MATHEMATICS

Section - A (Single Correct Answer)

1. B

Sol. $\tan^{-1}x + \tan^{-1}2x = \frac{\pi}{4};\ x > 0$

$\Rightarrow \tan^{-1}2x = \frac{\pi}{4} - \tan^{-1}x$

Taking tan both sides

$\Rightarrow 2x = \frac{1-x}{1+x}$

$\Rightarrow 2x^2 + 3x - 1 = 0$

$x = \frac{-3 \pm \sqrt{9+8}}{8} = \frac{-3 \pm \sqrt{17}}{8}$

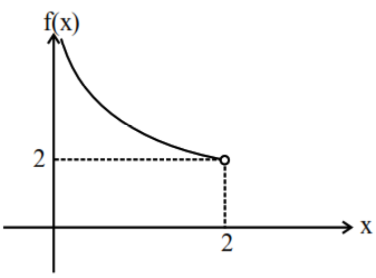
Only possible $x = \frac{-3+\sqrt{17}}{8}$

2. A

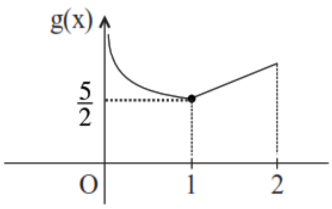
Sol. $f:(0,\ 2) \to R\ ;\ f(x) = \frac{x}{2} + \frac{2}{x}$

$f'(x) = \frac{1}{2} - \frac{2}{x^2}$

$\therefore f(x)$ is decreasing in domain.

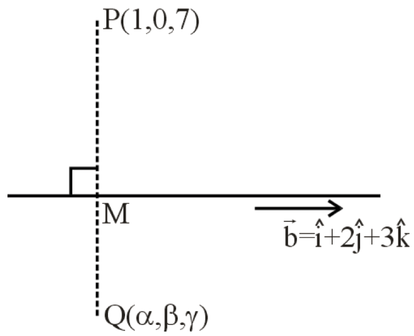


$$g(x) = \begin{cases} \frac{x}{2} + \frac{2}{x} & 0 < x \le 1 \\ \frac{3}{2} + x & 1 < x < 2 \end{cases}$$



3. C

Sol. $L_1 = \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$



$M(\lambda,\ 1+2\lambda,\ 2+3\lambda)$

$\overrightarrow{PM} = (\lambda-1)\hat{i} + (1+2\lambda)\hat{j} + (3\lambda-5)\hat{k}$

$\overrightarrow{PM}$ is perpendicular to line L_1

$\overrightarrow{PM} \cdot \vec{b} = 0 \quad (\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k})$

$\Rightarrow \lambda - 1 + 4\lambda + 2 + 9\lambda - 15 = 0$

$14\lambda = 14 \Rightarrow \lambda = 1$

$\vec{Q} = 2\vec{M} - \vec{P}$ [M is midpoint of $\vec{P}$ & $\vec{Q}$]

$\vec{Q} = 2\hat{i} + 6\hat{j} + 10\hat{k} - \hat{i} - 7\hat{k}$

$\vec{Q} = \hat{i} + 6\hat{j} + 3\hat{k}$

$\therefore (\alpha,\ \beta,\ \gamma) = (1,\ 6,\ 3)$

Required line having direction cosine (l, m, n)

$l^2\ \ m^2 + n^2 = 1$

$\Rightarrow l^2 + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 = 1$

$l^2 = \frac{1}{4}$

$\therefore l^2 = \frac{1}{4}$ [Line make acute angle with x-axis]

Equation of line passing through (1, 6, 3) will be

$$\vec{r} = (\hat{i} + 6\hat{j} + 3\hat{k}) + \mu\left(\frac{1}{2}\hat{i} - \frac{1}{2}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}\right)$$

Option (3) satisfying for $\mu = 4$

4. B

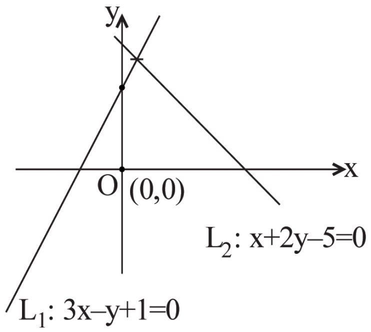
Sol. $P(a^2, a + 1)$

$L_1 = 3x - y + 1 = 0$

Origin and P lies same side w.r.t. L_1

$\Rightarrow L_1(0) \cdot L_1(P) > 0$

$\therefore 3(a^2) - (a + 1) + 1 > 0$



$\Rightarrow 3a^2 - a > 0$

$$a \in (-\infty, 0) \cup \left(\frac{1}{3}, \infty\right) \quad .....(1)$$

Let $L_2 : x + 2y - 5 = 0$

Origin and P lies same side w.r.t. L_2

$\Rightarrow L_2(0) \cdot L_2(P) > 0$

$\Rightarrow a^2 + 2(a + 1) - 5 < 0$

$\Rightarrow a^2 + 2a - 3 < 0$

$\Rightarrow (a + 3)(a - 1) < 0$

$\therefore a \in (-3, 1) \quad .....(2)$

Intersecting of (1) and (2)

$$a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$$

5. C

Sol. $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}.......,-129\frac{1}{4}$

This is A.P. with common difference

$$d_1 = -1 + \frac{1}{4} = -\frac{3}{4}$$

$$-129\frac{1}{4},.....,19\frac{1}{4}, 20$$

This is also A.P. $a = -129\frac{1}{4}$ and $d = \frac{3}{4}$

Required term =

$$-129\frac{1}{4} + (20-1)\left(\frac{3}{4}\right)$$

$$= -129 - \frac{1}{4} + 15 - \frac{3}{4} = -115$$

6. A

Sol. $f(x) = \frac{2x+3}{2x+1};\ x \neq -\frac{1}{2}$

$g(x) = \frac{|x|+1}{2x+5},\ x \neq -\frac{5}{2}$

Domain of f(g(x))

