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JEE ADVANCED | JEE MAIN | NEET | OLYMPIADS | MHT-CET | FOUNDATION

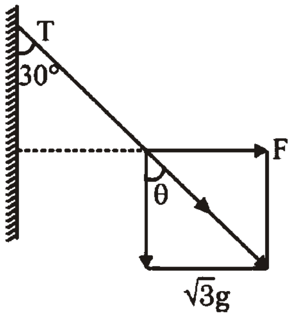
30-January-2023 (Evening Batch) : JEE Main Paper

PHYSICS

Section - A (Single Correct Answer)

1. A

Sol.



$\theta = 30°$

$\cos\theta = \frac{\sqrt{3}g}{T}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\sqrt{3}g}{T}$

$\Rightarrow T = 20N$

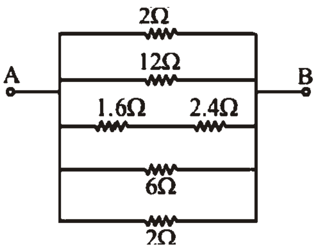
2. B

Sol. $K_{av} = \frac{5}{2}kT$

Ratio = 1 : 1

3. A

Sol.



$\frac{1}{R_{eq}} = \frac{1}{2} + \frac{1}{12} + \frac{1}{4} + \frac{1}{6} + \frac{1}{2}$

$= \frac{6+1+3+2+6}{12} = \frac{18}{12} = \frac{3}{2}$

$\Rightarrow R_{eq} = \frac{2}{3}\Omega$

4. B

Sol. Nuclear density is independent of A.

5. D

Sol. $\delta_1 = \delta_2$ [for no average deviation]

$\Rightarrow 6° (1.54 - 1) = A(1.72 - 1)$

$\Rightarrow A = \frac{6° \times 0.54}{0.72}$

$= \frac{18°}{4} = 4.5°$

6. D

Sol. Given circuit represent XOR.

7. D

Sol. $\frac{2}{V_{av}} = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$

$\Rightarrow V_{av} = \frac{15}{4} = 3.75$ km/h

8. B

Sol. Electric field inside material of conductor is zero.

9. D

Sol. d tan 60° = $2\sqrt{3}$

d = 2 cm

$B = 3 \times \frac{\mu_0 i}{2\pi d}\sin 60°$

$= 3 \times \frac{2\times10^{-7}\times 2}{2\times10^{-2}} \times \frac{\sqrt{3}}{2}$

$= 3\sqrt{3}\times10^{-5}$

10. A

Sol. $z = \sqrt{100^2 + (200-100)^2} = 100\sqrt{2}\Omega$

$i_{rms} = \frac{V_{rms}}{z} = \frac{200\sqrt{2}}{100\sqrt{2}}$

= 2A

11. D

Sol. $20\times10^{-3}\times\frac{180}{60}\times100=10V$

$\Rightarrow$ v = 0.6 m/s

12. D

Sol. Both A and R are true and R is the correct explanation of A

13. D

14. B

Sol. $\omega=\sqrt{\frac{k}{m}}$

$$\frac{\omega_2}{\omega_1}=\sqrt{\frac{m_1}{m_2}}=\sqrt{\frac{1}{2}}$$

15. B

Sol. $KE=\frac{P^2}{2m},P=\frac{h}{\lambda}$

$$eV_1=\frac{\left(\frac{h}{\lambda}\right)^2}{2m}$$

$$eV_2=\frac{\left(\frac{h}{1.5\lambda}\right)^2}{2m}$$

$$\frac{V_1}{V_2}=(1.5)^2=\frac{9}{4}$$

16. D

17. B

Sol. $F\propto I_1I_2$

$F_I : F_{2I} = 1 : 4$

18. D

Sol. $Y=\frac{F/A}{\frac{\Delta l}{l}}$

$$\Rightarrow \quad F=\frac{YA}{l}\Delta l$$

$$\left(\frac{A\Delta l}{l}\right)_1=\left(\frac{A\Delta l}{l}\right)_2$$

$$\Rightarrow\frac{\Delta l_2}{\Delta l_1}=\frac{A_1}{A_2}\times\frac{l_2}{l_1}$$

$$\Rightarrow\frac{\Delta l_2}{0.2}=\frac{1}{2.4\times2.4}\times\frac{2}{1}$$

$\Rightarrow \Delta l_2 = 6.9\times10^{-2}$ mm

19. B

Sol. Loss in PE = Gain in KE

$$\left(-\frac{GMm}{2R}\right)-\left(-\frac{GMm}{R}\right)=\frac{1}{2}mv^2$$

$$\Rightarrow v^2=\frac{GM}{R}=gR$$

$$\Rightarrow \ v=\sqrt{gR}$$

20. B

Sol. $I_{EF}=\frac{1}{2}\times\frac{5}{4\pi\times5^2}=\frac{1}{40\pi}W/m^2$

Section - B (Numerical Value)

