29-Jan.-2024 (Evening) : PCM [SOLUTIONS]

MATHEMATICS

Section - A (Single Correct Answer)

1. A

Sol. $|P^{-1}AP - 2I| = |P^{-1}AP - 2P^{-1}P|$

$= |P^{-1}(A - 2I)P|$

$= |P^{-1}|\,|A - 2I|\,|P|$

$= |A - 2I|$

$$= \begin{vmatrix} 0 & 1 & 2 \\ 6 & 0 & 11 \\ 3 & 3 & 0 \end{vmatrix} = 69$$

So, Prime factor of 69 is 3 & 23

So, sum = 26

2. D

Sol. 3 Shelf empty : (8, 0, 0, 0) $\rightarrow$ 1 way

2 shelf empty : $\left.\begin{matrix}(7, 1, 0, 0)\\(6, 2, 0, 0)\\(5, 3, 0, 0)\\(4, 4, 0, 0)\end{matrix}\right] \rightarrow 4$ ways

1 shelf empty :

$\left.\begin{matrix}(6, 1, 1, 0) & (3, 3, 2, 0)\\(5, 2, 1, 0) & (4, 2, 2, 0)\\(4, 3, 1, 0) & \end{matrix}\right] \rightarrow 5$ ways

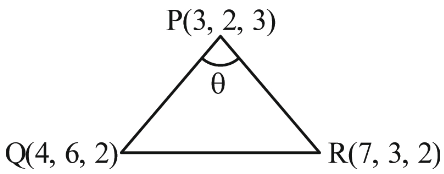
0 Shelf empty :

$\left.\begin{matrix}(1, 2, 3, 2) & (5, 1, 1, 1)\\(2, 2, 2, 2) & \\(3, 3, 1, 1) & \\(4, 2, 1, 1) & \end{matrix}\right] \rightarrow 5$ ways

Total = 15 ways

3. D

Sol.



Direction ratio of PR = (4, 1, –1)

Direction ratio of PQ = (1, 4, –1)

Now, $\cos\theta = \left|\dfrac{4+4+1}{\sqrt{18}\cdot\sqrt{18}}\right|$

$\theta = \dfrac{\pi}{3}$

4. C

Sol. $\overline{X} = \dfrac{24}{5};\ \sigma^2 = \dfrac{194}{25}$

Let first four observation be x_1, x_2, x_3, x_4

Here, $\dfrac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = \dfrac{24}{5}$ .....(1)

Also, $\dfrac{x_1 + x_2 + x_3 + x_4}{4} = \dfrac{7}{2}$

$\Rightarrow \boxed{x_1 + x_2 + x_3 + x_4 = 14}$

Now from equation - 1

$x_5 = 10$

Now, $\sigma^2 = \dfrac{194}{25}$

$\dfrac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2}{5} - \dfrac{576}{25} = \dfrac{194}{25}$

$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 = 54$

Now, variance of first 4 observations

$$\text{Var} = \frac{\sum_{i=1}^{4} x_i^2}{4} - \left(\frac{\sum_{i=1}^{4} x_i}{4}\right)^2$$

$$= \frac{54}{4} - \frac{49}{4} = \frac{5}{4}$$

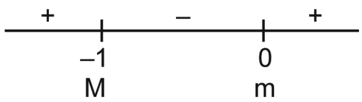
5. C

Sol. $f(x) = 2x + 3(x)^{\frac{2}{3}}$

$$f'(x) = 2 + 2x^{\frac{-1}{3}}$$

$$= 2\left(1 \frac{1}{x^{\frac{1}{3}}}\right)$$

$$= 2\left(\frac{x^{\frac{1}{3}} + 1}{x^{\frac{1}{3}}}\right)$$



So, maxima (M) at x = –1 & minima(m) at x = 0

6. A

Sol. $z = 2 - i\left(2\tan\frac{5\pi}{8}\right) = x + iy \text{ (let)}$

$$r = \sqrt{x^2 + y^2} \ \& \ \theta = \tan^{-1}\frac{y}{x}$$

$$r = \sqrt{(2)^2 + \left(2\tan\frac{5\pi}{8}\right)^2}$$

$$= \left|2\sec\frac{5\pi}{8}\right| = \left|2\sec\left(\pi - \frac{3\pi}{8}\right)\right|$$

$$= 2\sec\frac{3\pi}{8}$$

$$\& \ \theta = \tan^{-1}\left(\frac{-2\tan\frac{5\pi}{8}}{2}\right)$$

$$= \tan^{-1}\left(\tan\left(\pi - \frac{5\pi}{8}\right)\right)$$

$$= \frac{3\pi}{8}$$

7. C

Sol. $\dfrac{3\cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6$

$$\Rightarrow \frac{\cos 2x(3 + \cos^2 2x)}{\cos 2x(1 - \sin^2 x \cos^2 x)} = x^3 - x^2 + 6$$

$$\Rightarrow \frac{4(3 + \cos^2 2x)}{(4 - \sin^2 2x)} = x^3 - x^2 + 6$$

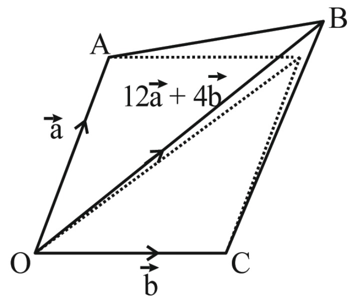
$$\Rightarrow \frac{4(3 + \cos^2 2x)}{(3 + \cos^2 2x)} = x^3 - x^2 + 6$$

$x^3 - x^2 + 2 = 0 \Rightarrow (x + 1)(x^2 - 2x + 2) = 0$

so, sum of real solutions = –1

8. D

Sol.



Area of parallelogram, $S = |\vec{a} \times \vec{b}|$

Area of quadrilateral =Area(ΔOAB) + Area (ΔOBC)

$$= \frac{1}{2}\{|\vec{a} \times (12\vec{a} + 4\vec{b})| + |\vec{b} \times (12\vec{a} + 4\vec{b})|\}$$

$$= 8|(\vec{a} \times \vec{b})|$$

$$\text{Ratio} = \frac{8|(\vec{a} \times \vec{b})|}{|\vec{a} \times \vec{b}|} = 8$$

9. A

Sol. $\log_e a, \log_e b, \log_e c$ are in A.P.

