1-Feb.-2024 (Evening) : PCM [SOLUTIONS]

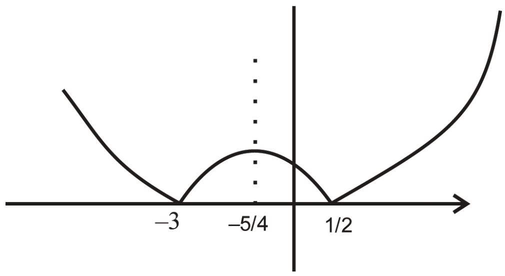
MATHEMATICS

Section - A (Single Correct Answer)

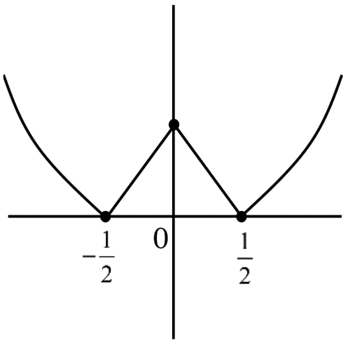
1. D

Sol. $f(x) = |2x^2 + 5|x| - 3|$

Graph of $y = |2x^2 + 5x - 3|$



Graph of f(x)



Number of points of discontinuit $y = 0 = m$

Number of points of non-differentiabilit $y = 3 = n$

2. A

Sol. $px^2 + qx - r = 0 <^{\alpha}_{\beta}$

$p = A,\ q = AR,\ r = AR^2$

$Ax^2 + ARx - AR^2 = 0$

$x^2 + Rx - R^2 = 0 <^{\alpha}_{\beta}$

$\because \frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{4}$

$\therefore \frac{\alpha+\beta}{\alpha\beta} = \frac{3}{4} \Rightarrow \frac{-R}{-R^2} = \frac{3}{4} \Rightarrow R = \frac{4}{3}$

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = R^2 - 4(-R^2) = 5\left(\frac{16}{9}\right)$

$= 80/9$

3. D

Sol. $4 \sin 2x - 4 \cos 3x + 9 - 4 \cos x = 0$; $x \in [-2\pi, 2\pi]$

$4 - 4 \cos^2 x - 4 \cos^3 x + 9 - 4 \cos x = 0$

$4 \cos^3 x + 4 \cos^2 x + 4 \cos x - 13 = 0$

$4 \cos^3 x + 4 \cos^2 x + 4 \cos x = 13$

L.H.S. $\le$ 12 can't be equal to 13.

4. A

Sol. $I = \int_0^1 (2x^3 - 3x^2 - x1)^{\frac{1}{3}} dx$

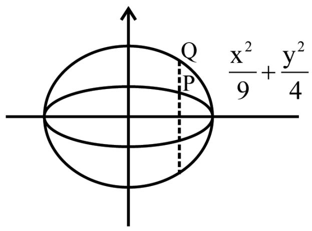
Using $\int_0^{2a} f(x)dx$ where $f(2a - x) = -f(x)$

Here $f(1 - x) = f(x)$

$\therefore I = 0$

5. D

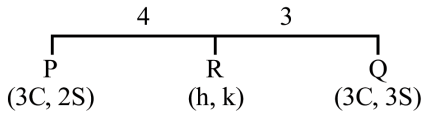
Sol.



$\frac{x^2}{9} + \frac{y^2}{4} = 1$

P(3cosθ, 2sinθ)

Q(3cosθ, 3sinθ)



$h = 3\cos\theta;$

$k = \frac{18}{7}\sin\theta$

$\therefore \text{locus} = \frac{x^2}{9} + \frac{49y^2}{324} = 1$

$e = \sqrt{1 - \frac{324}{49 \times 9}} = \frac{\sqrt{117}}{21} = \frac{\sqrt{13}}{7}$

6. D

Sol. $\left(\frac{x^{\frac{1}{3}}}{3} + x^{\frac{-2}{3}}\right)^{18}$

$$t_7 = {}^{18}c_6 \left(\frac{x^{\frac{1}{3}}}{3}\right)^{12} \left(\frac{x^{\frac{-2}{3}}}{2}\right) = {}^{18}c_6 \frac{1}{(3)^{12}} \cdot \frac{1}{2^6}$$

$$t_{13} = {}^{18}c_{12} \left(\frac{x^{\frac{1}{3}}}{3}\right)^{6} \left(\frac{x^{\frac{-2}{3}}}{2}\right)^{12} = {}^{18}c_{12} \frac{1}{(3)^{6}} \cdot \frac{1}{2^{12}} \cdot x^{-6}$$

$$m = {}^{18}c_6 \cdot 3^{-12} \cdot 2^{-6} \; : n = {}^{18}c_{12} \cdot 2^{-12} \cdot 3^{-6}$$

$$\left(\frac{n}{m}\right)^{\frac{1}{3}} = \left(\frac{2^{-12} \cdot 3^{-6}}{3^{12} \cdot 2^{-6}}\right)^{\frac{1}{3}} = \frac{3^2}{2} = \frac{9}{4}$$

7. 3 or Bonus

Sol. $f(0) = 2, \; \lim\limits_{x \to -\infty} f(x) = 1$

$f(x) - x \cdot f(x) = 3$

$I.F = e^{-\alpha x}$

$$y(e^{-\alpha x}) = \int 3 \cdot e^{-\alpha x} dx$$

$$f(x) \cdot (e^{-\alpha x}) = \frac{3e^{-\alpha x}}{-\alpha} + c$$

$$f(x) \cdot (e^{-\alpha x}) = \frac{3e^{-\alpha x}}{-\alpha} + c$$

$$x = 0 \Rightarrow 2 = \frac{-3}{\alpha} + c \Rightarrow \frac{3}{\alpha} = c - 2 \quad ....(1)$$

$$f(x) = \frac{-3}{\alpha} c.e^{\alpha x}$$

$$x \to -\infty \Rightarrow 1 = \frac{-3}{\alpha} + c(0)$$

$\alpha = -3 \;\; \therefore c = 1$

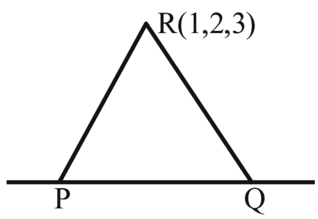
$$f(-\ln 2) = \frac{-3}{\alpha} + c \cdot e^{\alpha x}$$

$= 1 + e^{3\ln 2} = 9$

(But α should be greater than 0 for finite value of c)

8. C

Sol.