21. 313

Sol. $\frac{41°-5°}{95°-5°}=\frac{C-0°}{100°-0°}$

$$\Rightarrow \quad C=\frac{36}{90}\times100=40°C=313K$$

22. 2

Sol.

$$\frac{2}{3}=\frac{\frac{x}{x+1}}{x}\Rightarrow\frac{2}{3}=\frac{1}{x+1}\Rightarrow x=0.5=\frac{1}{2}$$

n = 2

23. 88

Sol. $4v^2 = 50 - x^2$

$$\Rightarrow \quad v=\frac{1}{2}\sqrt{50-x^2}$$

x = 88

24. 3

Sol. $I=4I_0\cos^2\left(\frac{\Delta\phi}{2}\right)$

$$I_1=4I_0\cos^2\left(\frac{\pi}{4}\right)=2I_0$$

$$I_2=4I_0\cos^2\left(\frac{2\pi}{3}\right)=I_0$$

$$\Rightarrow \frac{I_1 + I_2}{I_0} = 3$$

25. 300

Sol. $\frac{dN_1}{dt} = -\lambda_1 N \qquad \frac{dN_2}{dt} = -\lambda_2 N$

$$\frac{dN}{dt} = -(\lambda_1 + \lambda_2)N$$

$$\Rightarrow \quad I_{eq} = \lambda_1 + \lambda_2$$

$$\Rightarrow \frac{1}{t_{1/2}} = \frac{1}{300} + \frac{1}{30} = \frac{11}{300}$$

$$\Rightarrow t_{1/2} = \frac{300}{11}$$

26. 4

Sol. $\frac{1}{2}mV^2 = Pt$

$$V = \sqrt{\frac{2Pt}{m}}$$

$$\frac{dx}{dt} = \sqrt{\frac{2Pt}{m}}$$

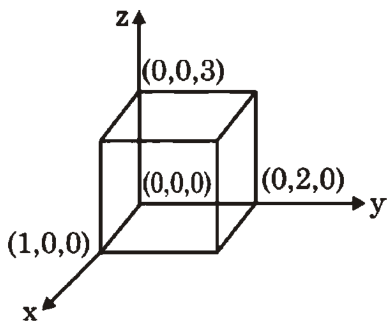
$$x = \sqrt{\frac{2P}{m}}\frac{2}{3}\left[t^{3/2}\right]_0^4$$

$$x = \frac{16\sqrt{P}}{3} = \frac{1}{3} \times 16\sqrt{P}$$

$\alpha = 4$

27. 12

Sol. $\vec{E} = 2x^2\hat{i} - 4y\hat{j} + 6\hat{k}$



$$\phi_{net} = -8 \times 3 + 2 \times 6 = -12$$

$$-12 = \frac{q}{\epsilon_0}$$

$$|q| = 12\,\epsilon_0$$

28. 1584

Sol. $\xi_{max} = NAB\omega$

$$= 100 \times 14 \times 10^{-2} \times 3 \times \frac{360 \times 2\pi}{60}$$

$= 1584V$

29. 125

Sol. $a = \omega^2 R = \left(\frac{28 \times 2\pi}{60}\right)^2 \times 1.8$

$$= \left(\frac{56}{60} \times \frac{22}{7}\right)^2 \times 1.8 = \frac{(44)^2}{225} \times 1.8 = \frac{1936 \times 1.8}{225}$$

$x = 125$

30. 54

Sol. $a = -\mu_k g = -3$

$V = 18 - 3 \times 2$

$V = 12$ m/s

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}\frac{mr^2}{2}\frac{v^2}{r^2}$$

$$KE = \frac{3}{4}mv^2$$

KE = 3 × 18 = 54 J

CHEMISTRY

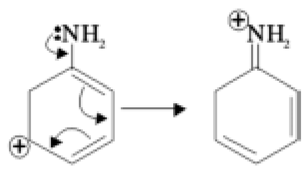
Section - A (Single Correct Answer)

31. C

Sol. $4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 4NO_2 + O_2$

32. A

Sol.



The +M effect of NH_2 is stabilizing the carbo-cation.