$\therefore b^2 = ac$ ....(i)

Also

$\log_e\left(\frac{a}{2b}\right), \ \log_e\left(\frac{2b}{3c}\right), \ \log_e\left(\frac{3c}{a}\right)$ are in A.P.

$$\left(\frac{2b}{3c}\right)^2 = \frac{a}{2b} \times \frac{3c}{a}$$

3S INSTITUTE
JEE ADVANCED | JEE MAIN | NEET | OLYMPIADS | MHT-CET | FOUNDATION

Putting in eq. (i) $b^2 = a \times \frac{2b}{3}$

$\frac{a}{b} = \frac{3}{2}$

$a : b : c = 9 : 6 : 4$

10. D

Sol. $\int \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}x}}{\sqrt{\sin^3 x \cos^3 x \sin(x-\theta)}} dx$

$$I = \int \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sqrt{\sin^3 x \cos^2 x(\sin x \cos\theta - \cos x \sin\theta)}} dx$$

$$= \int \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x \cos^2 x \sqrt{\tan x \cos\theta - \sin\theta}} dx$$

$$+ \int \frac{\cos^{\frac{3}{2}} x}{\sin^2 x \cos^{\frac{3}{2}} x \sqrt{\cos\theta - \cot x \sin\theta}} dx =$$

$$\int \frac{\sec^2 x}{\sqrt{\tan x \cos\theta - \sin\theta}} dx + \int \frac{\operatorname{cosec}^2 x}{\sqrt{\cos\theta - \cot x \sin\theta}}$$

$I = I_1 + I_2$ {Let}

for I_1, let $\tan x \cos\theta - \sin\theta = t^2$

$\sec^2 x \, dx = \frac{2t\,dt}{\cos\theta}$

For I_2, let $\cos\theta - \cot x \sin\theta = z^2$

$\operatorname{cosec}^2 x \, dx = \frac{2z\,dz}{\sin\theta}$

$I = I_1 + I_2$

$= \int \frac{2t\,dt}{\cos\theta\, t} + \int \frac{2z\,dz}{\sin\theta\, z}$

$= \frac{2t}{\cos\theta} + \frac{2z}{\sin\theta}$

$= 2\sec\theta\sqrt{\tan x \cos\theta - \sin\theta}$

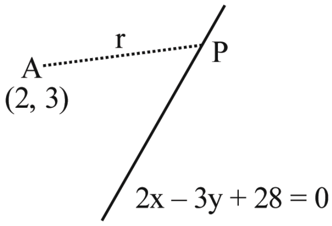
$+ 2\operatorname{cosec}\theta\sqrt{\cos\theta - \cot x \sin\theta}$

Comparing

$AB = 8\operatorname{cosec} 2\theta$

11. D

Sol.



Writing P in terms of parametric co-ordinates $2 + r\cos\theta,\ 3 + r\sin\theta$ as $\tan\theta = \sqrt{3}$

$P\left(2 + \frac{r}{2},\ 3 + \frac{\sqrt{3}r}{2}\right)$

P must satisfy $2x - 3y + 28 = 0$

So, $2\left(2 + \frac{r}{2}\right) - 3\left(3 + \frac{\sqrt{3}r}{2}\right) + 28 = 0$

We find $r = 4 + 6\sqrt{3}$

12. A

Sol. Differential equation :

$x \cos\frac{y}{x}\frac{dy}{dx} = y\cos\frac{y}{x} + x$

$\cos\frac{y}{x}\left[x\frac{dy}{dx} - y\right] = x$

Divide both sides by x^2

$\cos\frac{y}{x}\left(\frac{x\frac{dy}{dx} - y}{x^2}\right) = \frac{1}{x}$

Let $\frac{y}{x} = t$

$\cos t\left(\frac{dt}{dx}\right) = \frac{1}{x}$

$\cos t\, dt = \frac{1}{x} dx$

Integrating both sides

$\sin t = \ln|x| + c$

$\sin\frac{y}{x} = \ln|x| + c$

Using $y(1)=\frac{\pi}{3}$, we get $c=\frac{\sqrt{3}}{2}$

So, $\alpha=\sqrt{3}\Rightarrow\alpha^2=3$

13. D

Sol. Let r'th term of the GP be ar^{n-1}. Given,

$2a_r = ar + 1 + ar + 2$

$2ar^{n-1} = ar^n + ar^{n+1}$

$\frac{2}{r}=1+r$

$r^2 + r - 2 = 0$

Hence, we get, $r = -2$ (as $r \neq 1$)

So, $S_{20} - S_{18}$ = (Sum upto 20 terms) – (Sum upto 18 terms) = $T_{19} + T_{20}$

$T_{19} + T_{20} = ar^{18}(1+ r)$

Putting the values $a=\frac{1}{8}$ and $r = -2$;

we get $T_{19} + T_{20} = -2^{15}$

14. D

Sol. Solving lines L_1 $(3x + 2y = 14)$ and L_2 $(5x - y = 6)$ to get A(2, 4) and solving lines L_3 $(4x + 3y = 8)$ and L_4 $(6x + y = 5)$ to get $B\left(\frac{1}{2},\ 2\right)$

Finding Eqn. of AB : $4x - 3y + 4 = 0$

Calculate distance PM

$\Rightarrow\left|\frac{4(5)-3(-2)+4}{5}\right|=6$

15. D

Sol. Assume $\sin^{-1} x = 0$

$\cos(2\theta)=\frac{1}{9}$

$\sin\theta=\pm\frac{2}{3}$

as m and n are co-prime natural numbers,

$x=\frac{2}{3}$

i.e. $m = 2, n = 3$

So, the quadratic equation becomes $2x^2 - 3x + 1 = 0$ whose roots are $\alpha=1,\ \beta=\frac{1}{2}$

$\left(1,\ \frac{1}{2}\right)$ lies on $5x + 8y = 9$

16. B

Sol. $f(x)=\frac{x}{x^2-6x-16}$

Now,

$f'(x)=\frac{-(x^2+16)}{(x^2-6x-16)^2}$

$f'(x) < 0$

Thus f(x) is decreasing in

$(-\infty,\ -2)\cup(-2,\ 8)\cup(8,\ \infty)$

17. D

Sol. $y=\log_e\left(\frac{1-x^2}{1+x^2}\right)$

$\frac{dy}{dx}=y'=\frac{-4x}{1-x^4}$

Again,

$\frac{d^2y}{dx^2}=y''=\frac{-4(1+3x^4)}{(1-x^4)^2}$

Again

$y'-y''=\frac{-4x}{1-x^4}+\frac{4(1+3x^4)}{(1-x^4)^2}$

at $x=\frac{1}{2}$,

$y'-y''=\frac{736}{225}$

Thus $225(y'-y'')=225\times\frac{736}{225}=736$

18. A

Sol. Given set {1, 2, 3, 4}

Minimum order pairs are

(1, 1), (2, 2), (3, 3), (4, 4), (3, 1),(2, 1), (2, 3),(3,2), (1, 3), (1, 2)

