3S INSTITUTE
JEE ADVANCED | JEE MAIN | NEET | OLYMPIADS | MHT-CET | FOUNDATION

31-January-2023 (Morning Batch) : JEE Main Paper

PHYSICS

Section - A (Single Correct Answer)

1. A

Sol. u = – MB cosθ

W = Δu

W = – MB cos 180° (– mB cos 0°)

W = 2 MB = 2 × 5 × 0.4 = 4J

2. C

Sol. Energy of one photon $= \dfrac{\text{Power}}{\text{Photon frequency}}$

$$E = h\nu = \frac{15\times10^{3}}{10^{16}}$$

$$\nu = \frac{15\times10^{-13}}{6\times10^{34}} = 2.5\times10^{21}$$

So gamma Rays.

3. B

Sol. Carrier wave frequency

$$V_C = \frac{100\pi}{2\pi} = 500 \text{ Hz}$$

Modulating wave frequency

$$V_m = \frac{4\pi}{2\pi} = 2\text{Hz}$$

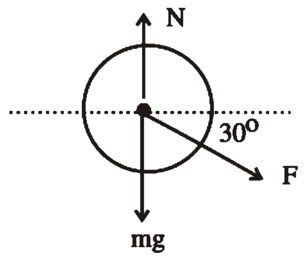
$$\therefore V_C - V_m, V_C, V_C + V_m$$

= 498 Hz. 500 Hz, 502 Hz.

4. C

Sol. N = mg + F sin 30°

$= 700 + 200 \times \dfrac{1}{2} = 800$ newton.



5. B

Sol. $u\cos\theta = \dfrac{\sqrt{3}u}{2} \Rightarrow \cos\theta = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad \theta = 30°$

$$T = \frac{2u\sin 30°}{g} = \frac{u}{g}$$

6. B

Sol. When metal is passing through magnetic field, eddy current will produce and it will oppose the motion, so it will take more time.

7. D

Sol. As neutron has more rest mass than proton it will require energy to decay proton into neutron.

8. C

Sol. As temperature increases, more electron excite to conduction band and hence conductivity increases, therefore resistance decreases.

9. C

Sol. $u_{max} = \dfrac{1}{2}m\omega^2A^2 = 25 \text{ J}$

$$\text{KE at } \frac{A}{2} = \frac{1}{2}mv_1^2 = \frac{1}{2}m\omega^2\left(A^2 - \frac{A^2}{4}\right)$$

$$\text{KE} = \frac{1}{2}m\omega^2\frac{3A^2}{4} = \frac{3}{4}\left(\frac{1}{2}m\omega^2A^2\right)$$

$$\text{KE} = \frac{3}{4}\times 25 = 18.75 \text{ J}$$

10. D

Sol. As Δq = 0

Δu = – W

$$W = \int P dV$$

$\Delta u = -W = 30 \times 10^3 \times 150 \times 10^{-6}$

$= 4500 \times 10^{-3} = 4.5$ J

11. C

Sol. Conceptual

12. D

Sol. Conceptual

13. A

Sol. $V_d = \frac{eE}{m}\tau$ that is independent of area

14. D

Sol. $\frac{GM}{R^2}\left[1-\frac{d}{R}\right] = \frac{4\times GM}{(4R)^2}$

$1-\frac{d}{R}=\frac{1}{4}\Rightarrow\frac{d}{R}=\frac{3}{4}\Rightarrow d\frac{3}{4}R$

d = 4800 km

15. B

Sol. $1000\times\frac{4\pi}{3}(1)^3=\frac{4\pi}{3}R^3$

R = 10 mm

$T \times 1000 \times 4\pi\,(10^{-3})^2 - T \times 4\pi\,(10 \times 10^{-3})^2 = \Delta E$

$\Delta E = 4 \times \pi \times 7 \times 10^{-2}\,[1000 - 100] \times 10^{-6}$

$\Delta E = 7.92 \times 10^{-4}$ J

16. C

Sol. $\phi = \mu_r\mu_0\frac{N}{l}I\times A$

$\mu_r = 125$

17. B

Sol. γ is independent of temperature

18. C

Sol. $I_A = \frac{I_0}{2}$

$IC = \frac{I_0}{2}\cos^2 45 = \frac{I_0}{4}$

$I_B = I_C\cos^2 45 = \frac{I_0}{8}$

19. B

Sol. All three have same dimension therefore

$\frac{R}{\sqrt{X_L X_C}}$ is dimensionless.

20. B

Sol. $P_i = Nm\upsilon\hat{i}$ $\quad\vec{P}_f = -Nm\upsilon\hat{i}$

N is Number of balls strikes with wall

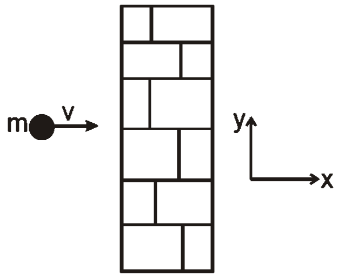
N = 100

$\Delta\vec{P} = \vec{P}_f - \vec{P}_i = -2Nm\upsilon\hat{i}$

$= -200\ Nm\upsilon\hat{i}$

$\vec{F}_{Total} = \frac{\Delta\vec{P}}{\Delta t} = -\frac{200mvt}{t}$

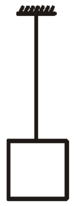
$|\vec{F}| = \frac{200mv}{t}$



Section - B (Numerical Value)

21. 60

Sol. If Δl is decease in length of rod due to decease in temperature



$\Delta l = l\,\alpha\Delta T$

$\alpha = 2\times10^{-5}K^{-1}, \Delta T = (210-160) = 50K$

$\Delta l = 1 \times 2 \times 10^{-5} \times 50 = 10^{-3}$ m

Young Modulus $= Y = \frac{F/A}{\Delta l/l}\ A = 3\times10^{-6}m^2$

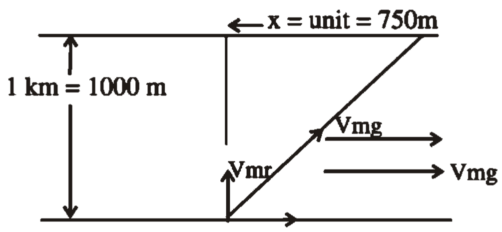
$2\times10^{11} = \frac{Mg/3\times10^{-6}}{10^{-3}/1}$

