3S INSTITUTE
JEE ADVANCED | JEE MAIN | NEET | OLYMPIADS | MHT-CET | FOUNDATION

30-January-2023 (Morning Batch) : JEE Main Paper

PHYSICS

Section - A (Single Correct Answer)

1. D

Sol. $Q=\left(\alpha t-\beta t^{2}+\gamma t^{3}\right)$

$$i=\frac{dQ}{dt}=\left(\alpha-2\beta t+3\gamma t^{2}\right)$$

$$\frac{di}{dt}=\left(3\gamma t-2\beta\right)=0$$

$$i=\left(\alpha-2\beta t+3\gamma t^{2}\right)=\left(\alpha-\frac{\beta^{2}}{3\gamma}\right)$$

2. D

Sol. PT^2 = constant, Using $PV = nRT$

$$P=\frac{nRT}{V}$$

$$PT^{2}=\frac{nRT}{V}\times T^{2}=\text{ constant}$$

$\Rightarrow \quad T^3 = KV$

So, $\frac{d}{dT}\left(KV\right)=3T^{2}$

$$\Rightarrow\frac{KdV}{dT}=3T^{2}$$

$$\Rightarrow dV=\frac{3T^{2}}{K}dT$$

$dV = V\,\gamma\, dT$

$$\Rightarrow\gamma V=\frac{3T^{2}}{K}\Rightarrow\gamma=\frac{3T^{2}}{KV}=\frac{3T^{2}}{T^{3}}=\frac{3}{T}$$

3. D

Sol. $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$,

$P = 2D = 2m^{-1}$

$$\Rightarrow\frac{1}{f}=\frac{2}{100}cm^{-1}$$

$$\frac{1}{V}-\left(-\frac{1}{25}\right)=\frac{2}{100}$$

$$\Rightarrow\frac{1}{V}=\frac{1}{50}-\frac{1}{25}$$

$$\Rightarrow V=-50cm$$

4. C

Sol. The velocities will be interchanged after collision.

Speed of P just before collision $=\sqrt{2gh}$

$$=\sqrt{2\times10\times0.2}=2\text{ m/s}$$

5. A

Sol.

$Y = 3h\,(1 + \sigma)$

$Y = 3h\,(1 - \sigma)$

$$\Rightarrow 2\eta\left(1+\sigma\right)=3K\left(1-2\sigma\right)$$

$$\Rightarrow\sigma=\left(\frac{3K-2\eta}{6K+2\eta}\right)$$

6. C

Sol. $M = NIA$

$M_A = M_B$

$\therefore \quad N_A I_A A_A = N_A I_B A_B$

$\therefore N_A I_A \pi\left(0.1\right)^{2}=N_B I_B \pi\left(0.2\right)^{2}$

$N_A I_A = 4N_B I_B$

7. B

Sol. Momentum $=\frac{\text{Energy}}{C}$

$$=\frac{\text{Power}\times\text{time}}{C}$$

$$=\frac{\left(20\times10^{-3}w\right)\left(300\times10^{-9}s\right)}{3\times10^{8}\text{ m/s}}$$

$= 2 \times 10^{-17}$ kg-m/s

8. D

Sol. $V_n \propto \frac{Z}{n}$

$Z = 1, \quad \therefore V_n \propto \frac{1}{n}$

$\therefore \frac{V_3}{V_7} = \frac{7}{3}$

$\therefore V_3 = \frac{7}{3} V_7$

$= \frac{7}{3} \times 3.6 \times 10^6 \text{ m/s} = 8.4 \times 10^6 \text{ m/s}$

9. D

Sol. $F_m = mg$

$\therefore \quad ILB = mg$

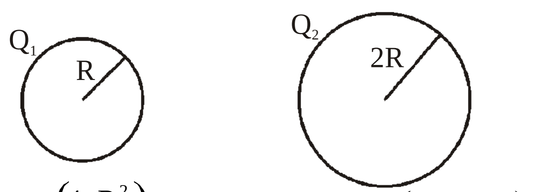
$\therefore \left(\frac{V}{R}\right) LB = mg$

$\therefore V = \frac{mgR}{LB}$

$= \frac{(1\times10^{-3}\text{kg})(10 \text{ m/s}^2)(10\Omega)}{(0.1\text{m})(10^3 \times 10^{-4}\text{T})} = 10\text{V}$

10. D

Sol.