$P(8\lambda - 3, \; 2\lambda + 4, \; 2\lambda - 1)$

$PR = 6$

$(8\lambda - 4)^2 + (2\lambda + 2)^2 + (2\lambda - 4)^2 = 36$

$\lambda = 0, \; 1$

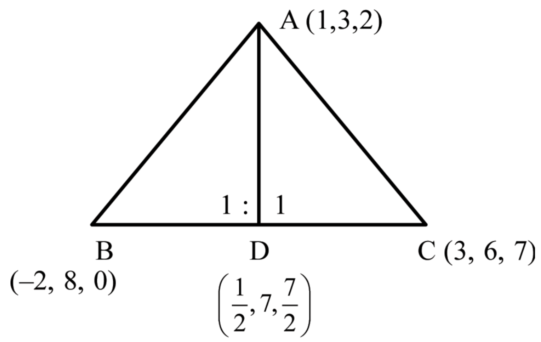
Hence P(–3, 4, –1) & Q(5, 6, 1)

Centroid $\Delta PQR = (1, \; 4, \; 1) \equiv (\alpha, \; \beta, \; \gamma)$

$\alpha^2 + \beta^2 + \gamma^2 = 18$

9. A

Sol.



A(l, 3, 2); B(–2, 8, 0); C(3, 6, 7);

$\overrightarrow{AC} = 2\hat{i} + 3\hat{j} + 5\hat{k}$

$AB = \sqrt{9 + 25 + 4} = \sqrt{38}$

$AC = \sqrt{4 + 9 - 25} = \sqrt{38}$

$$\overrightarrow{AD} = \frac{1}{2}\hat{i} - 4\hat{j} - \frac{3}{2}\hat{k} = \frac{1}{2}(\hat{i} - 8\hat{j} - 3\hat{k})$$

Letgth of projection of $\overrightarrow{AD}$ on $\overrightarrow{AC}$

10. C

Sol. $S_{10} = 390$

$$\frac{10}{2}[2a + (10-1)d] = 390$$

$\Rightarrow 2a + 9d = 78 \qquad .....(1)$

$$\frac{t_{10}}{t_5} = \frac{15}{7} \Rightarrow \frac{a + 9d}{a + 4d} = \frac{15}{7} \Rightarrow 8a = 3d \quad ....(2)$$

From (1) & (2) $\qquad$ a = 3 & d = 8

$$S_{15} - S_5 = \frac{15}{2}(6 + 14 \times 8) - \frac{5}{2}(6 + 4 \times 8)$$

$$= \frac{15 \times 118 - 5 \times 38}{2} = 790$$

11. A

Sol. $\displaystyle\int_0^{\pi/3} \cos^4 x \, dx$

$$= \int_0^{\pi/2} \left(\frac{1+\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{4}\int_0^{\pi/3} (1+2\cos 2x+\cos^2 2x)dx$$

$$= \frac{1}{4}\left[\int_0^{\pi/3} dx + 2\int_0^{\pi/3} \cos 2x\, dx + \int_0^{\pi/3} \frac{1+\cos 4x}{2} dx\right]$$

$$= \frac{1}{4}\left[\frac{\pi}{3} + (\sin 2x)_0^{\pi/3} + \frac{1}{2}\left(\frac{\pi}{3}\right) + \frac{1}{8}(\sin 4x)_0^{\pi/3}\right]$$

$$= \frac{1}{4}\left[\frac{\pi}{3} + (\sin 2x)_0^{\pi/3} + \frac{1}{2}\left(\frac{\pi}{3}\right) + \frac{1}{8}(\sin 4x)_0^{\pi/3}\right]$$

$$= \frac{1}{4}\left[\frac{\pi}{2} + \frac{\sqrt{3}}{2} + \frac{1}{8}\times\left(-\frac{\sqrt{3}}{2}\right)\right]$$

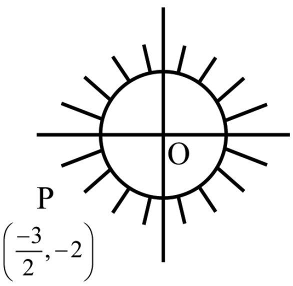
$$= \frac{\pi}{2} + \frac{7\sqrt{3}}{64}$$

$$\therefore a = \frac{1}{8};\ b = \frac{7}{64}$$

$$\therefore 9a + 8b = \frac{9}{8} + \frac{7}{8} = 2$$

12. Bonus

Sol. $|z| \geq 1$



Min. value of $\left|z + \frac{3}{2} + 2i\right|$ is actually zero.

13. C

Sol. $f(x) = \dfrac{\sqrt{x^2-25}}{4-x^2} + \log_{10}(x^2+2x-15)$

Domain : $x^2 - 25 \geq 0 \Rightarrow x \in (-\infty,\ -5] \cup [5,\ \infty)$

$4 - x^2 \neq 0 \Rightarrow x \neq \{-2,\ 2\}$

$x^2 + 2x - 15 > 0 \Rightarrow (x+5)(x-3) > 0$

$\Rightarrow x \in (-\infty,\ -5) \cup (3,\ \infty)$

$\therefore x \in (-\infty,\ -5) \cup [5,\ \infty)$

$\alpha = -5;\ \beta = 5$

$\therefore \alpha^2 + \beta^3 = 150$

14. B

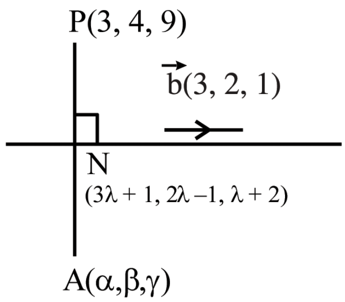
Sol. $aR_1b \Leftrightarrow a^2 + b^2 = 1;\ a, b \in R$

$(a,\ b)R_2(c,\ d) \Leftrightarrow a + d = b + c;\ (a,\ c),\ (c,\ d) \in N$

for R_1 : Not reflexive symmetric not transitive
for R_2 : R_2 is reflexive, symmetric and transitive
Hence only R_2 is equivalence relation.

15. C

Sol.



$\overrightarrow{PN} \cdot \vec{b} = 0$?