Thus no. of elements = 10

19. B

Sol. Given set = {1, 2, 3,..........50}

P(A) = Probability that number is multiple of 4

P(B) = Probability that number is multiple of 6

P(C) = Probability that number is multiple of 7

3S INSTITUTE
JEE ADVANCED | JEE MAIN | NEET | OLYMPIADS | MHT-CET | FOUNDATION

Now,

$$P(A)=\frac{12}{50},\ P(B)=\frac{8}{50},\ P(C)=\frac{7}{50}$$

again

$$P(A\cap B)=\frac{4}{50},\ P(B\cap C)=\frac{1}{50},\ P(A\cap C)=\frac{1}{50}$$

$$P(A\cap B\cap C)=0$$

Thus

$$P(A\cup B\cup C)=\frac{12}{50}+\frac{8}{50}+\frac{7}{50}-\frac{4}{50}-\frac{1}{50}-\frac{1}{50}+0$$

$$=\frac{21}{50}$$

20. B

Sol. Unit vector $\hat{u}=x\hat{i}+y\hat{j}+z\hat{k}$

$$\vec{p}_1=\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{k},\ \vec{p}_2=\frac{1}{\sqrt{2}}\hat{j}+\frac{1}{\sqrt{2}}\hat{k}$$

$$\vec{p}_3=\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}$$

Now angle between $\hat{u}$ and $\vec{p}_1=\frac{\pi}{3}$

$$\hat{u}\cdot\vec{p}_1=0\Rightarrow\frac{x}{\sqrt{2}}+\frac{z}{\sqrt{2}}=0$$

$$\Rightarrow x+z=0 \qquad ....(i)$$

Angle between $\hat{u}$ and $\vec{p}_2=\frac{\pi}{3}$

$$\hat{u}\cdot\vec{p}_3=|\hat{u}|\cdot|\vec{p}_2|\cos\frac{\pi}{3}$$

$$\Rightarrow\frac{y}{\sqrt{2}}+\frac{z}{\sqrt{2}}=\frac{1}{2}\Rightarrow y+z=\frac{1}{\sqrt{2}} \qquad .....(ii)$$

Angle between $\hat{u}$ and $\vec{p}_3=\frac{2\pi}{3}$

$$\hat{u}\cdot\vec{p}_3=|\hat{u}|\cdot|\vec{p}_3|\cos\frac{2\pi}{3}$$

$$\Rightarrow\frac{x}{\sqrt{2}}+\frac{4}{\sqrt{2}}=\frac{-1}{2}\Rightarrow x+y=\frac{-1}{\sqrt{2}} \qquad ....(iii)$$

from equation (i), (ii) and (iii) we get

$$x=\frac{-1}{\sqrt{2}} \qquad y=0 \qquad z=\frac{1}{\sqrt{2}}$$

Thus $\hat{u}-\vec{v}=\frac{-1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{k}-\frac{1}{\sqrt{2}}\hat{i}-\frac{1}{\sqrt{2}}\hat{j}-\frac{1}{\sqrt{2}}\hat{k}$

$$\hat{u}-\vec{v}=\frac{-2}{\sqrt{2}}\hat{i}-\frac{1}{\sqrt{2}}\hat{j}$$

$$\therefore|\hat{u}-\vec{v}|^2=\left(\sqrt{\frac{4}{2}+\frac{1}{2}}\right)^2=\frac{5}{2}$$

<u>Section - B (Numerical Value Type)</u>

21. $x^2-\sqrt{6}x+6=0 \begin{smallmatrix}\nearrow\alpha\\ \searrow\beta\end{smallmatrix}$

$$x=\frac{\sqrt{6}\pm i\sqrt{6}}{2}=\frac{\sqrt{6}}{2}(1\pm i)$$

$$\alpha=\sqrt{3}(e^{i\frac{\pi}{4}}),\ \beta=\sqrt{3}(e^{-i\frac{\pi}{4}})$$

$$\therefore\frac{\alpha^{99}}{\beta}+\alpha^{98}=\alpha^{98}\left(\frac{\alpha}{\beta}+1\right)$$

$$=\frac{\alpha^{98}(\alpha+\beta)}{\beta}=3^{49}\left(e^{i99\frac{\pi}{4}}\right)\times\sqrt{2}$$

$= 3^{49}(-1+i)$

$= 3^{n}(a+ib)$

$\therefore$ n = 49, a = –1, b = 1

$\therefore$ n + a + b = 49 – 1 + 1 = 49

22. 113

Sol. $\because$ a, b, c and in A.P.

$\Rightarrow$ 2b = a + c $\Rightarrow$ a – 2b + c = 0

$\therefore$ ax + by + c passes through fixed point (1, –2)

$\therefore$ P = (1, –2)

For infinite solution,

$D = D_1 = D_2 = D_3 = 0]$

$$D:\begin{vmatrix}1&1&1\\2&5&\alpha\\1&2&3\end{vmatrix}=0\ \Rightarrow\alpha=8$$

$$D_1:\begin{vmatrix}6&1&1\\\beta&5&\alpha\\4&2&3\end{vmatrix}=0\Rightarrow\beta=6$$

$$\therefore Q=(8,\ 6)$$

$$\therefore PQ^2=113$$

23. 192

Sol. Parabola is $x^2 = 8y$

Chord with mid point (x_1, y_1) is $T = S_1$

$$\therefore xx_1 - 4(y + y_1) = x_1^2 - 8y_1$$

$$\therefore (x_1, y_1) = \left(1, \frac{5}{4}\right)$$

$$\Rightarrow x - 4\left(y + \frac{5}{4}\right) = 1 - 8 \times \frac{5}{4} = 9$$

$$\therefore x - 4y + 4 = 0 \qquad .....(i)$$

(α, β) lies on (i) & also on $y^2 = 4x$

$$\therefore \alpha - 4\beta + 4 = 0 \qquad ......(ii)$$

$$\& \ \beta^2 = 4\alpha \qquad .....(iii)$$

Solving (ii) & (iii)

$$\beta^2 = 4(4\beta - 4) \Rightarrow \beta^2 - 16\beta + 16 = 0$$

$$\therefore \beta = 8 \pm 4\sqrt{3} \text{ and } \alpha = 4\beta - 4 = 28 \pm 16\sqrt{3}$$

$$\therefore (\alpha, \beta) = (28 + 16\sqrt{3}, 8 + 4\sqrt{3}) \ \&$$

$$(28 - 16\sqrt{3}, 8 - 4\sqrt{3})$$

$$\therefore (\alpha - 28)(\beta - 8) = (\pm 16\sqrt{3})(\pm 4\sqrt{3})$$

$$= 192$$

24. 6

Sol. $$= \int_{\pi/6}^{\pi/3} \sqrt{1 - \sin 2x}\, dx$$

$$= \int_{\pi/6}^{\pi/3} |\sin x - \cos x|\, dx$$

$$= \int_{\pi/6}^{\pi/3} (\cos x - \sin x) dx + \int_{\pi/4}^{\pi/3} (\sin x - \cos x) dx$$