33. B

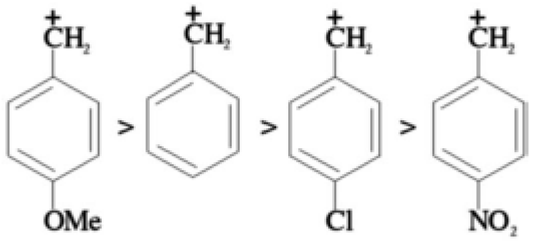
Sol. Due to –M effect of $-NO_2$ group, it increases acidity +M effect of $N(CH_3)_2$ decreases acidity. Hyperconjugation of isopropyl decrease acidity

∴ order of acidic strength

(c) > (a) > (d) > (b)

34. C

Sol. The rate of S_N1 reaction depends upon stability of carbocation which follows the order.

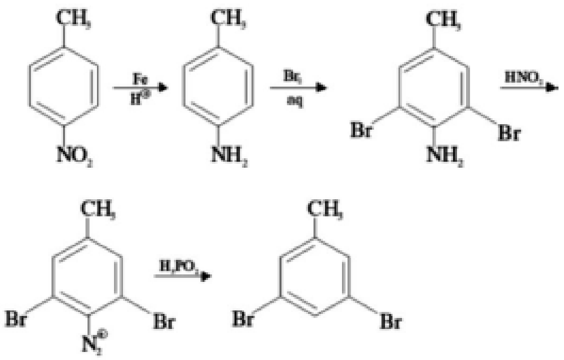


∴ Reactivity order

(b) > (d) > (c) > (a)

35. D

Sol.



36. B

Sol. The number of electrons in the orbitals of sub-shell of n = 4 are

4s	2
4p	6
4d	10
4f	14
(Total)	**32**

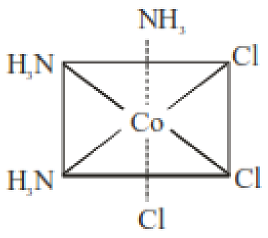
37. D

Sol. For $[Fe(NH_3)_6]^{+2}$, $\Delta_0 < P$, hence the pairing of electrons does not occur in t_{2g}. Therefore complex is outer orbital and its hybridisation is sp^3d^2.

List I (Complexes)	List II (Hybridisation)
$[Ni(CO)_4]$	sp^3
$[Cu(NH_3)_4]^{2+}$	dsp^2
$[Fe(NH_3)_6]^{2+}$	sp^3d^2
$[Fe(H_2O)_6]^{2+}$	sp^3d^2

38. B

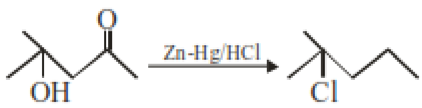
Sol.



The Cl – Co – Cl bond angle in above octahedral complex is 90°.

39. A

Sol.



The acid sensitive alcohol group reacts with HCl, hence Clemmenson reduction is not suitable for above conversion.

40. D

Sol. $BeCl_2$ having covalent nature is soluble in organic solvent.

41. B

Sol. Antiallergic and antacid drugs work on different receptors.

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42. D

Sol. At node $\psi_{2s} = 0$

$$\therefore\ 2 - \frac{r_0}{a_0} = 0$$

$$\therefore\ r_0 = 2a_0$$

43. A

Sol. In acidic medium

$$2MnO_4^- + 10I^- + 16H^+ \rightarrow 2Mn^{2+} + 5I_2 + 8H_2O$$

In neutral/faintly alkaline solution

$$2MnO_4^- + I^- + H_2O \rightarrow 2MnO_2 + 2OH^- + IO_3^-$$

44. A

Sol. Bond dissociation energy of E – H bond in hydrides of group 16 follows the order.

$H_2O > H_2S > H_2Se > H_2Te$

45. C

Sol. Clean water as BOD value of <5 while polluted water has BOD of 15 or more.

46. B

Sol.

List I (Mixture)	List II (Separation Technique)
$CHCl_3 + C_6H_5NH_2$	Distillation
$C_6H_{14} + C_5H_{12}$	Fractional distillation
$C_6H_5NH_2 + H_2O$	Steam distillation
Organic compound in H_2O	Differential extraction

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47. C

Sol. Boric acid has strong hydrogen bonding while BF_3 does not. Therefore boric acid is solid.

48. A

Sol. In Electrolytic refining, the pure metal is used as cathode and impure metal is used as anode.
Na_3AlF_6 is added during electrolysis of Al_2O_3 to lower the melting point and increase conductivity.

49. C

Sol. Nessler's reagent is K_2HgI_4.

