27-Jan.-2024 (Morning) : PCM [SOLUTIONS]

MATHEMATICS

Section - A (Single Correct Answer)

1. A

Sol. ${}^{n-1}C_r = (k^2 - 8)\, {}^{n}C_{r+1}$

$\underbrace{r+1 \geq 0,\ r \geq 0}_{r \geq 0}$

$\frac{{}^{n-1}C_1}{{}^{n}C_{r+1}} = k^2 - 8$

$\frac{r+1}{n} = k^2 - 8$

$\Rightarrow k^2 - 8 > 0$

$(k - 2\sqrt{2})(k + 2\sqrt{2}) > 0$

$k \in (-\infty,\ -2\sqrt{2}) \cup (2\sqrt{2},\ \infty)$ ....(I)

$\therefore n \geq r+1,\ \frac{r+1}{n} \leq 1$

$\Rightarrow k^2 - 8 \leq 1$

$k^2 - 9 \leq 0$

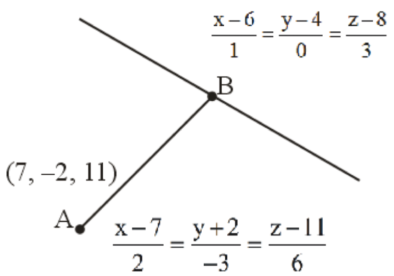
$-3 \leq k \leq 3$ ....(II)

From equation (I) and (II) we get

$k \in [-3,\ -2\sqrt{2}) \cup (2\sqrt{2},\ 3]$

2. B

Sol. $B = (2\lambda + 7, -3k - 2, 6\lambda + 11)$



Point B lies on $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$

$\frac{2\lambda+7-6}{1} = \frac{-3\lambda-2-4}{0} = \frac{6\lambda+11-8}{3}$

$-3\lambda - 6 = 0$

$\lambda = -2$

$B \Rightarrow (3,\ 4,\ -1)$

$AB = \sqrt{(7-3)^2 + (4+2)^2 + (11+1)^2}$

$= \sqrt{16 + 36 + 144}$

$= \sqrt{196} = 14$

3. D

Sol. $\frac{dx}{dt} + ax = 0$

$\frac{dx}{x} = -adt$

$\int \frac{dx}{x} = -a\int dt$

$\ln |x| = -at + c$

at $t = 0$, $x = 2$

$\ln 2 = 0 + c$

$\ln x = -at + \ln 2$

$\frac{x}{2} = e^{-at}$

$x = 2e^{-at}$ ....(i)

$\frac{dy}{dt} + by = 0$

$\frac{dy}{y} = -bdt$

$\ln |y| = bt + \lambda$

$t = 0$, $y = 1$

$0 = 0 + \lambda$

$y = e^{-bt}$ ....(ii)

According to question

$3y(1) = 2x(1)$

$3e^{-b} = 2(2\, e^{-a})$

$$e^{a-b} = \frac{4}{3}$$

For x(t) = y(t)

$$\Rightarrow 2e^{-at} = e^{-bt}$$

$2 = e^{(a-b)t}$

$$2 = \left(\frac{4}{3}\right)^t$$

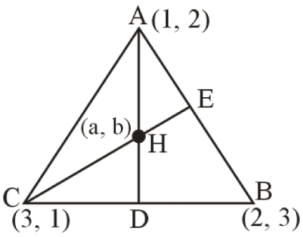
$$\log_{\frac{4}{3}} 2 = t$$

4. A

Sol. Equation of CE

$y - 1 = -(x - 3)$

$x + y = 4$



orthocentre lies on the line x + y = 4

so, a + b = 4

$$I_1 = \int_a^b x\sin(x(4-x))dx \quad ....(i)$$

Using king rule

$$I_1 = \int_a^b (4-x)\sin(x(4-x))dx \quad ....(ii)$$

(i) + (ii)

$$2I_1 = \int_a^b 4\sin(x(4-x))dx$$

$2I_1 = 4I_2$

$I_1 = 2I_2$

$$\frac{I_1}{I_2} = 2$$

$$\frac{36I_1}{I_2} = 72$$

5. A

Sol. Sum of coefficients in the expansion of $(1 - 3x + 10x^2)^n = A$

then $A = (1 - 3 + 10)^n = 8^n$ (put x = 1)

and sum of coefficients in the expansion of $(1 + x^2)^n = B$ then $B = (1 + 1)^n = 2^n$

$A = B^3$

6. C

Sol. 4, 9, 14, 19, ..., up to 25th term

$T_{25} = 4 + (25 - 1)\,5 = 4 + 120 = 124$

3, 6, 9, 12, ., up to 37th term

$T_{37} = 3 + (37 - 1)3 = 3 + 108 = 111$

Common difference of Ist series $d_1 = 5$

Common difference of IInd series $d_2 = 3$

First common term = 9, and

their common difference = 15 (LCM of d_1 & d_2)

then common terms are

9, 24, 39, 54, 69, 84, 99

7. C

Sol. Equation of normal to parabola

$y = mx - 2m - m^3$

this normal passing through center of circle (2, 8)

