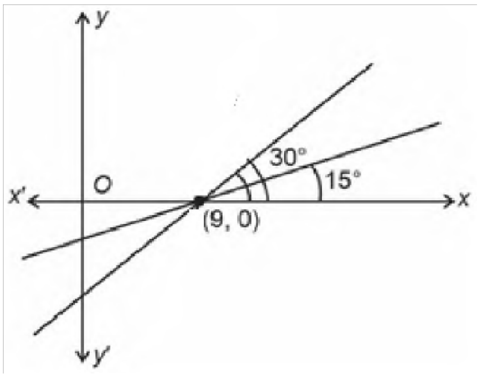
30-Jan.-2024 (Morning) : PCM [SOLUTIONS]

MATHEMATICS

Section - A (Single Correct Answer)

1. A

Sol.



Eq^n : $y - 0 = \tan 15^\circ (x-9) \Rightarrow y = (2-\sqrt{3})(x-9)$

2. A

Sol. $S_{20} = \frac{20}{2}[2a+19d] = 790$

$2a + 19d = 79$ .....(1)

$S_{10} = \frac{10}{2}[2a+9d] = 145$

$2a + 9d = 29$ ......(2)

From (1) and (2) $a = -8$, $d = 5$

$S_{15} - S_5 = \frac{15}{2}[2a+14d] - \frac{5}{2}[2a+4d]$

$= \frac{15}{2}[-16+70] - \frac{5}{2}[-16+20]$

$= 405 - 10$

$= 395$

3. B

Sol. $z^2 = -i\bar{z}$

$|z^2| = |i\bar{z}|$

$|z^2| = |z|$

$|z|^2 - |z| = 0$

$|z|(|z| - 1) = 0$

$|z| = 0$ (not acceptable)

$\therefore |z| = 1$ $\qquad \therefore |z|^2 = 1$

4. C

Sol. Given $|\vec{a}| = 1,\ |\vec{b}| = 4,\ \vec{a}\cdot\vec{b} = 2$

$\vec{c} = 2(\vec{a}\times\vec{b})$

Dot product with $\vec{a}$ on both sides

$\vec{c}\cdot\vec{a} = -6$ ....(1)

Dot product with $\vec{b}$ on both sides

$\vec{b}\cdot\vec{c} = -48$ ....(2)

$\vec{c}\cdot\vec{c} = 4|\vec{a}\times\vec{b}|^2 + 9|\vec{b}|^2$

$|\vec{c}|^2 = 4[|a|^2|b|^2 - (a\cdot\vec{b})^2] + 9|\vec{b}|^2$

$|\vec{c}|^2 = 4[(1)(4)^2 - (4)] + 9(16)$

$|\vec{c}|^2 = 4[12] + 144$

$|\vec{c}|^2 = 48 + 144$

$|\vec{c}|^2 = 192$

$\therefore \cos\theta = \frac{\vec{b}\cdot\vec{c}}{|\vec{b}||\vec{c}|}$

$\therefore \cos\theta = \frac{-48}{\sqrt{192}.4}$

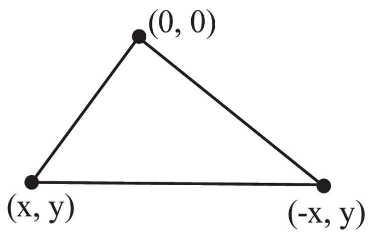
$\therefore \cos\theta = \frac{-48}{8\sqrt{3}.4}$

$\therefore \cos\theta = \frac{-3}{2\sqrt{3}}$

$\therefore \cos\theta = \frac{-\sqrt{3}}{2} \Rightarrow \theta = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

5. D

Sol.



Area of Δ

$$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ x & y & 1 \\ -x & y & 1 \end{vmatrix}$$

$$\Rightarrow \left|\frac{1}{2}(xy + xy)\right| = |xy|$$

Area (Δ) = |xy| = $|x(-2x^2 + 54)|$

$$\frac{d(\Delta)}{dx} = \left|(-6x^2 + 54)\right| \Rightarrow \frac{d\Delta}{dx} = 0 \text{ at } x = 3$$

Area = 3 (–2 × 9 + 54) = 108

6. B

Sol. $$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n^3}{n^4\left(1+\frac{k^2}{n^2}\right)\left(1\frac{3k^2}{n^2}\right)}$$

$$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{n^3}{\left(1+\frac{k^2}{n^2}\right)\left(1+\frac{3k^2}{n^2}\right)}$$

$$= \int_0^1 \frac{dx}{3(1+x^2)\left(\frac{1}{3}+x^2\right)}$$

$$= \int_0^1 \frac{1}{3}\times\frac{3}{2}\frac{(x^2+1)-\left(x^2+\frac{1}{3}\right)}{(1+x^2)\left(x^2+\frac{1}{3}\right)}dx$$

$$= \frac{1}{2}\left[\sqrt{3}\tan^{-1}\left(\sqrt{3}x\right)\right]_0^1 - \frac{1}{2}(\tan^{-1}x)_0^1$$

$$= \frac{\sqrt{3}}{2}\left(\frac{\pi}{3}\right) - \frac{1}{2}\left(\frac{\pi}{4}\right) = \frac{\pi}{2\sqrt{3}} - \frac{\pi}{8}$$

$$= \frac{13\pi}{8\cdot\left(4\sqrt{3}+3\right)}$$

7. A

Sol. $$f'(x) = \frac{g'(x) - g'(2-x)}{2},$$

$$f'\left(\frac{3}{2}\right) = \frac{g'\left(\frac{3}{2}\right) - g'\left(\frac{1}{2}\right)}{2} = 0$$