$Mg = 2 \times 10^{11} \times 3 \times 10^{-9} = 6 \times 10^{-2}$

M = 60 kg

22. 3

Sol.



time to cross the River width ω = 1000 m

is $= \frac{1km}{4km/h}$

Drift x = Vm/g × t

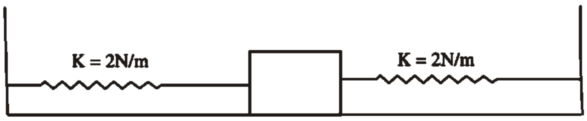
Where Vm/g is velocity of River w.r. to ground.

$$x = Vm/g \times \frac{1}{4} = 750m = \frac{3}{4}km$$

Vm / g = 3km / hr

23. 20

Sol.



Keff = K + K as both springs are in use in parallel

= 2k

= 2 × 2 = 4 N/m m = 490 gm = 0.49 kg

$$T = 2\pi\sqrt{\frac{m}{Keff}} = 2\pi\sqrt{\frac{0.49kg}{4}}$$

$$= 2\pi\sqrt{\frac{49}{400}} = 2\pi\frac{7}{20} = \frac{7\pi}{10}$$

No. of oscillation in the 14π is

$$N = \frac{time}{T} = \frac{14\pi}{7\pi/10} = 20$$

24. 5

Sol. $V = \frac{C}{\mu} \Rightarrow \mu = \frac{C}{V} = \frac{C}{0.2C}$

$$\boxed{\mu = 5}$$

$$\mu = \sqrt{\in_r \mu_r}$$

$$\therefore \frac{\in_r}{\mu} = \frac{\mu}{\mu_r} = 5$$

25. 10

Sol. $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = 7\times10^{-3}$

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{V}{R}\right)^2 = 7\times10^{-3}$$

$$\frac{1}{2}MV^2\left[1+\frac{2}{5}\right] = 7\times10^{-3}$$

$$\frac{1}{2}(1)(V^2)\left(\frac{7}{5}\right) = 7\times10^{-3}$$

$V^2 = 10^{-2}$

$V = 10^{-1}$ = 0.1 m/s = 10 cm/s

26. 242

Sol. $X_L = X_C$

So, Z = R = 20 Ω

$$i_{rms} = \frac{220}{20} = 11$$

$$i_{max} = 11\sqrt{2} = \sqrt{242}$$

Ans: 242

27. 640

Sol. Flux = $\vec{E}\cdot\vec{A}$

$$= 4000(0\cdot2)^2\frac{V}{m}\cdot(0\cdot2)^2 m^2$$

$= 4000 \times 16 \times 10^{-4}$ Vm

= 640 Vcm

28. 7

Sol. $v^2 = u^2 + 2as$

$= 2^2 + 2\,(2)(6)$

= 4 + 24 = 28

$$KE = \frac{1}{2}mv^2$$

$$= \frac{1}{2}(500)28$$

= 7000 J = 7 kJ

29. 27

Sol. $\frac{1}{\lambda} = Rz^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$$\frac{1}{\lambda_1} = Rz^2\left[\frac{1}{1^2} - \frac{1}{3^2}\right] = \frac{8}{9}Rz^2 \qquad ....(1)$$

$$\frac{1}{\lambda_2} = Rz^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3}{4}Rz^2 \qquad ....(2)$$

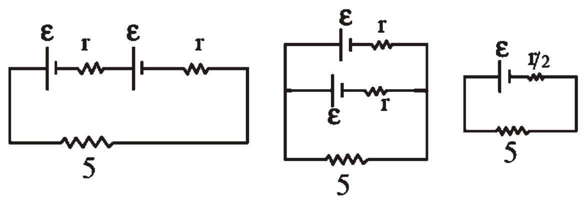
$$\frac{1}{2} \Rightarrow \frac{\lambda_2}{\lambda_1} = \frac{8}{9} \times \frac{4}{3} = \frac{32}{27}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{27}{32}$$ Ans : 27

30. 5

Sol.

Parallel



$$i = \frac{2\varepsilon}{5+2r} \quad ......(1) \qquad i = \frac{\varepsilon}{\frac{r}{2}+5} \quad .....(2)$$

Equating (1) and (2)

$$\frac{2\varepsilon}{5+2r} = \frac{\varepsilon}{\frac{r}{2}+5} \Rightarrow r+10 = 5+2r$$

r = 5

CHEMISTRY

Section - A (Single Correct Answer)

31. B

Sol. Nd(60) = [Xe] $4f^4$ $5d^0$ $6s^2$

Nd^{2+} = [Xe] $4f^4$ $5d^0$ $5s^0$

32. C

Sol. Methods involved in concentration of one are

(i) Hydraulic Washing

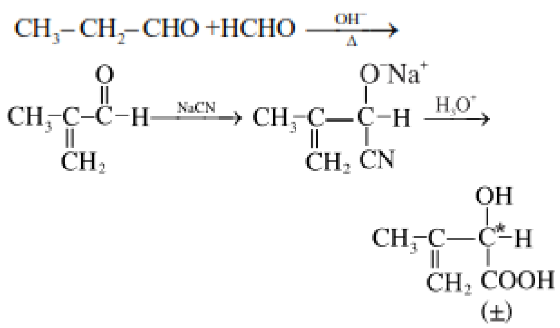
(ii) Froth Flotation

(iii) Magnetic Separation

(iv) Leaching

33. C

Sol.



Carboxylic acid will give CO_2 gas, with $NaHCO_3$ solution.

34. A

Sol. With increase in % of oxygen acidic nature of oxide of an element increase and basic nature decreases.