$8 = 2m - 2m - m^3$

$m = -2$

So point P on parabola $\Rightarrow (am^2, -2am) = (4, 4)$

And C = (2, 8)

$$PC = \sqrt{4+16} = \sqrt{20}$$

$d^2 = 20$

8. B

Sol. $$\frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3}$$

$$\frac{x-\lambda}{2} = \frac{y+1}{4} = \frac{z-2}{-5}$$

the shortest distance between the lines

$$= \left|\frac{(\vec{a}-\vec{b})\cdot(\vec{d}_1 \times \vec{d}_2)}{|\vec{d}_1 \times \vec{d}_2|}\right|$$

$$= \left|\frac{\begin{vmatrix} \lambda-4 & 0 & 2 \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}}\right|$$

$$= \left| \frac{(\lambda - 4)(-10+12) - 0 + 2(4-4)}{|2\hat{i} - 1\hat{j} + 0\hat{k}|} \right|$$

$$\frac{6}{\sqrt{5}} = \left| \frac{2(\lambda - 4)}{\sqrt{5}} \right|$$

$$3 = |\lambda - 4|$$

$$\lambda - 4 = \pm 3$$

$$\lambda = 7,\ 1$$

Sum of all possible values of λ is = 8

9. D

Sol. $\int_0^1 \frac{1}{\sqrt{3+x} + \sqrt{1+x}} dx = \int_0^1 \frac{\sqrt{3+x} - \sqrt{1+x}}{(3+x) - (1+x)} dx$

$$\frac{1}{2}\left[\int_0^1 \sqrt{3+x}\ dx - \int_0^1 (\sqrt{1+x}) dx \right]$$

$$\frac{1}{2}\left[2\frac{(3+x)^{\frac{3}{2}}}{3} - \frac{2(1+x)^{\frac{3}{2}}}{3} \right]_0^1$$

$$\frac{1}{2}\left[\frac{2}{3}(8 - 3\sqrt{3}) - \frac{2}{3}\left(2^{\frac{3}{2}} - 1\right) \right]$$

$$\frac{1}{3}[8 - 3\sqrt{3} - 2\sqrt{2} + 1]$$

$$= 3 - \sqrt{3} - \frac{2}{3}\sqrt{2} = a + b\sqrt{2} + c\sqrt{3}$$

$$a = 3,\ b = -\frac{2}{3},\ c = -1$$

2a + 3b – 4c = 6 – 2 + 4 = 8

10. D

Sol. Let S = {1, 2, 3, ..., 10}

$R = \{(A,\ B) : A \cap B \neq \phi;\ A, B \in M\}$

For Reflexive,

M is subset of 'S'

So $\phi \in M$

for $\phi \cap \phi = \phi$

$\Rightarrow$ but relation is $A \cap B \neq \phi$

So it is not reflexive.

For symmetric,

ARB $\quad A \cap B \neq \phi$,

$\Rightarrow$ BRA $\quad \Rightarrow B \cap A \neq \phi$,

So it is symmetric.

For transitive,

If A = {(1, 2), (2, 3)}

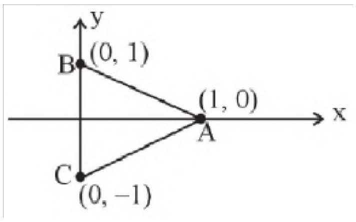
B = {(2, 3), (3, 4)}

C = {(3, 4), (5, 6)}

ARB & BRC but A does not relate to C So it not transitive

11. A

Sol. |z – i| = |z + i| = |z – 1|

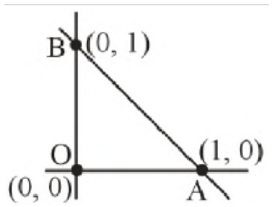


ABC is a triangle. Hence its circum-centre will be the only point whose distance from A, B, C will be same.

So n(S) = 1

12. C

Sol. (2k, 3k) will lie on circle whose diameter is AB.



(x – 1) (x) + (y – 1) (y) = 0

$x^2 + y^2 - x - y = 0$ .....(i)

Satisfy (2k, 3k) in (i)

$(2k)^2 + (3k)^2 - 2k - 3k = 0$

$13k^2 - 5k = 0$

$$k = 0,\ k = \frac{5}{13}$$

hence $k = \frac{5}{13}$

13. D

Sol. $f(3^-) = \frac{a}{b} \frac{(7x - 12 - x^2)}{|x^2 - 7x + 12|}$ (for f(x) to be cont.)

$$\Rightarrow f(3^-) = \frac{-a}{b}\frac{(x-3)(x-4)}{(x-3)(x-4)};\ x<3 \Rightarrow \frac{-a}{b}$$

Hence $f(3^-) = \frac{-a}{b}$

Then $f(3^+) = 2^{\lim_{x\to 3^+}\left(\frac{\sin(x-3)}{x-3}\right)} = 2$ and

$f(3) = b$.

Hence $f(3) = f(3^+) = f(3^-)$

$$\Rightarrow b = 2 = -\frac{a}{b}$$

$b = 2,\ a = -4$

Hence only 1 ordered pair $(-4, 2)$

14. B

Sol. $\sum_{k=1}^{10} a_k = 50$

$a_1 + a_2 + \ldots\ldots + a_{10} = 50$ ......(i)

$\sum_{\forall x<j} a_k a_j = 1100$ .....(ii)

If $a_1 + a_2 + \ldots\ldots. + a_{10} = 50$.