$$f(g(x)) = \frac{2g(x)+3}{2g(x)+1}$$

$$x \neq -\frac{5}{2} \text{ and } \frac{|x|+1}{2x+5} \neq -\frac{1}{2}$$

$$x \in R - \left\{-\frac{5}{2}\right\} \text{ and } x \in R$$

7. C

Sol. $I = \int_0^{\pi} \frac{dx}{1 - 2a\cos x + a^2};\ 0 < a < 1$

$$I = \int_0^{\pi} \frac{dx}{1 + 2a\cos x + a^2}$$

$$2I = 2\int_0^{\pi/2} \frac{2(1+a^2)}{(1+a^2)^2 - 4a^2\cos^2 x}dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{2(1+a^2)\cdot\sec^2 x}{(1+a^2)^2\cdot\sec^2 x - 4a^2}dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{2\cdot(1+a^2)\cdot\sec^2 x}{(1+a^2)^2\cdot\tan^2 x + (1-a^2)^2}dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\frac{2\cdot\sec^2 x}{1+a^2}dx}{\tan^2 x + \left(\frac{1-a^2}{1+a^2}\right)^2}$$

$$\Rightarrow I=\frac{2}{(1-a^2)}\left[\frac{\pi}{2}-0\right]$$

8. C

Sol. $g(x)=3f\left(\frac{x}{3}\right)+f(3-x)$ and $f''(x)>0\ \forall\ x\in(0,\ 3)$

$\Rightarrow$ $f'(x)$ is increasing function

$$g'(x)=3\times\frac{1}{3}\cdot f'\left(\frac{x}{3}\right)-f'(3-x)$$

$$\Rightarrow f'\left(\frac{x}{3}\right)-f'(3-x)$$

If g is decreasing in $(0,\ \alpha)$

$g'(x)<0$

$$f'\left(\frac{x}{3}\right)-f'(3-x)<0$$

$$f'\left(\frac{x}{3}\right)<f'(3-x)$$

$$\Rightarrow \frac{x}{3}<3-x$$

$$\Rightarrow x<\frac{9}{4}$$

Therefore $\alpha=\frac{9}{4}$

Then $8\alpha=8\times\frac{9}{4}=18$

9. C

Sol. $\lim_{x\to0}\frac{3+\alpha\sin x+\beta\cos x+\log_e(1-x)}{3\tan^2 x}=\frac{1}{3}$

$$\Rightarrow \lim_{x\to0}\frac{3+\alpha\left[x-\frac{x^3}{3!}+....\right]+\beta\left[1-\frac{x^2}{2!}+\frac{x^2}{2!}....\right]+\left(-x-\frac{x^2}{2}-\frac{x^3}{3}\right)}{3\tan^2 x}=\frac{1}{3}$$

$$\Rightarrow \lim_{x\to0}\frac{(3+\beta)(\alpha-1)x+\left(-\frac{1}{2}-\frac{\beta}{2}\right)x^2+....}{3x^2}\times\frac{x^2}{\tan^2 x}=\frac{1}{3}$$

$$\Rightarrow \beta+3=0,\ \alpha-1=0 \text{ and } \frac{-\frac{1}{2}-\frac{\beta}{2}}{3}=\frac{1}{3}$$

$$\Rightarrow \beta+3=0,\ \alpha=1$$

$$\Rightarrow 2\alpha-\beta=2+3=5$$

10. B

Sol. $x^2-x-1=0$

$S_n=2023\alpha^n+2024\beta^n$

$S_n+S_{n-2}=2023\alpha^{n-1}+2024\beta^{n-1}+2023\alpha^{n-2}+2024\alpha^{n-2}$

$=2023\alpha^{n-2}[1+\alpha]+2024\beta^{n-2}[1+\beta]$

$=2023\alpha^{n-2}[\alpha^2]+2024\beta^{n-2}[\beta^2]$

$=2023\alpha^n+2024\beta^n$

$S_{n-1}+S_{n-2}=S_n$

Put n = 12

$S_{11}+S_{10}=S_{12}$

11. $2^m-2^n=56$

$2^n(2^{m-n}-1)=2^3\times7$

$2^n=2^3$ and $2^{m-n}-1=7$

$\Rightarrow$ n = 3 and $2^{m-n}=8$

$\Rightarrow$ n = 3 and m – n = 3

$\Rightarrow$ n = 3 and m = 6

P(6, 3) and Q(–2, –3)

$$PQ=\sqrt{8^2+6^2}=\sqrt{100}=10$$

Hence option (A) is correct

12. B

Sol. $\begin{vmatrix}1 & \frac{3}{2} & \alpha+\frac{3}{2}\\ 1 & \frac{1}{3} & \alpha+\frac{1}{3}\\ 2\alpha+3 & 3\alpha+1 & 0\end{vmatrix}=0$

$$\Rightarrow(2\alpha+3)\left\{\frac{7\alpha}{6}\right\}-(3\alpha+1)\left\{\frac{-7}{6}\right\}=0$$

$$\Rightarrow(2\alpha+3)\cdot\frac{7\alpha}{6}+(3\alpha+1)\cdot\frac{7}{6}=0$$

$$\Rightarrow 2\alpha^2+3\alpha+3\alpha+1=0$$

$$\Rightarrow\alpha=\frac{-3+\sqrt{7}}{2}\cdot\frac{-3-\sqrt{7}}{2}$$

Hence option (B) is correct.

13. C

Sol. $\frac{{}^6C_4}{{}^{15}C_4}\times\frac{{}^9C_4}{{}^{11}C_4}=\frac{3}{715}$

Hence option (C) is correct.