$3(3\lambda - 2) + 2(2\lambda - 5) + (\lambda - 7) = 0$

$14\lambda = 23 \Rightarrow \lambda = \dfrac{23}{14}$

$N\left(\dfrac{83}{14}, \dfrac{32}{14}, \dfrac{51}{14}\right)$

$\therefore \dfrac{\alpha+3}{2} = \dfrac{83}{14} \Rightarrow \alpha = \dfrac{62}{7}$

$\dfrac{\beta+4}{2} = \dfrac{32}{14} \Rightarrow \beta = \dfrac{4}{7}$

$\dfrac{\gamma+9}{2} = \dfrac{51}{14} \Rightarrow \gamma = \dfrac{-12}{7}$

Ans. $14(\alpha + \beta + r) = 108$

16. B

Sol. $f(x) = \begin{cases} x-1; & x = \text{even} \\ 2x; & x = \text{odd} \end{cases}$

f(f(f(a))) = 21

C–1 : If a = even

f(a) = a – 1 = odd

f(f(a)) = 2(a – 1) = even

f(f(f(a))) = 2a – 3 = 21 $\Rightarrow$ a = 12

C–2 : If a = odd

f(a) = 2a = even

f(f(a)) = 2a – 1 = odd

f(f(f(a))) = 4a – 2 = 21 (Not possible)

Hence a = 12

Now

$$\lim_{x\to 12^-}\left(\frac{|x|^3}{2}-\left[\frac{x}{12}\right]\right)$$

$$= \lim_{x\to 12^-}\frac{|x|^3}{12}-\lim_{x\to 12^-}\left[\frac{x}{12}\right]$$

= 144 – 0 = 144.

17. B

Sol. $x + 2y + 3z = 5$

$2x + 3y + z = 9$

$4x + 3y + \lambda z = \mu$

for infinite following $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 4 & 3 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = -13$$

$$\Delta_1 = \begin{vmatrix} 5 & 2 & 3 \\ 9 & 3 & 1 \\ \mu & 3 & -13 \end{vmatrix} = 0 \Rightarrow \mu = 15$$

$$\Delta_2 = \begin{vmatrix} 1 & 5 & 3 \\ 2 & 9 & 1 \\ 4 & 15 & -13 \end{vmatrix} = 0$$

$$\Delta = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 9 \\ 4 & 3 & 15 \end{vmatrix} = 0$$

for $\lambda = -13$, $\mu = 15$ system of equation has infinite solution hence $\lambda + 2\mu = 17$

18. A

Sol. $x_1, x_2, \ldots\ldots x_{10}$

$$\sum_{i=1}^{10}(x_i-\alpha) = 2 \Rightarrow \sum_{i=1}^{10} x_i - 10\alpha = 2$$

$$\text{Mean } \mu = \frac{6}{5} = \frac{\Sigma x_i}{10}$$

$\therefore \Sigma x_i = 12$

$10\alpha + 2 = 12 \quad \therefore \alpha = 1$

Now $\sum_{i=1}^{10}(x_i-\beta)^2 = 40$ Let $y_i = x_i - \beta$

$$\therefore \sigma_y^2 = \frac{1}{10}\sum y_i^2 - (\bar{y})^2$$

$$\sigma_x^2 = \frac{1}{10}\sum (x_i-\beta)^2 - \left(\frac{\sum_{i=1}^{10}(x_i-\beta)}{10}\right)^2$$

$$\frac{84}{25} = 4 - \left(\frac{12-10\beta}{10}\right)^2$$

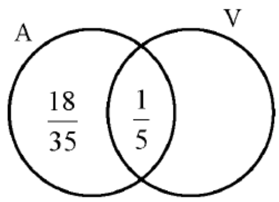
$$\therefore \left(\frac{6-5\beta}{5}\right)^2 = 4 - \frac{84}{25} = \frac{16}{25}$$

$6 - 5\beta = \pm 4 \Rightarrow \beta = \frac{2}{5}$ (not possible) or $\beta = 2$

Hence $\frac{\beta}{\alpha} = 2$

19. B

Sol.



$P(\bar{A}) = f27 = p$

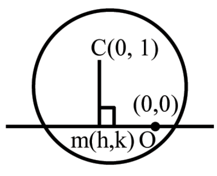
$P(A \cap V) = \frac{1}{5} = q$

$P(A) = \frac{5}{7}$

Ans. $P(A \cap \vec{V}) = \frac{18}{35}$

20. A

Sol.

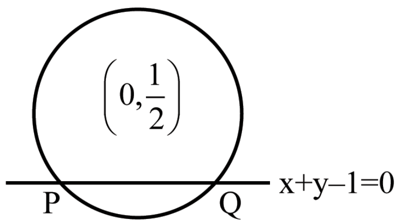


$m_{OM} \cdot m_{CM} = -1$

$$\frac{k}{h} \cdot \frac{k-1}{h} = -1$$

$\therefore$ locus is $x^2 + y(y-1) = 0$

$x^2 + y^2 - y = 0$



$$p = \left|\frac{1/2}{\sqrt{2}}\right| \quad p = \frac{1}{2\sqrt{2}}$$

$$PQ = 2\sqrt{r^2 - p^2}$$

$$= 2\sqrt{\frac{1}{4} - \frac{1}{8}} = \frac{1}{\sqrt{2}}$$

Section - B (Numerical Value Type)

21. 1

Sol. a, ar, $ar^2 \rightarrow$ G.P.

Sum of any two sides > third side

$a + ar > ar^2$, $a + ar^2 > ar$, $ar + ar^2 > a$

$r^2 - r - 1 < 0$

$$r \in \left(\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right) \quad \text{....(1)}$$

$r^2 + r + 1 > 0$

alway true

$r^2 + r - 1 > 0$

$$r \in \left(-\infty, \frac{-1-\sqrt{5}}{2}\right) \cup \left(\frac{-1+\sqrt{5}}{2}, \infty\right) \quad \text{......(2)}$$

$$r \in \left(\frac{-1+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$$

As $r > 1$

$$r \in \left(1, \frac{1+\sqrt{5}}{2}\right)$$

$[r] = 1 \quad [-r] = -2$

$3[r] + [-r] = 1$

22. B

Sol. $A = I_2 - 2\,MM^T$

$A^2 = (I_2 - 2MM^T)(I_2 - 2MM^T)$

$= I_2 - 2MM^T - 2MM^T + 4MM^TMM^T$

$= I_2 - 4MM^T + 4MM^T$

$= I_2$

$AX = \lambda X$

$A^2X = \lambda AX$

$X = \lambda(\lambda X)$

$X = \lambda^2 X$

$X(\lambda^2 - 1) = 0$

$\lambda^2 = 1$

$\lambda = \pm 1$

Sum of square of all possible values = 2

23. 219

Sol. $F(x) = \int_0^x t \cdot f(t)dt$

$F^1(x) = xf(x)$

Given $\quad F(x^2) = x^4 + x^5, \quad$ let $x^2 = t$

$F(t) = t^2 + t^{5/2}$

$F'(t) = 2t + 5/2\, t^{3/2}$

$t \cdot f(t) = 2t + 5/2\, t^{3/2}$

$f(t) = 2 + 5/2\, r^{1/2}$

$$\sum_{r=1}^{12} f(r^2) = \sum_{r=1}^{12} 2 + \frac{5}{2}r$$

$$= 24 + 5/2\left[\frac{12(13)}{2}\right]$$

$= 219$

24. 105

Sol. $$y = \frac{(\sqrt{x}+1)(x^2 - \sqrt{x})}{x\sqrt{x} + x + \sqrt{x}} + \frac{1}{15}(3\cos^2 x - 5)\cos^3 x$$