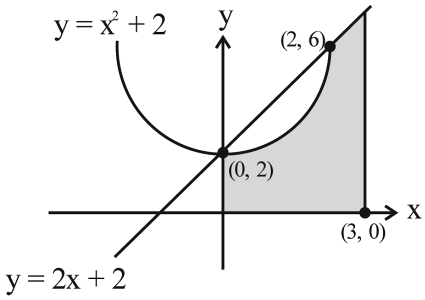
$$= -1 + 2\sqrt{2} - \sqrt{3}$$

$$= \alpha + \beta\sqrt{2} + \gamma\sqrt{3}$$

$$\alpha = -1, \ \beta = 2, \ \gamma = -1$$

$$3\alpha + 4\beta - \gamma = 6$$

25. 164



$$A = \int_0^2 (x^2 + 2) dx + \int_2^3 (2x + 2) dx$$

$$A = \frac{41}{3}$$

$$12A = 41 \times 4 = 164$$

26. 9

Sol. $L_1 : \frac{x - 5}{4} = \frac{y - 4}{1} = \frac{z - 5}{3} = \lambda$ drs$(4, 1, 3) = b_1$

$$M(4\lambda + 5, \lambda + 4, 3\lambda + 5)$$

$$L_2 : \frac{x + 8}{12} = \frac{y + 2}{5} = \frac{z + 11}{9} = \mu$$

$$N(12\mu - 8, 5\mu - 2, 9\mu - 11)$$

$$\overrightarrow{MN} = (4\lambda - 12\mu + 13, \lambda - 5\mu + 6, 3\lambda - 9\lambda + 16) \qquad ......(1)$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & 3 \\ 12 & 5 & 9 \end{vmatrix} = -6\hat{i} + 8\hat{k} \qquad .....(2)$$

Equation (1) and (2)

$$\therefore \frac{4\lambda - 12\mu + 13}{-6} = \frac{\lambda - 5\mu + 6}{0} = \frac{3\lambda - 9\mu + 16}{8}$$

I and II

$$\lambda - 5\mu + 6 = 0 \qquad ......(3)$$

I and III

$$\lambda - 3\mu + 4 = 0 \qquad .....(4)$$

Solve (3) and (4) we get

$$\lambda = -1, \ \mu = 1$$

$$\therefore M(1, 3, 2)$$

$$M(4, 3, -2)$$

$$\therefore \overline{OM} \cdot \overline{ON} = 4 + 9 - 4 = 9$$

3S INSTITUTE
JEE ADVANCED | JEE MAIN | NEET | OLYMPIADS | MHT-CET | FOUNDATION

27. 2

Sol. f(1) = 1, f(a) = 0

$$f^2(x)=\lim_{r\to x}\left(\frac{2r^2(f^2(r)-f(x)f(r))}{r^2-x^2}-r^3e^{\frac{f(t)}{t}}\right)$$

$$=\lim_{r\to x}\left(\frac{2r^2f(r)(f(r)-f(x))}{r+x\quad r-x}-r^3e^{\frac{f(t)}{t}}\right)$$

$$f^2(x)=\frac{2x^2f(x)}{2x}f'(x)-x^3e^{\frac{f(x)}{x}}$$

$$y^2=xy\frac{dy}{dx}-xe^{\frac{y}{x}}$$

$$\frac{y}{x}=\frac{dy}{dx}-\frac{x^2}{y}e^{\frac{y}{x}}$$

Put $\boxed{y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}}$

$$v=v+x\frac{dv}{dx}-\frac{x}{v}e^v$$

$$\frac{dv}{dx}=\frac{e^v}{v}\Rightarrow e^{-v}vdv=dx$$

Integrating both side

$e^v(x+c)+1+v=0$

$f(1)=1 \Rightarrow x=1, y=1$

$$\Rightarrow c=-1-\frac{2}{e}$$

$$e^v\left(-1-\frac{2}{e}+x\right)+1+v=0$$

$$e^{\frac{y}{x}}\left(-1-\frac{2}{e}+x\right)+1+\frac{y}{x}=0$$

$$x=a,\ y=0\Rightarrow a=\frac{2}{e}$$

ae = 2

28. 1

Sol. Let $32^{32}=t$

$$64^{32^{32}}=64^t=8^{2t}=(9-1)^{2t}$$

= 9k + 1

Hence remainder = 1

29. 46

Sol. $x^2-2^y=2023$

$$\Rightarrow\boxed{x=45,\ y=1}$$

$$\sum_{(x,\,y)\in C}(x+y)=46.$$

30. 12

Sol. According to the question,

$$27r_1+\frac{9r_2}{2}=-9$$

$$\lim_{x\to3}\frac{\int_3^x 8t^2dt}{\frac{3r_2x}{2}-r_2x^2-r_1x^3-3x}$$

$$=\lim_{x\to3}\frac{8x^2}{\frac{3r_2}{2}-2r_2x-3r_1x^2-3}\quad\text{(using LH}'\text{ Rule)}$$

$$=\frac{72}{\frac{3r_2}{2}-6r_2-27r_1-3}$$

$$=\frac{72}{-\frac{9r_2}{2}-27r_1-3}$$

$$=\frac{72}{9-3}=12$$

PHYSICS

Section - A (Single Correct Answer)

31. D

Sol. $n_1\times\frac{hc}{\lambda_1}=200$

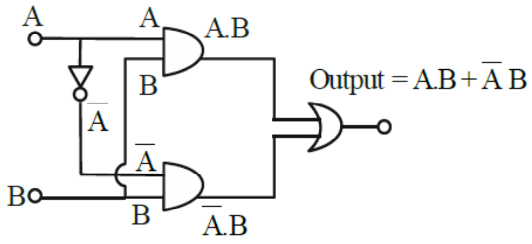
$$n_2\times\frac{hc}{\lambda_2}=200$$

$$\frac{n_1}{n_2}=\frac{\lambda_1}{\lambda_2}=\frac{300}{500}$$

$$\frac{n_1}{n_2}=\frac{3}{5}$$

32. B

Sol.