Also $$f'\left(\frac{1}{2}\right) = \frac{g'\left(\frac{1}{2}\right) - g'\left(\frac{3}{2}\right)}{2} = 0,\ f'\left(\frac{1}{2}\right) = 0$$

$$\Rightarrow f'\left(\frac{3}{2}\right) = f'\left(\frac{1}{2}\right) = 0$$

$\Rightarrow$ roots in $\left(\frac{1}{2}, 1\right)$ and $\left(1, \frac{3}{2}\right)$

$\Rightarrow$ f "(x) is zero at least twice in $\left(\frac{1}{2}, \frac{3}{2}\right)$

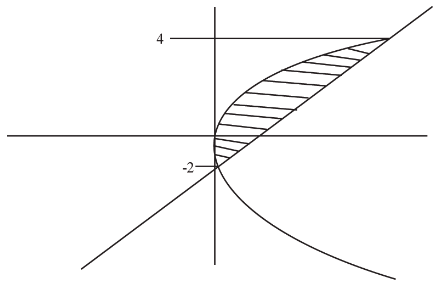
8. B

Sol. Let X = x – 2

$y^2 = 4x$, y = 2 (x + 2) – 8

$y^2 = 4x$, y = 2x – 4

$$A = \int_{-2}^{4}\frac{y^2}{4} - \frac{y+4}{2}$$



= 9

9. A

Sol. $$\frac{dy}{dx} = 2(x-1)\sin x + (x^2 - 2x)\cos x$$

Now both side integrate

$$y(x) = \int 2(x-1)\sin x\,dx + [(x^2-2x)(\sin x) - \int(2x-2)\sin x\,dx]$$

$$y(x) = (x^2 - 2x)\sin x + \lambda$$

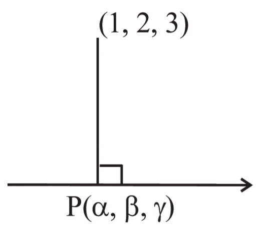
$$y(0) = 0 + \lambda \Rightarrow 2 = \lambda$$

y(x) = $(x^2 - 2x)$ sin x + 2

y(2) = 2

10. B

Sol.



Let foot P (5k – 3, 2k + 1, 3k – 4)

DR's $\rightarrow$ AP : 5k – 4, 2k – 1, 3k – 7

DR's $\rightarrow$ Line : 5, 2, 3

Condition of perpendicular lines (25k – 20) + (4k – 2) + (9k – 21) = 0

Then $k = \frac{43}{38}$

Then $19(\alpha+\beta+\gamma) = 101$

11. A

Sol. If x = 0, y = 6, 7, 8, 9, 10

If x = 1, y = 7, 8, 9, 10

If x = 2, y = 8, 9, 10

If x = 3, y = 9, 10

If x = 4, y = 10

If x = 5, y = no possible value

Total possible ways = (5 + 4 + 3 + 2 + 1) × 2

$= 30$ Required probability $= \frac{30}{11\times 11} = \frac{30}{121}$

12. C

Sol. $-1 \le \left|\frac{2-|x|}{4}\right| \le 1$

$\Rightarrow \left|\frac{2-|x|}{4}\right| \le 1$

$-4 \le 2-|x| \le 4$

$-6 \le -|x| \le 2$

$-2 \le |x| \le 6$

$|x| \le 6$

$\Rightarrow x \in [-6, 6]$ ....(1)

Now, $3 - x \ne 1$

And $x \ne 2$ ....(2)

and 3 – x > 0

x < 3 .....(3)

From (1), (2) and (3)

$\Rightarrow x \in [-6, 3) - \{2\}$

$\alpha = 6$

$\beta = 3$

$\gamma = 2$

$\alpha+\beta+\gamma = 11$

13. B

Sol. $x + y + z = 4\mu$, $x + 2y + 2\lambda z = 10\mu$, $x + 3y + 4\lambda^2 z = \mu^2 + 15$,

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2\lambda \\ 1 & 3 & 4\lambda^2 \end{vmatrix} = (2\lambda-1)^2$$

For unique solution $\Delta \ne 0,\ 2\lambda - 1 \ne 0,\ \left(\lambda \ne \frac{1}{2}\right)$

Let $\Delta \ne 0,\ \lambda = \frac{1}{2}$

$$\Delta_y = 0,\ \Delta_x = \Delta_z = \begin{vmatrix} 4\mu & 1 & 1 \\ 10\mu & 2 & 1 \\ \mu^2+15 & 3 & 1 \end{vmatrix}$$

$= (\mu-15)(\mu-1)$

For infinite solution $\lambda = \frac{1}{2},\ \mu = 1$ or 15

14. C

Sol. If two circles intersect at two distinct points

$\Rightarrow |r_1 - r_2| < C_1C_2 < r_1 + r_2$

$|r-2| < \sqrt{9+16} < r+2$

|r – 2| < 5 and r + 2 > 5

–5 < r – 2 < 5 r > 3 ....(2)

–3 < r < 7 .....(1)

From (1) and (2)

3 < r < 7

15. D

Sol. 2b = ae $\quad \frac{b}{a} = \frac{e}{2}$

$e = \sqrt{1 - \frac{e^2}{4}} \quad e = \frac{2}{\sqrt{5}}$

16. D

Sol.

