$

$c = 21 - 40 + 12 = -7$

$a + b + c - 8$

$|a + b + c| = 8$

13. A

Sol. $L_1 \perp L_2$ $\qquad L_3 \perp L_1, L_2$

$3 - 1 + 2P = 0$

$P = -1$

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 2\\ 3 & 1 & -1\end{vmatrix} = -\hat{i}+7\hat{j}+4\hat{k}$$

$\therefore (-\delta, 7\delta, 4\delta)$ will lie on L_3

For $\delta = 1$ the point will be $(-1, 7, 4)$

14. D

Sol. f is continuous $\qquad f'(x) = 2x + 3,\ k < 1$

$\therefore 4 + a = b + 2$ $\qquad$ b, $\qquad x > 1$

$a = b = 2$ $\qquad$ f is differentiable

$\therefore b = 5$

$\therefore a = 3$

$$\int_{-2}^{1}(x^2+3x3)dx+\int_1^2(5x+2)dx$$

$$=\left[\frac{x^3}{3}+\frac{3x^2}{2}+3x\right]_{-2}^{1}+\left[\frac{5x^2}{2}+2x\right]_1^2$$

$$=\left(\frac{1}{3}+\frac{3}{2}+3\right)-\left(\frac{-8}{3}+6-6\right)+\left(10+4-\frac{5}{2}-2\right)$$

$$=6+\frac{3}{2}+12-\frac{5}{2}=17$$

15. A

Sol. $f\left(\frac{x}{y}\right)=\frac{f(x)}{f(y)}$ $\qquad f'(1) = 2024$, $\quad f(1) = 1$

Partially differentiating w.r.t. x

$$f'\left(\frac{x}{y}\right)\cdot\frac{1}{y}=\frac{1}{f(y)}f'(x)$$

$y\to x$

$$f'(1)\cdot\frac{1}{x}=\frac{f'(x)}{f(x)}$$

$$2024f(x)=xf'(x)\Rightarrow xf'(x)-2024f(x)=0$$

16. B

Sol. $z^{1985}+z^{100}+1=0$ & $z^3+2z^2+2z+1=0$

$(z+1)(z^2-z+1)+2z(z+1)=0$

$(z+1)(z^2+z+1)=0$

$\Rightarrow z=-1, z=w, w^2$

Now putting $z=-1$ not satisfy

Now put $z=w$

$\Rightarrow w^{1985}+w^{100}+1$

$\Rightarrow w^2+w+1=0$

Also, $z=w^2$

$\Rightarrow w^{3970}+w^{200}+1$

$\Rightarrow w+w^2+1=0$

Two common root

17. Bonus

Sol. $2-p, p, 2-\alpha, \alpha$

Binomial coefficients are

${}^nC_r, {}^nC_{r+1}, {}^nC_{r+2}, {}^nC_{r+3}$ respectively

$\Rightarrow {}^nC_r+{}^nC_{r+1}=2$

$\Rightarrow {}^{n+1}C_{r+1}=2$ ....(1)

Also, ${}^nC_{r+2}+{}^nC_{r+3}=2$

$\Rightarrow {}^{n+1}C_{r+3}=2$ ....(2)

From (1) and (2)

${}^{n+1}C_{r+1}={}^{n+1}C_{r+3}$

$\Rightarrow 2r+4=n+1$

$n=2r+3$

${}^{2r+4}C_{r+1}=2$

Data Inconsistent

18. B

Sol. $\dfrac{2x+3}{4x^2+x-3}>0$ and $-1\le\dfrac{2x-1}{x+2}\le1$

$$\frac{2x+3}{(4x-3)(x+1)}>0\quad \frac{3x+1}{x+2}\ge0\ \&\ \frac{x-3}{x+2}\le0$$

− + − +
–3/2 –1 3/4

$(-\infty, -2)\cup\left[\frac{-1}{3},\infty\right)$ ....(1)

$(-2, 3]$ ......(2)