50. B

Sol. $\left[Co(NH_3)_5SO_4\right]Br + AgNO_3 \rightarrow AgBr\downarrow$
0.01 mol excess 0.01 Mol

$\left[Co(NH_3)_5Br\right]SO_4 + BaCl_2 \rightarrow BaSO_4\downarrow$
0.01 mol excess 0.01 Mol

Section - B (Numerical Value)

51. 150

Sol. q = 0

$\Delta U = w$

$1 \times 20 \times [T_2 - 300] = -3000$

$T_2 - 300 = -150$

$T_2 = 150$ K

52. 4

Sol. $$d = \frac{Z \times M}{N_0 \times a^3}$$

$$4 = \frac{Z \times 72}{6 \times 10^{23} \times 125 \times 10^{-24}}$$

$$Z = 4.166 \simeq 4$$

53. 1350

Sol. $$\frac{t_1}{t_2} = \frac{\frac{1}{K}\ln\frac{a_0}{0.4a_0}}{\frac{1}{K}\ln\frac{a_0}{0.1a_0}}$$

$$\frac{540}{t_2} = \frac{\ln\frac{10}{4}}{\ln 10}$$

$$\frac{540}{t_2} = \frac{\log 10 - \log 4}{\log 10}$$

$$\frac{540}{t_2} = \frac{1 - 0.6}{1}$$

$$\Rightarrow \frac{540}{t_2} = 0.4$$

$$\Rightarrow t_2 = \frac{540}{0.4} = 1350 \text{ sec}$$

54. 243

Sol. $\Delta T_f = i.K_f.m$

$$\Rightarrow \Delta T_f = 2.67 \times 1.8 \times \frac{38}{98} \times \frac{1000}{62}$$

$$\Rightarrow \Delta T_f = 30.05$$

$\therefore$ F.P. = 243 K

55. 3

Sol. The yield of SO_3 at equilibrium will be due to :

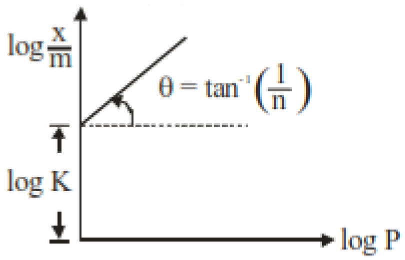
B. Increasing pressure

C. Adding more SO_2

D. Adding more O_2

56. 16

Sol.



$$\log\frac{x}{m} = \log k + \frac{1}{n}\log P$$

$$\frac{1}{n} = \tan 45^0 = 1$$

$$\log k = 0.6020 = \log 4$$

$$\Rightarrow K = 4$$

$$\therefore \frac{x}{m} = K.P^{1/n}$$

$$\frac{x}{m} = 4(0.4) = 1.6$$

$$\frac{x}{m} = 1.6 = 16\times 10^{-1}$$

57. 3

Sol. Compound 2, 3, 7.

58. 6

Sol. Number of peptide linkage = (amino acid – 1)

= 7 – 1

= 6

59. 150

Sol. Molarity $= \frac{50}{11.35}$

$\therefore$ Strength in gm/L $= \frac{50}{11.35}\times 34$

60. 275

Sol. $X + Y^{2+} \longrightarrow Y + X^{2+}$

$$E^o_{cell} = 0.36 - (-2.36) = 2.72\ V$$

$$E_{cell} = 2.72 - \frac{0.06}{2}\log\frac{0.001}{0.01}$$

$$= 2.72 + 0.03 = 2.75\ V$$

$$= 275\times 10^{-2}\ V$$

MATHEMATICS

Section - A (Single Correct Answer)

61. A

Sol. $P \to (\sim Q \wedge R)$

$\sim P \vee (\sim Q \wedge R)$

$(\sim P \vee \sim Q) \wedge (\sim P \vee R)$

62. D

Sol. $y = mx + \frac{4}{m}$

$$\frac{\left|\frac{4}{m}\right|}{\sqrt{1+m^2}} = 2\sqrt{2} \quad \therefore m = \pm 1$$

$y = \pm x \pm 4.$ Point of contact on parabola

Let m = 1, $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

R (4, 8)

Point of contact on circle Q(–2, 2)

$\therefore (QR)^2 = 36 + 36 = 72$

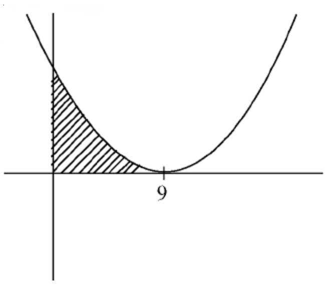
63. A

Sol. $x^2 - px + \frac{5p}{4} = 0$

D = p^2 – 5p = p (p – 5)