Class	Frequency	Comulative frequency
0-4	3	**3**
4-8	9	**12**
8-12	10	**22**
12-16	8	**30**
16-20	6	**36**

$$M = 1 + \left(\frac{\frac{N}{2} - C}{f}\right) h$$

$$M = 8 + \frac{18 - 12}{10} \times 4$$

$M = 10.4$

$20M = 208$

17. A

Sol. $\begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3 + \sin^2 2x \\ 3 + 2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3 + 2\sin^2 4x & \sin^2 2x \end{vmatrix}$

$R_2 \to R_2 - R_1,\ R_3 - R_1$

$$\begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3 + \sin^2 2x \\ 3 & 0 & -3 \\ 0 & 3 & -3 \end{vmatrix}$$

$f(x) = 45$

$f'(x) = 0$

18. B

Sol. $\text{Area} = |\overrightarrow{AC} \times \overrightarrow{BD}|$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -1 & 7 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= \frac{1}{2} |-17\hat{i} - 8\hat{j} + 11\hat{k}| = \frac{1}{2}\sqrt{474}$$

19. B

Sol. $2 \sin x + 2 \sin x \cdot \cos^2 x + 4 \sin x - 4 = 0$

$2 \sin^3 x + 2 \sin x \cdot (1 - \sin^2 x) + 4 \sin x - 4 = 0$

$6 \sin x - 4 = 0$

$$\sin x = \frac{2}{3}$$

n = 5 (in the given interval)

$x^2 + 5x + 2 = 0$

$$x = \frac{-5 \pm \sqrt{17}}{2}$$

Required interval $(-\infty, 0)$

20. B

Sol. $$\lim_{x \to 0} \frac{x\int_0^x f(t)dt}{\left(\frac{e^{x^2} - 1}{x^2}\right) \times x^2}$$

$$\lim_{x \to 0} \frac{\int_0^x f(t)dt}{x} \quad \left(\lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} = 1\right)$$

$$= \lim_{x \to 0} \frac{f(x)}{1} \text{ (using L Hospital)}$$

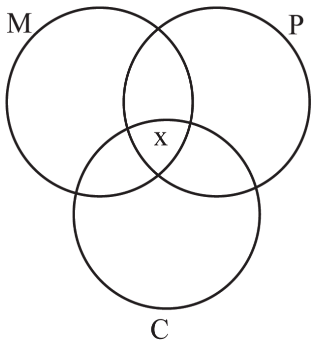
$f(0) = \frac{1}{2}$ $\qquad$ $\alpha = \frac{1}{2}$

$8\alpha^2 = 2$

<u>Section - B (Numerical Value Type)</u>

21. 10

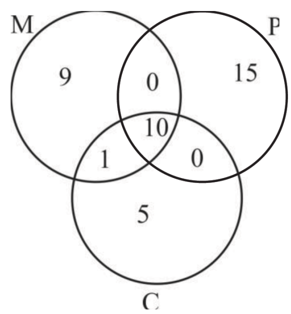
Sol.



$11 - x > 0$ (Maths and Physics) $\quad x \le 11$

$x = 11$ does not satisfy the data.

For $x = 10$



Hence maximum number of students passed in all the three subjects is 10.

22. 16

Sol. $L_1 : \frac{x+1}{1} = \frac{y}{1/2} = \frac{z}{-1/12}, \; L_2 : \frac{x}{1} = \frac{y+2}{1} = \frac{z-1}{\frac{1}{6}}$

d_1 = shortest distance between L_1 & L_2

$$= \left| \frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|(\vec{b}_1 \times \vec{b}_2)|} \right|$$

$d_1 = 2$

$L_3 : \frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5}, \; L_4 : \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$

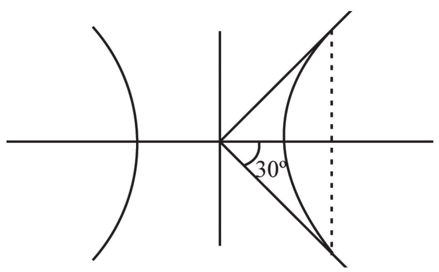
d_2 = shortest distance between L_3 & L_4

$d_2 = \frac{12}{\sqrt{3}}$ Hence

$$= \frac{32\sqrt{3}d_1}{d_2} = \frac{32\sqrt{3}\times 2}{\frac{12}{\sqrt{3}}} = 16$$

23. 182

Sol. LR subtends 60° at centre



$$\Rightarrow \tan 30^\circ = \frac{b^2/a}{ae} = \frac{b^2}{a^2e} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow e = \frac{\sqrt{3}b^2}{9}$$

Also, $e^2 = 1 + \frac{b^2}{9} \Rightarrow 1 + \frac{b^2}{9} = \frac{3b^4}{81}$

$$\Rightarrow b^4 = 3b^2 + 27$$

$$\Rightarrow b^4 - 3b^2 - 27 = 0$$

$$\Rightarrow b^2 = \frac{3}{2}(1+\sqrt{13})$$

$$\Rightarrow \ell = 3,\; m = 2,\; n = 13$$

$$\Rightarrow \ell^2 + m^2 + n^2 = 182$$

24. 44

Sol. $f : A \to P(A)$

$a \in f(a)$

That means 'a' will connect with subset which contain element 'a'.