35. B

Sol. $Cu^{2+} + 2KI \longrightarrow \underset{\text{Unstable}}{CuI_2} \downarrow + 2K^+$

I^- is strong R.A it reduces Cu^{2+} to Cu^+

$2CuI_2 \longrightarrow \underset{\text{(White) 'X'}}{Cu_2I_2} \downarrow + I_2$

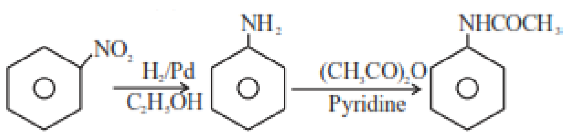
$KI + I_2 \longrightarrow K^+I_3^-$ (Brown solution)

$I_3^- \rightleftharpoons I_2 + I^-$

$KI_3 + Na_2S_2O_3 \rightarrow KI + \underset{(Y)}{Na_2S_4O_6}$

36. D

Sol.



37. A

Sol. $CoCl_2 + 6H_2O \longrightarrow [Co(H_2O)_6]Cl_2$

Pink(X)

octahedral

$\downarrow$ +HCl(conc.)

$[CoCl_4]^{2-}$

(Y)Blue solution

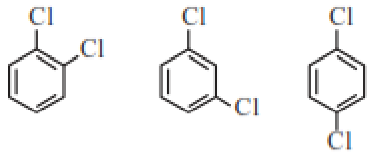
(Z)Tetrahedral

38. C

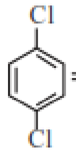
Sol. $Cl\dot{O} + NO_2 \rightarrow \underset{(X)}{ClONO_2} \xrightarrow{+H_2O} \underset{(Y)}{HOCl} + \underset{(Z)}{HNO_3}$

39. D

Sol.

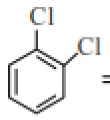


b.p / K	453	446	448
m.p / K	256	249	323



$\Rightarrow$ More Symmetrical & better Crystal fitting

M.P a Packing efficiency



$\Rightarrow \mu_{max}$

40. B

Sol. Only in option (2) α-Amino acid is given all the other options are not α-Amino acids.

41. A

Sol. He^{+} ion :

$$\frac{1}{\lambda(H)} = R(1)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\frac{1}{\lambda(He^{+})} = R(2)^2\left[\frac{1}{2^2} - \frac{1}{4^2}\right]$$

Given $\lambda(H) = \lambda(He^{+})$

$$R(1)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = R(4)\left[\frac{1}{2^2} - \frac{1}{4^2}\right]$$

$$\frac{1}{n_1^2} - \frac{1}{n_2^2} = \frac{1}{1^2} - \frac{1}{2^2}$$

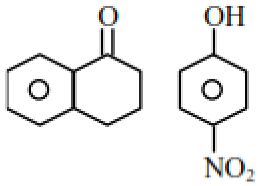
On comparing $n_1 = 1$ and $n_2 = 2$.

Ans. 1

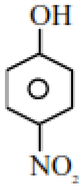
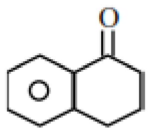
42. C

Sol. **A.** $H_2O/CH_2Cl_2 \rightarrow$ ii, $CH_2Cl_2 > H_2O$ (density) so they can be separated by differential solvent extraction.

B.



iii. column chromatography

Due to H-bonding in  it can be separated from  by column chromatography.

C. Kerosene / Naphthalene $\rightarrow$ iv. Fractional distillation.

Due to different B.P. of kerosene and Naphthalene it can be separated by fractional distillation.

D. $C_6H_{12}O_6/NaCl \rightarrow$ i. Crystallization.

NaCl (ionic compound) can be crystallized.

43. D

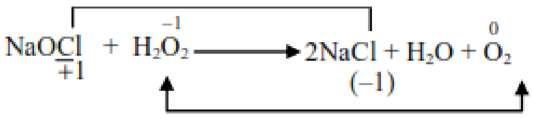
Sol. In isoelectronic species size $\propto \frac{1}{Z}$

$Ca^{2+} < K^{+} < Cl^{-} < S^{2-}$: Size

Z : 20 19 17 18

44. A

Sol.



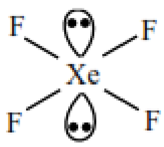
45. C

Sol. Sweetness value order wrt cane sugar –

Alitame > Sucralose > Saccharin > Aspartame

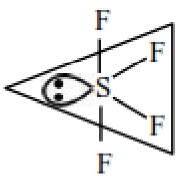
46. D

Sol. (A)



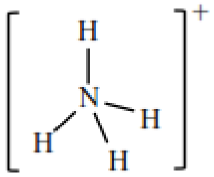
Square planer

(B) SF_4



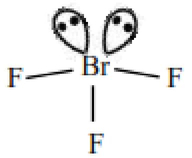
See-Saw

(C) NH_4^+



Tetrahedral

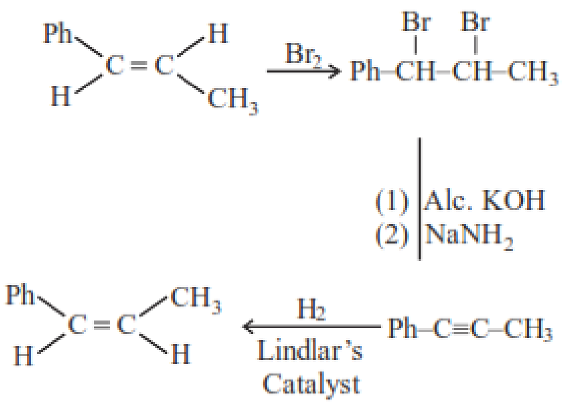
(D) Br F_3



Bent T- Shaped

47. B

Sol.



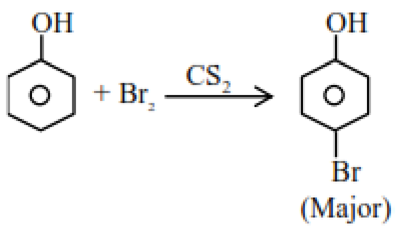
48. C

Sol. Non-Polar tail towards non-polar solvent.