$Y = A \cdot B + \overline{A} \cdot B$

$= (A + \overline{A}) \cdot B$

Y = 1.B

Y = B

33. C

Sol. $Q = \frac{a^4 b^3}{c^2}$

$$\frac{\Delta Q}{Q} = 4\frac{\Delta a}{a} + 3\frac{\Delta b}{b} + 2\frac{\Delta c}{c}$$

$$\frac{\Delta Q}{Q} \times 100 = 4\left(\frac{\Delta a}{a} \times 100\right) + 3\left(\frac{\Delta b}{b} \times 100\right) + 2\left(\frac{\Delta c}{c} \times 100\right)$$

% error in Q = 4 × 3% + 3 × 4% + 2 × 5%

= 12% + 12% + 10% = 34%

34. C

Sol. $P_{avg} = V_{rms} = I_{rms} \cos(\Delta\phi)$

$$= \frac{100}{\sqrt{2}} \times \frac{100 \times 10^{-3}}{\sqrt{2}} \times \cos\left(\frac{\pi}{3}\right)$$

$$= \frac{10^4}{2} \times \frac{1}{2} \times 10^{-3} = \frac{10}{4} = 2.5W$$

35. A

Sol. PV = nRT

$$PV = \frac{N}{N_A} RT$$

N = Total no. of molecules

$$P = \frac{N}{V} kT$$

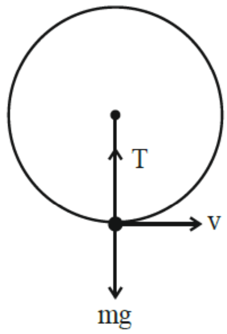
$1.38 \times 1.01 \times 10^5 = 2 \times 10^{25} \times 1.38 \times 10^{-23} \times T$

$1.01 \times 10^5 = 2 \times 10^2 \times T$

$$T = \frac{1.01 \times 10^3}{2} \approx 500K$$

36. B

Sol. Given that



$$m = 900gm = \frac{900}{1000} kg = \frac{9}{10} kg$$

r = 1m

$$\omega = \frac{2\pi N}{60} = \frac{2\pi(10)}{60} = \frac{\pi}{3} rad/sec$$

$T - mg = mr\omega^2$

$T = mg + mr\omega^2$

$$= \frac{9}{10} \times 9.8 + \frac{9}{10} \times 1\left(\frac{\pi}{3}\right)^2$$

$$= 8.82 + \frac{9}{10} \times \frac{\pi^2}{9}$$

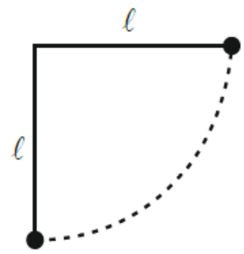
= 8.82 + 0.98

= 9.80 N

37. A

Sol. $l = 10$ m,

Initial energy = mg$l$



$$\text{So, } \frac{9}{10} mgl = \frac{1}{2} mv^2$$

$$\Rightarrow \frac{9}{10} \times 10 \times 10 = \frac{1}{2} v^2$$

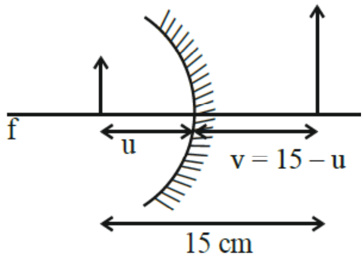
$v^2 = 180$

$v = \sqrt{180} = 6\sqrt{5}$ m/s

3S INSTITUTE
JEE ADVANCED | JEE MAIN | NEET | OLYMPIADS | MHT-CET | FOUNDATION

38. C

Sol.



$$m = 2 = \frac{-v}{u}$$

$$2 = \frac{-(15-u)}{-u}$$

$2u = 15 - u$

$3u = 15 \Rightarrow u = 5$ cm

$v = 15 - u = 15 - 5 = 10$ cm

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$f = -10$ cm

39. B

Sol. $R = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2m(KE)}}{qB} = \frac{\sqrt{2mqV}}{qB}$

$R \propto \sqrt{m}$

$m \propto R^2$

$$\frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^2$$

40. A

Sol. $\Delta x = \frac{7\lambda}{4}$

$$\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda} \times \frac{7\lambda}{4} = \frac{7\pi}{2}$$

$$I = I_{max}\cos^2\left(\frac{\phi}{2}\right)$$

$$\frac{I}{I_{max}} = \cos^2\left(\frac{\phi}{2}\right) = \cos^2\left(\frac{7\pi}{2\times 2}\right) = \cos^2\left(\frac{7\pi}{4}\right)$$

$$= \cos^2\left(2\pi - \frac{\pi}{4}\right) = \cos^2\frac{\pi}{4} = \frac{1}{2}$$

41. A

Sol. Volume constant

$$\frac{4}{3}\pi R^3 = 27 \times \frac{4}{3} \times \pi r^3$$

$R^3 = 27r^3$

$R = 3r$

$$r = \frac{R}{3}$$

$$r^2 = \frac{R^2}{9}$$

Work done = $T.\Delta A$

$= 27\ T(4\pi r^2) - T\ 4\pi R^2$

$$= 27T4\pi\frac{R^2}{9} - 4\pi R^2 T$$

$= 8\pi R^2 T$

42. B

Sol. Apply energy conservation between A & B

$$\frac{1}{2}mV_L^2 = \frac{1}{2}mV_H^2 + mg(2L)$$

$\because V_L = \sqrt{5gL}$

So, $V_H = \sqrt{gL}$

$$\frac{(K.E.)_A}{(K.E.)_B} = \frac{\frac{1}{2}m\left(\sqrt{5gL}\right)^2}{\frac{1}{2}m\left(\sqrt{gL}\right)^2} = \frac{5}{1}$$