Total options for 1 will be 2^6. (Because 2^6 subsets contains 1)

Similarly, for every other element

Hence, total is $2^6 \times 2^6 \times 2^6 \times 2^6 \times 2^6 \times 2^6 \text{ x } 2^6 = 2^{42}$

Ans. 2 + 42 = 44

25. 155

Sol. $\frac{10x}{x+1} = 1 \qquad \Rightarrow x = \frac{1}{9}$

$\frac{10x}{x+1} = 4 \qquad \Rightarrow x = \frac{2}{3}$

$\frac{10x}{x+1} = 9 \qquad \Rightarrow x = 9$

$$I = 9\left(\int_0^{1/9} 0\,dx + \int_{1/9}^{2/3} t\cdot dt + \int_{2/3}^{9} 2dx \right)$$

$= 155$

26. 138

Sol. General term in expansion of $((7)^{1/2} + (11)^{1/6})^{824}$ is

$$t_{r+1} = {}^{824}C_r (7)^{\frac{824-r}{2}} (11)^{r/6}$$

For integral term, r must be multiple of 6.

Hence r = 0, 6, 12, ... 822

27. 97

Sol. $\frac{dy}{dx} - \frac{xy}{1-x^2} = \frac{(x^3+2)\sqrt{3(1-x^2)}}{1-x^2}$

$$IF = e^{-\int \frac{x}{1-x^2}dx} = e^{+\frac{1}{2}\ln(1-x^2)} = \sqrt{1-x^2}$$

$$y\sqrt{1-x^2} = \sqrt{3}\int (x^3+2)dx$$

$$y\sqrt{1-x^2} = \sqrt{3}\left(\frac{x^4}{4} + 2x\right) + c$$

$\Rightarrow y(0) = 0 \qquad \therefore c = 0$

$$y\left(\frac{1}{2}\right) = \frac{65}{32} = \frac{m}{n}$$

$m + n = 97$

28. 60

Sol. $x^2 - 70x + \lambda = 0$

$\alpha + \beta = 70$

$\alpha\beta = \lambda$

$\therefore \alpha(70 - \alpha) = \lambda$

Since, 2 and 3 does not divide λ

$\therefore \alpha = 5,\ \beta = 65,\ \lambda = 325$

By putting value of α, β, λ we get the required value 60.

29. 15

Sol. $f(x) = \begin{cases} \frac{1}{x}; & x \geq 2 \\ ax^2 + 2b; & -2 < x < 2 \\ -\frac{1}{x}; & x \leq -2 \end{cases}$

Continuous at $x = 2 \Rightarrow \frac{1}{2} = \frac{\pi}{4} + 2b$

Continuous at $x = -2 \Rightarrow \frac{1}{2} = \frac{\pi}{4} + 2b$

Since, it is differentiable at x = 2

$-\frac{1}{x^2} = 2ax$

Differentiable at $x = 2 \Rightarrow \frac{-1}{4} = 4a \Rightarrow a = \frac{-1}{16}$, b

$= \frac{3}{8}$

30. 353

$\alpha = 1^2 + 4^2 + 8^2 ......$

$t_n = an^2 + bn + c$

$1 = a + b + c$

$4 = 4a + 2b + c$

$8 = 9a + 3b + c$

On solving we get, $a = \frac{1}{2},\ b = \frac{3}{2},\ c = -1$

$$\alpha = \sum_{n=1}^{10} \left(\frac{n^2}{2} + \frac{3n}{2} - 1 \right)^2$$

$$4\alpha = \sum_{n=1}^{10} (n^2 + 3n - 2)^2,\ \beta = \sum_{n=1}^{10} n^4$$

$$4\alpha - \beta = \sum_{n=1}^{10} (6n^3 - 5n^2 - 12n + 4) = 55(353) + 40$$

PHYSICS

Section - A (Single Correct Answer)

31. C

Sol. $F = \eta A \frac{dv}{dy}$

$[MLT^{-2}] = \eta [L^2][T^{-1}]$

$\eta = [ML^{-1}T^{-1}]$

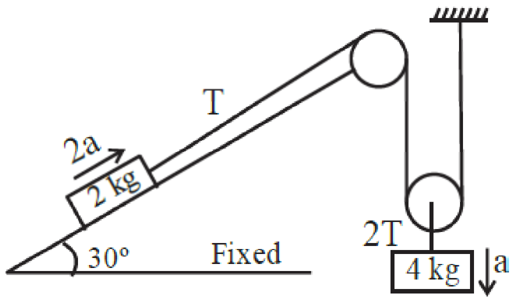
$S.T. = \frac{F}{l} = \frac{[MLT^{-2}]}{[L]} = [ML^0T^{-2}]$

$L = mvr = [ML^2T^{-1}]$

$K.E. = \frac{1}{2} I\omega^2 = [ML^2T^{-2}]$

32. B

Sol.