Ans. 3

49. A

Sol. Aromatic compounds burns with sooty flame



50. D

Sol. Electrolysis of brine solution.

$NaCl(aq) \longrightarrow Na^+_{(aq)} + Cl^+_{(aq)}$

At anode :

$2Cl^+_{(aq)} \longrightarrow \underset{(Major)}{Cl_2(g)} + 2e^-$

$2H_2O_{(\ell)} \longrightarrow \underset{(Minor)}{O_{2(g)}} + 4H^+_{(aq)} + 4e^-$

At cathode :

$2H_2O_{(\ell)} + 2e^- \longrightarrow H_{2(g)} \uparrow + 2OH^-_{(aq)}$

$2Na^+ + 2OH^- \longrightarrow 2NaOH$

Section - B (Numerical Value)

51. 6

Sol. $\Delta G^o = -RT\ell n K$

$-nFE^o_{cell} = -RT \times 2.303(\log_{10} K)$

$\frac{E^o_{cell}}{0.06} \times n = \log K$ ... (1)

$Pd^{+2}(aq) + \cancel{2}e^- \rightleftharpoons Pd(s), E^o_{cat, red^n} = 0.83$

$Pd(s) + 4Cl^-(aq) \rightleftharpoons PdCl_4^{2-}(aq) + 2e^-,$

$E^o_{Anode, Oxid^n} = 0.65$

Net reaction $\rightarrow$

$Pd^{2+}(aq) + 4Cl^-(aq) \rightleftharpoons PdCl_4^{2-}(aq)$

$E^o_{cell} = E^o_{cat, red^n} - E^o_{Anode, Oxid^n}$

$E^o_{cell} = 0.83 - 0.65$

$E^o_{cell} = 0.18$ ... (2)

Also, n = 2 ... (3)

Using equation (1), (2) & (3),

log K = 6

52. 2520

Sol. $\log \frac{K_{300}}{K_{200}} = \frac{E_a}{2.3 \times 8.314}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$\log \frac{0.05}{0.03} = \frac{Ea}{2.305 \times 8.314} \times \left[\frac{1}{200} - \frac{1}{300}\right]$

$E_a = 2519.88\ J \Rightarrow E_a = 2520\ J$

53. 610

Sol. $\frac{1}{2}Cl_{2(g)} \rightarrow Cl_{(g)} \rightarrow Cl^-_{(g)} \rightarrow Cl^-_{(aq.)}$

$$\Delta H^o = \frac{1}{2} \times 240 + (-350) + (-380)$$

$= -610$

54. 44

Sol. weight of C in 0.792 gm CO_2

$$= \frac{12}{44} \times 0.792 = 0.216$$

% of C in compound = $\frac{0.216}{0.492} \times 100$

= 43.90 %

Ans : 44

55. 62250

Sol. $\pi = CRT$

$$\frac{400Pa}{10^5} = \frac{\frac{2.5\ g}{M_0}}{250/1000\ L} \times 0.83 \frac{L-bar}{K.mol} \times 300\ K$$

$M_0 = 62250$

56. 4

Sol. $Zn + 2HCl \rightarrow ZnCl_2 + H_2 \uparrow$

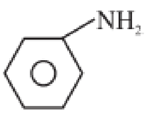
Moles of Zn used = $\frac{11.5}{65.4}$ = Moles of H_2 evolved

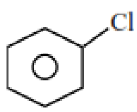
Volume of H_2 = $\frac{11.5}{65.4} \times 22.7\ L$ = 3.99 L

Ans : 4

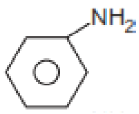
57. 3

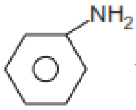
Sol. Product in the given reactions are as follow –

1. 

2. No reactions will be observed as in Gabriel phthalimide synthesis 

is poor substrate for SN^2.

3. 

4. 

+ CH_3COOH

Aromatic amines will he formed in 1, 3 & 4.

Ans : 3

58. 1

Sol. $SO_{2(g)} + \frac{1}{2}O_{2(g)} \rightleftharpoons SO_{3(g)}$

$K_P = 2 \times 10^{12}$ at 300 K

$K_P = K_C \times (RT)^{\Delta n_g}$

$2 \times 10^{12} = K_c \times (0.082 \times 300)^{-1/2}$

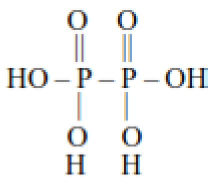
$K_C = 9.92 \times 10^{12}$

$K_C = 0.992 \times 10^{13}$

Ans. 1

59. 4

Sol. $H_4P_2O_6$



O.S. of P is +4.

60. 555

Sol. $P_X = \chi_X\ P_T$

$$= \frac{\frac{0.6}{20}}{\frac{0.6}{20} + \frac{0.45}{45}} \times 740$$

$P_X = 555$ mm Hg

MATHEMATICS

Section - A (Single Correct Answer)

61. B

Sol. Equation of normal is

$$2x\sec\theta - by\,\mathrm{cosec}\theta = 4 - b^2$$

Distance from (0, 0) $= \dfrac{4-b^2}{\sqrt{4\sec^2\theta + b^2\,\mathrm{cosec}^2\theta}}$

Distance is maximum if

$4\sec^2\theta + b^2\,\mathrm{cosec}^2\theta$ is minimum

$$\Rightarrow \tan^2\theta = \frac{b}{2}$$

$$\Rightarrow \frac{4-b^2}{\sqrt{4\cdot\frac{b+2}{2} + b^2\cdot\frac{b+2}{b}}} = 1$$

$$\Rightarrow 4 - b^2 = b + 2 \Rightarrow b = 1 \Rightarrow e = \frac{\sqrt{3}}{2}$$

62. A

Sol. Let $w = z + \dfrac{1}{z} = 4e^{i\theta} + \dfrac{1}{4}e^{-i\theta}$

$$\Rightarrow w = \frac{17}{4}\cos\theta + i\frac{15}{4}\sin\theta$$

So locus of w is ellipse $\dfrac{x^2}{\left(\frac{17}{4}\right)^2} + \dfrac{y^2}{\left(\frac{15}{4}\right)^2} = 1$