$$y = \frac{(\sqrt{x}+1)(\sqrt{x})((\sqrt{x})^3 - 1)}{(\sqrt{x})((\sqrt{x})^2 + (\sqrt{x})1)} + \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x$$

$$y = (\sqrt{x}+1)(\sqrt{x}-1) + \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x$$

$$y'\left(\frac{\pi}{6}\right) = 1 - \frac{9}{16} \times \frac{1}{2} + \frac{3}{4} \times \frac{1}{2}$$

$$= \frac{32 - 9 + 12}{32} = \frac{35}{32}$$

$$= 96y'\left(\frac{\pi}{6}\right) = 105$$

25. 38

Sol. $\vec{a} = \hat{i} + \hat{j} + k$

$\vec{b} = \hat{i} + 8\hat{j} + 2k$

$\vec{c} = 4\hat{i} + c_2\hat{j} + c_3 k$

$\vec{b} \times \vec{a} = \vec{c} \times \vec{a}$

$(\vec{b} - \vec{c}) \times \vec{a} = 0$

$\vec{b} - \vec{c} = \lambda\vec{\alpha}$

$\vec{b} = \vec{c} + \lambda\vec{\alpha}$

$-\hat{i} - 8\hat{j} + 2k = (4\hat{i} + c_2\hat{j} + c_3 k) + \lambda(\hat{i} + \hat{j} + k)$

$\lambda + 4 = -1 \Rightarrow \lambda = -5$

$\lambda + c_2 = 8 \Rightarrow c_2 = -3$

$\lambda + c_3 = 2 \Rightarrow c_3 = 7$

$\vec{c} = 4\hat{i} - 3\hat{j} + 7\hat{k}$

$$\cos\theta = \frac{12 - 12 + 7}{\sqrt{26}\cdot\sqrt{74}} = \frac{7}{\sqrt{26}\cdot\sqrt{74}} = \frac{7}{2\sqrt{481}}$$

$$\tan^2\theta = \frac{625 \times 3}{49}$$

$[\tan^2\theta] = 38$

26. 101

Sol. $L_1, L_3, L_5, - - L_{19}$ are Parallel

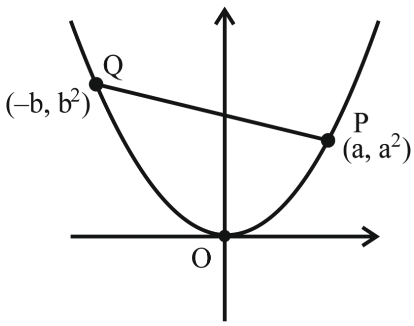
$L_2, L_4, L_6, - - L_{20}$ are Concurrent

Total points of intersection $= {}^{20}C_2 - {}^{10}C_2 - {}^{10}C_2 + 1$

$= 101$

27. 7

Sol.



$$S_2 = 1/2\left|\begin{vmatrix} 0 & 0 & 1 \\ a & a^2 & 1 \\ -b & b^2 & 1 \end{vmatrix}\right| = 1/2(ab^2 + a^2 b)$$

$$PQ : -y - a^2 = \frac{a^2 - b^2}{a + b}(x - a)$$

$y - a^2 = (a - b)x - (a - b)a$

$y = (a - b)\,x + ab$

$$S_1 = \int_{-b}^{a} ((a - b)x + ab - x^2)dx$$

$$= (a - b)\frac{x^2}{2} + (ab)x - \frac{x^3}{3}\Big|_{-b}^{a}$$

$$= \frac{(a - b)^2(a + b)}{2} + ab(a + b) - \frac{(a^3 + b^3)}{3}$$

$$\frac{S_1}{S_2} = \frac{\frac{(a - b)^2}{2} + ab - \frac{(a^2 + b^2 - ab)}{3}}{\frac{ab}{2}}$$

$$= \frac{3(a - b)^2 + 6ab - 2(a^2 + b^2 - ab)}{3ab}$$

$$= \frac{1}{3}\Bigg[\underbrace{\frac{a}{b} + \frac{b}{a}}_{\min = 2} + 2\Bigg]$$

$$= \frac{4}{3} = \frac{m}{n} \qquad m + n = 7$$

28. 8

Sol. $ky^2 = 2(y - x) \qquad 2y^2 = kx$

Point of intersection $\rightarrow$

$$ky^2 = \left(\frac{y - 2y^2}{k}\right)$$

$$y = 0 \qquad k = 2\left(\frac{1 - 2y}{k}\right)$$

$$ky + \frac{4y}{k} = 2$$

$$y = \frac{2}{k + \frac{4}{k}} = \frac{2k}{k^2 + 4}$$

$$A=\int_0^{\frac{2k}{k^2+4}}\left(\left(y-\frac{ky^2}{2}\right)-\left(\frac{2y^2}{k}\right)\right)\cdot dy$$

$$=\frac{y^2}{2}-\left(\frac{k}{2}+\frac{2}{k}\right)\cdot\frac{y^3}{3}\Bigg|_0^{\frac{2k}{k^2+4}}$$

$$=\left(\frac{2k}{k^2+4}\right)^2\left[\frac{1}{2}-\frac{k^2+4}{2k}\times\frac{1}{3}\times\frac{2k}{k^2+4}\right]$$

$$=\frac{1}{6}\times 4\times\left(\frac{1}{k+\frac{4}{k}}\right)^2$$

$$A\cdot M\ge G\cdot M\frac{\left(k+\frac{4}{k}\right)}{2}\ge 2$$

$$k+\frac{4}{k}\ge 4$$

Area if maximum when $k=\frac{4}{k}$

$k=2,-2$

29. 5

Sol. $\frac{dx}{dy}-\frac{x}{y}=\frac{1-y^2}{y}$

Integrating factor $=e^{\int-\frac{1}{y}dy=\frac{1}{y}}$

$$x\cdot\frac{1}{y}=\int\frac{1-y^2}{y^2}dy$$

$$\frac{x}{y}=\frac{-1}{y}-y+c$$

$x=-1-y^2+cy$

$x(1)=1$

$1=-1-1+c\Rightarrow c=3$

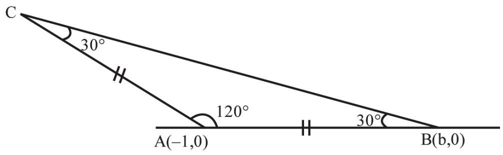
$x=-1-y^2+3y$

$5x(2)=5(-1-4+6)$

$=5$

30. 36

Sol.