$40 - 2T = 4a$

$T - 10 = 4a \Rightarrow 20 = 12a$

$\Rightarrow a = 5/3 \Rightarrow 2a = g/3$

33. C

Sol. $R_{eq} = 4000\ \Omega$

$i = \frac{4}{4000} = \frac{1}{1000} A$

$V_0 = i.R = \frac{1}{1000} \times 500 = 0.5V$

34. C

Sol. Young's modulus depends on the material not length and cross sectional area. So young's modulus remains same.

35. B

Sol. For P.E.E. $\lambda \le \dfrac{hc}{W_e}$

$$\lambda \le \frac{1240\text{nm} - \text{eV}}{3\text{eV}}$$

$\lambda \le 413.33\text{nm}$

$\lambda_{max} \approx 414\text{nm}$ for P.E.E.

36. C

Sol. $\dfrac{1}{2}|\text{P.E}| = \text{KE}$ for each value of n (orbit)

$$\therefore \frac{\text{KE}}{|\text{PE}|} = \frac{1}{2}$$

37. B

Sol. By COME

$KE_A + U_A = KE_B + U_B$

$$0 + mg(1) = \frac{1}{2}mv^2 + mg \times 0.5$$

$$v = \sqrt{g} = \sqrt{10}\ \text{m/s}$$

38. B

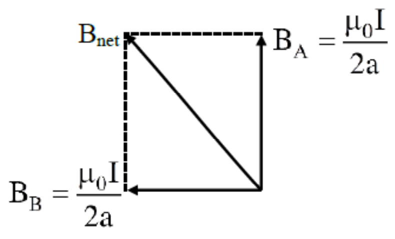
Sol. Given $\vec{E} = E_0 \cos(\omega t - kz)\hat{i}$

$$\vec{B} = \frac{E_0}{C}\cos(\omega t - kz)\hat{j}$$

$$\hat{C} = \hat{E} \times \hat{B}$$

39. C

Sol.



$$\because B_{net} = \frac{\sqrt{2}\mu_0 I}{2a}$$

40. A

Sol. Width of 1st secondary maxima $= \dfrac{\lambda}{a} \cdot D$

Here

$a = 0.2 \times 10^{-3}$ m

$\lambda = 400 \times 10^{-9}$ m

$D = 100 \times 10^{-2}$

Width of 1st secondary maxima

$$= \frac{400 \times 10^{-9}}{0.2 \times 10^{-3}} \times 100 \times 10^{-2} = 2\text{mm}$$

41. A

Sol. $\dfrac{\varepsilon_1}{\varepsilon_2} = \dfrac{N_1}{N_2} = \dfrac{100}{10} \Rightarrow \varepsilon_2 = 22\text{V}$

$$I = \frac{22}{22 \times 10^3} = 1\ \text{mA},\ V_0 = 7\text{V}$$

42. A

Sol. $-\dfrac{GM_E}{R_E + h} = -5.12 \times 10^{-7}$ .... (i)

$\dfrac{GM_E}{(R_E + h)^2} = 6.4$ ... (ii)

By (i) and (ii)

$\Rightarrow h = 16 \times 10^5\ \text{m} = 1600\text{km}$

43. C

Sol. $R_{T-27} = 60\Omega, R_T = \dfrac{220}{2.75} = 80\Omega$

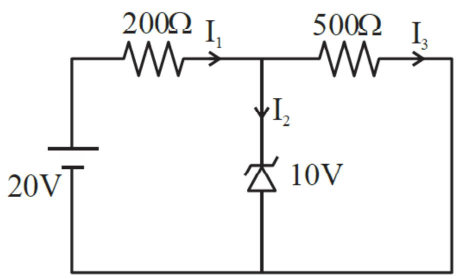
$R = R_0(1 + \alpha\Delta T)$

$80 = 60[1 + 2 \times 10^{-4}(T - 27)]$

$T \approx 1694°\text{C}$

44. C

Sol.



Zener is in breakdown region.

$$I_3 = \frac{10}{500} = \frac{1}{50}$$

$$I_1 = \frac{10}{200} = \frac{1}{20}$$

$I_2 = I_1 - I_3$

$$I_2 = \left(\frac{1}{20} - \frac{1}{50}\right) = \left(\frac{3}{100}\right) = 30\text{mA}$$

45. Bonus

Sol. For process A

$\log P = \gamma \log V \Rightarrow P = V^{\gamma}\ (\gamma > 1)$

$PV^{-\gamma}$ = Constant

$C_A = C_V + \dfrac{R}{1+\gamma}$ .... (i)

Likewise for process B $\rightarrow PV^{-1}$ Constant

$$C_B = C_v + \frac{R}{1+1}$$

$$C_B = C_v + \frac{R}{2} \quad .... \text{(ii)}$$

$C_P = C_v + R$ .... (iii)

By (i), (ii) & (iii)

$C_P > C_B > C_A > C_v$ [No answer matching]

46. B

Sol. $V = \frac{kP\cos\theta}{r^2}$

& can also checked dimensionally

47. D

Sol. $\vec{I} = \Delta\vec{P} = \vec{P}_f - \vec{P}_i$

M = 0.1 kg

$$I = \Delta P = 0.1\left(\sqrt{2\times9.8\times5} - \left(-\sqrt{2\times9.8\times10}\right)\right)$$

$$= 0.1\left(14 + 7\sqrt{2}\right) \approx 2.39 \text{ kg ms}^{-1}$$

48. A

Sol. L = mu cos θ H

$$= mu\cos\theta \times \frac{u^2\sin^2\theta}{2g}$$

$$= \frac{mu^3}{2g}\times\frac{\sqrt{3}}{2}\times\left(\frac{1}{2}\right)^2 = \frac{\sqrt{3}mu^3}{16g}$$