$(a_1 + a_2 + \ldots\ldots\ldots + a_{10}) = 2500$

$$\Rightarrow \sum_{i=1}^{10} a_i^2 + 2\sum_{k<j} a_k a_j = 2500$$

$$\Rightarrow \sum_{i=1}^{10} a_i^2 = 2500 - 2(1100)$$

$\sum_{i=1}^{10} a_i^2 = 300$, Standard deviation '$\sigma$'

$$= \sqrt{\frac{\sum a_i^2}{10} - \left(\frac{\sum a_i}{10}\right)^2} = \sqrt{\frac{300}{10} - \left(\frac{50}{10}\right)^2}$$

$$= \sqrt{30-25} = \sqrt{5}$$

15. A

Equation of chord with given middle point.

$T = S_1$

$$\frac{x}{25} + \frac{y}{40} = \frac{1}{25} + \frac{1}{100}$$

$$\frac{8x+5y}{200} = \frac{8+2}{200}$$

$$y = \frac{10-8x}{5} \qquad \text{....(i)}$$

$\frac{x^2}{25} + \frac{(10-8x)^2}{400} = 1$ (put in original equation)

$$\frac{16x^2 + 100 + 64x^2 - 160x}{400} = 1$$

$4x^2 - 8x - 15 = 0$

$$x = \frac{8 \pm \sqrt{304}}{8}$$

$$x_1 = \frac{8+\sqrt{304}}{8};\ x_2 = \frac{8-\sqrt{304}}{8}$$

Similarly, $y = \frac{10 - 18 \pm \sqrt{304}}{5} = \frac{2 \pm \sqrt{304}}{5}$

$$y_1 = \frac{2-\sqrt{304}}{5};\ y_2 = \frac{2+\sqrt{304}}{5}$$

Distance $= \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$

$$= \sqrt{\frac{4\times 304}{64} + \frac{4\times 304}{25}} = \frac{\sqrt{1691}}{5}$$

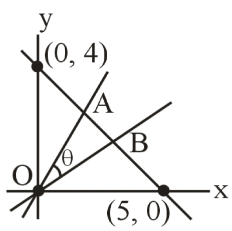
16. D

Co-ordinates of $A = \left(\frac{5}{3}, \frac{8}{3}\right)$

Co-ordinate of $B = \left(\frac{10}{3}, \frac{4}{3}\right)$

Slope of $OA = m_1 = \frac{8}{5}$

Slope of $OB = m_2 = \frac{2}{5}$



$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \frac{\frac{6}{5}}{1+\frac{16}{25}} = \frac{30}{41}$$

$$\tan\theta = \frac{30}{41}$$

17. B

Sol. $\vec{a}\cdot[(\vec{c}\times\vec{b})-\vec{b}-\vec{c}]$

$\vec{a}\cdot(\vec{c}\times\vec{b})-\vec{a}\cdot\vec{b}-\vec{a}\cdot\vec{c}$ .....(i)

given $\vec{a}\times\vec{c}=\vec{b}$

$\Rightarrow (\vec{a}\times\vec{c})\cdot\vec{b}=\vec{b}\cdot\vec{b}=|\vec{b}|^2=27$

$\Rightarrow \vec{a}\cdot(\vec{c}\times\vec{b})=[\vec{a}\ \vec{c}\ \vec{b}]=(\vec{a}\times\vec{c})\cdot\vec{b}=27$ ....(ii)

Now $\vec{a}\cdot\vec{b}=3-6+3=0$ ....(iii)

$\vec{a}\cdot\vec{c}=3$ ....(iv) (given)

By (i), (ii), (iii) and (iv)

27 – 0 – 3 = 24

18. B

Sol. $$a=\lim_{x\to0}\frac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4}$$

$$=\lim_{x\to0}\frac{\sqrt{1+x^4}-1}{x^4\left(\sqrt{1+\sqrt{1+x^4}}+\sqrt{2}\right)}$$

$$=\lim_{x\to0}\frac{x^4}{x^4\left(\sqrt{1+\sqrt{1+x^4}}+\sqrt{2}\right)\left(\sqrt{1+x^4}+1\right)}$$

Applying limit $a=\frac{1}{4\sqrt{2}}$

$$b=\lim_{x\to0}\frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$$

$$=\lim_{x\to0}\frac{(1+\cos x)(\sqrt{2}+\sqrt{1+\cos x})}{2-(1+\cos x)}$$

$$b=\lim_{x\to0}(1+\cos x)(\sqrt{2}+\sqrt{1+\cos x})$$

Applying limits $b=2(\sqrt{2}+\sqrt{2})=4\sqrt{2}$

Now, $ab^3=\frac{1}{4\sqrt{2}}\times(4\sqrt{2})^3=32$

19. D

Sol. $$f(-x)=\begin{bmatrix}\cos x & \sin x & 0\\ -\sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$f(x)\cdot f(-x)=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$$

Hence statement - I is correct

Now, checking statement - II

$$f(y)=\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$f(x)\cdot f(y)=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}$$

$\Rightarrow f(x)\cdot f(y)=f(x+y)$

Hence statement - II is also correct.

20. D

Sol. $f:N\{1\}\to N$

f(n) = The highest prime factor of n.

f(2) = 2

f(4) = 2

$\Rightarrow$ many one

4 is nt image of any element

$\Rightarrow$ into

Hence many one and into

Neither one-one nor onto.