$\left[\frac{-1}{3}, 3\right]$ .....(3) $(1)\cap(2)\cap(3)$

$\left(\frac{3}{4}, 3\right]$

$\alpha=\frac{3}{4}\ \beta=3$

$5\beta-4\alpha=15-3=12$

19. D

Sol. $f(x)=\dfrac{x}{(1+x^4)^{1/4}}$

$$fof(x)=\frac{f(x)}{(1+f(x)^4)^{1/4}}=\frac{\frac{x}{(1+x^4)^{1/4}}}{\left(1+\frac{x^4}{1+x^4}\right)}=\frac{x}{(1+2x^4)^{1/4}}$$

$$f(f(f(f)x))=\frac{x}{(1+4x^4)^{1/4}}$$

$$18\int_0^{\sqrt{2\sqrt{5}}}\frac{x^3}{(1+4x^4)^{1/4}}dx$$

Let $1+4x^4=t^4$

$16x^3\,dx=4t^3\,dt$

$$\frac{18}{4}\int_1^3\frac{t^3dt}{t}$$

$$=\frac{9}{2}\left(\frac{t^3}{3}\right)_1^3$$

$$=\frac{3}{2}[26]=39$$

20. B

Sol. $x=\sin\theta=y\sin\left(\theta+\frac{2\pi}{3}\right)=z\sin\left(\theta+\frac{4\pi}{3}\right)\ne0$

$\Rightarrow x, y, z\ne0$

Also,

$$\sin\theta+\sin\left(\theta+\frac{2\pi}{3}\right)+\sin\left(\theta+\frac{4\pi}{3}\right)=0\ \forall\theta\in R$$

$$\Rightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$$

$\Rightarrow xy + yz + zx = 0$

(i) Trace (R) = $x + y + z$

If $x + y + z = 0$ and $xy + yz + zx = 0$

$\Rightarrow$ $x = y = z = 0$

Statement (i) is False

(ii) $Adj(Adj(R)) = |R|\,R$

Trace $(Adj(Adj(R)))$

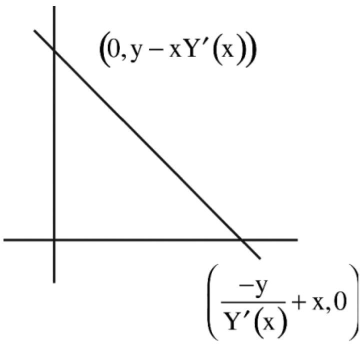
$= xyz\,(x + y + z) \neq 0$

Statement (ii) is also False

Section - B (Numerical Value Type)

21. 20

Sol. $A=\frac{1}{2}\left(\frac{-y}{Y'(x)}+x\right)(y-xY/x)=\frac{-y^2}{2Y'(x)}+1$



$\Rightarrow (-y + xY'(x))\,(y - xY'(x)) = -y^2 + 2Y'(x)$

$-y^2 + xyY'(x) + xyY'(x) - x^2\,[Y'(x)]^2$

$= -y^2 + 2Y'(x)$

$2xy - x^2\,Y'(x) = 2$

$$\frac{dy}{dx}=\frac{2xy-2}{x^2}$$

$$I.F.=e^{-2\ln x}=\frac{1}{x^2}$$

$$y\cdot\frac{1}{x^2}=\frac{2}{3}x^{-3}+c$$

Put $x = 1$, $y = 1$

$$1=\frac{2}{3}+c\Rightarrow c=\frac{1}{3}$$

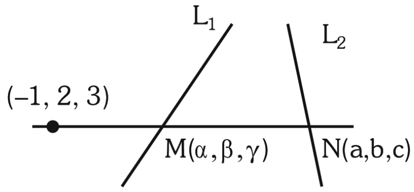
$$Y=\frac{2}{3}\cdot\frac{1}{x}+\frac{1}{3}X^2$$

$$\Rightarrow 12Y(2)=\frac{5}{3}\times 12=20$$

22. 196

Sol. $M(3\lambda+1,\ 2\lambda+2,\ -2\lambda-1)\ \therefore \alpha+\beta+\gamma=3\lambda+2$

$N(-3\mu-2,\ -2\mu+2,\ 4\mu+1)\ \therefore a+b+c=-\mu+1$



$$\frac{3\lambda+2}{-3\mu-1}=\frac{2\lambda}{-2\mu}=\frac{-2\lambda-4}{4\mu-2}$$

$3\lambda\mu+2\mu=3\lambda\mu+\lambda$

$2\mu=\lambda$

$2\lambda\mu-\lambda=\lambda\mu+2\mu$

$\lambda\mu=\lambda+2\mu$

$\Rightarrow \lambda\mu=2\lambda$

$\Rightarrow \mu=2\quad(\lambda\neq 0)$

$\therefore \lambda=4$

$\alpha+\beta+\gamma=14$

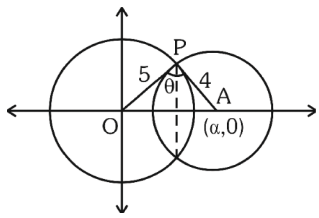
$a + b + c = -1$

$$\frac{(\alpha+\beta+\gamma)^2}{(a+b+c)^2}=196$$

23. 1575

Sol. $C_1 : x^2 + y^2 = 25$, $C_2 : (x - \alpha)^2 + y^2 = 16$

$5 < \alpha < 9$



$$\theta=\sin^{-1}\left(\frac{\sqrt{63}}{8}\right)$$

$$\sin\theta = \frac{\sqrt{63}}{8}$$

$$\text{Area of } \Delta OAP = \frac{1}{2}\times\alpha\left(\frac{\beta}{2}\right) = \frac{1}{2}\times 5\times 4\sin\theta$$