49. D

Sol. $\sqrt{\frac{3RT}{2}} = \sqrt{\frac{3R(320)}{32}}$

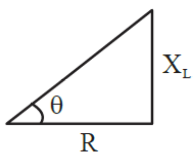
$$T = \frac{320}{16} = 20K$$

50. C

Sol. E = 25 sin (1000 t)

$$\cos\theta = \frac{1}{\sqrt{2}}$$

LR circuit



Initially $\frac{R}{\omega_1 L} = \frac{1}{\tan\theta} = \frac{1}{\tan 45^\circ} = 1$

$X_L = \omega_1 L$

$\omega_2 = 2\omega_1$, given

$$\tan\theta' = \frac{\omega_2 L}{R} = \frac{2\omega_1 L}{R}$$

$\tan\theta' = 2$

$$\cos\theta' = \frac{1}{\sqrt{5}}$$

Section - B (Numerical Value Type)

51. 35

Sol. $B_H = 3.5 \times 10^{-5}$ T

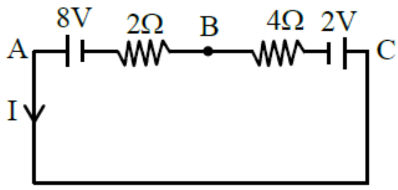
$F = ilB\sin\theta$, $i = \sqrt{2}A$

$$\frac{F}{l} = iB\sin\theta = \sqrt{2}\times3.5\times10^{-5}\times\frac{1}{\sqrt{2}}$$

$= 35\times10^{-6}$ N/m

52. 6

Sol.



$$I = \frac{8-2}{2+4} = \frac{6}{6} = 1A$$

Applying Kirchhoff from C to B

$V_C - 2 - 4\times1 = V_B$

$V_C - V_B = 6V$

53. 6

Sol. $E_n = -\frac{13.6}{n^2} = -0.85$

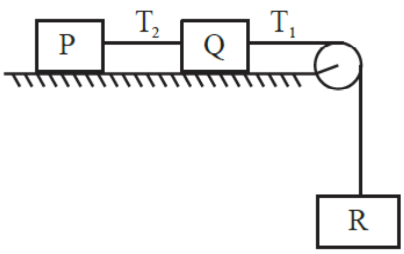
$\Rightarrow \quad n = 4$

No of transition

$$= \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

54. B

Sol.



$a = \frac{10}{3}$ m/s^2

$30 - T_1 = 3 \times a$
$T_1 = 20$ N

$\text{strain} = \frac{\text{stress}}{Y}$

$= 2 \times 10^{-4}$

55. 10

Sol. $\frac{v}{u} = -2$

$v = -2u$ .....(i)

$v - u = 45$ .....(ii)

$\Rightarrow$ $u = -15$ cm

$v = 30$ cm

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$f = +10$ cm

56. 175

Sol. Considering acceleration is constant

$v = u + at$

$u + 50 = u + a \Rightarrow a = 50$ m/s^2

$125 = ut + \frac{1}{2}at^2$

$125 = u + \frac{a}{2}$

$\Rightarrow u = 100$ m/s

$\therefore S_{n^{th}} = u + \frac{a}{2}[2n - 1]$ $= 175$ m

57. B

Sol. Energy loss $= \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$

$= \frac{2}{3} \cdot E$

$\therefore x = 2$

58. 250

Sol. $\vec{L}_i = I\omega_i = \frac{MR^2}{2} \cdot \omega = 100$ kgm^2/s

$E_i = \frac{1}{2} \cdot \frac{MR^2}{2} \cdot \omega^2 = 500$ J

$\vec{L}_i = \vec{L}_f \Rightarrow 100 = 2I\omega_f$

$\omega_f = 5$ rad/sec

$E_f = 2 \times \frac{1}{2} \cdot \frac{5(2)^2}{2} \cdot (5)^2 = 250$ J

$\Delta E = 250$ J

59. 400

Sol. $\frac{V}{4l_1} = 30 \Rightarrow l_1 = \frac{11}{4}$m

$\frac{V}{4l_2} = 110 \Rightarrow l_2 = \frac{3}{4}$m

$\Delta l = 2$m

Change in volume = $A\Delta l$ = 400cm^3

$M = 400$ g ; $(\because \rho = 1 g/cm^3)$

60. 32

Sol. $B_v = B \sin 30 = \frac{1}{4} \times 10^{-4}$

$\omega = 2\pi \times f = \frac{2\pi}{60} \times 1200$ rad/s

$\varepsilon = \frac{1}{2} B_V \omega l^2 = 32\pi \times 10^{-5}$ V

CHEMISTRY

Section - A (Single Correct Answer)

61. **(C)**

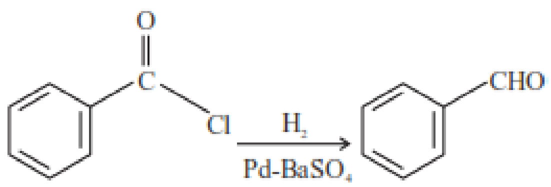
Sol. $Na_2S + H_2SO_4 \rightarrow Na_2SO_4 + H_2S$

$(CH_3COO)_2Pb + H_2S \rightarrow PbS + 2CH_3COOH$

(Black lead sulphide)

62. **(A)**

Sol.