$$\frac{c}{\sin 30^o}=\frac{4\sqrt{3}}{\sin 120^o}$$ [By sine rule]

$2c=8\Rightarrow c=4$

$AB=|(b+1)|=4$

$b=3,\ m_{AB}=0$

$$m_{BC}=\frac{-1}{\sqrt{3}}$$

$$BC:-y=\frac{-1}{\sqrt{3}}(x-3)$$

$$\sqrt{3}y+x=3$$

Point of intersection : $y=x+3,\ \sqrt{3}y+x=3$

$$(\sqrt{3}+1)y=6$$

$$y=\frac{6}{\sqrt{3}+1}$$

$$x=\frac{6}{\sqrt{3}+1}3$$

$$=\frac{6-3\sqrt{3}-3}{\sqrt{3}+1}$$

$$=3\frac{(1-\sqrt{3})}{(1+\sqrt{3})}=\frac{-6}{(1+\sqrt{3})^2}$$

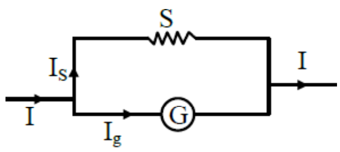
$$\frac{\beta^4}{\alpha^2}=36$$

PHYSICS

Section - A (Single Correct Answer)

31. Bonus

Sol.



$I_S S = I_g G$

$\frac{95}{100} IS = \frac{5I}{100} G$

$S = \frac{G}{19}$

$R_A = \frac{SG}{S+G} = \frac{\frac{G^2}{19}}{\frac{20G}{19}}$

$R_A = \frac{G}{20}$

32. C

Sol. For no deflection $= \frac{0.8}{1} = \frac{R}{3}$

$\Rightarrow$ R = 2.4mΩ

Temperature fall in 10s = 20°C

ΔR = R α Δt

$\alpha = \frac{\Delta R}{R\Delta t} = \frac{-0.6}{3 \times 20}$

$= -10^{-2}C^{-1}$

33. C

Sol. Part of theory

34. D

Sol. $\gamma = 1 + 2/f = 1.4 \Rightarrow 2/f = 0.4$

$\Rightarrow f = 5$

W = n R ΔT = 200J

$Q = \left(\frac{f+2}{2}\right) nR\Delta T$

$= \frac{7}{2} \times 200 = 700J$

35. C

Sol. Only the translational kinetic energy of disc changes into gravitational potential energy. And rotational KE remains unchanged as there is no friction.

$\frac{1}{2}mv^2 = mgh$

$h = \frac{v^2}{2g}$

36. A

Sol. $E_g = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{660 \times 10^{-9}} J$

$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{660 \times 10^{-9} \times 1.6 \times 10^{-19}} eV = \frac{15}{8} eV$

So x = 15

37. D

Sol. Lets say radius of small droplets is r and that of big drop is R

$\frac{4}{3}\pi R^3 = 1000 \frac{4}{3}\pi r^3$

R = 10r

$U_i = 1000\ (4\pi r^2 S)$

$U_f = 4\pi R^2 S$

$= 100\ (4\pi r^2 S)$

$U_f = \frac{1}{10} U_i$

38. C

Sol. $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{60 \times 10^6} = 5m$

39. D

Sol. $F_{av} = \frac{\Delta p}{\Delta t}$

$= \frac{0.12 \times 25}{0.1} = 30N$

40. C

Sol. P = nhν

$n = \frac{P}{h\nu} = \frac{2 \times 10^{-3}}{6.63 \times 10^{-34} \times 6 \times 10^{14}}$

$= 5 \times 10^{15}$

41. C

Sol. For first minima a sin θ = λ

$\sin\theta = \frac{\lambda}{a} = \frac{1}{2}$

$\theta = 30°$

Angular spread = 60°

42. A

Sol. $\phi_{\text{smaller cube}} = \frac{2Q}{\epsilon_0}$

$\phi_{\text{bigger cube}} = \frac{5Q}{\epsilon_0}$

$\frac{\phi_{\text{smaller cube}}}{\phi_{\text{bigger cube}}} = \frac{2}{5}$

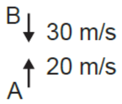
43. B

Sol. $V_{rms} = \sqrt{\dfrac{3RT}{M}}$

$$\frac{V_1}{V_2} = \sqrt{\frac{M_2}{M_1}} \Rightarrow \frac{2}{V_2} = \sqrt{\frac{32}{2}}$$

$V_2 = 0.5$ km/s

44. C

Sol. 

$V_A = 20$ m/s

$V_B = -30$ m/s

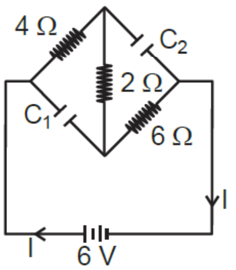
Velocity of B w.r.t. A

$V_{B/A} = -50$ m/s

Velocity of ground w.r.t. B

$V_{G/B} = 30$ m/s

45. D

Sol. 

In steady state

Req = 12Ω

$$I = \frac{6}{12} = 0.5A$$

P.D across C_1 = 3V

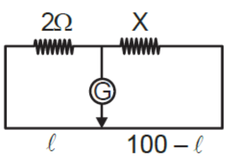
P.D acoross C_2 = 4V

$q_1 = C_1V_1 = 12\ \mu C$

$q_2 = C_2V_2 = 24\ \mu C$

$$\frac{q_1}{q_2} = \frac{1}{2}$$

46. A

Sol. 