$\therefore q = 9$

$0 \le y \le (x-9)^2$



Area $= \int_0^9 (x-9)^2 dx = 243$

64. D

Sol. f '(x) = x^2 + 2b + ax

g '(x) = x^2 + a + 2bx

(2b – a) – x (2b – a) = 0

$\therefore$ x = 1 is the common root

Put x = 1 in f '(x) = 0 or g '(x) = 0

1 + 2b + a = 0

7 + 2b + a = 6

65. A

Sol. $y^2 = 3 - x + 2 + x + 2\sqrt{(3-x)(2+x)}$

$= 5 + 2\sqrt{6 + x - x^2}$

$y^2 = 5 + 2\sqrt{\frac{25}{4} - \left(x - \frac{1}{2}\right)^2}$

$y_{max} = \sqrt{5+5} = \sqrt{10}$

$y_{min} = \sqrt{5}$

66. C

Sol. Put y = vx

$v + x\frac{dv}{x} = -\left(\frac{1+3v^2}{3+v^2}\right)$

$x\frac{dv}{dx} = -\frac{(v+1)^3}{3+v^2}$

$\frac{(3+v^2)dv}{(v+1)^3} + \frac{dx}{x} = 0$

$\int\frac{4dv}{(v+1)^3} + \int\frac{dv}{v+1} - \int\frac{2dv}{(v+1)^2} + \int\frac{dx}{x} = 0$

$\frac{-2}{(v+1)^2} + \ln(v+1) + \frac{2}{v+1} + \ln x = c$

$\frac{-2x^2}{(x+y)^2} + \ln\left(\frac{x+y}{x}\right) + \frac{2x}{x+y} + \ln x = c$

$\frac{2xy}{(x+y)^2} + \ln(x+y) = c$

$\therefore$ c = 0, as x = 1, y = 0

$\therefore \frac{2xy}{(x+y)^2} + \ln(x+y) = 0$

67. A

Sol. $x = (8\sqrt{3}+13) = {}^{13}C_0 \cdot (8\sqrt{3})^{13} + {}^{13}C_1(8\sqrt{3})^2 + ...$

$x' = (8\sqrt{3}-13)^{13} = {}^{13}C_0(8\sqrt{3})^{13} - {}^{13}C_1(8\sqrt{3})^{12}(13)^1 + ...$

$x - x' = 2[{}^{13}C_r \cdot (8\sqrt{3})^{12}(13^1) + {}^{13}C_3(8\sqrt{3})^{10} \cdot (3)^3....]$

therefore, x – x' is even integer, hence [x] is even

Now, $y = (7\sqrt{2}+9) = {}^9C_0(7\sqrt{2})^9 + {}^9C_1(7\sqrt{2})^8(9)^1$

$+ {}^9C_2(7\sqrt{2})^7(9)^2......$

$y' = (7\sqrt{2}-9)^9 = {}^9C_0(7\sqrt{2})^9 - {}^9C_1(7\sqrt{2})^8(9)^1$

$+ {}^9C_2(7\sqrt{2})^7(9)^2.....$

$y - y' = 2[{}^9C_1(7\sqrt{2})^8(9)^1 + {}^9C_3(7\sqrt{2})^6(9)^3 + ...]$

y – y' = Even integer, hence [y] is even

68. C

Sol. $\hat{v} = \cos 60^\circ\hat{i} + \cos 45^\circ\hat{j} + \cos\gamma\hat{k}$

$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2\gamma = 1$ ($\gamma \to$ Acute)

$\Rightarrow \cos\gamma = \frac{1}{2}$

$\Rightarrow \gamma = 60^\circ$

Equation of plane is

$\frac{1}{2}(x - \sqrt{2}) + \frac{1}{\sqrt{2}}(y+1) + \frac{1}{2}(z-1) = 0$

$\Rightarrow x + \sqrt{2}y + z = 1$

(a, b, c) lies on it.