$$\Rightarrow \alpha\beta = 40\times\frac{\sqrt{63}}{8}$$

$$\alpha\beta = 5\times\sqrt{63}$$

$$(\alpha\beta)^2 = 25\times 63 = 1575$$

24. 10

Sol. $\alpha = \sum_{k=0}^{n} \frac{{}^nC_k \cdot {}^nC_k}{k+1}\cdot\frac{n+1}{n+1}$

$$= \frac{1}{n+1}\sum_{k=0}^{n} {}^{n+1}C_{k+1}\cdot {}^nC_{n-k}$$

$$\alpha = \frac{1}{n+1}\cdot {}^{2n+1}C_{n+1}$$

$$\beta = \sum_{k=0}^{n-1} {}^nC_k \cdot \frac{{}^nC_{k+1}}{k+2}\,\frac{n+1}{n+1}$$

$$\frac{1}{n+1}\sum_{k=0}^{n-1} {}^nC_{n-k}\cdot {}^{n+1}C_{k+2}$$

$$= \frac{1}{n+1}\cdot {}^{2n+1}C_{n+2}$$

$$\frac{\beta}{\alpha} = \frac{{}^{2n+1}C_{n+2}}{{}^{2n+1}C_{n+1}} = \frac{2n+1-(n+2)+1}{n+2}$$

$$\frac{\beta}{\alpha} = \frac{n}{n+2} = \frac{5}{6}$$

$n = 10$

25. 9

Sol. $S_n = 3 \;\; 7 + 11 + \ldots..$ n terms

$$= \frac{n}{2}(6+(n-1)4) = 3n + 2n^2 - 2n$$

$= 2n^2 + n$

$$\sum_{k=1}^{n} S_k = 2\sum_{k=1}^{n} K^2 + \sum_{k=1}^{n} K$$

$$= 2\cdot\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= n(n+1)\left[\frac{2n+1}{3} + \frac{1}{2}\right]$$

$$= \frac{n(n+1)(4n+5)}{6}$$

$$\Rightarrow 40 < \frac{6}{n(n+1)}\sum_{k=1}^{n} S_k < 42$$

$40 < 4n + 5 < 42$

$35 < 4n < 37$

$n = 9$

26. 11376

Sol. If 4 questions from each section are selected Remaining 3 questions can be selected either in (1, 1, 1) or (3, 0, 0) or (2, 1, 0)

$\therefore$ Total ways $= {}^8c_5 \cdot {}^6c_5 + {}^6c_5 + {}^8c_6\, {}^6c_5 \cdot {}^6c_4 \times 2 + {}^8c^6 \cdot {}^6c_6 \cdot {}^6c_4 \times 2 + {}^8c_4 \cdot {}^6c_6 \cdot {}^6c_5 \times 2 + {}^8c_7 \cdot {}^6c_4 \cdot {}^6c_4$

$= 56 \cdot 6 \cdot 6 + 28 \cdot 6 \cdot 15 \cdot 2 + 56 \cdot 15 \cdot 2 + 70 \cdot 6 \cdot 2 + \cdot 15 \cdot 15$

$= 2016 + 5040 + 1680 + 840 + 1800 = 11376$

27. 960

Sol. Total number of relation both symmetric and reflexive $= 2^{\frac{n^2-n}{2}}$

Total number of symmetric relation $= 2^{\left(\frac{n^2+n}{2}\right)}$

$\Rightarrow$ Then number of symmetric relation which are not reflexive

$$\Rightarrow 2^{\frac{n(n+1)}{2}} - 2^{\frac{n(n-1)}{2}}$$

$$\Rightarrow 2^{10} - 2^{6}$$

$$\Rightarrow 1024 - 64$$

$$= 960$$

28. 1

Sol. $x = 0$ and $x^2 + 3\,|x| + 5\,|x - 1| + 6|x - 2| = 0$

Here all terms are +ve except at $x = 0$

So there is no value of x

Satisfies this equation

Only solution $x = 0$

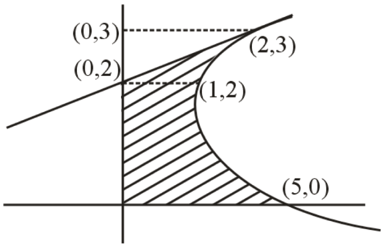
No of solution 1.