It is known as rosenmund reduction that is the partial reduction of acid chloride to aldehyde.

63. **(A)**

Sol. Sucrose do not contain hemiacetal group.

Hence it does not give test with Fehling solution. While all other give positive test with Fehling solution.

64. **(B)**

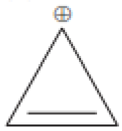
Sol. According to NCERT,

Statement-I : Factual data,

Statement-II is true.

But correct explanation is presence of completely filled d and f-orbitals of heavier members.

65. **(A)**

Sol.  it is aromatic species.

66. **(B)**

Sol. Ce : [Xe] $4f^{1}\ 5d^{1}\ 6s^{2}$; Ce^{4+} diamagnetic

La : [Xe] $4f^{0}\ 5d^{1}\ 6s^{2}$; La^{3+} diamagnetic

67. **(A)**

Sol. $AlCl_3$ in acidified aqueous solution forms octahedral geometry $[Al(H_2O)_6]^{3+}$.

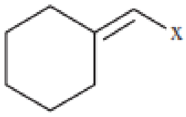
68. **(A)**

Sol. For single electron species the energy depends upon principal quantum number 'n' only. So, statement-II is false.

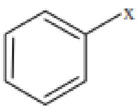
Statement-I is correct definition of degenerate orbitals.

69. **(A)**

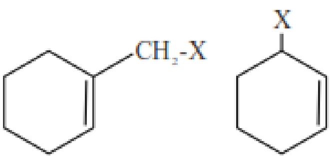
Sol. Vinyl carbon is sp^2-hybridized aliphatic carbon



is vinyl halide

While 

is aryl halide

and 

are allyl halide

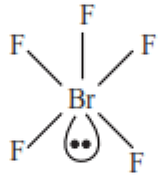
70. **(D)**

Sol. $\overset{5}{C}H_3 - \overset{4}{C}H(CH_3) - \underset{3}{C}H = \underset{2}{C}H - \underset{1}{C}(=O) - H$

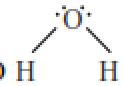
4-Methylpent-2-enal

71. **(D)**

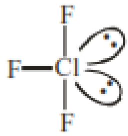
Sol.

BrF_5 

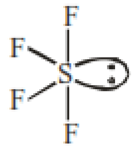
Square pyramidal

H_2O 

Bent

ClF_3 

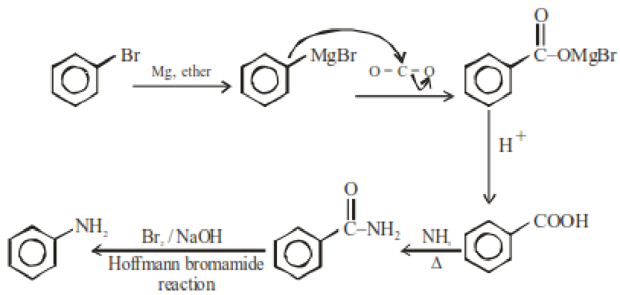
T-shape

SF_4, 

See-saw

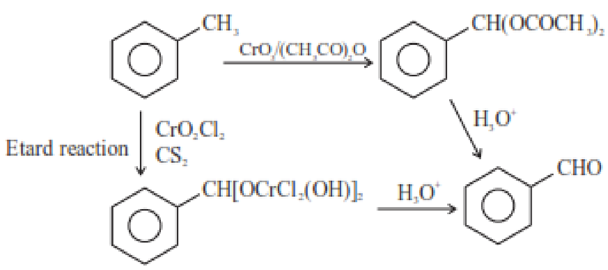
72. **(B)**

Sol.



73. **(B)**

Sol.



74. **(A)**

Sol. $CH_2 = CH - CH_2 - Cl$

$\uparrow$

It is allyl carbon and sp^3-hybridized

75. **(D)**

Sol. On addition of naphthalene to benzene there is depression in freezing point of benzene.

76. **(B)**

Sol. ${}_{24}Cr \rightarrow [Ar]\ 3d^5\ 4s^1$; $Cr^{2+} \rightarrow [Ar]\ 3d^4$

${}_{25}Mn \rightarrow [Ar]\ 3d^5\ 4s^2$; $Mn^{+} \rightarrow [Ar]\ 3d^5\ 4s^1$

${}_{28}Ni \rightarrow [Ar]\ 3d^8\ 4s^2$; $Ni^{2+} \rightarrow [Ar]\ 3d^8$

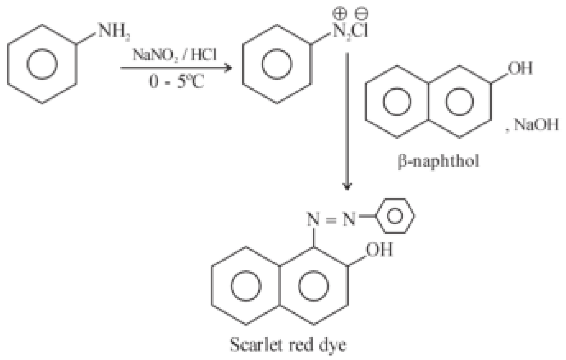
${}_{23}V \rightarrow [Ar]\ 3d^3\ 4s^2$; $V^{+} \rightarrow [Ar]\ 3d^3\ 4s^1$

77. **(A)**

Sol.

$$CH_3-C\equiv CH \xrightarrow{Na} CH_3-C\equiv C^{-}Na^{+} \xrightarrow{CH_3CH_2CH_2-Br} NaBr + CH_3-C\equiv C-CH_2CH_2CH_3$$

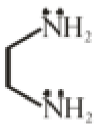
78. **(D)**

Sol. 