<u>Section - B (Numerical Value Type)</u>

21. 5

Sol. $$\cos\theta=\frac{(\alpha\hat{i}-2\hat{j}+2\hat{k})\cdot(\alpha\hat{i}+2\alpha\hat{j}-2\hat{k})}{\sqrt{\alpha^2+4+4}\sqrt{\alpha^2+4\alpha^2+4}}$$

$$\cos\theta = \frac{\alpha^2 - 4\alpha - 4}{\sqrt{\alpha^2+8}\sqrt{5\alpha^2+4}}$$

$$\Rightarrow \alpha^2 - 4\alpha - 4 > 0$$

$$\Rightarrow \alpha^2 - 4\alpha + 4 > 8$$

$$\Rightarrow (\alpha - 2)^2 > 8$$

$$\Rightarrow \alpha - 2 > 2\sqrt{2} \text{ or } \alpha - 2 < -2\sqrt{2}$$

$$\alpha > 2 + 2\sqrt{2} \text{ or } \alpha < 2 - 2\sqrt{2}$$

$$\alpha \in (-\infty, \ -0.82) \cup (4.82, \ \infty)$$

Least positive integral value of $\alpha \Rightarrow 5$

22. 2890

Sol. $f(x) - f(y) \geq \ln x - \ln y + x - y$

$$\frac{f(x)-f(y)}{x-y} \geq \frac{\ln x - \ln y}{x-y} + 1$$

Let x > y

$$\lim_{y \to x} f'(x^-) \geq \frac{1}{x} + 1 \qquad .....(1)$$

Let x < y

$$\lim_{y \to x} f'(x) \leq \frac{1}{x} + 1 \qquad .....(2)$$

$f^1(x^-) = f^1(x^+)$

$$f'(x) = \frac{1}{x} + 1$$

$$f'\left(\frac{1}{x^2}\right) = x^2 + 1$$

$$\sum_{x=1}^{20}(x^2+1) = \sum_{x=1}^{20} x^2 + 20$$

$$= \frac{20 \times 21 \times 41}{6} + 20$$

$= 2890$

23. 29

Sol. $2x + 3y - 2 = 1 \qquad 4x + 6y - 4 = 2t$

$$2 + \frac{dy}{dx} = \frac{dt}{dx} \qquad 4x + 6y - 7 = 2t - 3$$

$$\frac{dy}{dx} = \frac{-(2x+3y-2)}{4x+6y-7}$$

$$\frac{dt}{dx} = \frac{-3t+4t-6}{2t-3} = \frac{t-6}{2t-3}$$

$$\int \frac{2t-3}{t-6} dt = \int dx$$

$$\int \left(\frac{2t-12}{t-6} + \frac{9}{t-6}\right) \cdot dt = x$$

$2t + 9 \ln (t - 6) = x + c$

$2(2x + 3y - 2) + 9\ln(2x + 3y - 8) = x + c$

$x = 0, y = 3$

$c = 14$

$4x + 6y - 4 + 9\ln (2x + 3y - 8) = x + 14$

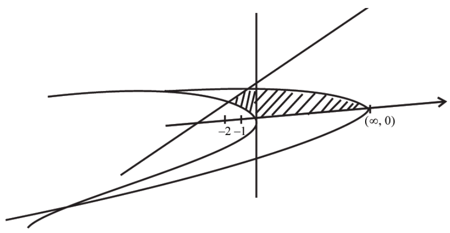
$x + 2y + 3 \ln (2x + 3y - 8) = 6$

$\alpha = 1, \beta = 2, \gamma = 8$

$\alpha + 2\beta + 3\gamma = 1 + 4 + 24 = 29$

24. 119

Sol.



$$A = \int_0^1 [(8-4y^2) - (-2y^2)]dy +$$

$$\int_1^{3/2} [(8-4y^2) - (2y-4)]dy$$

$$= \left[8y - \frac{2y^3}{3}\right]_0^1 \left[12y - y^2 - \frac{4y^3}{3}\right]_1^{3/2} = \frac{107}{12} = \frac{m}{n}$$

$$\therefore m + n = 119$$

25. 9

Sol. $8 = \dfrac{3}{1-\frac{1}{4}} + \dfrac{p \cdot \frac{1}{4}}{\left(1-\frac{1}{4}\right)^2}$

(sum of infinite terms of A.G.P $= \dfrac{a}{1-r} + \dfrac{dr}{(1-r)^2}$)

$$\Rightarrow \frac{4p}{9} = 4 \Rightarrow p = 9$$

26. 12

Sol. $a = P(X=3) = \frac{5}{6}\times\frac{5}{6}\times\frac{1}{6} = \frac{25}{216}$

$$b = P(X \ge 3) = \frac{5}{6}\times\frac{5}{6}\times\frac{1}{6} + \left(\frac{5}{6}\right)^3\cdot\frac{1}{6} + \left(\frac{5}{6}\right)^4\cdot\frac{1}{6} + ...$$

$$= \frac{\frac{25}{216}}{1-\frac{5}{6}} = \frac{25}{216}\times\frac{6}{1} = \frac{25}{36}$$

$$P(X \ge 6) = \left(\frac{5}{6}\right)^5\cdot\frac{1}{6} + \left(\frac{5}{6}\right)^6\cdot\frac{1}{6} + ...$$

$$= \frac{\left(\frac{5}{6}\right)^5\cdot\frac{1}{6}}{1-\frac{5}{6}} = \left(\frac{5}{6}\right)^5$$

$$c = \frac{\left(\frac{5}{6}\right)^5}{\left(\frac{5}{6}\right)^3} = \frac{25}{36}$$

$$\frac{b+c}{a} = \frac{\left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^2}{\left(\frac{5}{6}\right)^2\cdot\frac{1}{6}} = 12$$