29. 5

Sol. Solving the equations

$(y - 2)^2 = x - 1$ and $x - 2y + 4 = 0$

$X = 2(y - 2)$



$\frac{x^2}{4} = x - 1$

$x^2 - 4x + 4 = 0$

$(x - 2)^2 = 0$

$x = 2$

Exclose area (w.r.t. y-axis) $= \int_0^3 x\,dy -$ Area of Δ.

$= \int_0^3 ((y-2)^2 + 1)dy - \frac{1}{2} \times 1 \times 2$

$= \int_0^3 (y^2 - 4y + 5)dy - 1$

$= \left[\frac{y^3}{3} - 2y^2 + 5y\right]_0^3 - 1$

$= 9 - 18 + 15 - 1 = 5$

30. 29

Sol.

x_i	f_i	f_ix_i	$f_ix_i^2$
0	3	0	0
1	2	2	2
5	3	15	75
6	2	12	72
10	6	60	600
12	3	36	432
17	3	51	867
	$\Sigma f_i = 22$		$\Sigma f_ix_i^2 = 2048$

$\therefore \Sigma f_ix_i = 176$

So $\bar{x} = \frac{\Sigma f_ix_i}{\Sigma f_i} = \frac{176}{22} = 8$

for $\sigma^2 = \frac{1}{N}\Sigma f_ix_i^2 - (\bar{x})^2$

$= \frac{1}{22} \times 2048 - (8)^2$

$= 93.090964$

$= 29.0909$

PHYSICS

Section - A (Single Correct Answer)

31. D

Sol. 50 V + S = 49S + S

S = 50 (S – V)

.5 = 50 (S – V)

$S - V = \frac{0.5}{50} = \frac{1}{100} = 0.01$ mm

32. B

Sol. Work done again frictional force

$= \mu N \times 10$

$= 0.1 \times 5 \times 10 = 5J$

33. D

Sol. K.E. = hf - ϕ

tan θ = h

34. D

35. C

Sol. (X) $\rightarrow$ (Y) + (Z) + (P)

M M/3 M/3 M/3

$\Delta Mc^2 = \frac{1}{2}\frac{M}{3}V^2 + \frac{1}{2}\frac{M}{3}V^2 + \frac{1}{2}\frac{M}{3}V^2$

$V = c\sqrt{\frac{2\Delta M}{M}}$

36. A & C

Sol. Steeper curve (B) is adiabatic

Adiabatic $\Rightarrow PV^v$ = const.

Or $P\left(\frac{T}{P}\right)^v$ = const.

$\frac{T^v}{P^{v-1}}$ = const.

Curve (A) is isothermal

T = const.

PV = const.

37. B

Sol. Magnetic moment = $i\pi r^2$

$$\mu = \frac{evr}{2}$$

$$\mu \alpha \left(\frac{1}{n}\right) n^2$$

$\mu \ \alpha \ n$

$x = 1$

38. B

Sol. Rising half to peak

$t = T/6$

$$t = \frac{2\pi}{6\omega} = \frac{\pi}{3\omega} = \frac{\pi}{300\pi} = \frac{1}{300} = 3.33 \text{ ms}$$