79. **(D)**

Sol. If nitrogen or sulphur is also present in the compound, the sodium fusion extract is first boiled with concentrated nitric acid to decompose cyanide or sulphide of sodium during Lassaigne's test.

80. **(C)**

Sol. 

Bidentate, chelating

Based on Elall-Eleroults process

$[Rh(PPh_3)_3Cl]$ Wilkinson's catalyst

$Ag_2S + NaCN \xrightleftharpoons{Air} Na[Ag(CN)_2] + Na_2S$

Ca^{++} ion forms more stable complex with EDTA.

Section - B (Numerical Value Type)

81. **(24)**

Sol. $0.04 = k[A]_0\ e^{-k \times 10 \times 60}$ ... (1)

$0.03 = k[A]_0\ e^{-k \times 20 \times 60}$ ... (2)

(1)/(2)

$$\frac{4}{3} = e^{600\,k(2-1)}$$

$$\frac{4}{3} = e^{600k}$$

$$\ln\frac{4}{3} = 600k$$

$$\ln\frac{4}{3} = 600 \times \frac{\ln 2}{t_{1/2}}$$

$$t_{1/2} = 600\frac{\ln 2}{\ln\frac{4}{3}}\text{sec}$$

$$t_{1/2} = 600 \times \frac{\log 2}{\log 4 - \log 3}\text{sec.} = 10 \times \frac{0.3010}{0.6020 - 0.477}\text{min.}$$

$$t_{1/2} = 24.08 \text{ min}$$

Ans. 24

82. **(9)**

Sol. Precipitation when $Q_{sp} = K_{sp}$

$[Mg^{2+}]\,[OH^-]^2 = 10^{-11}$

$0.1 \times [OH^-]^2 = 10^{-11} \Rightarrow [OH^-] = 10^{-5}$

$\Rightarrow$ pOH = 5 $\Rightarrow$ pH = 9

83. **(200)**

Sol. Work done is given by area enclosed in the P *vs* V cyclic graph or V *vs* P cyclic graph.

Sign of work is positive for clockwise cyclic process for V *vs* P graph.

$W = 1 \times (30 - 10) \times (30 - 10) = 200$ kPa – dm^3

$= 200 \times 1000$ Pa – L = 2 L-bar = 200 J

84. **(11)**

Sol. 111 belongs to 11th group.

85. **(8)**

Sol. Two molecular orbitals $\sigma2s$ and σ^*2s.

Six molecular orbitals $\sigma2p_z$ and σ^*2p_z

$\pi2p_x$, $\pi2p_y$ and π^*2p_x, π^*2p_y

86. **(7)**

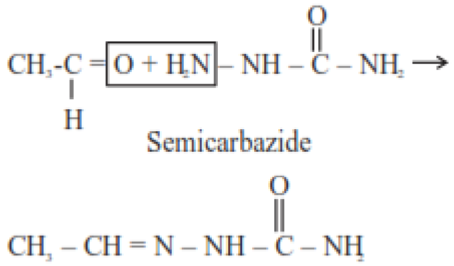
Sol. Retardation factor =

$$\frac{\text{Distance travelled by sample/organic compound}}{\text{Distance travelled by solvent}}$$

$$= \frac{3.5}{5} = 7 \times 10^{-1}$$

87. **(3)**

Sol.



88. **(11)**

Sol. Volume of silver coating = 0.05 × 0.05 × 10000 = 25 cm^3

Mass of silver deposited = 25 × 7.9 g

Moles of silver atoms

$$= \frac{25 \times 7.9}{108}$$

Number of silver atoms

$$= \frac{25 \times 7.9}{108} \times 6.023 \times 10^{23}$$

$= 11.01 \times 10^{23}$

Ans. 11

89. **(8)**

Sol. Reduction Half

$2MnO_4^- \rightarrow 2MnO_2$

$2MnO_4^- + 4H_2O + 6e^- \rightarrow 2MnO_2 + 8OH^-$

Oxidation Half

$2I^- \rightarrow I_2 + 2e^-$

$6I^- \rightarrow 3I_2 + 6e^-$

Adding oxidation half and reduction half, net reaction is,

$2MnO_4^- + 6I^- + 4H_2O \rightarrow 3I_2 + 2MnO_2 + 8OH^-$

$\Rightarrow z = 8$

$\Rightarrow$ Ans 8

90. **(7)**

Sol. Moles = Molarity × Volume in litres

= 0.35 × 0.25

Mass = moles × molar mass

= 0.35 × 0.25 × 82.02 = 7.18 g

Ans. 7

● ● ●