43. D

Sol. $Y = \frac{\text{stress}}{\text{strain}}$

$$Y = \frac{\frac{F}{\pi r^2}}{\frac{l}{L}}$$

$$F = Y\pi r^2 \times \frac{l}{L} \qquad .....(i)$$

$$Y = \frac{\frac{F/2}{\pi r^2/4}}{\frac{\Delta l}{L}}$$

$$F = Y\frac{\Delta l}{L} \times 2 \times \frac{\pi r^2}{4}$$

From (i)

$$Y\pi r^2 \frac{l}{L} = Y\frac{\Delta l}{L}\frac{\pi r^2}{2}$$

$\Delta l = 2l$

44. A

Sol. $T^2 \propto r^3$

$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$$

$$\frac{(200)^2}{r^3} = \frac{T_2^2}{\left(\frac{r}{4}\right)^3}$$

$$\frac{200\times 200}{4\times4\times4} = T_2^2$$

$$T_2 = \frac{200}{4\times 2}$$

$T_2 = 25$ days

45. A

Sol. $\frac{E}{B} = C$

$$\frac{E}{B} = 3\times10^8$$

$$B = \frac{E}{3\times10^8} = \frac{9.6}{3\times10^8}$$

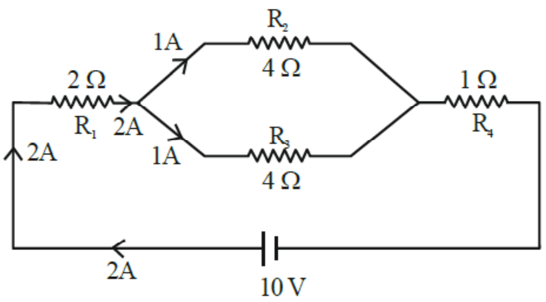
$B = 3.2 \times 10^{-8}$ T

$$\hat{B} = \hat{v}\times\hat{E} = \hat{i}\times\hat{j} = \hat{k}$$

So, $\vec{B} = 3.2\times10^{-8}\hat{k}T$

46. A

Sol.



$R_{eq} = 2\Omega + 2\Omega + 1\Omega = 5\Omega$

$$i = \frac{V}{R_{eq}} = \frac{10}{5} = 2A$$

Current in resistance $R_3 = 2\times\left(\frac{4}{4+4}\right)$

$$= 2\times\frac{4}{8} = 1A$$

47. B

Sol. $x = t^3 - 6t^2 + 20t + 15$

$$\frac{dx}{dt} = v = 3t^2 - 12t + 20$$

$$\frac{dv}{dt} = a = 6t - 12$$

When $a = 0$

$6t - 12 = 0$; $t = 2$ sec

At $t = 2$ sec

$v = 3(B)^2 - 12(B) + 20$

$v = 8$ m/s

48. C

Sol. $f_{eq} = \frac{n_1f_1 + n_2f_2}{n_1 + n_2}$

For diatomic gas $f_{eq} = 5$

$$5 = \frac{(N)(6)+(2)(3)}{N+2}$$

$5N + 10 = 6N + 6$

$N = 4$

49. C

Sol. According to Rutherford atomic model, most of mass of atom and all its positive charge is concentrated in tiny nucleus & electron revolve around it.

According to Thomson atomic model, atom is spherical cloud of positive charge with electron embedded in it.

Hence, Statement I is true but statement II false.

50. C

Sol. $\vec{E} = 6\hat{i} + 5\hat{j} + 3\hat{k}$

$$\vec{A} = 30\hat{i}$$

$$\phi = \vec{E}\cdot\vec{A}$$

$$\phi = (6\hat{i}+5\hat{j}+3\hat{k})\cdot(30\hat{i})$$

$$\phi = 6\times30 = 180$$

<u>Section - B (Numerical Value Type)</u>

51. 16

Sol.

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta l/l} = \frac{Fl}{A\Delta l}$$

3S INSTITUTE
JEE ADVANCED | JEE MAIN | NEET | OLYMPIADS | MHT-CET | FOUNDATION

$$\Delta l = \frac{Fl}{AY}$$

$$V = Al \Rightarrow l = \frac{V}{A}$$

$$\Delta l = \frac{FV}{A^2Y}$$

Y & V is same for both the wires

$$\Delta l \propto \frac{F}{A^2}$$

$$\frac{\Delta l_1}{\Delta l_2} = \frac{F_1}{A_1^2} \times \frac{A_2^2}{F_2}$$

$$\Delta l_1 = \Delta l_2$$

$$F_1A_2^2 = F_2A_1^2$$

$$\frac{F_1}{F_2} = \frac{A_1^2}{A_2^2} = \left(\frac{4}{1}\right)^2 = 16$$

52. 3

Sol. $B_H = 0.60 \times 10^{-4}$ Wb/m^2

Induced emf H e = $B_H\, vl$

$= 0.60 \times 10^{-4} \times 10 \times 5 = 3 \times 10^{-3}$ V

53. 121

Sol. For minimum potential difference electron has to make transition from n = 3 to n = 2 state but first electron has to reach to n = 3 state from ground state. So, energy of bombarding electron should be equal to energy difference of n = 3 and n = 1 state.

$$\Delta E = 13.6\left[1 - \frac{1}{3^2}\right]e = eV$$

$$\frac{13.6 \times 8}{9} = V$$

V = 12.09 V ≈ 12.1V

So, $\alpha = 121$

54. 32

Sol. $q = 4\ \mu C,\ \vec{v} = 4 \times 10^6\,\hat{j}$ m/s

$$\vec{B} = 2\hat{k}T$$

$$\vec{F} = q\left(\vec{v} \times \vec{B}\right)$$

$$= 4 \times 10^{-6}\left(4 \times 10^6\,\hat{j} \times 2\hat{k}\right)$$

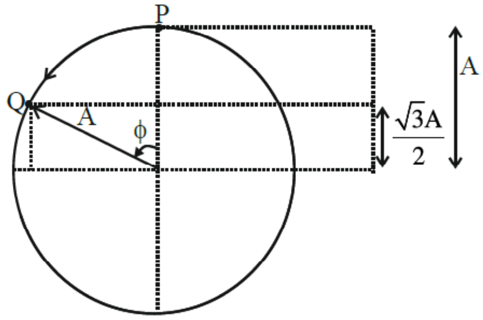
$$= 4 \times 10^{-6} \times 8 \times 10^6\,\hat{i}$$

$$\vec{F} = 32\hat{i}N$$

x = 32

55. 2

Sol.



From phasor diagram particle has to move from P to Q in a circle of radius equal to amplitude of SHM.

$$\cos\phi = \frac{\frac{\sqrt{3}A}{2}}{A} = \frac{\sqrt{3}}{2}$$

$$\phi = \frac{\pi}{6}$$

$$\text{Now, } \frac{\pi}{6} = \omega t$$

$$\frac{\pi}{6} = \frac{2\pi}{T}t$$

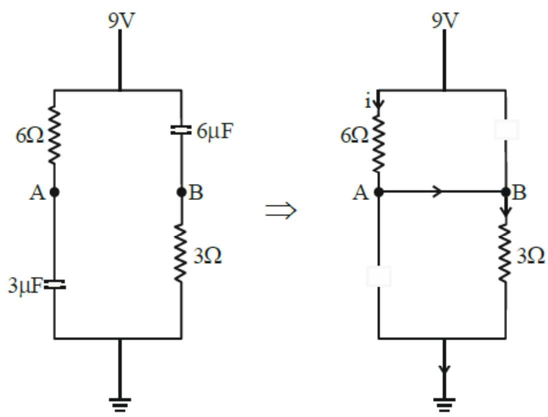
$$\frac{\pi}{6} = \frac{2\pi}{6\pi}t$$

$$t = \frac{\pi}{2}$$

So, x = 2

56. 36

Sol. At steady state, capacitor behaves as an open circuit and current flows in circuit as shown in the diagram.