14. A

Sol. $I=\int \frac{x^8-x^2}{(x^{12}+3x^6+1)\tan^{-1}\left(x^3+\frac{1}{x^3}\right)}dx$

Let $\tan^{-1}\left(x^3+\frac{1}{x^3}\right)=t$

$\Rightarrow \frac{1}{1+\left(x^3+\frac{1}{x^3}\right)^2}\cdot\left(3x^2-\frac{3}{x^4}\right)dx=dt$

$\Rightarrow \frac{x^6}{x^{12}+3x^6+1}\cdot\frac{3x^6-3}{x^4}dx=dt$

$I=\frac{1}{3}\int\frac{dt}{t}=\frac{1}{3}\ln|t|+C$

$I=\frac{1}{3}\ln\left|\tan^{-1}\left(x^3+\frac{1}{x^2}\right)\right|+C$

$I=\ln\left|\tan^{-1}\left(x^2+\frac{1}{x^3}\right)\right|^{1/3}+C$

Hence option (A) is correct

15. D

Sol. $2\tan^2\theta-5\sec\theta-1=0$

$\Rightarrow 2\sec^2\theta-5\sec\theta-3=0$

$\Rightarrow (2\sec\theta+1)(\sec\theta-3)=0$

$\Rightarrow \sec\theta=-\frac{1}{2},\ 3$

$\Rightarrow \cos\theta=-2,\ \frac{1}{3}$

$\Rightarrow \cos\theta=\frac{1}{3}$

For 7 solutions $n=13$

So, $\sum_{k=1}^{13}\frac{k}{2^k}=S$ (say)

$S=\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^2}+\ldots\ldots+\frac{13}{2^{13}}$

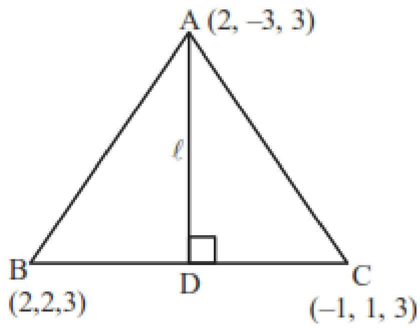
$\frac{1}{2}S=\frac{1}{2^2}+\frac{1}{2^3}+\ldots\ldots\ldots+\frac{12}{2^{13}}+\frac{13}{2^{14}}$

$\Rightarrow \frac{S}{2}=\frac{1}{2}\cdot\frac{1-\frac{1}{2^{13}}}{1-\frac{1}{2}}\Rightarrow S=2\cdot\left(\frac{2^{13}-1}{2^{13}}\right)-\frac{13}{2^{13}}$

16. D

Sol. AB = 5

AC = 5



$\therefore$ D is midpoint of BC

$D\left(\frac{1}{2},\ \frac{3}{2},\ 3\right)$

$\therefore l=\sqrt{\frac{45}{2}}$

$\therefore 2l^2=45$

17. A

Sol. $(x^2-4)dy-(y^2-3y)dx=0$

$\Rightarrow \int\frac{dy}{y^2-3y}=\int\frac{dx}{x^2-4}$

$\Rightarrow \frac{1}{3}\int\frac{y-(y-3)}{y(y-3)}dy=\int\frac{dx}{x^2-4}$

$\Rightarrow \frac{1}{3}(\ln|y-3|-\ln|y|)=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+C$

$\Rightarrow \frac{1}{3}\ln\left|\frac{y-3}{y}\right|=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+C$

At $x=4,\ y=\frac{3}{2}$

$\therefore C=\frac{1}{4}\ln 3$

$\therefore \frac{1}{3}\ln\left|\frac{y-3}{y}\right|=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+\frac{1}{4}(\ln 3)$

At x = 0

$$\frac{1}{3}\ln\left|\frac{y-3}{y}\right| = \frac{1}{4}\ln\left|\frac{2}{3}\right| + \frac{1}{4}\ln(3)$$

$$\ln\left|\frac{y-3}{y}\right| = \ln 2^{3/4},\ \forall x > 2,\ \frac{dy}{dx} < 0$$

$$\text{as } y(4) = \frac{3}{2} \Rightarrow y \in (0,\ 3)$$

$-y + 3 = 8^{1/4} \cdot y$

$$y = \frac{3}{1+8^{1/4}}$$

18. C

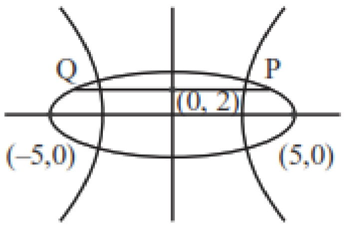
Sol. $H: \frac{x^2}{16} - \frac{y^2}{9} = 1 \qquad e_1 = \frac{5}{4}$

$$\therefore e_1 e_2 = 1 \Rightarrow e_2 = \frac{4}{5}$$

Also, ellipse is passing through $(\pm 5,\ 0)$

$\therefore$ a = 5 and b = 3

$$E: \frac{x^2}{25} + \frac{y^2}{9} = 1$$



End point of chord are $\left(\pm\frac{5\sqrt{5}}{3},\ 2\right)$

$$\therefore L_{PQ} = \frac{10\sqrt{5}}{3}$$

19. C

Sol. $\alpha = \frac{(4!)!}{(4!)^{3!}},\ \beta = \frac{(5!)!}{(5!)^{4!}}$

$$\alpha = \frac{(24)!}{(4!)^6},\ \beta = \frac{(120)!}{(5!)^{24}}$$

Let 24 distinct objects are divided into 6 groups of 4 objects in each group.

No. of ways of formation of group

$$= \frac{24!}{(4!)^6 \cdot 6!} \in N$$

Similarly,

Let 120 distinct objects are divided into 24 groups of 5 objects in each group.

No. of ways of formation of groups

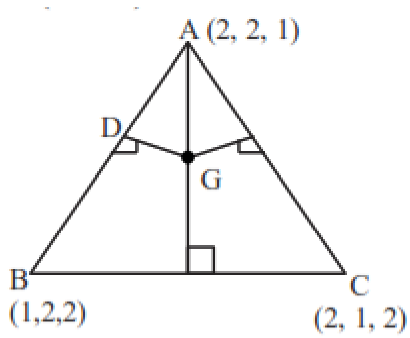
$$= \frac{(120)!}{(5!)^{24} \cdot 24!} \in N$$

20. B

Sol. ΔABC is equilateral

Orthocentre and centroid will be same

$$G\left(\frac{5}{3},\ \frac{5}{3},\ \frac{5}{3}\right)$$



Mid-point of AB is $D\left(\frac{3}{2},\ 2,\ \frac{3}{2}\right)$

$$\therefore \ell_1 = \sqrt{\frac{1}{36} + \frac{1}{9} + \frac{1}{36}}$$

$$\ell_1 = \sqrt{\frac{1}{6}} = \ell_2 = \ell_3$$

$$\therefore e_1^2 + \ell e_2^2 + \ell_3^2 = \frac{1}{2}$$

Section - B (Numerical Value Type)