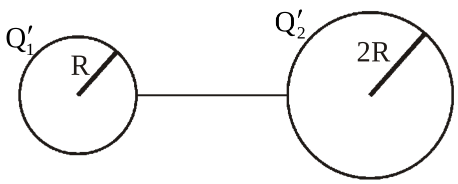


$Q_1 = \sigma(4\pi R^2)$

$= 4\pi R^2 \sigma$

$Q_2 = \sigma(4\pi(2R^2))$

$= 16\pi R^2 \sigma$



$\frac{Q_1'}{4\pi\varepsilon_0 R} = \frac{Q_2'}{4\pi\varepsilon_0 (2R)}$

$\therefore Q_2' = 2Q_1'$

$Q_1' + Q_2' = Q_1 + Q_2$

$\therefore \frac{Q_2'}{2} + Q_2' = 20\pi R^2 \sigma$

$\frac{3}{2} Q_2' = 20\pi R^2 \sigma$

$\therefore \frac{Q_2'}{4\pi(2R)^2} = \frac{2}{3} \cdot \frac{20\pi R^2 \sigma}{16\pi R^2}$

$\therefore \frac{\sigma'}{\sigma} = \frac{5}{6}$

11. D

Sol. $dQ = dU + dW \Rightarrow dU = nC_V \, dT$

$dU = 0$ (for isothermal)

$\therefore U$ = constant

Also $dQ > 0$ (supplied)

Hence $dW > 0$

12. B

Sol. $\vec{E} = \frac{A}{x^2}\hat{i} + \frac{B}{y^3}\hat{j}$

$\left[\frac{A}{x^2}\right] = NC^{-1} \Rightarrow [A] = Nm^2C^{-1}$

$\left[\frac{B}{y^3}\right] = NC^{-1} \Rightarrow [B] = Nm^3C^{-1}$

13. D

Sol. $\left(\overline{A \cdot A}\right) = \overline{A}$

$\overline{B \cdot B} = \overline{B}$

$\left(\overline{\overline{A} \cdot \overline{B}}\right) = A + B$

OR Gate

14. C

Sol. $h = \frac{2S\cos\theta}{r\rho g}$

$\therefore \frac{h_1}{h_2} = \frac{S_1}{S_2}\frac{\rho_2}{\rho_1}$

$\frac{5}{h_2} = \left[\frac{1}{2}\right]\left[\frac{2}{1}\right] \Rightarrow h_2 = 5cm = 0.05m$

{Info about angle of contact not there so most appropriate is C}

15. B

Sol. $A_c + A_m = 120$

$A_c - A_m = 80$

$\therefore \overline{A_c = 100}$

$A_m = 20$

Modulation index $= \frac{20}{100} = \frac{1}{5}$

Amplitude of each sideband

$= A_c \frac{(\text{mod ulation index})}{2}$

$= 100 \times \frac{1}{10} = 10$ volt

16. B

Sol. $P = \frac{R}{Z} \Rightarrow P_1 = \frac{R}{\sqrt{R^2 + X_L^2}} = \frac{R}{R\sqrt{2}} (\text{as } X_L = R)$

$P_1 = \frac{1}{\sqrt{2}}$

$P_2 = \frac{R}{\sqrt{R^2 + (X_L - X_L)^2}} = P_2 = 1$

$\frac{P_1}{P_2} = \frac{1}{\sqrt{2}}$

17. B

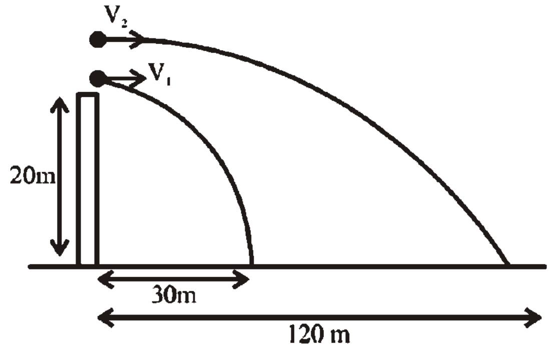
Sol. $-\frac{dV}{dr} = -\frac{k}{r^2} \Rightarrow \int_{10}^{V} dV = \int_{2}^{3} \frac{k}{r^2} dr$

$V - 10 = k\left[\frac{1}{2} - \frac{1}{3}\right]$

$V - 10 = \frac{k}{6} \Rightarrow V = 11$ volts

18. D

Sol.