27. 48

Sol. $\cos 2x + a \sin x = 2a - 7$

$a(\sin x - 2) = 2(\sin x - 2)(\sin x + 2)$

$\sin x = 2,\ a = 2(\sin x + 2)$

$\Rightarrow a \in [2,\ 6]$

$p = 2 \quad q = 6$

$r = \tan 9^\circ + \cot 9^\circ - \tan 27 - \cot 27$

$$r = \frac{1}{\sin 9\cdot\cos 9} - \frac{1}{\sin 27\cdot\cos 27}$$

$$= 2\left[\frac{4}{\sqrt{5}-1} - \frac{4}{\sqrt{5}+1}\right]$$

$r = 4$

$p\cdot q\cdot r = 2\times 6\times 4 = 48$

28. 202

Sol. $f(x) = x^3 + x^2\cdot f'(1) + x\cdot f''(2) + f'''(3)$

$f'(x) = 3x^2 + 2xf'(1) + f''(2)$

$f''(x) = 6x + 2f'(1)$

$f'''(x) = 6$

$f(1) = -5,\ f''(2) = 2,\ f'''(3) = 6$

$f(x) = x^3 + x^2\cdot(-5) + x\cdot(2) + 6$

$f(x) = 3x^2 - 10x + 2$

$f(10) = 300 - 100 + 2 = 202$

29. 28

Sol. $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ $\qquad B = [B_1, B_2, B_3]$

$$B_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix},\ B_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix},\ B_3 = \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix}$$

$$AB_1 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = 1,\ y_1 = -1,\ z_1 = -1$

$$AB_2 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

$x_3 = 2,\ y_3 = 0,\ z_3 = -1$

$$B = \begin{bmatrix} 1 & 2 & 2 \\ -1 & 1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$\alpha = |B| = 3$

$\beta = 1$

$\alpha^3 + \beta^3 = 27 + 1 = 28$

30. 5

Sol. $x^2 + x + 1 = 0 \Rightarrow x = \omega,\ \omega^2 = \alpha$

Let $\alpha = \omega$

Now $(1+\alpha)^7 = -\omega^{14} = -\omega^2 = 1+\omega$

$A = 1, B = 1, C = 0$

$\therefore 5(3A - 2B - C) = 5(3 - 2 - 0) = 5$

PHYSICS

Section - A (Single Correct Answer)

31. D

Sol. $\vec{v} = \frac{d\vec{s}}{dt} = 4t\hat{j}$

At t = 1 sec $\vec{v} = 4\hat{j}$

32. B

Sol. Gases have less viscosity.

Due to insoluble impurities like detergent surface tension decreases

33. A

Sol. $\cot\frac{A}{2} = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\frac{A}{2}}$

$\Rightarrow \cot\frac{A}{2} = \sin\left(\frac{A+\delta_{min}}{2}\right)$

$\frac{A+\delta_{min}}{2} = \frac{\pi}{2} - \frac{A}{2}$

$\delta_{min} = \pi - 2A$

34. C

Sol. Net force on particle must be zero i.e.

$q\vec{E} + q\vec{V}\times\vec{B} = 0$

Possible cases are

(i) $\vec{E}$ & $\vec{B} = 0$

(ii) $\vec{V}\times\vec{B} = 0,\ \vec{E} = 0$

(iii) $q\vec{E} = -q\vec{V}\times\vec{B}$

$\vec{E} \neq 0$ & $\vec{B} \neq 0$

35. D

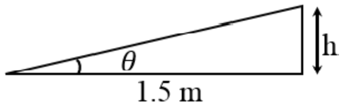
Sol. $g = \frac{GM}{R^2} \Rightarrow g \propto \frac{1}{R^2}$

$\frac{g_2}{g_1} = \frac{R_1^2}{R_2^2}$

$g_2 = 4g_1 \left(R_2 = \frac{R_1}{2}\right)$

36. B

Sol. $\tan\theta = \frac{v^2}{Rg} = \frac{12\times12}{10\times400}$



$\tan\theta = \frac{h}{1.5}$

$\Rightarrow \frac{h}{1.5} = \frac{144}{4000}$

h = 5.4 cm

37. D

Sol. P end should be at higher potential for forward biasing.

38. B

Sol. Spherometer can be used to measure curvature of surface.