21. 2521

Sol. Let the incorrect mean be μ' and standard deviation be σ'

We have

$$\mu' = \frac{\Sigma x_i}{15} = 12 \Rightarrow \Sigma x_i = 180$$

As per given information correct

$$\Sigma x_i = 180 - 10 + 12$$

$$\Rightarrow \mu \text{ (correct mean) } = \frac{182}{15}$$

Also

$$\sigma' \sqrt{\frac{\Sigma x_i^2}{15} - 144} = 3 \Rightarrow \Sigma x_i^2 = 2295$$

Correct $\Sigma x_i^2 = 2295 - 100 + 144 = 2339$

$$\sigma^2 \text{ (correct variance) } = \frac{2339}{15} - \frac{182 \times 182}{15 \times 15}$$

Required value

$$= 15(\mu + \mu^2 + \sigma^2)$$

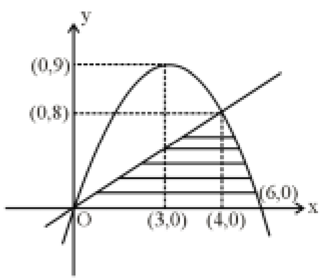
$$= 15\left(\frac{182}{15} + \frac{182 \times 182}{15} + \frac{2339}{15} - \frac{182 \times 182}{15 \times 15}\right)$$

$$= 15\left(\frac{182}{15} + \frac{2339}{15}\right)$$

$$= 2521$$

22. 304

Sol. We have



$$A = \frac{1}{2} \times 4 \times 8 + \int_4^6 (6x - x^2)dx$$

$$A = \frac{76}{3}$$

$$12A = 304$$

23. 10

Sol. $|A - xI| = 0$

Roots are –1 and 3

Sum of roots = tr(A) = 2

Product of roots = |A| = –3

$$\text{Let } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

We have a + d = 2

ad – bc = –3

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix}$$

We need $a^2 + bc + bc + d^2$

$= a^2 + 2bc + d^2$

$= (a + d)^2 - 2ad + 2bc$

$= 4 - 2(ad - bc)$

$= 4 - 2(-3)$

$= 4 + 6$

$= 10$

24. 32

Sol. $2x - y + 3 = 0$

$6x + 3y + 1 = 0$

$\alpha x + 2y - 2 = 0$

Will not form a Δ if $ax + 2y - 2 = 0$ is concurrent with $2x - y + 3 = 0$ and $6x + 3y + 1 = 0$ or parallel to either of them so

Case-1: Concurrent lines

$$\begin{vmatrix} 2 & -1 & 3 \\ 6 & 3 & 1 \\ \alpha & 2 & -2 \end{vmatrix} = 0 \Rightarrow \alpha = \frac{4}{5}$$

Case-2 : Parallel lines

$$-\frac{\alpha}{2} = \frac{-6}{3} \text{ or } -\frac{\alpha}{2} = 2$$

$$\Rightarrow \alpha = 4 \text{ or } \alpha = -4$$

$$P = 16 + 16 + \frac{16}{25}$$

$$[P] = \left[32 + \frac{16}{25}\right] = 32$$

25. 0

Sol. $(1 - x)(1 - x)^{2007}(1 + x + x^2)^{2007}$

$(1 - x)(1 - x^3)^{2007}$

$(1 - x)({}^{2007}C_0 - {}^{2007}C_1 (x^3) + ...)$

General term

$(1 - x)((-1)^r\ {}^{2007}C_r\ x^{3r})$

$(-1)^r\ {}^{2007}C_r\ x^{3r} - (-1)^r\ {}^{2007}C_r\ x^{3r+1}$

$3r = 2012$

$r \neq \frac{2012}{3}$

$3r + 1 = 2012$

$3r = 2011$

$r \neq \frac{2011}{3}$

Hence there is no term containing x^{2012}.

So coefficient of $x^{2012} = 0$

26. 11

Sol. $\frac{dy}{dx} = \frac{x+y-2}{x-y}$

$x = X + h,\ y = Y + k$

$\frac{dY}{dX} = \frac{X+Y}{X-Y}$

$\left.\begin{matrix} h+k-2=0 \\ h-k=0 \end{matrix}\right\} h = k = 1$

$Y = vX$

$v + \frac{dv}{dX} = \frac{1+v}{1-v} \Rightarrow X - \frac{dv}{dX} = \frac{1+v^2}{1-v}$

$\frac{1-v}{1+v^2}dv = \frac{dX}{X}$

$\tan^{-1} v - \frac{1}{2}\ln(1+v^2) = \ln|X| + C$

As curve is passing through (2, 1)

$\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{2}\ln\left(1+\left(\frac{y-1}{x-1}\right)^2\right) = \ln|x-1|$

$\therefore \alpha = 1 \text{ and } \beta = 2$

$\Rightarrow 5\beta + \alpha = 11$

27. 2

Sol. $f(x) = \int_0^x g(t)\ln\left(\frac{1-t}{1+t}\right)dt$

$f(-x) = \int_0^{-x} g(t)\ln\left(\frac{1-t}{1+t}\right)dt$

$f(-x) = -\int_0^x g(-y)\ln\left(\frac{1+y}{1-y}\right)dy$

$= -\int_0^x g(y)\ln\left(\frac{1-y}{1+y}\right)dy$ (g is odd)

$f(-x) = -f(x) \Rightarrow$ f is also odd

Now,

$I = \int_{-\pi/2}^{\pi/2}\left(f(x) + \frac{x^2\cos x}{1+e^x}\right)dx$ ....(1)

$I = \int_{-\pi/2}^{\pi/2}\left(f(-x) + \frac{x^2e^x\cos x}{1+e^x}\right)dx$ ....(2)

$2I = \int_{-\pi/2}^{\pi/2} x^2\cos x\,dx = 2\int_0^{\pi/2} x^2\cos x\,dx$

$I = (x^2\sin x)_0^{\pi/2} - \int_0^{\pi/2} 2x\sin x\,dx$

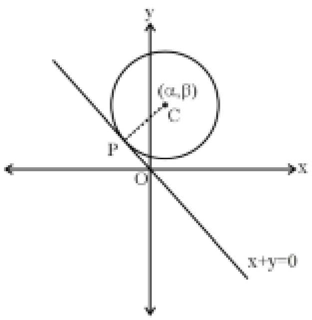
$= \frac{\pi^2}{4} - 2(-x\cos x + \int\cos x\,dx)_0^{\pi/2}$

$= \frac{\pi^2}{4} - 2(0+1) = \frac{\pi^2}{4} - 2 \Rightarrow \left(\frac{\pi}{2}\right)^2 - 2$

$\therefore \alpha = 2$

28. 100

Sol.