$V_1 = \frac{30}{\sqrt{\frac{2h}{g}}}, V_2 = \frac{120}{\sqrt{\frac{2h}{g}}}$

$(0.01)u = (0.2)\frac{30\sqrt{g}}{\sqrt{2h}} + (0.01)\frac{120\sqrt{g}}{\sqrt{2h}}$

$u = 300 + 60 = 360 \text{ ms}^{-1}$

19. A

Sol. $\frac{dx}{dt}$ = slope ≥ 0 always increasing

(A - II)

$\frac{dx}{dt} < 0$; and at $t \to \infty \frac{dx}{dt} \to 0$

(B - IV)

$\frac{dx}{dt} > 0$ for first half $\frac{dx}{dt} < 0$ for second half.

(C - III)

$\frac{dx}{dt}$ = constant

(D - I)

20. C

Sol. $\left|\frac{d\vec{p}}{dt}\right| = |\vec{F}| \Rightarrow \frac{d\vec{p}}{dt}$ = Slope of curve

max slope (c)

min slope (b)

Section - B (Numerical Value)

21. 8

Sol. $x = A\sin(\omega t)$

$$U_{(x)} = \frac{1}{2}kx^2,$$

$$\frac{dU}{dt} = \frac{1}{2}k2x\frac{dx}{dt}$$

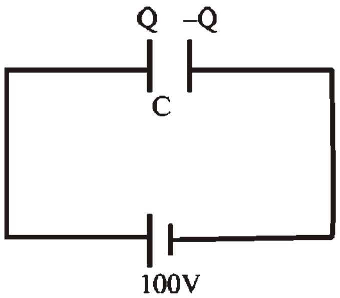
$$= kA^2\omega \sin \omega t \cos \omega t \times \frac{2}{2}$$

$$\left(\frac{dU}{dt}\right)_{max} = \frac{kA^2\omega}{2}(\sin 2\omega t)_{max}$$

$$2\omega t = \frac{\pi}{2} \Rightarrow t = \frac{\pi}{4}\omega = \frac{T}{8} \Rightarrow \beta = 8$$

22. 225

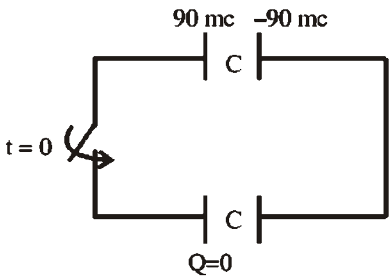
Sol.

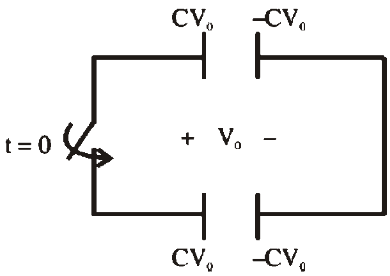


$C = 900\ \mu F$

$Q = CV = 900 \times 10^{-6} \times 100 = 9 \times 10^{-2} = 90$ MC

Now



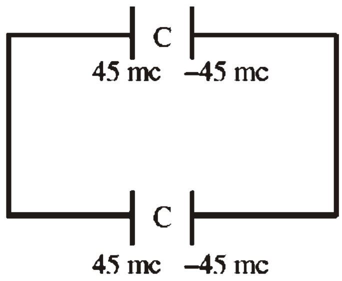


Common potential will be developed across both capacitors by kVL

Total charge on left plates of capacitors should be conserved.