Locus of z is circle $x^2 + y^2 = 16$

So intersect at 4 points

63. A

Sol. $\ell_1 + \ell_2 = 20 \Rightarrow \dfrac{d\ell_2}{d\ell_1} = -1$

$A_1 = \left(\dfrac{\ell_1}{4}\right)^2$ and $A_2 = \pi\left(\dfrac{\ell_2}{2\pi}\right)^2$

Let $S = 2A_1 + 3A_2 = \dfrac{\ell_1^2}{8} + \dfrac{3\ell_2^2}{4\pi}$

$$\frac{ds}{d\ell} = 0 \Rightarrow \frac{2\ell_1}{8} + \frac{6\ell_1}{4\pi}\cdot\frac{d\ell_2}{d\ell_1} = 0$$

$$\Rightarrow \frac{\ell_1}{4} = \frac{6\ell_2}{4\pi} \Rightarrow \frac{\pi\ell_1}{\ell_2} = 6$$

64. D

Sol. By equation 1 and 3y + 2z = 8

y = 8 – 2z

And x = –2 + z

Now putting in equation 2

$\alpha(z-2) + \beta(-2z+8) + 7z = 3$

$\Rightarrow (\alpha - 2\beta + 7)\,z = 2\alpha - 8\alpha + 3$

So equations have unique solution if

$\alpha - 2\beta + 7 \neq 0$

And equations have no solution if

$\alpha - 2\beta + 7 = 0$ and $2\alpha - 8\beta + 3 \neq 0$

And equations have infinite solution if

$\alpha - 2\beta + 7 = 0$ and $2\alpha - 8\beta + 3 = 0$

65. A

Sol. $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix} = -\hat{i} - \hat{j} - 2\hat{k}$

$$\vec{a}_2 - \vec{a}_1 = 6\hat{i} + (\lambda - 1)\hat{j} + (-\lambda - 4)\hat{k}$$

$$2\sqrt{6} = \left|\frac{-6-\lambda+1+2\lambda+8}{\sqrt{1+1+4}}\right|$$

$$|\lambda + 3| = 12 \Rightarrow \lambda = 9,\ -15$$

$\alpha = -2k + 5,\ \lambda = k - \lambda$ where $k \in R$

$$\Rightarrow \alpha + 2\gamma = 5 - 2\lambda = -13,\ 35$$

66. A

Sol. $\left(x + \dfrac{1}{2}\right)^2 = \left(y + \dfrac{1}{4}\right)$

$y = (x^2 + x)$

$$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2 + x + 1} = \pi/2$$

$$0 \le x^2 + x + 1 \le 1$$

$x^2 + x \le 0$ ....(1)

Also $x^2 + x \ge 0$ .....(2)

$\therefore x^2 + x = 0 \Rightarrow x = 0,\ -1$

So contains 2 element.

67. C

Sol. $A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} = A$$

$\Rightarrow A^3 = A^4 = ... = A$

$(A + I)^{11} = {}^{11}C_0A^{11} + {}^{11}C_1A^{10} + ... + {}^{11}C_{10}A + {}^{11}C_{11}I$

$= ({}^{11}C_0 + {}^{11}C_1 + ... {}^{11}C_{10})\,A + I$

$= (2^{11} - 1)\,A + I = 2047A + I$

$\therefore$ Sum of diagonal elements $= 2047(1 + 4 - 3) + 3$

$= 4094 + 3 = 4097$

68. A

Sol. $(a, b)\,R\,(c, d) \Rightarrow ad(b - c) = bc(a - d)$

Symmetric :

$(c, d)\,R\,(a, b) \Rightarrow cb(d - a) = da(c - b) \Rightarrow$ Symmetric

Reflexive :

$(a, b)\,R\,(a, b) \Rightarrow ab(b - a) \Rightarrow ba(a - b) \Rightarrow$ Not reflexive

Transitive : (2, 3) R (3, 2) and (3, 2) R (5, 30) but $((2, 3), (5, 30)) \notin R \Rightarrow$ Not transitive

69. B

Sol. $y = \sin^3(\pi/3\cos g(x))$

$$g(x) = \frac{\pi}{3\sqrt{2}}(-4x^3 + 5x^2 + 1)^{3/2}$$

$$g(1) = 2\pi/3$$

$$y'3\sin^2\left(\frac{\pi}{3}\cos g(x)\right)\times\cos\left(\frac{\pi}{3}\cos g(x)\right)$$

$$\times\frac{\pi}{3}(-\sin g(x))g'(x)$$

$$y'(1) = 3\sin^2\left(-\frac{\pi}{6}\right)\cdot\cos\left(\frac{\pi}{6}\right)\cdot\frac{\pi}{3}\left(= -\sin\frac{2\pi}{3}\right)g'(1)$$

$$g'(x) = \frac{\pi}{3\sqrt{2}}(-4x^3 + 5x^2 + 1)^{1/2}(12x^2 + 10x)$$

$$g'(1) = \frac{\pi}{2\sqrt{2}}(\sqrt{2})(-2) = -\pi$$

$$y'(1) = \frac{\not{3}}{4}\cdot\frac{\sqrt{3}}{2}\cdot\frac{\pi}{\not{3}}\left(\frac{-\sqrt{3}}{2}\right)(-\pi) = \frac{3\pi^2}{16}$$

$$y(1) = \sin^3(\pi/3\cos 2\pi) = -\frac{1}{8}$$

$$2y'(1) + 3\pi^2 y(1) = 0$$

70. A

Sol. a, ar, ar^2, ar^3 $(a, r > 0)$

$a^4r^6 = 1296$

$a^2r^3 = 36$

$$a = \frac{6}{r^{3/2}}$$

$a + ar + ar^2 + ar^3 = 126$

$$\frac{1}{r^{3/2}} + \frac{r}{r^{3/2}} + \frac{r^2}{r^{3/2}} + \frac{r^3}{r^{3/2}} = \frac{126}{6} = 21$$