39. C

Sol. $\frac{P_1^2}{2m_1} = \frac{P_2^2}{2m_2}$

$\frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \frac{2}{5}$

40. C

Sol. Q = ΔU as work done is zero [constant volume]

ΔU= ms ΔT

$= 0.08 \times (170 \times 4.18) \times 5 \simeq 284$ J

41. B

Sol. $r \propto \frac{n^2}{Z}$

$\frac{r_4}{r_3} = \frac{4^2}{3^2}$

$r_4 = \frac{16}{9}R$

42. C

Sol. Average emf = $\frac{\text{Change in flux}}{\text{Time}}$

$= -\frac{\Delta\phi}{\Delta t}$

$= -\frac{0-\left(4\times(2.5\times2)\cos60°\right)}{10}$

$= +1V$

43. C

Sol. Potential difference $= \frac{KQ}{r_1} - \frac{KQ}{r_2}$

$$r_1 = \sqrt{(\sqrt{3})^2 + (\sqrt{3})^2}$$

$$r_2 = \sqrt{(\sqrt{6})^2 + 0}$$

As $r_1 = r_2 = \sqrt{6}$ m

So potential difference = 0

44. D

Sol. As amount of energy incident on cell is same so current will remain same.

45. D

Sol. Momentum will remain conserve

1000 × 6 = 1200 × v

v = 5 m/s

46. B

Sol. $I = \frac{1}{2}\varepsilon_0 E_0^2 \times c$

$$I = \frac{1}{2} \times 8.85 \times 10^{-12} \times 4 \times 10^4 \times 3 \times 10^8$$

I = 53.1 W/m^2

47. A

Sol. [h] = ML^2T^{-1}

[L] = ML^2T^{-1}

[P] = MLT^{-1}

[τ] = ML^2T^{-2}

(Here h is Planck's constant, L is angular momentum, P is linear momentum and τ is moment of force)

48. C

Sol. For null point,

$$\frac{4.5}{60} = \frac{R}{40}$$

Also, $R = \frac{\rho l}{A} = \frac{\rho l}{\pi r^2}$

$$4.5 \times 40 = \rho \times \frac{0.1}{\pi \times 7 \times 10^{-8}} \times 60$$

$$\rho = 66 \times 10^{-7} \Omega \times m$$

49. A

Sol. Resistance of each part = R/5

Total resistance $= \frac{1}{5} \times \frac{R}{5} = \frac{R}{25}$

50. B

Sol. For monoatomic molecule degree of freedom = 3.

$$\therefore K_{avg} = \frac{3}{2} K_B T$$

$$T = \frac{0.414 \times 1.6 \times 10^{-19} \times 2}{3 \times 1.38 \times 10^{-23}}$$

= 3200 K

Section - B (Numerical Value Type)

51. 673

Sol. u_x = 5 m/s

a_x = 3 m/s^2

x = 84 m

$$v_x^2 - u_x^2 = 2ax$$

$$v_x^2 - 25 = 2(3)(84)$$

V_x = 23 m/s

$$t = \frac{23 - 5}{3} = 6s$$

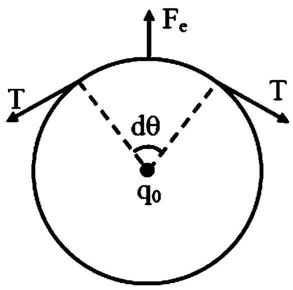
$v_y = 0 + a_y t = 0 + 2 \times (6) = 12$ m/s

$$v^2 = v_x^2 + v_y^2 = 23^2 + 12^2 = 673$$

$$v = \sqrt{673} \text{ m/s}$$

52. 3

Sol.



$$2T\sin\frac{d\theta}{2} = \frac{kq_0}{R^2} \cdot \lambda R d\theta$$

$$\left[\lambda = \frac{Q}{2\pi R}\right]$$

$$\Rightarrow T = \frac{Kq_0 Q}{(R^2) \times 2\pi}$$

$$= \frac{(9 \times 10^9)(2\pi \times 30 \times 10^{-12})}{(0.30)^2 \times 2\pi}$$

$$= \frac{9 \times 10^{-3} \times 30}{9 \times 10^{-2}} = 3N$$

53. 2

Sol. $\phi = Mi = Mi_0 \sin\omega t$

$$EMF = -M\frac{di}{dt} = -0.002(i_0\omega\cos\omega t)$$

$$EMF_{max} = i_0\omega(0.002) = (5)(50\pi)(0.002)$$

$$EMF_{max} = \frac{\pi}{2}V$$

54. 31

Sol. $h_{app} = \frac{h_1}{\mu_1} + \frac{h_2}{\mu_2} = \frac{6}{3/2} + \frac{6}{8/5} = 4 + \frac{15}{4} = \frac{31}{4}cm$

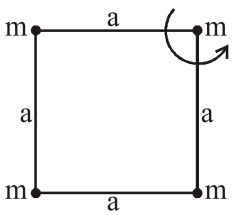
55. 236

Sol. $Q = BE_{Product} - BE_{Rectant}$

$= 2(118)(8.6) - 236(7.6) = 236 \times 1 = 236$ MeV

56. 16

Sol.