First case $\dfrac{2}{40} = \dfrac{X}{60} \Rightarrow X = 3\Omega$

In second case $X' = \dfrac{2\times 3}{2+3} = 1.2\Omega$

$200 - 2l = 1.2l$

$$l = \frac{200}{3.2} = 62.5 \text{ cm}$$

Balance length changes by 22.5 cm

47. C

Sol. Theoretical

48. B

Sol. Efficiency $= \dfrac{E_S I_S}{E_P I_P}$

$$0.8 = \frac{240 I_S}{4000}$$

$$I_S = \frac{3200}{240} = 13.33A$$

49. A

Sol. $F = \dfrac{GMm}{R^{3/2}} = m\omega^2 R$

$\omega^2 \propto \dfrac{1}{R^{5/2}}$ $\quad \because T = \dfrac{2\pi}{\omega}$ so

$T^2 \propto R^{5/2}$

50. C

Sol. Net force $= 8\hat{i} + 4\hat{j} + 4\hat{k}$

$$\vec{a} = \frac{\vec{F}}{m} = 2\hat{i} + \hat{j} + \hat{k}$$

Section - B (Numerical Value Type)

51. 3

Sol. $f_1 = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{k}{9m}}$$

$$\frac{f_1}{f_2} = \sqrt{\frac{9}{1}} = \frac{3}{1}$$

52. 8

Sol. $V = 4\sqrt{x}$

$$a = V\frac{dv}{dx} = 4\sqrt{x} \times 4 \times \frac{1}{2} x^{-1/2} = 8 \text{ m/s}^2$$

53. 4

Sol. τ = BINA sin ϕ

$C\theta$ = BINA sin 90°

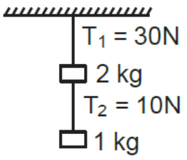
$$C = \frac{BINA}{\theta} = \frac{0.1\times10\times10^{-3}\times100\times2\times10^{-4}}{0.05}$$

$= 4 \times 10^{-5}$ N-m/rad.

x = 4

54. 3

Sol.



$$\Delta L = \frac{FL}{AY}$$

$$\frac{\Delta L}{L} = \frac{F}{AY}$$

$$\frac{\frac{\Delta L_1}{L_1}}{\frac{\Delta L_2}{L_2}} = \frac{F_1}{F_2} = \frac{30}{10} = 3$$

55. 32

Sol. $$E = -13.6z^2\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

$$E = C\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$h\nu = C\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$\frac{v_1}{v_2} = \frac{\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]_{2-1}}{\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]_{3-1}} = \frac{\left[\frac{1}{1} - \frac{1}{4}\right]}{\left[\frac{1}{1} - \frac{1}{9}\right]} = \frac{\frac{3}{4}}{\frac{8}{9}}$$

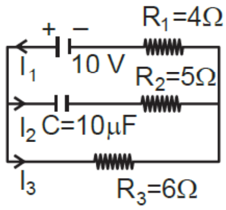
$$= \frac{3}{4}\times\frac{9}{8}$$

$$\frac{v_1}{v_2} = \frac{27}{32}$$

$$v_2 = \frac{32}{27}v_1 = \frac{32}{27}\times3\times10^{15}\,Hz = \frac{32}{9}\times10^{15}\,Hz$$

56. 60

Sol.



In steady state there will be no current in branch of capacitor, so no voltage drop across R_2 = 5Ω

$I_2 = 0$

$$I_1 = I_3 = \frac{10}{4+6} = 1A$$

$V_{R_3} = V_c + V_{R_2}$ $V_{R_2} = 0$

$I_3R_3 = V_c$

$V_c = 1 \times 6 = 6$ volt

$q_c = CV_c = 10 \times 6 = 60\ \mu C$

57. 5

Sol. ϕ = NAB cos(ωt)

$$\varepsilon = -\frac{d\phi}{dt} = NAB\omega\sin(\omega t)$$

ε_{max} = NABω

$$= 200 \times 0.2 \times 0.01 \times \pi = \frac{4\pi}{10} = \frac{2\pi}{5}\text{volt}$$

58. 125

Sol. Let intensity of light on screen due to each slit is I_0

So internity at centre of screen is $4I_0$

Intensity at distance y from centre-

$$I = I_0 + I_0 + 2\sqrt{I_0I_0}\cos\phi$$

$I_{max} = 4I_0$

$$\frac{I_{max}}{2} = 2I_0 = 2I_0 + 2I_0\cos\phi$$

$\cos\phi = 0$

$\phi = \pi/2$

$$K\Delta x = \frac{\pi}{2}$$

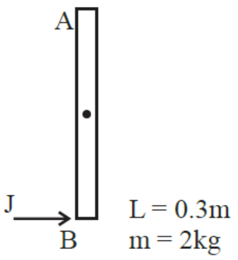
$$\frac{2\pi}{\lambda}d\sin\theta = \frac{\pi}{2}$$

$$\frac{2}{\lambda}d\times\frac{y}{D} = \frac{1}{2}$$

$$y = \frac{\lambda D}{4d} = \frac{5\times10^{-7}\times1}{4\times10^{-3}} = 125\times10^{-6} = 125$$

59. 4

Sol.



Impulse J = 0.2 N-S

$J = \int F dt = 0.2$ N-s

Angular impuls $(\vec{M})$

$$\vec{M}_c = \int \tau dt = \int F\frac{L}{2}dt = \frac{L}{2}\int F dt = \frac{L}{2}\times J$$

$$= \frac{0.3}{2}\times 0.2 = 0.03$$

$$I_{cm} = \frac{ML^2}{12} = \frac{2\times(0.3)^2}{12} = \frac{0.09}{6}$$

$$M = I_{cm}(\omega_f - \omega_i)$$

$$0.03 = \frac{0.09}{6}(\omega_f)$$

$$\omega_f = 2 \text{rad/s}$$

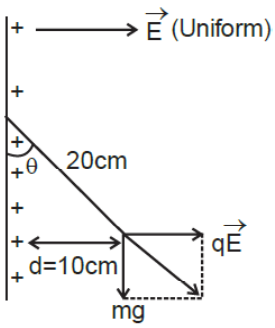
$$\theta = \omega t$$

$$t = \frac{\theta}{\omega} = \frac{\pi}{2\times 2} = \frac{\pi}{4} \text{ sec.}$$

X = 4

60. 3

Sol.



$$\sin\theta = \frac{10}{20} = \frac{1}{2}$$

$$\theta = 30°$$

$$\tan\theta = \frac{qE}{mg}$$

$$\tan 30° = \frac{q\times 2\times 10^4}{1\times 10^{-3}\times 10}$$

$$\frac{1}{\sqrt{3}} = q\times 10^6$$

$$q = \frac{1}{\sqrt{3}}\times 10^{-6} C$$

x = 3

CHEMISTRY

Section - A (Single Correct Answer)