$(r^{-3/2} + r^{3/2}) + (r^{1/2} + r^{-1/2}) = 21$

$r^{1/2} + r^{-1/2} = A$

$r^{-3/2} + r^{3/2} + 3A = A^3$

$A^3 - 3A + A = 21$

$A^3 - 2A = 21$

$A = 3$

$$\sqrt{r} + \frac{1}{\sqrt{r}} = 3$$

$$r + 1 = 3\sqrt{r}$$

$r^2 + 2r + 1 = 9r$

$r^2 - 7r + 1 = 0$

71. B

Sol. $\sqrt{(x-1)(x-3)} + \sqrt{(x-3)(x+3)}$

$$= \sqrt{4\left(x - \frac{12}{4}\right)\left(x - \frac{2}{4}\right)}$$

$\Rightarrow \sqrt{x-3} = 0 \Rightarrow x = 3$ which is in domain

or

$$\sqrt{x-1}+\sqrt{x+3}=\sqrt{4x-2}$$

$$2\sqrt{(x-1)(x+3)}=2x-4$$

$x^2 + 2x - 3 = x^2 - 4x + 4$

$6x = 7$

$x = 7/6$ (rejected)

72. C

Sol. Differentiate w.r.t. x

$$f'(x)+\frac{f(x)}{x}=\frac{1}{2\sqrt{x+1}}$$

$$I.F.=e^{\int\frac{1}{x}dx}=e^{\ln x}=x$$

$$xf(x)=\int\frac{x}{2\sqrt{x+1}}dx$$

$x + 1 = t^2$

$$=\int\frac{t^2-1}{2t}2tdt$$

$$xf(x)=\frac{t^2-1}{3}t+c$$

Aslo putting $x = 3$ in given equation $f(3)+0=\sqrt{4}$

$f(3) = 2$

$$\Rightarrow C=8-\frac{8}{3}=\frac{16}{3}$$

$$f(x)=\frac{\frac{(x+1)^{3/2}}{3}\sqrt{x+1}+\frac{16}{3}}{x}$$

$$f(8)=\frac{9-3+\frac{16}{3}}{8}=\frac{34}{24}$$

$\Rightarrow$ 12 f(8) = 17

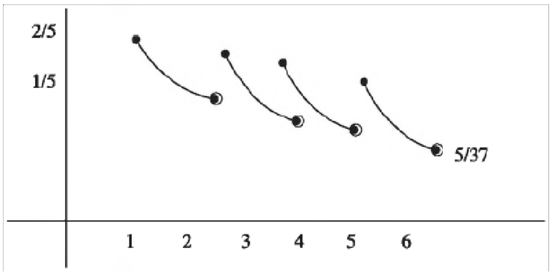
73. D

Sol. $f(x)=\frac{2}{1+x^2}$ $x\in[2,3)$

$f(x)=\frac{3}{1+x^2}$ $x\in[3,4)$

$f(x)=\frac{4}{1+x^2}$ $x\in[4,5)$

$f(x)=\frac{5}{1+x^2}$ $x\in[5,6)$



$$\left(\frac{5}{37},\frac{2}{5}\right]$$

74. C

Sol. $\left|\vec{a}+\vec{b}+\vec{c}\right|^2=\left|\vec{a}+\vec{b}-\vec{c}\right|^2$

$2\vec{a}\cdot\vec{b}+2\vec{b}\cdot\vec{c}+2\vec{c}\cdot\vec{a}\cdot\vec{b}-2\vec{b}\cdot\vec{c}-2\vec{c}\cdot\vec{a}$

$4\vec{a}\cdot\vec{c}=0$

B is incorrect

$\left|\vec{a}+\lambda\vec{c}\right|^2\ge\left|\vec{a}\right|^2$

$\lambda^2c^2\ge0$

True $\forall\lambda\in R$ (A) is correct.

75. B

Sol. $$\int_0^\alpha\frac{t^{50}-1+1}{1-t}=-\int_0^\alpha(1+t+.....+t^{49})+\int_0^\alpha\frac{1}{1-t}dt$$

$$=-\left(\frac{\alpha^{50}}{50}+\frac{\alpha^{49}}{49}+....+\frac{\alpha^1}{1}\right)+\left(\frac{\ln(1-f)}{-1}\right)_0^\alpha$$

$$=-P_{50}(\alpha)-\ln(1-\alpha)$$

$$=-P_{50}(\alpha)-\beta$$

76. A

Sol. $\cos^{-1}\frac{4}{5}=\tan^{1}\frac{3}{4}$

$$\therefore\sin^{-1}\frac{\alpha}{17}=\tan^{-1}\frac{77}{36}-\tan^{-1}\frac{3}{4}=\tan^{-1}\left(\frac{\frac{77}{36}-\frac{3}{4}}{1+\frac{77}{36}\cdot\frac{3}{4}}\right)$$

$$\sin^{-1}\frac{\alpha}{17}=\tan^{-1}\frac{8}{15}=\sin^{-1}\frac{8}{17}$$

$$\Rightarrow\frac{\alpha}{17}=\frac{8}{17}\Rightarrow\alpha=8$$

$$\therefore\sin^{-1}(\sin 8)+\cos^{-1}(\cos 8)$$

$$=3\pi-8+8-2\pi$$

$$=\pi$$

77. B

Sol. $C=(2,\ 3),\ r=\sqrt{2}$

Centre of $G=A=2+4\frac{1}{\sqrt{2}},$

$$3+\frac{4}{\sqrt{2}}=(2+2\sqrt{2},\ 3+2\sqrt{2})$$

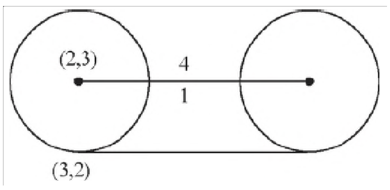
$$A(2+2\sqrt{2},\ 3+2\sqrt{2})$$

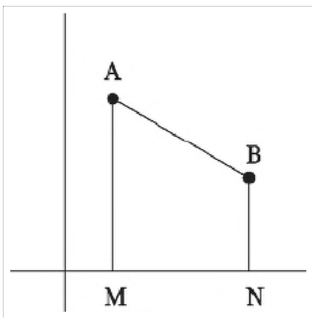
$$B(4+2\sqrt{2},\ 1+2\sqrt{2})$$

$$\frac{x(2+2\sqrt{2})}{1}=\frac{y-(3+2\sqrt{2})}{-1}=2$$

$\therefore$ area of trapezium :

$$\frac{1}{2}(4+4\sqrt{2})2=4(1+\sqrt{2})$$





78. D

Sol.