$$I = ma^2 + ma^2 + m(\sqrt{2}a)^2$$

$= 4ma^2$

$= 4 \times 1 \times (2)^2 = 16$

57. 12

Sol. $V_{at\ mean\ position} = A\omega \Rightarrow 10 = 4\omega$

$$\omega = \frac{5}{2}$$

$$v = \omega\sqrt{A^2 - x^2}$$

$$5 = \frac{5}{2}\sqrt{4^2 - x^2} \Rightarrow x^2 = 16 - 4$$

$$x = \sqrt{12}\ cm$$

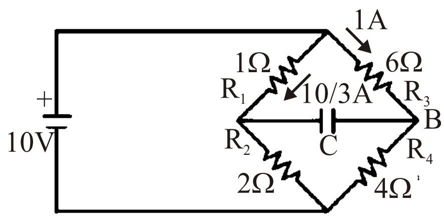
58. 160

Sol. $B = \left(\frac{\mu_0 i}{2\pi a}\right) \times 2 = \frac{4\pi \times 10^{-7} \times 10}{\pi \times \left(\frac{5}{2} \times 10^{-2}\right)}$

$= 16 \times 10^{-5} = 160\ \mu T$

59. 400

Sol.



$$V_A + \frac{10}{3}(1) - 6(1) = V_B$$

$$V_A - V_B = 6 - \frac{10}{3} = \frac{8}{3}volt$$

$$Q = C(V_A - V_B)$$

$$= 150 \times \frac{8}{3} = 400\mu C$$

60. 2

Sol. $B = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)}$

$$-\left(\frac{\Delta V}{V}\right) = \frac{\rho gh}{B} = \frac{1000 \times 10 \times 4000}{2 \times 10^9}$$

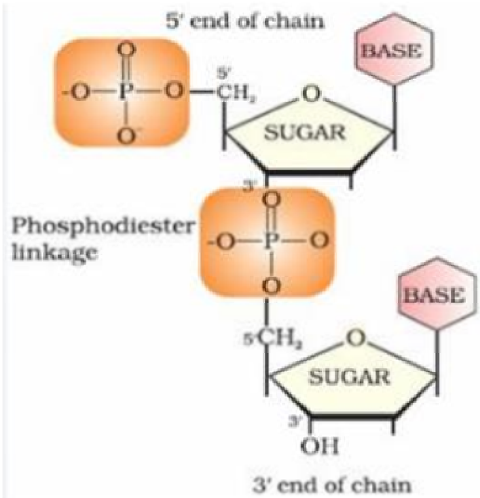
$= 2 \times 10^{-2}$ [– ve sign represent compression]

CHEMISTRY

Section - A (Single Correct Answer)

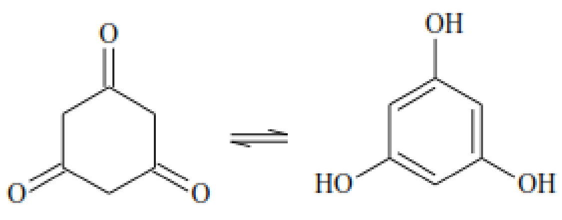
61. **(A)**

Sol. Phosphodiester linkage



62. **(B)**

Sol.



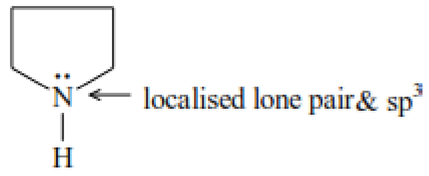
Aromatic

63. **(D)**

Sol. Fluorine does not show variable oxidation state.

64. **(D)**

Sol.



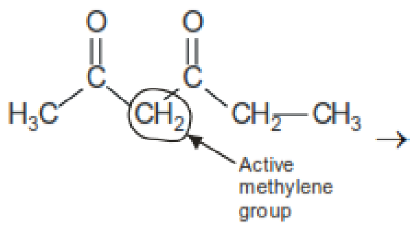
65. **(D)**

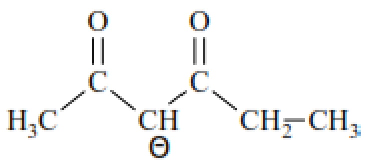
Sol.

	$3d^7$	$3d^8$	$3d^3$	$3d^6$
No. of unpaired e^-	3	2	3	4
Spin only magnetic moment	$\sqrt{15}$ BM	$\sqrt{8}$ BM	$\sqrt{15}$ BM	$\sqrt{24}$ BM

66. **(D)**

Sol.





Conjugate base is more stable due to more resonance of negative charge.

67. **(D)**

Sol. Solution with negative deviation has

$$P_T < P_{A^0}X_A + P_{B^0}X_B$$

$$P_A < P_{A^0}X_A$$

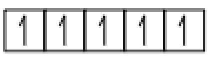
$$P_B < P_{B^0}X_B$$

If vapour pressure decreases so boiling point increases.

68. **(C)**

Sol. $[FeF_6]^{3-}$: Fe^{+3} : [Ar] $3d^5$

F : Weak field Ligand

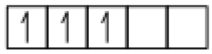


No. of unpaired electron's = 5

$$\mu = \sqrt{5(5+2)}$$

$$\mu = \sqrt{35}\ \text{BM}$$

$[V(H_2O)_6]^{+2}$: V^{+2} : $3d^3$



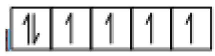
No. of unpaired electron's = 3

$$\mu = \sqrt{3(3+2)}$$

$$\mu = \sqrt{15}\ \text{BM}$$

$[Fe(H_2O)_6]^{+2}$: Fe^{+2} : $3d^6$

H_2O : Weak field Ligand



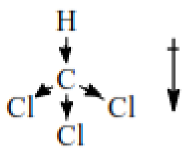
No. of unpaired electron's = 4

$$\mu = \sqrt{4(4+2)}$$

$$\mu = \sqrt{24}\ \text{BM}$$

69. **(D)**

Sol.