61. **(B)**

Sol. 3rd Ionisation energy : [NCERT Data]

V : 2833 KJ/mol

Cr : 2990 KJ/mol

Mn : 3260 KJ/mol

Fe : 2962 KJ/mol

alternative

Mn : $3d^5\ 4s^2$

Fe : $3d^6\ 4s^2$

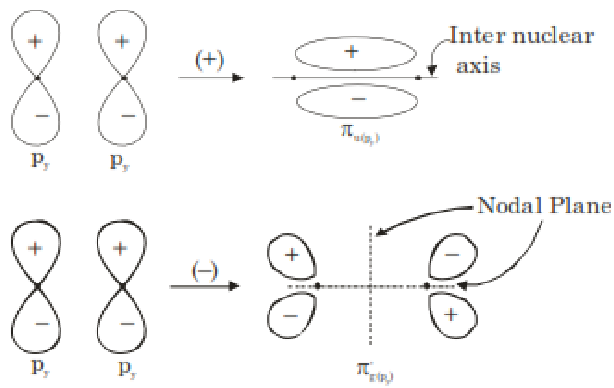
Cr : $3d^5\ 4s^1$

V : $3d^3\ 4s^2$

So Mn has highest 3rd IE among all the given elements due to d^5-configuration.

62. **(C)**

Sol. A π bonding molecular orbital has higher electron density above and below inter nuclear axis.

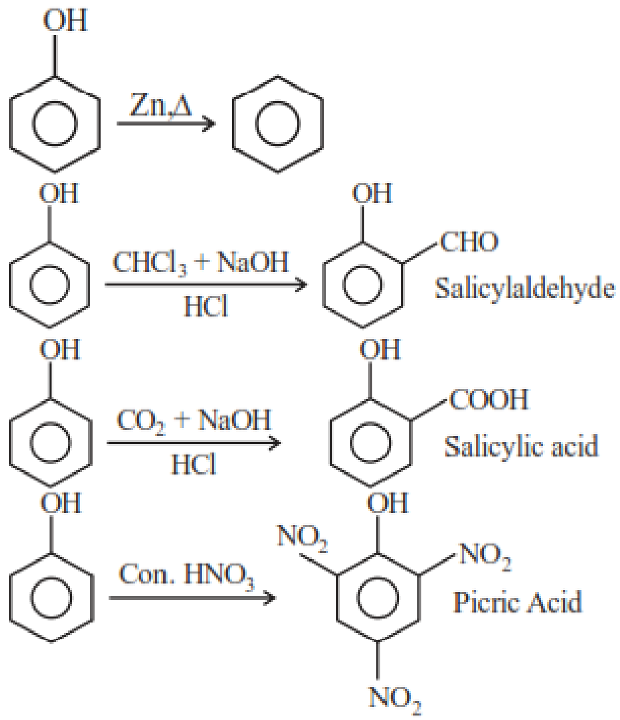


63. **(A)**

Sol. Cr^{2+} is reducing as it configuration changes from d^4 to d^3 due to formation of Cr^{3+}, which has half filled t_{2g} level, on other hand, the change Mn^{3+} to Mn^{2+} result half filled d^5 configuration which has extra stability.

64. **(C)**

Sol.



65. **(B)**

Sol. **I.** In p-Block both metals and non metals are present but in d-Block only metals are present.

II. EN and IE of non-metals are greater than that of metals.

I - False, **II** - True

66. **(C)**

Sol. Strongest reducing agent : BiH_3 explained by its low bond dissociation energy.

67. **(A)**

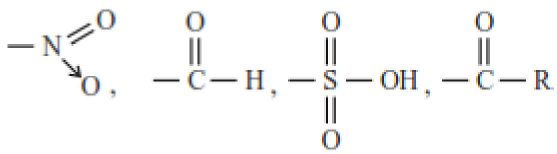
Sol. $CuSO_4 \cdot 5H_2O$

Cu^{2+} : $3d^9\ 4s^0$

Unpaired electron present so it show colour due to d-d transition.

68. **(C)**

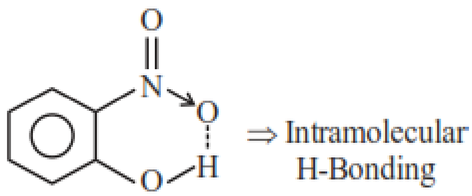
Sol.



All are –M, hence meta directing groups.

69. **(D)**

Sol. H_2O, NH_3, C_2H_5OH ⇒ Intermolecular H-Bonding



70. **(D)**

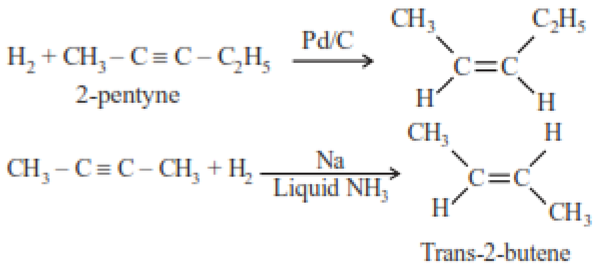
Sol. Lassaigne's test is used for detection of all element N, S, P, X.

71. **(B)**

Sol. Due to H-bonding boiling point of alcohol is High.

72. **(A)**

Sol.



73. **(A)**

Sol. For 3p : n = 3, ℓ = 1

Number of radial node = n – ℓ – 1

= 3 – 1 – 1

= 1

74. **(B)**

Sol. CCl_4 used in fire extinguisher. CH_2Cl_2 used as paint remover. Freons used in refrigerator and AC.

DDT used as non Biodegradable insecticide.

75. **(C)**

Sol. $-\overset{\overset{\displaystyle O}{||}}{C}-OH$ shows –R effect, while rest 3 groups shows +R effect via lone pair.

76. **(B)**

Sol. $[Co(NH_3)_6]^{3+}$

Co^{3+} (strong field ligand) ⇒ $3d^6$ (t_{2g}^6, e_g^0)

Hybridisation : d^2sp^3

Inner obital complex(spin paired complex)

Pairing will take place.