39. A

Sol. $\frac{dy}{dx} = \tan\theta = \frac{x}{2} = \mu = \frac{1}{2}$

$x = 1$, $y = 1/4$

40. B

Sol. $p = \frac{E}{C} = \frac{6.48\times10^5}{3\times10^8} = 2.16\times10^{-3}$

41. A

Sol. Intensity of emergent light

$$= \frac{I_0}{2}\cos^2 45° = \frac{I_0}{4}$$

42. D

Sol. $R_P = \frac{R_E}{3}$, $M_P = \frac{M_E}{6}$

$$V_e = \sqrt{\frac{2GM_e}{R_e}} \qquad \text{...........(i)}$$

$$V_P = \sqrt{\frac{2GM_P}{R_P}} \qquad \text{...........(ii)}$$

$$\frac{V_e}{V_P} = \sqrt{2}$$

$$V_P = \frac{V_e}{\sqrt{2}} = \frac{11.2}{\sqrt{2}} = 7.9 \text{ km/sec}$$

43. B

Sol. $\frac{2k\lambda q}{r} = m\omega^2 r$

$$\omega^2 = \frac{2k\lambda q}{mr^2}$$

$$\left(\frac{2\pi}{T}\right)^2 = \frac{2k\lambda q}{mr^2}$$

$$T = 2\pi r\sqrt{\frac{m}{2k\lambda q}}$$

44. A

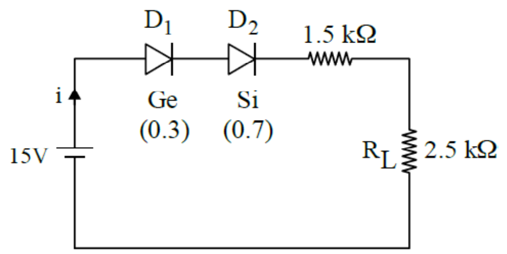
Sol. $m = k\, c^P\, G^{-1/2}\, h^{1/2}$

$M^1L^0T^0 = [LT^{-1}]^P\ [M^{-1}L^3T^{-2}]^{-1/2}\ [ML^2T^{-1}]^{1/2}$

By comparing P = 1/2

45. A

Sol.



$$i = \frac{14}{4} = 3.5 \text{ mA}$$

$V_L = iR_L = 3.5 \times 2.5$ volt

$= 8.75$volt

46. C

Sol. $f_1 = 3$, $f_2 = 5$

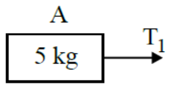
$n_1 = 3$, $n_2 = 2$

$$f_{mixture} = \frac{n_1f_1 + n_2f_2}{n_1 + n_2} = \frac{9+10}{f} = \frac{19}{5}$$

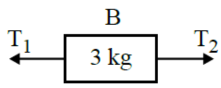
$$\gamma_{mixture} = 1 + \frac{2\times5}{19} = \frac{29}{19} = 1.52$$

47. A

Sol. $a_A = a_B = a_C = \frac{F}{5+3+2} = \frac{80}{10} = 8 m/s^2$



$T_1 = 5 \times 8 = 40$



$T_2 - T_1 = 3 \times 8 \Rightarrow T_2 = 64$

48. D

Sol. $\frac{v^2}{R} = W$ ....(i)

$\frac{v^2}{\frac{1}{2}\left(\frac{R}{2}\right)} = W'$ ....(ii)

From (i) & (ii), we get

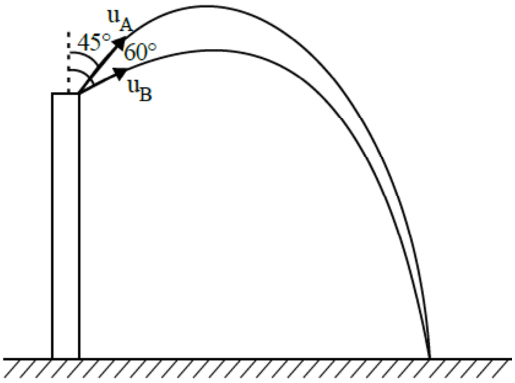
W' = 4W

49. D

Sol. Maxwell's equation

50. Bonus

Sol.



For u_A & u_B time of flight and range can not be same. So above options are incorrect.

Section - B (Numerical Value Type)

51. 45

Sol. $P_i = 2300 \times 5$ watt

$P_0 = 2300 \times 5 \times 0.9 = 230 \times I_2$

$I_2 = 45A$

52. 10

Sol. $\rho\left(\frac{4}{3}\pi r^3\right)1000 = \frac{4}{3}\pi R^3\rho$

R = 10r

$E_1 = 1000 \times 4\pi r^2 \times S$

$E_2 = 4\pi (10r)^2 S$

$\frac{E_1}{E_2} = \frac{10}{1}, x = 10$

53. 24

Sol. $I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_0$ (C.O.A.M.)

gives $\omega_0 = 8$ rad/s

$E_1 = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 = 216$ J

$E_2 = \frac{1}{2}(I_1 + I_2)\omega_0^2 = 192$ J

$\therefore \quad \Delta E = 24J$

54. 20

Sol. $\frac{1}{f+20} - \frac{1}{-(f+20)} = \frac{1}{f}$

$\frac{2}{f+20} = \frac{1}{f} \quad f = 20$ cm

Or $\quad x_1x_2 = f^2$ gives f = 20 cm

55. 4

Sol. $\bar{N} = |\bar{A}|\hat{B} = \frac{5(4\hat{i} + 3\hat{j})}{5} = 4\hat{i} + 3\hat{j}$

$\therefore \quad x = 4$

56. 40

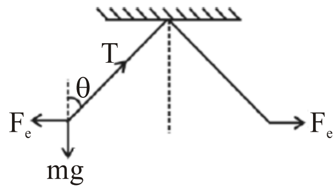
Sol. $B = 4 \times \frac{\mu_0 i}{4\pi\left(\frac{1}{2}\right)}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$

$= 4 \times 10^{-7} \times 5 \times 2 \times \sqrt{2}$

$= 40\sqrt{2} \times 10^{-7}$ T

57. 2

Sol.