$\Rightarrow a + \sqrt{2}b + c = 1$

69. A

Sol. $LHL = \lim_{k\to 0} g(h(-k)),\ k > 0$

$= \lim_{k\to 0} g(-2+1)\ \because f(x) = -1 \forall x < 0$

= g(–1) = 1

$RHL = \lim_{k\to 0} g(h(k)),\ k > 0$

$= \lim_{k\to 0} g(-1),\ \because f(x) = 1,\ \forall x > 0$

= 1

70. C

Sol. $a \in \{2, 4, 6, 8, 10,......,100\}$

$b \in \{1, 3, 5, 7, 9,......,99\}$

Now, $a + b \in \{25, 71, 117, 163\}$

(i) a + b = 25, no. of ordered pairs (a, b) is 12

(ii) a + b = 71, no. of ordered pairs (a, b) is 35

(iii) a + b = 117, no. of ordered pairs (a, b) is 42

(iv) a + b = 163, no. of ordered pairs (a, b) is 19

$\therefore$ total = 108 pairs

71. D

Sol. $P^T = aP + (a-1)I$

$\Rightarrow P = aP^T + (a-1)I$

$\Rightarrow P^T - P = a(P - P^T)$

$\Rightarrow P^T$, as $a \neq -1$

Now, $P = aP + (a-1)I$

$\Rightarrow P = -I \Rightarrow |P = 1$

$\Rightarrow |\text{Adj } P| = 1$

72. A

Sol. $\vec{a} = \lambda\hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{b} = \hat{i} - \lambda\hat{j} + 2\hat{k}$

$\Rightarrow (\vec{b} - \vec{a}) \times ((\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b})) = 8\hat{i} - 40\hat{j} - 24\hat{k}$

$\Rightarrow ((\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}))(\vec{a} \times \vec{b}) = 8\hat{i} - 40\hat{j} - 24\hat{k}$

$\Rightarrow 8(\vec{a} \times \vec{b}) = 8\hat{i} - 40\hat{j} - 24\hat{k}$

$$\text{Now, } \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \lambda & 2 & -3 \\ 1 & -\lambda & 2 \end{vmatrix}$$

$= (4 - 3\lambda)\hat{i} - (2\lambda + 3)\hat{j} + (-\lambda^2 - 2)\hat{k}$

$\Rightarrow \lambda = 1$

$\therefore \vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$

$\Rightarrow \vec{a} + \vec{b} = 3\hat{i} + \hat{j} - \hat{k},\ \vec{a} - \vec{b} = 3\hat{j} - 5\hat{k}$

$$\Rightarrow (\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 0 & 3 & -5 \end{vmatrix} = 2\hat{i} + 10\hat{j} + 6\hat{k}$$

$\therefore$ required answer = 4 + 100 + 36 = 140

73. B

Sol. $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$

$\vec{b} \cdot \vec{c} = \vec{b}(2\vec{a} \times \vec{b}) - 3\vec{b} \cdot \vec{b}$

$= -3|b|^2$

$= -48$

74. C

Sol. $a_2 - a_1 = a_3 - a_2 = \ldots\ldots\ldots = a_{2022}\ a_{2021} = 1.$

$$\therefore \tan^{-1}\left(\frac{a_2 - a_1}{1 + a_1a_2}\right) + \tan^{-1}\left(\frac{a_3 - a_2}{1 + a_2a_3}\right) + \ldots.$$

$$+ \tan^{-1}\left(\frac{a_{2022} - a_{2021}}{1 + a_{2021}a_{2022}}\right)$$

$= [(\tan^{-1} a_2) - \tan^{-1} a_1] + [\tan^{-1} a_3 - \tan^{-1} a_2 \ldots. + [\tan^{-1} a_{2022} - \tan^{-1} a_{2021}]$

$= \tan^{-1} a_{2022} - \tan^{-1} a_1$

$= \tan^{-1}(2022) - \tan^{-1}1 = \tan^{-1}2022 - \dfrac{\pi}{4}$ (option C)

$$= \left(\frac{\pi}{2} - \cot^{-1}(2022)\right) - \frac{\pi}{4}$$

$$= \frac{\pi}{4} - \cot^{-1}(2022) \text{ (option A)}$$

75. C

Sol. $ax^2 + 2bx + c = 0$

$\Rightarrow ax^2 + 2\sqrt{ac}x + c = 0\ (\because b^2 = ac)$

$\Rightarrow (x\sqrt{a} + \sqrt{c})^2 = 0$

$$x^2 - \frac{\sqrt{c}}{\sqrt{a}} \qquad \ldots.(1)$$

Now, $dx^2 + 2ex + f = 0$

$$\Rightarrow d\left(\frac{c}{a}\right) + 2c\left[-\frac{\sqrt{c}}{\sqrt{a}}\right] + f = 0$$

$$\Rightarrow \frac{dc}{a} + f = 2e\sqrt{\frac{c}{a}}$$

$$\Rightarrow \frac{d}{a} + \frac{f}{c} = 2e\sqrt{\frac{1}{ac}}$$

$$\Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{b} \ [\text{as } b = \sqrt{ae}]$$

$$\therefore \frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in A.P.}$$

76. D

Sol. $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$

$\frac{x-1}{1}=\frac{y+\frac{1}{2}}{1}=\frac{z+1}{-1}$

Points : A(–1, k, 0), B(2, k, –1), C(1, 1, 2)