$S: (x-\alpha)^2 + (y-\beta)^2 = 50$

$CP = r$

$\left|\frac{\alpha+\beta}{\sqrt{2}}\right| = 5\sqrt{2}$

$\Rightarrow (\alpha+\beta)^2 = 100$

29. 108

Sol. $\frac{x-2}{1}=\frac{y}{-1}=\frac{z-7}{8}=\lambda$

$\frac{x+3}{4}=\frac{y+2}{3}=\frac{z+2}{1}=k$

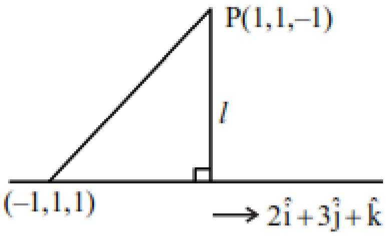
$\Rightarrow \lambda+2=4k-3$

$-\lambda=3k-2$

$\Rightarrow k=1,\ \lambda=-1$

$8\lambda+7=k-2$

$\therefore P=(1,\ 1,\ -1)$



Projection of $2\hat{i}-2\hat{k}$ on $2\hat{i}+3\hat{j}+\hat{k}$ is

$=\frac{4-2}{\sqrt{4+9+1}}=\frac{2}{\sqrt{14}}$

$\therefore l^2=8-\frac{4}{14}=\frac{108}{14}$

$\Rightarrow 14l^2=108$

30. 20

Sol. $|z-z_0|^2=4$

$\Rightarrow (\alpha-z_0)(\bar{\alpha}-\bar{z}_0)=4$

$\Rightarrow \alpha\bar{\alpha}-\alpha\bar{z}_0-z_0\bar{\alpha}+|z_0|^2=4$

$\Rightarrow |\alpha|^2-\alpha\bar{z}_0-z_0\bar{\alpha}=2$ .....(1)

$|z-z_0|^2=16$

$\Rightarrow \left(\frac{1}{\alpha}-z_0\right)\left(\frac{1}{\alpha}-\bar{z}_0\right)=16$

$\Rightarrow (1-\bar{\alpha}z_0)(1-\alpha\bar{z}_0)=16|\alpha|^2$

$\Rightarrow 1-\bar{\alpha}z_0-\alpha\bar{z}_0+|\alpha|^2|z_0|^2=16|\alpha|^2$

$\Rightarrow 1-\bar{\alpha}z_0-\alpha\bar{z}_0=14|\alpha|^2$ ....(2)

From (1) and (2)

$\Rightarrow 5|\alpha|^2=1$

$\Rightarrow 100|\alpha|^2=20$

PHYSICS

Section - A (Single Correct Answer)

31. B

Sol. $[P]=\left[\frac{a}{V^2}\right]\Rightarrow[a]=\left[PV^2\right]$

And [V] = [b]

$\frac{[a]}{[b^2]}=\frac{[PV^2]}{[V^2]}=[P]$

32. D

Sol. As specific resistance does not depends on dimension of wire so, it will not change.

33. B

Sol. $\omega=\frac{2\pi}{T}\Rightarrow\omega\propto\frac{1}{T}$

T_{moon} = 27 days

T_{earth} = 365 days 4 hour

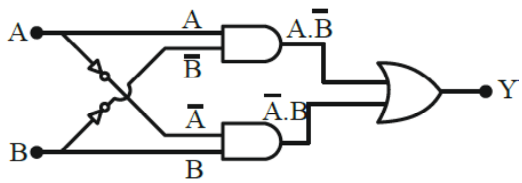
$\Rightarrow \omega_{moon}>\omega_{earth}$

34. B

Sol. Co-efficient of friction depends on surface in contact So, depends on material of object.

35. B

Sol.



$Y=A\cdot\bar{B}+\bar{A}\cdot B$

This is XOR GATE

36. C

Sol. $i\propto\theta$ (angle of deflection)

$\therefore \frac{i_2}{i_1}=\frac{\theta_2}{\theta_1}\Rightarrow\frac{i_2}{200\mu A}=\frac{\pi/10}{\pi/3}=\frac{3}{10}$

$\Rightarrow i_2=60\mu A$

37. C

Sol. ${}_6C^{13}$ + Energy $\rightarrow {}_6C^{12}+{}_0n^1$

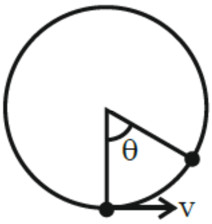
$\Delta m=(12.000000+1.008665)-13.003354$

= – 0.00531 u

$\therefore$ Energy required = 0.00531 × 931.5 MeV = 4.95 MeV

38. B

Sol.