T cos θ = mg

T sin θ = F_e

$\tan\theta = \frac{F_e}{mg}$

$\tan\theta = \frac{F_e}{\rho_B Vg}$ ....(i)

$\tan\theta = \frac{\frac{F_e}{k}}{(\rho_B - \rho_L)Vg}$ ....(ii)

From Eq. (i) & (ii)

$\rho_B Vg = (\rho_B - \rho_L) kVg$

$1.4 = 0.7\, k$

$k = 2$

58. 8

Sol. Acceleration due to gravity $g' = \frac{g}{4}$

$$T = 2\pi\sqrt{\frac{4l}{g}}$$

$$T = 2\pi\sqrt{\frac{4\times 4}{g}}$$

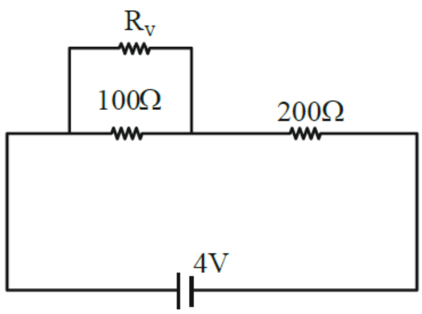
$$T = 2\pi\frac{4}{\pi} = 8s$$

59. Bonus

Sol. Question is wrong as data is incomplete.

60. 200

Sol.



$$\frac{R_v 100}{R_v + 100} = \frac{200}{3}$$

$3R_v = 2R_v + 200$

$R_v = 200$

CHEMISTRY

Section - A (Single Correct Answer)

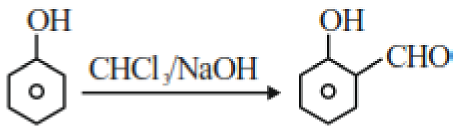
61. **(D)**

Sol. Different Extraction

Different layers are formed which can be separated in funnel. (Theory based)

62. **(D)**

Sol.



63. **(D)**

Sol. **Statement-I :** Rate of $S_N2 \propto$ [R–X] [Nu^-]

S_N2 reaction is favoured by high concentration of nucleophile (Nu^-) and less crowding in the substrate molecule.

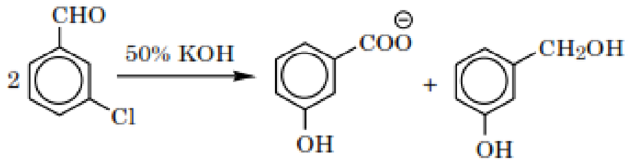
Statement-II : Solvolysis follows S_N1 path.

Both are correct Statements.

64. **(B)**

Sol. Meta–chlorobenzaldehyde will undergo

Cannizzaro reaction with 50% KOH to give m–chlorobenzoate ion and m–chlorobenzyl alcohol.

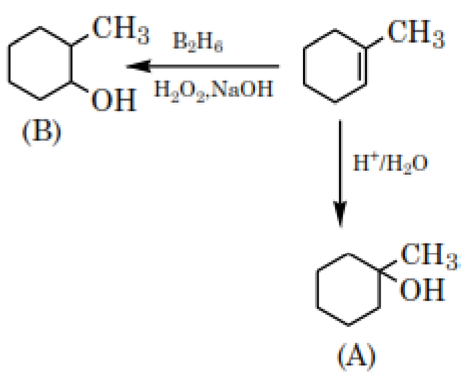


65. **(B)**

Sol. Due to lower Bond dissociation enthalpy of H_2Te it ionizes to give H^+ more easily than H_2S.

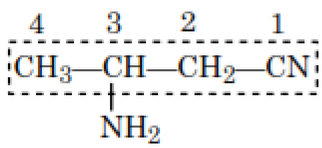
66. **(B)**

Sol.



67. **(C)**

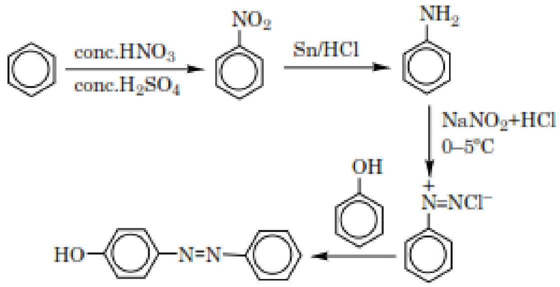
Sol.