$\overrightarrow{CA}=\hat{i}+(k-1)\hat{j}-2\hat{k}$

$\overrightarrow{CB}=\hat{i}+(k-1)\hat{j}-3\hat{k}$

$$\overrightarrow{CA}\times\overrightarrow{CB}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -2 & k-1 & -2\\ 1 & k-1 & -3\end{vmatrix}$$

$=\hat{i}(-3\hat{k}+3+2k-2)-\hat{j}(6+2)+\hat{k}(-2k+2-k+1)$

$=(1-k)\hat{i}-8\hat{j}+(3-3k)\hat{k}$

The line $\frac{x-1}{1}=\frac{y+\frac{1}{2}}{1}=\frac{z+1}{-1}$ is perpendicular to normal vector.

$\therefore$ 1 ·(1 – k) +1(–8) + (–1)(3 – 3k) = 0

$\Rightarrow$ 1 – k – 8 – 3 + 3k = 0

$\Rightarrow$ 2k = 10 $\Rightarrow$ k = 5

$\therefore \frac{k^2+1}{(k-1)(k-2)}=\frac{26}{4\cdot 3}=\frac{13}{6}$

77. B

Sol. As a^3, b^3, c^3 be in A.P. $\rightarrow$ $a^3 + c^3 = 2b3$ .....(1)

$\log_a^b, \log_c^a, \log_b^c$ are in G.P.

$\therefore \frac{\log b}{\log a}\cdot\frac{\log c}{\log b}=\left(\frac{\log a}{\log c}\right)^2$

$\therefore (\log a)^3=(\log c)^3 \Rightarrow a=c$ ....(2)

a = b = c

$T_1=\frac{a+4b+c}{3}=2a;\ d=\frac{a-8b+c}{10}=\frac{-6a}{10}=\frac{-3}{5}a$

$\therefore S_{20}=\frac{20}{2}\left[4a+19\left(-\frac{3}{5}a\right)\right]$

$=10\left[\frac{20a-57a}{5}\right]$

$=-74a$

$\therefore -74a=-444 \Rightarrow a=6$

$\therefore abc=6^3=216$

78. C

Sol. let a_1 be any natural number

a_1, a_1 + 1, a_1 + 2, ..., a_1 + 99 are values of a_i 'S

$\bar{x}=\frac{a_1+(a_1+1)+(a_1+2)+......+a_1+99}{100}$

$=\frac{100a_1+(1+2+....+99)}{100}=a_1+\frac{99\times 100}{2\times 100}$

$=a_1+\frac{99}{2}$

Mean deviation about mean $=\frac{\sum_{1=1}^{100}|x_i-\bar{x}|}{100}$

$=\frac{2\left(\frac{99}{2}+\frac{97}{2}+\frac{95}{2}+.....+\frac{1}{2}\right)}{100}$

$=\frac{1+3+....+99}{100}$

$=\frac{\frac{50}{2}[1+99]}{100}$

= 25 So, it is true for every natural no. 'a_1'

79. D

Sol. $\lim_{n\to\infty}\frac{3}{n}\sum_{r=0}^{n-1}\left(2+\frac{r}{n}\right)^2$

$=3\int_0^1(2+x)^2dx=27-8=19$

80. C

Sol. $\begin{vmatrix}1 & -1 & 1\\ 2 & 2 & \alpha\\ 3 & -1 & 4\end{vmatrix}=0;\ 8+\alpha-2(4+1)+3(-\alpha-2)=0$

$8+\alpha+6-3\alpha-6=0$

$\alpha=4$

Section - B (Numerical Value)