Loss in kinetic energy = Gain in potential energy

$$\Rightarrow \frac{1}{2}mv^2 = mgl(1-\cos\theta)$$

$$\Rightarrow \frac{v^2}{l} = 2g(1-\cos\theta)$$

Acceleration at lowest point $\frac{v^2}{l}$

Acceleration at extreme point = g sin θ

Hence, $\frac{v^2}{l} = g\sin\theta$

$$\therefore \sin\theta = 2(1-\cos\theta)$$

$$\Rightarrow \tan\frac{\theta}{2} = \frac{1}{2} \Rightarrow \theta = 2\tan^{-1}\left(\frac{1}{2}\right)$$

39. D

Sol. From KVL,

$V_1 + V_2 - V_3 = 0 \Rightarrow V_1 + V_2 = V_3$

40. C

Sol. Kinetic energy $= \frac{f}{2}nRT$

$$= \frac{5}{2} \times 1 \times 8.31 \times 300\text{J}$$

= 6232.5 J

41. B

Sol. Assertion & Reason both are correct Theory

42. C

Sol. $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

$$\frac{230}{V_2} = \frac{10}{1}$$

$V_2 = 23\text{V}$

Power consumed $= \frac{V_2^2}{R}$

$$= \frac{23 \times 23}{46} = 11.5\text{W}$$

43. B

Sol. $P \propto T^3 \Rightarrow PT$ constant

PV^γ = const nRT

$$P\left(\frac{nRT}{P}\right)^\gamma = \text{const}$$

$$P^{1-\gamma}T^\gamma = \text{const}$$

$$PT^{\frac{\gamma}{1-\gamma}} = \text{const}$$

$$\frac{\gamma}{1-\gamma} = -3$$

$$\gamma = -3 + 3\gamma$$

$$3 = 2\gamma$$

$$\gamma = \frac{3}{2}$$

44. D

Sol. $\phi_0 = h\nu_0$

$6.63 \times 1.6 \times 10^{-19} = 6.63 \times 10^{-34}\ \nu_0$

$$\nu_0 = \frac{1.6 \times 10^{-19}}{10^{-34}}$$

$\nu_0 = 1.6 \times 10^{15}$ Hz

45. C or D

Sol. Theory

46. D

Sol. Let I_0 be intensity of unpolarised light incident on first polaroid.

I_1 = Intensity of light transmitted from 1st polaroid

$$= \frac{I_0}{2}$$

θ be the angle between 1st and 2nd polaroid

ϕ be the angle between 2nd and 3rd polaroid

θ + ϕ = 90° (as 1st and 3rd polaroid are crossed)

ϕ = 90° – θ

I_2 = Intensity from 2nd polaroid

$$I_2 = I_1\cos^2\theta = \frac{I_0}{2}\cos^2\theta$$

I_3 = Intensity from 3rd polaroid

$I_3 = I_2\cos^2\phi$

$I_3 = I_1\cos^2\theta\cos^2\phi$

$$I_3 = I_1\cos^2\theta\cos^2\phi$$

$$I_3 = \frac{I_0}{2}\cos^2\theta\cos^2\phi$$

$$\phi = 90 - \theta$$

$$I_3 = \frac{I_0}{2}\cos^2\theta\sin^2\theta$$

$$I_3 = \frac{I_0}{2}\left[\frac{2\sin\theta\cos\theta}{2}\right]^2$$

I_3 will be maximum when $\sin 2\theta = 1$

$2\theta = 90°$

$\theta = 45°$

47. C

Sol. Radiation pressure = I/v

$$= \frac{I\cdot\mu}{c} = \frac{6\times10^8\times3}{3\times10^8}$$

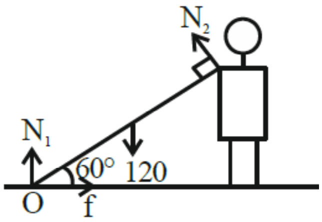
$= 6\ N/m^2$

48. D

Sol. Electric line of force are always perpendicular to equipotential surface so angle between farce and displacement will always be 90°. So work done equal to 0.

49. C

Sol.



Torque about O = 0

$$120\left(\frac{L}{2}\cos 60°\right) - N_2L = 0$$

$N_2 = 30\ N$

50. Bonus

Sol. $v^2 - u^2 = 2aS$

$$\left(\frac{2u}{3}\right)^2 = u^2 + 2(-a)(4\times10^{-2})$$

$$\frac{4u^2}{9} = u^2 - 2a(4\times10^{-2})$$

$$-\frac{5u^2}{9} = -2a(4\times10^{-2}) \quad .....(1)$$

$$0 = \left(\frac{2u}{3}\right)^2 + 2(-a)(x)$$

$$-\frac{4u^2}{9} = -2ax \quad ....(2)$$

(1) /(2)

$$\frac{5}{4} = \frac{4\times10^{-2}}{x}$$

$$x = \frac{16}{5}\times10^{-2}$$

$x = 3.2 \times 10^{-2}$ m

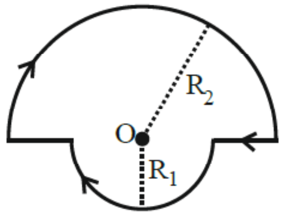
$x = 32 \times 10^{-3}$ m

Note : Since no option is matching, Question should be bonus.

Section - B (Numerical Value Type)

51. 3

Sol.



$$\frac{\mu_0 i}{2R_2}\left(\frac{\pi}{2\pi}\right)\otimes + \frac{\mu_0 i}{2R_1}\left(\frac{\pi}{2\pi}\right)\otimes$$

$$\left(\frac{\mu_0 i}{4R_2} + \frac{\mu_0 i}{4R_1}\right)\otimes$$

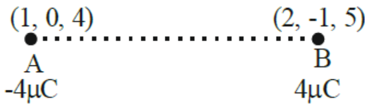
$$\frac{4\pi\times10^{-7}\times4}{4\times4\pi} + \frac{4\pi\times10^{-7}\times4}{4\times2\pi}$$

$= 3 \times 10^{-7} = \alpha \times 10^{-7}$

$\alpha = 3$

52. 2

Sol.



$$\vec{\tau} = \vec{p}\times\vec{E}$$

$$\vec{p} = q\vec{l}$$

$$\vec{E} = 0.2\frac{V}{cm} = 20\frac{V}{m}$$

$$\vec{p} = 4\times(\hat{i} - \hat{j} + \hat{k})$$

$$= (4\hat{i} - 4\hat{j} + 4\hat{k})\mu C - m$$

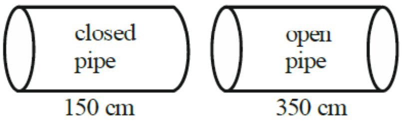
$$\vec{\tau} = (4\hat{i} - 4\hat{j} + 4\hat{k})\times(20\hat{i})\times10^{-6}\,Nm$$

$$= (8\hat{k} + 8\hat{j})\times10^{-5} = 8\sqrt{2}\times10^{-5}$$

$\alpha = 2$

53. 294

Sol.



$$f_c = \frac{v}{4l_1} \quad f_0 = \frac{v}{2l_2}$$

$$|f_c - f_0| = 7$$

$$\frac{v}{4\times150} - \frac{v}{2\times350} = 7$$

$$\frac{v}{600\text{ cm}} - \frac{v}{700\text{ cm}} = 7$$

$$\frac{v}{6m} - \frac{v}{7m} = 7$$

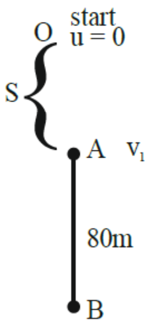
$$v\left(\frac{1}{42}\right) = 7$$

$$v = 42\times7$$

$= 294$ m/s

54. 45

Sol.