3–Aminobutanenitrile

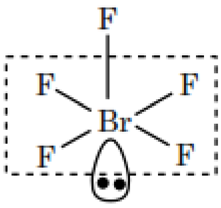
68. **(C)**

Sol.



69. **(C)**

Sol. BrF_5



Square Pyramidal.

70. **(A)**

Sol.

$$\left.\begin{array}{l} K_2Cr_2O_7 \rightarrow Cr^{+6} \rightarrow \text{No d} - \text{d transition} \\ KMnO_4 \rightarrow Mn^{7+} \rightarrow \text{No d} - \text{d transition} \end{array}\right\} \text{Charge transfer}$$

71. **(B)**

Sol. Alkaline oxidative fusion of MnO_2 :

$2MnO_2 + 4OH^- + O_2 \rightarrow 2MnO_4^{2-} + 2H_2O$

Electrolytic oxidation of MnO_4^{2-} in alkaline medium.

$$MnO_4^{2-} \rightarrow MnO_4^- + e^-$$

72. **(B)**

Sol. Mole fraction of C = $\dfrac{n_C}{n_A + n_B + n_C}$

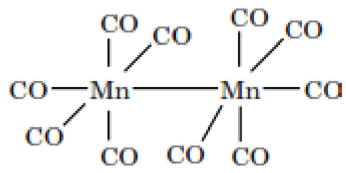
73. **(C)**

Sol. Chemical reactivity of elements decreases along the period therefore statement –I is false.

Group – 1 elements from basic nature oxides while group – 17 elements form acidic oxides therefore statement-II is true.

74. **(A)**

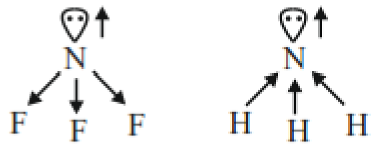
Sol. $Mn_2(CO)_{10}$



Octahedral around Mn

75. **(B)**

Sol.



76. **(C)**

Sol. More number of hyperconjugable Hydrogens, more stable is the carbocations.

77. **(A)**

Sol. ΔT_f is maximum when i × m is maximum.

(1) $m_1 = \dfrac{180}{60} = 3,\ i = 1 + \alpha$

Hence

$\Delta T_f = (1 + \alpha) \cdot k_f = 3 \times 1.86 = 5.58\ °C\ (\alpha << 1)$

(2) $m_2 = \dfrac{180}{60} = 3,\ i = 0.5,\ \Delta T_f = \dfrac{3}{2} \times k_f' = 7.68\ °C$

(3) $m_3 = \dfrac{180}{122} = 1.48,\ i = 0.5,\ \Delta T_f = \dfrac{1.48}{2} \times k_f' = 3.8\ °C$

(4) $m_4 = \dfrac{180}{180} = 1,\ i = 1,\ \Delta T_f = 1 \cdot k_f' = 1.86\ °C$

As per NCERT,

k_f' (H_2O) = 1.86 k· kg mol^{-1}

k_f' (Benzene) = 5.12 k · kg mol^{-1}

78. **(A)**

Sol.

$$\underset{(A)}{CrO_2Cl_2} + 4NaOH \rightarrow Na_2CrO_4 + 2NaCl + 2H_2O$$

$$Na_2CrO_4 + 2H_2O_2 + 2HCl \rightarrow \underset{(B)}{CrO_5} + \underset{\text{Missing from balanced eqaution}}{2NaCl} + 3H_2O$$

79. **(C)**

Sol. On moving down the group, bond strength of M–H bond decreases, which reduces the thermal stability but increases reducing nature of hydrides, hence A and B are correct statements.

80. **(B)**

Sol. Higher the value of ⊕ve SRP (Std. reduction potential) more is tendency to undergo reduction, so better is oxidising power of reactant.

Hence, ox. Power : $BrO_4^- > IO_4^- > ClO_4^-$

Section - B (Numerical Value Type)

81. **(4)**

Sol. cis – $[Cr(ox)_2Cl_2]^{3-}$ → can show optical isomerism (no POS & COS)

$[Co(en)_3]^{3+}$ → can show (no POS & COS)

cis – $[Pt(en)_2Cl_2]^{2+}$ → can show (no POS & COS)

cis – $[Co(en)_2Cl_2]^{+}$ → can show (no POS & COS)

trans – $[Pt(en)_2Cl_2]^{2+}$ → can't show (contains POS & COS)

trans – $[Cr(ox)_2Cl_2]^{3-}$ → can't show (contains POS & COS)