p	q	$p\Rightarrow q$	$\sim q$	$p\wedge\sim q$	$(p\Rightarrow q)\vee(p\wedge\sim q)$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	T	F	T

$\sim p$	$\sim q$	$\sim p\Rightarrow\sim q$	$\sim p\vee q$	$((\sim p)\Rightarrow(\sim q))\wedge(\sim p)\vee q)$
F	F	T	T	T
F	T	T	F	F
T	F	F	T	F
T	T	T	T	T

79. C

Sol. $$\int_{\pi/3}^{\pi/2}\left(\frac{2+3\sin x}{\sin x(1+\cos x)}\right)dx=2\int_{\pi/3}^{\pi/2}\frac{dx}{\sin x+\sin x\cos x}+3$$

$$3\int_{\pi/3}^{\pi/2}\frac{dx}{1+\cos x}$$

$$\int_{\pi/3}^{\pi/2}\frac{dx}{1+\cos x}=\int_{\pi/3}^{\pi/2}\frac{1\cos x}{\sin^2 x}dx$$

$$=\int_{\pi/3}^{\pi/2}(\operatorname{cosec}^2 x-\cot x\operatorname{cosec} x)dx$$

$$=(\operatorname{cosec} x-\cot x)\int_{\pi/3}^{\pi/2}=(1)-\left(\frac{2}{\sqrt{3}}-\frac{1}{\sqrt{3}}\right)=1-\frac{1}{\sqrt{3}}$$

$$\int_{\pi/3}^{\pi/2}\frac{dx}{\sin x(1+\cos x)}=$$

$$\int\frac{dx}{(2\tan x/2)(1+1-\tan^2 x/2)}$$

$$=\int\frac{(1+\tan^2 x/2)\sec^2 x/2\,dx}{2\tan x/2\,2}$$

$$\tan x/2=t\qquad\tan x/2\,\frac{1}{2}dx=dt$$

$$\frac{1}{2}\int\left(\frac{1+t^2}{t}\right)dt=\frac{1}{2}\left[\ell nt+\frac{t^2}{2}\right]_{\frac{1}{\sqrt{3}}}^{1}$$

$$=\frac{1}{2}\left[\left(0+\frac{1}{2}\right)-\left(\ell n\frac{1}{\sqrt{3}}+\frac{1}{6}\right)\right]=\left(\frac{1}{3}+\ell n\sqrt{3}\right)\frac{1}{2}$$

$$=\left(\frac{1}{6}+\frac{1}{2}\ell n\sqrt{3}\right)$$

$$2\left(\frac{1}{6}+\frac{1}{2}\ell n\sqrt{3}\right)+3\left(1-\frac{1}{\sqrt{3}}\right)$$

$$=\frac{1}{3}+\ell n\sqrt{3}+3\sqrt{3}=\frac{10}{3}+\ell n\sqrt{3}-\sqrt{3}$$

80. A

Sol. $\dfrac{{}^5C_2+{}^6C_2}{{}^2C_2+{}^3C_2+{}^4C_2+{}^5C_2+{}^8C_2}=\dfrac{10+15}{1+3+6+10+15}$

$$=\frac{25}{35}=\frac{5}{7}$$

Section - B (Numerical Value)

81. 2997

Sol. $2\underset{6}{+}\underset{6}{+}\underset{6}{+}\underset{6}{+}=1296$

$3\underset{6}{+}\underset{6}{+}\underset{6}{+}\underset{6}{+}=1296$

$40\underset{6}{+}\underset{6}{+}\underset{6}{+}=216$

$420\underset{6}{+}\underset{6}{+}=36$

$\underline{422}\underset{6}{+}\underset{6}{+}=36$

$423\underset{6}{+}\underset{6}{+}=36$

$424\underset{6}{+}\underset{6}{+}=36$

$427\underset{6}{+}\underset{6}{+}=36$

$429\underline{0}\underset{6}{+}=6$

429 2 0 = 1

429 2 2 = 1

429 2 3 = 1

= 2997

82. 8

Sol. $2a_7 = a_5$ (given)

$2(a_1 + 6d) = a_1 + 4d$

$a_1 + 8d = 0$ ....(1)

$a + 10d = 18$ ....(2)

By (1) and (2) we get $a_1 = -72$, $d = 9$

$a_{18} = a_1 + 17d = -72 + 153 = 81$

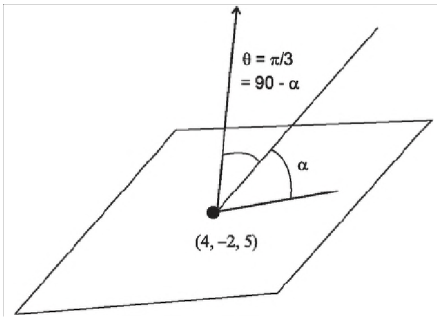
$a_{10} = a_1 + 9d = 9$

$$12\left(\frac{\sqrt{a_{11}}-\sqrt{a_{10}}}{d}+\frac{\sqrt{a_{12}}-\sqrt{a_{11}}}{d}+\ldots\frac{\sqrt{a_{18}}-\sqrt{a_{17}}}{d}\right)$$

$$12\left(\frac{\sqrt{a_{18}}-\sqrt{a_{10}}}{d}\right)=\frac{12(9-3)}{9}=\frac{12\times 6}{6}=8$$

83. 9

Sol.