$R_{eq} = 9\Omega$

$$i = \frac{9V}{9\Omega} = 1A$$

$\Delta V_{6\Omega} = 1 \times 6 = 6V$

$V_A = 3$ V

So, potential difference across 6μF is 6 V.

Hence Q = CΔV

$= 6 \times 6 \times 10^{-6}$ C

= 36 μC

57. 2

Sol. For n^{th} minima

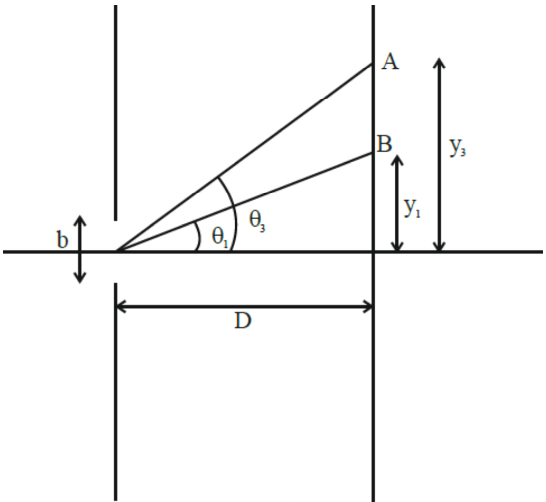
b sin θ = nλ

(λ is small so sin θ is small, hence sin θ ≃ tan θ)

b tan θ = nλ

$$b\frac{y}{D} = n\lambda$$

$$\Rightarrow y_n = \frac{n\lambda D}{b} \text{(Position of } n^{th} \text{ minima)}$$



B → 1^{st} minima, A → 3^{rd} minima

$$y_3 = \frac{3\lambda D}{b}, y_1 = \frac{\lambda D}{b}$$

$$\Delta y = y_3 - y_1 = \frac{2\lambda D}{b}$$

$$3 \times 10^{-3} = \frac{2 \times 6000 \times 10^{-10} \times 0.5}{b}$$

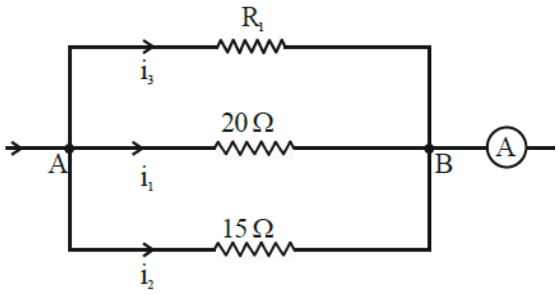
$$b = \frac{2 \times 6000 \times 10^{-10} \times 0.5}{3 \times 10^{-3}}$$

$b = 2 \times 10^{-4}$ m

x = 2

58. 30

Sol.



Given, $i_1 = 0.3$ A, $i_1 + i_2 + i_3 = 0.9$ A

So, $V_{AB} = i_1 \times 20\Omega = 20 \times 0.3$ V = 6 V

$$i_2 = \frac{6V}{15\Omega} = \frac{2}{5}A$$

$$i_1 + i_2 + i_3 = \frac{9}{10}A$$

$$\frac{3}{10} + \frac{2}{5} + i_3 = \frac{9}{10}$$

$$\frac{7}{10} + i_3 = \frac{9}{10}$$

$i_3 = 0.2$ A

So, $i_3 \times R_1 = 6$ V

$(0.2)R_1 = 6$

$$R_1 = \frac{6}{0.2} = 30\Omega$$

59. 8

Sol. $|\vec{a}_C| = |\vec{a}_t|$

$$\frac{v^2}{r} = \frac{dv}{dt}$$

$$\Rightarrow \int_4^v \frac{dv}{v^2} = \int_0^t \frac{dt}{r}$$

$$\Rightarrow \left[\frac{-1}{v}\right]_4^v = \frac{t}{r}$$

$$\Rightarrow \frac{-1}{v} + \frac{1}{4} = 2t$$

$$\Rightarrow v = \frac{4}{1-8t} = \frac{ds}{dt}$$

$$4\int_0^t \frac{dt}{1-8t} = \int_0^s ds$$

(r = 0.5 m

s = 2πr = π)

3S INSTITUTE
JEE ADVANCED | JEE MAIN | NEET | OLYMPIADS | MHT-CET | FOUNDATION

$$4\times\frac{\left[ln(1-8t)\right]_0^t}{-8}=\pi$$

$$ln(1-8t)=-2\pi$$

$$1-8t=e^{-2\pi}$$

$$t=\left(1-e^{-2\pi}\right)\frac{1}{8}s$$

So, $\alpha = 8$

60. 60

Sol. $y - x - 4 = 0$

d_1 is perpendicular distance of given line from origin.

$$d_1=\left|\frac{-4}{\sqrt{1^2+1^2}}\right|\Rightarrow 2\sqrt{2}m$$

So, $|\vec{L}| = mvd_1 = 5\times 3\sqrt{2}\times 2\sqrt{2}\text{ kg m}^2/s$

$= 60\text{ kg m}^2/s$

CHEMISTRY

Section - A (Single Correct Answer)

61. **(A)**

Sol. $CH\equiv C^{\ominus} > CH_2=\overset{\ominus}{C}H > H_3C-\overset{\ominus}{C}H_2 > (CH_3)_3C^{\ominus}$

Stability of conjugate base a acidic strength

C < D < B < A.

62. **(D)**

Sol. A-II, B-III, C-I, D-IV

Fact based.

63. **(D)**

Sol. Ethanol $\rightarrow$ 15.9

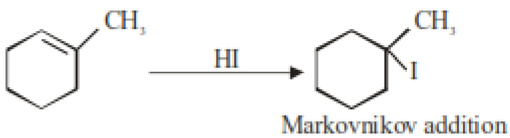
Phenol $\rightarrow$ 10

M-Nitrophenol $\rightarrow$ 8.3

P-Nitrophenol $\rightarrow$ 7.1

64. **(B)**

Sol.