From A → B

$$-80 = -v_1 t - \frac{1}{2}\times10t^2$$

$$-80 = -2v_1 - \frac{1}{2}\times10\times2^2$$

$-80 = -2v_1 - 20$

$-60 = -2v_1$

$v_1 = 30$ m/s

From O to A

$v^2 = u^2 + 2gS$

$30^2 = 0 + 2\times(-10)(-S)$

$900 = 20$ S

S = 45 m

55. 50

Sol. Change in pressure $= \frac{1}{2}\rho v^2$

$v^2 = 50$

$v = \sqrt{50}$

Velocity of water $= \sqrt{V} = \sqrt{50}$

$= V = 50$

56. 7

Sol. In pure rolling work done by friction is zero. Hence potential energy is converted into kinetic energy. Since initially the ring and the sphere have same potential energy, finally they will have same kinetic energy too.

∴ Ratio of kinetic energies = 1

$$\Rightarrow \frac{7}{x} = 1 \Rightarrow x = 7$$

57. 30

Sol. For first minima

$a \sin\theta = \lambda$

$$\Rightarrow \sin\theta = \frac{\lambda}{a} = \frac{5000\times10^{-10}}{1\times10^{-6}} = \frac{1}{2}$$

$$\Rightarrow \theta = 30°$$

58. 8

Sol. Potential $= \frac{kQ}{R} = \frac{k.Ze}{R}$

$$= \frac{9\times10^9\times50\times1.6\times10^{-19}}{9\times10^{-13}\times10^{-2}}$$

$= 8\times10^6$ V

59. 144

Sol. Longest wavelength corresponds to transition between n = 3 and n = 4

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = RZ^2\left(\frac{1}{9} - \frac{1}{16}\right)$$

$$= \frac{7RZ^2}{9\times16}$$

$$\Rightarrow \lambda = \frac{144}{7R} \text{ for } Z = 1 \quad \therefore \alpha = 144$$

60. 1

Sol. $X_c = \frac{1}{\omega C} = \frac{\pi}{2\pi\times50\times10^{-3}} = 10\Omega$

$$X_L = \omega L = 2\pi\times50\times\frac{100}{\pi}\times10^{-3}$$

$= 10\Omega$

$\because X_C = X_L$, Hence, circuit is in resonance

$\therefore$ power factor $= \frac{R}{Z} = \frac{R}{R} = 1$

CHEMISTRY

Section - A (Single Correct Answer)

61. **(A)**

Sol. I > II > III, since neutral resonating structures are more stable than charged resonating structure.

II > III, since stability of structure with –ve charge on more electronegative atom is higher.

62. **(D)**

Sol. Since $CH_3 - CH = \overset{+}{C}H$ is very unstable, $CH_3 - CH = CH - Cl$ cannot give S_{N^1} reaction.

63. **(A)**

Sol. As tin coating is peeled off, then iron is exposed to atmosphere.

64. **(B)**

Sol. Statement (1) is true, Ce^{+4} has noble gas electronic configuration.

Statement (2) is also true due to high reduction potential for Ce^{4+}/Ce^{3+} (+1.74 V), and stability of Ce^{3+}, Ce^{4+} acts as strong oxidizing agent.

65. **(B)**

Sol. $[Cr] = [Ar]\ 4s^1\ 3d^5$

$[Cd] = [Kr]\ 5s^2\ 4d^{10}$

$[Cu] = [Ar]\ 4s^1\ 3d^{10}$

$[Ag] = [Kr]\ 5s^1\ 4d^{10}$

$[Zn] = [Ar]\ 4s^2\ 3d^{10}$

66. **(A)**

Sol. Carbylamine Test – Identification of primary amines

Lucas Test – Differentiation between 1°, 2° and 3° alcohols

Tollen's Test – Identification of Aldehydes

Phthalein Dye Test – Identification of phenols

67. **(B)**

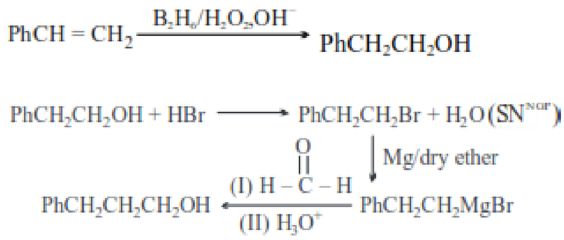
Sol. HCOOH, CH_3COOH
$\uparrow$
Second homologue

68. **(D)**

Sol. Steam distillation is used for those liquids which are insoluble in water, containing non-volatile impurities and are steam volatile.

69. **(D)**

Sol.



70. **(D)**

Sol. (A) $\rightarrow$ Kolbe Schmidt Reaction

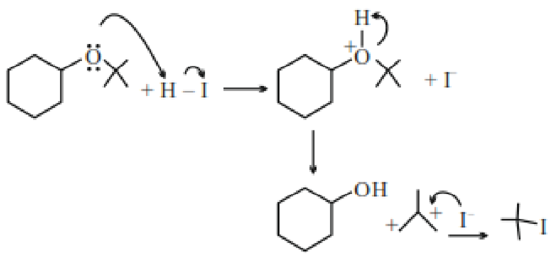
(B) $\rightarrow$ Reimer Tiemann Reaction

(C) $\rightarrow$ Oxidation of phenol to p-benzoquinone

(D) $\rightarrow$ PhOH + NaOH $\rightarrow$ H_2O + PhO^-

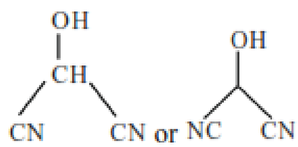
71. **(D)**

Sol.