81. 23

Sol. $x + y = 12^{50} + 18^{50} = (150 - 6)^{25} + (325 - 1)^{25}$

$= 25\ K - (6^{25} + 1) = 25\ K - ((5 + 1)^{25} + 1)$

$= 25K_1 - 2$ Remainder = 23

82. 432

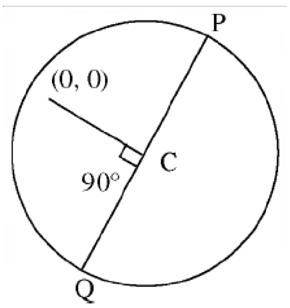
Sol. f(1) = 1; f(9) = f(3) × f(3)

i.e., f(3) = 1 or 3

Total function = 1 × 6 × 2 × 6 × 6 × 1 = 432

83. 24

Sol. $\frac{1}{2}\times PC\times\sqrt{5}=\frac{\sqrt{35}}{2};\ PC=\sqrt{7}$



$a_1^2+b_1^2+a_2^2+b_2^2=OP^2+OQ^2$

= (5 + 7) = 24

84. 151

Sol. $T_8 = 11 + (8 - 1) \times 20$

= 11 + 140 = 151

85. 158

Sol. $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 3 & -2\\ 1 & -1 & 2\end{vmatrix}=4\hat{i}-4\hat{j}-4\hat{k}$

$\therefore$ Equation of line is $\frac{x-2}{1}=\frac{y-3}{-1}=\frac{z-1}{-1}$

Let Q be (5, 3, 8) and foot of T from Q on this line be R.

Now, R ≡ (k + 2, –k + 3, –k + 1)

DR of QR are (k – 3, –k, –k – 7)

$\therefore$ (1)(k – 3) + (–1)(–k) + (–1)(–k – 7) = 0

$\Rightarrow k=-\frac{4}{3}$

$\therefore \alpha^2=\left(\frac{13}{3}\right)^2+\left(\frac{4}{3}\right)^2+\left(\frac{17}{3}\right)^2=\frac{474}{9}$

$\therefore 3\alpha^2=158$

86. 1

Sol. $\int\sqrt{\sec 2x 1}dx=\int\sqrt{\frac{1-\cos 2x}{\cos 2x}}dx$

$=\sqrt{2}\int\frac{\sin x}{\sqrt{2\cos^2 x-1}}dx$

put $\cos x=t\Rightarrow -\sin x dx=dt$

$=-\sqrt{2}\int\frac{dt}{\sqrt{2t^2-1}}$

$=-\ln\left|\sqrt{2}\cos x\right|+\sqrt{\cos 2x}+c$

$=-\frac{1}{2}\ln\left|2\cos^2 x+\cos 2x+2\sqrt{\cos 2x}\cdot\sqrt{2}\cos x\right|+c$

$=-\frac{1}{2}\ln\left|\cos 2x+\frac{1}{2}+\sqrt{\cos 2x}\cdot\sqrt{1+\cos 2x}\right|+c$

$\because \beta=\frac{1}{2},\ \alpha=-\frac{1}{2}\Rightarrow\beta-\alpha=1$

87. 13

Sol. Two equations have common root

$\therefore$ (4a)(26a) = $(-6)^2$ = 36

$\Rightarrow a^2=\frac{9}{26}\therefore a=\frac{3}{\sqrt{26}}\Rightarrow\beta=13$

88. 240

Sol. If unit digit 5, then total numbers $=\frac{6!}{3!2!}$

If unit digit 3, then total numbers $=\frac{6!}{3!}$

If unit digit 1, then total numbers $=\frac{6!}{3!2!}$

$\therefore$ total numbers = 60 + 60 + 120 = 240

89. 14

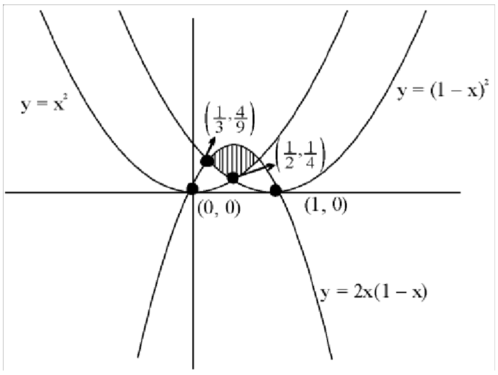
Sol. $p = \frac{^6C_1}{6\times 6} = \frac{1}{6}$

$q = \frac{^6C_1 \times {}^5C_1 \times 4}{6\times 6\times 6\times 6} = \frac{5}{54}$

$\therefore p:q = 9:5 \Rightarrow m+n = 14$

90. 25

Sol.



$$A = 2\int_{\frac{1}{3}}^{\frac{1}{2}} (2x - 2x^2 - (1-x)^2)dx$$

$$= 2[2x^2 - x^2 - x]_{1/3}^{1/2}$$

$$\therefore A = \frac{5}{108} \Rightarrow 540A = \frac{5}{108}\times 540 = 25$$

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