$$\cos\theta\frac{(\hat{i}+\hat{j}+2\hat{k})\cdot(2\hat{i}-\hat{j}+\hat{k})}{6}=\frac{2-1+2}{6}=\frac{1}{2}$$

$\theta=\pi/3$ $\alpha=\pi/6$

$(\tan^2\theta)(\cot^2\alpha)$

(3) (3) = 9

84. 2

Sol. $T_{r+1} = {}^{30}C_r(x^{2/3})^{30-r}\left(\dfrac{2}{x^3}\right)^r$

$$= {}^{30}C_r\cdot 2^r\cdot x^{\frac{60-11r}{3}}$$

$$\frac{60-11r}{3}<0\Rightarrow 11r>60\Rightarrow r>\frac{60}{11}\Rightarrow r=6$$

$T_7 = {}^{30}C_6\cdot 2^6\, x^{-2}$

We have also observed $\beta = {}^{30}C_6\,(2)^6$ is a natural number.

$\therefore\ \alpha = 2$

85. 36

Sol. $|\vec{a}|=\sqrt{14},\ |\vec{b}|=\sqrt{6}\ |\vec{a}\times\vec{b}|=\sqrt{48}$

$$|\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2=|\vec{a}|^2\times|\vec{b}|^2$$

$$\Rightarrow(\vec{a}\cdot\vec{b})^2=84-48=36$$

86. 180

Sol. Any point on L $((2\lambda+1),\ (-\lambda-1),\ (\lambda+3))$

$$2(2\lambda+1)+(-\lambda-1)+3(\lambda+3)=16$$

$$6\lambda+10=16\Rightarrow\lambda=1$$

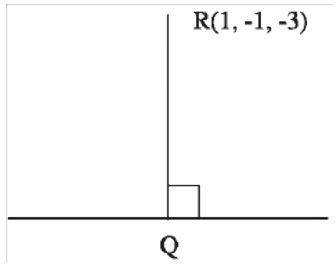
$\therefore$ P(3, –2, 4)

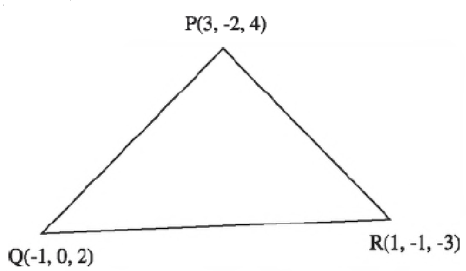
Dr of QR $=\langle 2\lambda,\ -\lambda,\ \lambda+6\rangle$

Dr of L $=\langle 2,\ -1,\ 1\rangle$

$$4\lambda+\lambda+6=0\ \ 6\lambda+6=0\Rightarrow\lambda=-1$$

Q = (–1, 0, 2)





$\overrightarrow{QR}=2\hat{i}-\hat{j}-5\hat{k}\ \ \overrightarrow{QP}=4\hat{i}-2\hat{j}+2\hat{k}$

$$\overrightarrow{QR}\times\overrightarrow{QP}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & -5\\ 4 & -2 & 2\end{vmatrix}=-12\hat{i}-24\hat{j}$$

$$\alpha=\frac{1}{2}\times\sqrt{144+576}\Rightarrow\alpha^2=\frac{720}{4}=180$$

87. 9

Sol.

x_i	f_i	$d_i=x_i-5$	$f_id_i^2$	f_id_i
2	3	–3	27	–9
3	6	–2	24	–12
4	16	–1	16	–16
5	α	0	0	0
6	9	1	9	9
7	5	2	20	10
8	6	3	54	18

$$\sigma_x^2=\sigma_d^2=\frac{\sum f_id_i^2}{\sum f_i}-\left(\frac{\sum f_id_i}{\sum f_i}\right)^2$$

$$=\frac{150}{45+\alpha}-0=3$$

$$\Rightarrow 150=135+3\alpha$$

88. 5

89. 72

Sol. $f(x)=\frac{x+|x|}{2}=\begin{bmatrix}x & x\geq 0\\ 0 & x<0\end{bmatrix}$

$$g(x)=\begin{bmatrix}x^2 & x\geq 0\\ 0 & x<0\end{bmatrix}$$

$$fog(x)=f[g(x)]=\begin{bmatrix}g(x) & g(x)\geq 0\\ 0 & g(x)<0\end{bmatrix}$$

$$fog(x)=\begin{bmatrix}x^2 & x\geq 0\\ 0 & x<0\end{bmatrix}$$

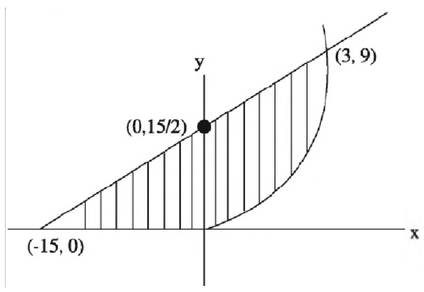
2y – x = 15

$$A=\int_0^3\left(\frac{x+15}{2}-x^2\right)dx+\frac{1}{2}\times\frac{15}{2}\times 15$$

$$\left.\frac{x^2}{4}+\frac{15x}{2}-\frac{x^3}{3}\right|_0^3+\frac{225}{4}$$

$$=\frac{9}{4}+\frac{45}{2}-9+\frac{225}{4}=\frac{99-36+225}{4}$$

$$=\frac{288}{4}=72$$



90. 710

Sol. 1000 – 2799

Divisible by 3

1002 + (n – 1) 3 = 2799

n = 600

Divisible by 11

$$1-2799 \to \left[\frac{2799}{11}\right] = [254] = 254$$

$$1-999 = \left[\frac{999}{11}\right] = 90$$

1000 – 2799 = 254 – 90 = 164

Divisible by 33

$$1-2799 \to \left[\frac{2799}{33}\right] = 84$$

$$1-999 \to \left[\frac{999}{33}\right] = 30$$

1000 – 2799 $\to$ 54

$\therefore$ n(3) + n(11) – n(33)

600 + 164 – 54 = 710

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