72. **(D)**

Sol. CH (OH) $(CN)_2$ is



73. **(C)**

Sol. Statement-I : Oxygen can have oxidation state from –2 to +2, so statement I is incorrect.

Statement-II : On moving down the group stability of +4 oxidation state increases whereas stability of +6 oxidation state decreases down the group, according to inert pair effect.

So both statements are wrong.

74. **(A)**

Sol. $[Co(NH_3)_6]^{+3}$ – d^2sp^3 hybridization

BrF_5 – sp^3d^2 hybridization

$[PtCl_4]^{-2}$ – dsp^2 hybridization

SF_6 – sp^3d^2 hybridization

75. **(B)**

Sol. $Cl-(CH_2)_4-Cl \xrightarrow[NH_3]{excess} Cl^- \overset{\oplus}{N}H_3-(CH_2)_4-\overset{\oplus}{N}H_3Cl^-$ (A)

$\downarrow$ NaOH

$2NaCl + 2H_2O + NH_2-(CH_2)_4-NH_2$

76. **(A)**

Sol. $\text{Molarity} = \dfrac{\text{Moles of solute}}{\text{Volume of solution}}$

Since volume depends on temperature, molarity will change upon change in temperature.

77. **(A)**

Sol. Boiling an egg causes denaturation of its protein resulting in loss of its quarternary, tertiary and secondary structures.

78. **(A)**

Sol. In N^{3-} ion 'N' is present in its lowest possible oxidation state, hence it cannot be reduced further because of which it cannot act as an oxidizing agent.

79. **(C)**

Sol. Eclipsed conformation is the least stable conformation of ethane.

80. **(D)**

Sol. The catalyst used in Wacker's process is $PdCl_2$.

Section - B (Numerical Value Type)

81. **(3)**

Sol. Noble gas configuration = $ns^2\ np^6$

$[Sr^{2+}] = [Kr]$

$[Cs^+] = [Xe]$

$[Yb^{2+}] = [Kr]\ 4d^{10}\ 4f^{14}\ 5s^2\ 5p^6$

$[La^{2+}] = [Xe]\ 5d^1$

$[Pb^{2+}] = [Xe]\ 4f^{14}\ 5d^{10}\ 6s^2$

$[Fe^{2+}] = [Ar]\ 3d^6$

82. **(4)**

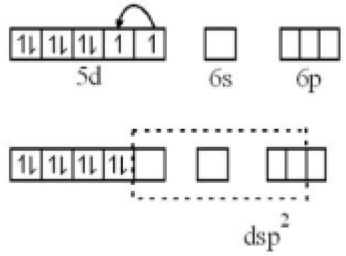
Sol. The non-polar molecules are CO_2, H_2, CH_4 and BF_3.

83. **(10)**

Sol. $\dfrac{t_{99.9\%}}{t_{1/2}} = \dfrac{\frac{2.303}{k}\left(\frac{a}{a-x}\right)}{\frac{2.303}{k}\log 2} = \dfrac{\log\left(\frac{100}{100-99.9}\right)}{\log 2} = \dfrac{\log 10^3}{\log 2} = \dfrac{3}{0.3} = 10$

84. **(0)**

Sol. Pt^{2+} (d^8)



$Pt^{2+} \rightarrow dsp^2$ hybridization and have no unpaired e^-s.

$\therefore$ Magnetic moment = 0

85. **(37)**

Sol. $\Delta G° = \Delta H° - T\Delta S°$

$= 77.2 \times 10^3 - 400 \times 122 = 28400$ J

$\Delta G° = -2.303\ RT \log K$

$\Rightarrow 28400 = -2.303 \times 8.314 \times 400 \log K$

$\Rightarrow \log K = -3.708 = -37.08 \times 10^{-1}$

86. **(7)**

Sol. $M = \dfrac{n_{NaOH}}{V_{sol}(\text{in L})} \Rightarrow 3 = \dfrac{(84/40)}{V} \Rightarrow V = 0.7\ L = 7\times10^{-1}\ L$

87. **(8)**

Sol. $PbS + 4O_3 \rightarrow PbSO_4 + 4O_2$

$x = 4, y = 4$

88. **(18)**

Sol. $2H^+_{(aq)} + 2e^- \rightarrow H_2(g)$

$$E_{cell} = E^o_{cell} - \frac{0.059}{2}\log\frac{P_{H_2}}{[H^+]^2}$$

$= 0 - 0.059 \times 3 = -0.177$ volts. $= -17.7 \times 10^{-2}$ V.

89. **(135)**

Sol. $C_6H_5NH_2 + CH_3-\overset{\overset{O}{||}}{C}-O-\overset{\overset{O}{||}}{C}-CH_3 \rightarrow$
$\underset{(\text{Aniline MM}=93)}{}$

$$C_6H_5NH-\overset{\overset{O}{||}}{C}-CH_3 + CH_3COOH$$
$\underset{(\text{Acetanilide MM}=135)}{}$

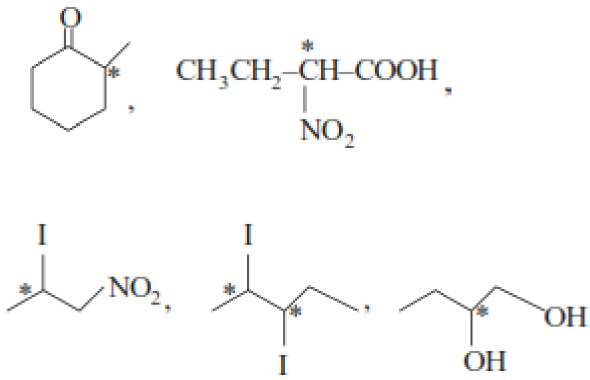
$n_{\text{Acetanilide}} = n_{\text{Aniline}}$

$\Rightarrow \quad \dfrac{m}{135} = \dfrac{9.3}{93}$

$\Rightarrow \quad m = 13.5\ g$

90. **(5)**

Sol. Chiral carbons are marked by,



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