82. **(17)**

Sol. Rate of reaction (ROR)

$$= -\frac{1}{2}\frac{\Delta[N_2O_5]}{\Delta t} = \frac{1}{4}\frac{[NO_2]}{\Delta t} = \frac{\Delta[O_2]}{\Delta t}$$

$$ROR = -\frac{1}{2}\frac{\Delta[N_2O_5]}{\Delta t} = -\frac{1}{2}\frac{(2.75-3)}{30}\text{mol L}^{-1}\text{min}^{-1}$$

$$ROR = -\frac{1}{2}\frac{(-0.25)}{30}\text{mol L}^{-1}\text{min}^{-1}$$

$$ROR = \frac{1}{240}\text{mol L}^{-1}\text{min}^{-1}$$

$$\text{Rate of formation } NO_2 = \frac{\Delta[NO_2]}{\Delta t} = 4\times ROR$$

$$= \frac{4}{240} = 16.66\times10^{-3}\text{mol L}^{-1}\text{min}^{-1} \simeq 17\times10^{-3}.$$

83. **(492)**

Sol.

$$2Fe_{(s)} + \frac{3}{2}O_{2(g)} \rightarrow Fe_2O_{3(s)};\ \Delta H^o = -822 \text{ kJ/mol} \quad \text{..... (1)}$$

$$C_{(s)} + \frac{1}{2}O_{2(g)} \rightarrow CO_{(g)};\ \Delta H^o = -110 \text{ kJ/mol} \quad \text{..... (2)}$$

$3C_{(s)} + Fe_2O_{3(s)} \rightarrow 2Fe_{(s)} + 3CO_{(g)}$; ΔH_3 = ?

(3) = 3 × (2) – (1)

$\Delta H_3 = 3 \times \Delta H_2 - \Delta H_1$

$= 3(-110) + 822$

$= 492$ kJ/mole

84. **(3)**

Sol. A. RNA is regarded as the reserve of genetic information. (False)

B. DNA molecule self-duplicates during cell division. (True)

C. DNA synthesizes proteins in the cell. (False)

D. The message for the synthesis of particular proteins is present in DNA. (True)

E. Identical DNA strands are transferred to daughter cells. (True)

85. **(100)**

Sol.

	1M Benzoic acid ($V_a$ml)	+	1M Sodium Benzoate ($V_s$ml)
Millimole	$V_a \times 1$		$V_s \times 1$

pH = 4.5

$$pH = pKa + \log\frac{[salt]}{[acid]}$$

$$4.5 = 4.2 + \log\left(\frac{V_S}{V_a}\right)$$

$$\frac{V_S}{V_a} = 2 \quad \text{..... (1)}$$

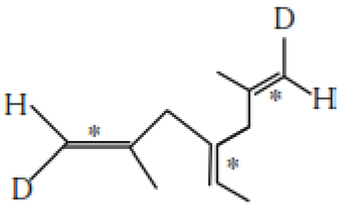
$$V_S + V_a = 300 \quad \text{..... (2)}$$

$$V_a = 100 \text{ ml}$$

86. **(4)**

Sol. 3 stereocenteres, symmetrical

Total Geometrical isomers $\rightarrow$ 4. EE, ZZ, EZ (two isomers)



87. **(6)**

Sol. Intermediate oxidation state of element can undergo disproportionation.

H_2O_2, ClO_3^-, P_4, Cl_2, Cu^{+1}, NO_2

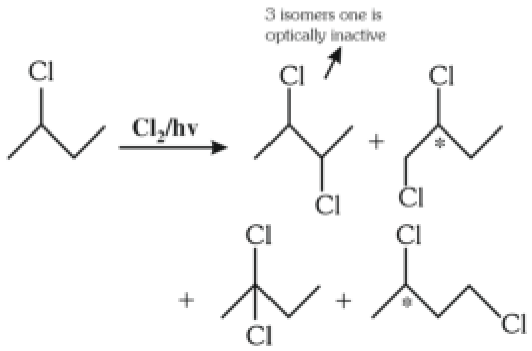
88. **(4)**

Sol. All the following metal ions will respond to flame test.

Sr^{2+}, Ba^{2+}, Ca^{2+}, Cu^{2+}

89. **(6)**

Sol.



90. **(10)**

Sol. 5^{th} excited state $\Rightarrow n_1 = 6$

1^{st} excited state $\Rightarrow n_2 = 2$

$\Delta n = n_1 - n_2 = 6 - 2 = 4$

Maximum number of spectral lines

$$= \frac{\Delta n(\Delta n + 1)}{2} = \frac{4(4+1)}{2} = 10$$

● ● ●