$\mu \neq 0$

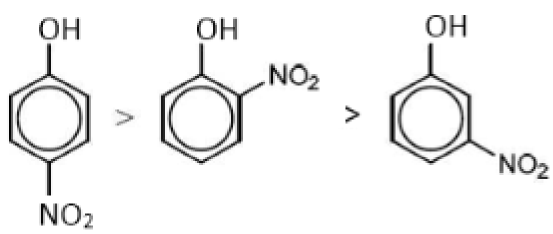
$CHCl_3$ is polar molecule and rest all molecules are non-polar.

70. **(C)**

Sol. s-block elements are highly reactive and found in combined state.

71. **(A)**

Sol. Acidic strength

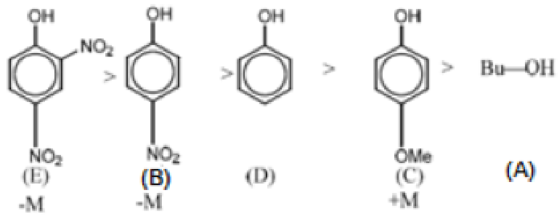


Ethanol give lucas test after long time.

Statement (I) → correct

Statement (II) → incorrect

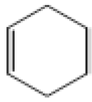
72. **(D)**

Sol. 

73. **(B)**

Sol. Solid Boron has very strong crystalline lattice so its melting point unusually high in group 13 elements.

74. **(D)**

Sol.  is Alicyclic

75. **(D)**

Sol. $PbCrO_4$ + NaOH (hot excess) $\rightarrow$

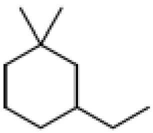
$[Pb(OH)_4]^{-2} + Na_2CrO_4$

Dianionic complex with coordination number four.

76. **(A)**

Sol. Aqueous solution of $(NH_4)_2CO_3$ is Basic pH of salt of weak acid and weak base depends on K_a and K_b value of acid and the base forming it.

77. **(B)**

Sol. 

3-ethy 1, 1 -dimethylcyclohexane

78. **(A)**

Sol. NaCl + conc. $H_2SO_4 + K_2Cr_2O_7 \rightarrow$

$CrO_2Cl_2 + KHSO_4 + NaHSO_4 + H_2O$

(B)

Reddish brown

$CrO_2Cl_2 + NaOH \rightarrow Na_2CrO_4 + NaCl + H_2O$

(C)

Yellow colour

79. **(D)**

Sol. SN^1 – Racemisation

SN^2 – Inversion

80. **(A)**

Sol. Electronic configuration of Nd(Z = 60) is ;

$[Xe]\ 4f^4\ 6s^2$

Section - B (Numerical Value Type)

81. **(107 gm or 108)**

Sol. Eq. of Ag = Eq. of O_2

Let x gm silver displaced,

$$\frac{x \times 1}{108} = \frac{5.6}{22.7} \times 4$$

(Molar volume of gas at STP = 22.7 lit)

x = 106.57 gm

Ans. 107

OR,

as per old STP data, molar volume = 22.4 lit

$$\frac{x \times 1}{108} = \frac{5.6}{22.4} \times 4, \quad x = 108 \text{ gm}$$

Ans. 108

82. **(2)**

Sol. Let, R = $k[HI]^n$

using any two of given data,

$$\frac{3 \times 10^{-3}}{7.5 \times 10^{-4}} = \left(\frac{0.01}{0.005}\right)^n$$

n = 2

83. **(8)**

Sol. $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$

Moles of $CO_2 = \frac{22}{44} = 0.5$

So, required moles of $CH_4 = 0.5$

Mass = 0.5 × 16 = 8 gm

84. **(1200)**

Sol. Using, first law of thermodynamics,

$\Delta U = Q + W$,

$\Delta U = 0$: Process is isothermal

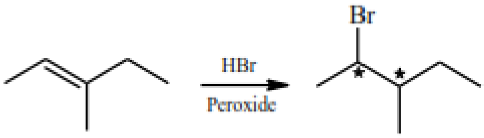
$Q = -W$

$W = -P_{ext}\Delta V$: Irreversible

$= -80 \times 10^3 (45 - 30) \times 10^{-3}$

$= -1200$ J

85. **(4)**

Sol. 

2chiral centres

No. of stereoisomers = 4.

86. **(3)**

Sol. B, C and D are Aromatic.

87. **(4)**

Sol. $-NO_2$, $-C\equiv N$, $-COR$, $-COOH$ are meta directing.

88. **(16)**

Sol. n = 4 can have,

	4s	4p	4d	4f
Total e^-	2	6	10	14
Total e^- with $S=+\frac{1}{2}$	1	3	5	7

So, Ans. 16

89. **(6)**

Sol. $CO \Rightarrow \bar{C}\equiv\overset{+}{O}$: BO = 3

$NO^+ \Rightarrow N\equiv O^+$: BO = 3

90. **(4)**

Sol.

Compounds	SO_3	H_2SO_3	$SOCl_2$	SF_4	$BaSO_4$	$H_2S_2O_7$
O.S. of Sulphur	+6	+4	+4	+4	+6	+6

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