$[CoF_6]^{3-}$

Co^{3+} (weak field ligand) $\Rightarrow$ $3d^6$ (t_{2g}^4, e_g^2)

Hybridisation : sp^3d^2

Outer orbital complex (spin free complex)

no pairing will take place

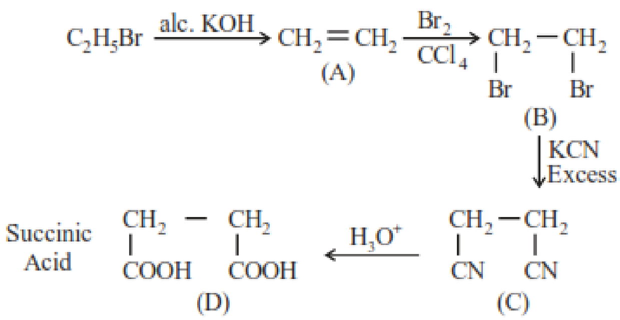
77. **(C)**

Sol. SiO_2 and GeO_2 are acidic and SnO, PbO are amphoteric.

Carbon does not have d-orbitals so can not form $p\pi$-$d\pi$ bond with itself. Due to properties of catenation and $p\pi$-$p\pi$ bond formation. carbon is able to show allotropic forms.

78. **(B)**

Sol.



79. **(B)**

Sol. $S = \frac{W \times 10}{M}$

$$Ca_3(PO_4)_2(s) \rightleftharpoons 3Ca^{2+}(aq.) + 2PO_4^{3-}(aq.)$$

3s 2s

$$S = \frac{W \times 1000}{M \times 100} = \frac{W \times 10}{M}$$

$$K_{sp} = (3s)^3(2s)^2$$

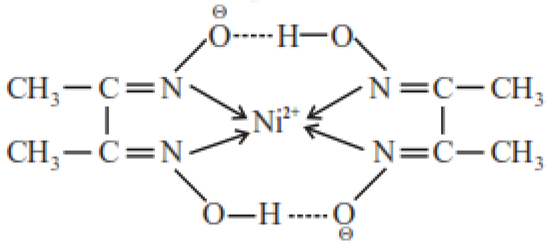
$$= 108\ s^5$$

$$= 108 \times 10^5 \times \left(\frac{W}{M}\right)^5$$

$$= 1.08 \times 10^7 \left(\frac{W}{M}\right)^5$$

80. **(A)**

Sol. $Ni^{2+} + NH_4OH + dmg \rightarrow$



2 Five member ring

III II

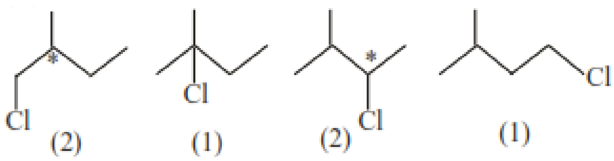
$Fe_4\ [Fe(CN)_6]_3$

Prussian Blue

Section - B (Numerical Value Type)

81. **(6)**

Sol.



82. **(2)**

Sol.

	A(g)	$\rightarrow$	**2B(g)**	+	**C(g)**
t = 0	0.1				
t = 115 sec.	0.1 – x		2x		x

$0.1 + 2x = 0.28$

$2x = 0.18$

$x = 0.09$

$$K = \frac{1}{115} \ell n \frac{0.1}{0.1 - 0.09}$$

$= 0.0200\ sec^{-1}$

$= 2 \times 10^{-2}\ sec^{-1}$

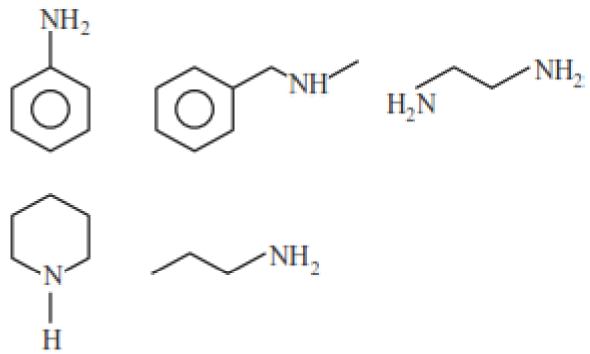
83. **(6)**

Sol. Let 3 different amino acid are A, B, C then following combination of tripeptides can be formed.

ABC, ACB, BAC, BCA, CAB, CBA

84. **(5)**

Sol.



85. **(15)**

Sol. $\Delta T_f = iK_f \times$ molality

$$24 = (1) \times 1.86 \times \frac{W}{62 \times 18.6}$$

W $= 14880$ gm

$= 14.880$ kg

86. **(56)**

Sol. $H_2SO_4 + 2NH_3 \rightarrow (NH_4)_2\, SO_4$

Millimole of $H_2SO_4 \rightarrow 10 \times 2$

So Millimole of $NH_3 = 20 \times 2 = 40$

Organic Compound $\rightarrow$ NH^3 40 Millimole

$$\therefore \text{ Mole of N} = \frac{40}{1000}$$

$$\text{wt. of N} = \frac{40}{1000} \times 14$$

% composition of N in organic compound

$$= \frac{40 \times 14}{1000 \times 1} \times 100$$

$= 56\%$

87. **(2)**

Sol. $2H_2O \rightarrow O_2 + 4H^+ + 4e^-$

$$\frac{W}{E} = \frac{Q}{96500}$$

$$\text{mole} \times \text{n-factor} = \frac{Q}{96500}$$

$$1 \times 2 = \frac{Q}{96500}$$

Q $= 2 \times 96500$ C

$= 1.93 \times 10^5$ C

88. **(57)**

Sol. $\Delta G° = -RT \text{ in } K$

$= -8.314 \times 300\ \ell n\,(10)$

$= 5744.14$ J/mole

$= 57.44 \times 10^{-1}$ kJ/mole

89. **(338 or 339)**

Sol. Cell Rx^n ; $MnO_4^- + H_2C_2O_4 \rightarrow Mn^{2+} + CO_2$

$$E^o_{cell} = E^o_{op} \text{ of anode} + E^o_{RP} \text{ of cathode}$$

$= 0.49 + 1.51 = 2.00$ V

At equilibrium,

$E_{cell} = 0,$

$$E^o_{cell} = \frac{0.059}{n} \log K$$

(As per NCERT $\frac{RT}{F} = 0.059$ But $\frac{RT}{F} = 0.0591$ can also be taken)

$$2 = \frac{0.059}{10} \log K$$

$\log K = 338.98$

90. **(14)**

Sol. $\boxed{\begin{matrix} CxHy \\ 10 \text{ ml} \end{matrix}} + O_2 \rightarrow CO_2 + H_2O$

$$CxHy + \left(x + \frac{y}{4}\right) O_2 \rightarrow xCO_2 + \frac{y}{2} H_2O$$

10x 5y

$10x = 40$

$x = 4$

$5y = 50$

$y = 10$

C_4H_{10}

• • •