65. **(D)**

Sol. Cyclohex-2-en-1-ol

66. **(B)**

Sol. K_2MnO_4

$2 + x - 8 = 0$

$\Rightarrow x = +6$

O.S. of Mn = +6

IUPAC Name = Potassium tetraoxidomanganate (VI)

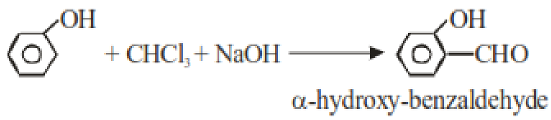
67. **(D)**

Sol. $Ni^{2+} + 2\ dmg^- \rightarrow [Ni(dmg)_2]$

Rosy red / Bright Red precipitate

68. **(D)**

Sol.



It is Reimer Tiemann Reaction.

69. **(C)**

Sol. A - II, B - IV, C - III, D - I

Fact based.

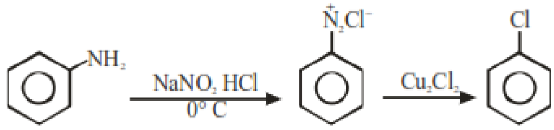
70. **(C)**

Sol. Nessler's Reagent Reaction :

$$\underset{\text{(Nessler's Reagent)}}{2K_2HgI_4} + NH_3 + 3KOH \rightarrow \underset{\text{(Iodine of Millon's base Brown precipitate)}}{HgO.Hg(NH_2)I} + 7KI + 2H_2O$$

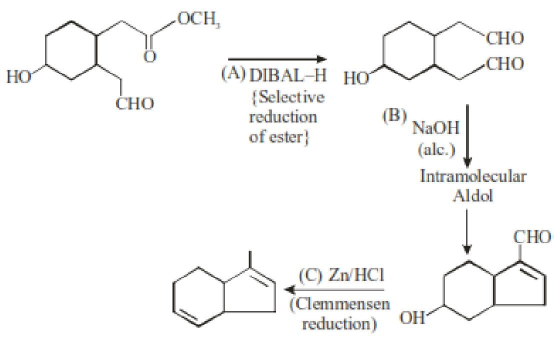
71. **(C)**

Sol.



72. **(D)**

Sol.



73. **(C)**

Sol. $Eu^{+2} \longrightarrow Eu^{+3} + 1e^-$

$[Xe]4f^7\ 6s^0 \qquad [Xe]\ 4f^6\ 6s^0$

74. **(C)**

Sol. Memory Based

75. **(A)**

Sol. (A) Zn, Cd, Hg exhibit lowest enthalpy of atomization in respective transition series.

(C) Compounds of Zn, Cd and Hg are diamagnetic in nature.

76. **(C)**

Sol. Al < Si < C < N ; IE_1 order.

77. **(D)**

Sol. Covalent character of AgCN.

78. **(C)**

Sol. Due to unsymmetrical.

79. **(D)**

Sol. Statement-1 is false because chlorine has most negative electron gain enthalpy in its group.

80. **(C)**

Sol. Fact Based.

<u>Section - B (Numerical Value Type)</u>

81. **(4)**

Sol. Antibonding molecular orbital from 2s = 1

Antibonding molecular orbital from 2p = 3

Total = 4

82. **(1)**

Sol. $[Fe(H_2O)_5(NO)]^{2+}$,

Oxidation no. of Fe = +1

83. **(417)**

Sol. $$K_C = \frac{[NH_3]^2}{[N_2][H_2]^3}$$

$$K_C = \frac{(1.5\times10^{-2})^2}{(2\times10^{-2})\times(3\times10^{-2})^3}$$

$$K_C = 417$$

84. **(815)**

Sol. $$m = \frac{M\times1000}{d_{sol}\times1000 - M\times \text{Molar mass}_{solute}}$$

$$= 815\times10^{-3}\,m$$

85. **(4)**

Sol. Equivalent of Oxalic acid = Equivalents of NaOH

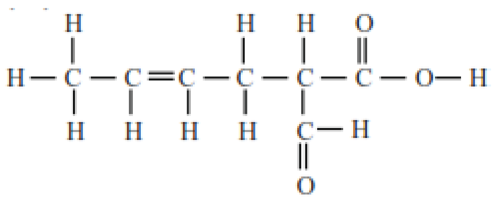
$50 \times 0.5 \times 2 = 25 \times M \times 1$

$M_{NaOH} = 2$ M

W_{NaOH} in 50 ml = $2 \times 50 \times 40 \times 10^{-3}$ g = 4 g

86. **(22)**

Sol.



22 bounds.

87. **(63)**

Sol. Half life of bromine – 82 = 36 hours

$$t_{1/2} = \frac{0.693}{K}$$

$$K = \frac{0.693}{36} = 0.01925\ hr^{-1}$$

1^{st} order rxn kinetic equation,

$$t = \frac{2.303}{K}\log\frac{a}{a-x}$$

$$\log\frac{a}{a-x} = \frac{t\times K}{2.303}\quad (t = 1\text{ day} = 24\text{ hr})$$

$$\log\frac{a}{a-x} = \frac{24\ hr\times0.01925\ hr^{-1}}{2.303}$$

$$\log\frac{a}{a-x} = 0.2006$$

$$\frac{a}{a-x} = \text{antilog}\ (0.2006)$$

$$\frac{a}{a-x} = 1.587$$

If a = 1

$$\frac{1}{1-x} = 1.587 \Rightarrow 1-x = 0.6301 =$$

Fraction remain after one day.

3S INSTITUTE
JEE ADVANCED | JEE MAIN | NEET | OLYMPIADS | MHT-CET | FOUNDATION

88. **(56)**

Sol. ΔH^o_{vap} $CCl_4 = 30.5$ kJ/mol

Mass of $CCl_4 = 284$ gm

Molar mass of $CCl_4 = 154$ g/mol

Moles of $CCl_4 = \frac{284}{154} = 1.844$ mol

ΔH^o_{vap} for 1 mole = 30.5 kJ/mol

ΔH^o_{vap} for 1.844 mol = 30.5×1.844

= 56.242 kJ

89. **(2)**

Sol. $\frac{W}{E} = \frac{\text{charge}}{1\text{ F}}$

$$\frac{1.314}{\frac{197}{3}} = \frac{Q}{1\text{ F}}$$

$Q = 2 \times 10^{-2}$ F

90. **(3)**

Sol. Molecules with zero dipole moment = CO_2, CH_4, BF_3.

● ● ●