$\therefore$ 90 mc + 0 = $2cv_0$

cv_0 = 45 mc



Heat dissipated = $U_i - U_f$ [Change in energy stored in the capacitors]

$$= \frac{1}{2}\frac{(90mc)^2}{900\mu F} - 2\times\frac{1}{2}\frac{(45mc)^2}{900\mu F}\left[U = \frac{Q^2}{2c}\right]$$

$$= \frac{1}{2\times 900\times 10^{-6}}(8100 - 4050)\times 10^{-6}$$

$= 2.25$ Joule

OR

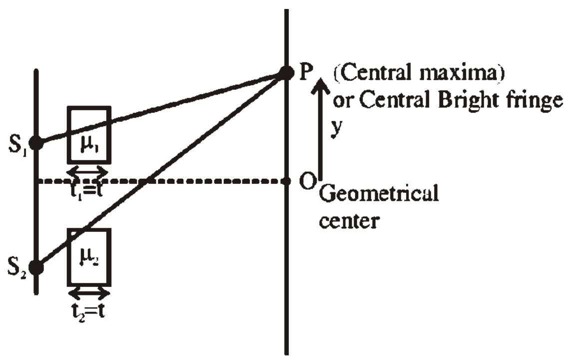
$$\text{Heat} = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

$$= \frac{1}{2}\frac{C^2}{2C}(100 - 0)^2$$

$$= \frac{1}{2}\frac{900\times 10^{-6}}{2}\times 10^4 = \frac{9}{4}\text{ Joule} = 2.25\text{ Joule}$$

23. 10

Sol.



Path difference at P be Δx

$\Delta x = (\mu_2 - \mu_1)t$

$= (1.55 - 1.51)0.1$ mm

$= 0.04 \times 10^{-4}$

$\Delta x = 4 \times 10^{-6} = 4\ \mu m$

$$y = \frac{\Delta x D}{d} = 4 \times 10^{-6}\frac{D}{d}$$

{y is the distance of central maxima from geometric center}

$$\text{fringe width } = \frac{\lambda D}{d} = 4 \times 10^{-6} m \frac{D}{d} = 4\mu m \frac{D}{d}$$
(β)

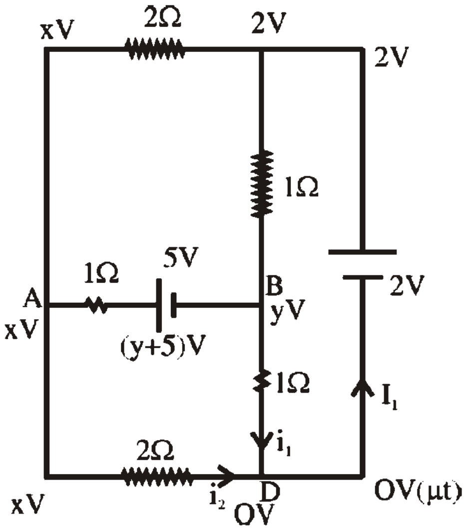
$\therefore$ Central bright fringe spot will shift by 'x'

$$\text{Number of shift } = \frac{y}{\beta}$$

$$= \frac{4 \times 10^{-6} D/d}{4 \times 10^{-7} D/d} = 10 \text{ Ans}$$

24. 1.5

Sol.



Junction law at A,

$$\frac{x - (y+5)}{1} + \frac{x-2}{2} + \frac{x-0}{2} = 0 \qquad \text{......(1)}$$

Junction law at B,

$$\frac{y+5-x}{1} + \frac{y-0}{1} + \frac{y-2}{1} = 0 \qquad \text{.....(2)}$$

On solving equation (A) and Equation (B)

$x = 3$

& $y = 0$

At D junction

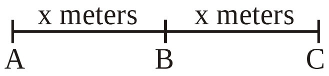
$I_1 = i_1 + i_2$

$$I_1 = \frac{y-0}{1} + \frac{x-0}{2} = \frac{0-0}{1} + \frac{3-0}{2}$$

$I_1 = 1.5$ A

25. 50

Sol.



$$t_{AB} = \frac{x}{5 \text{ m/s}}$$

In motion BC

$x = d_1 + d_2$

where d_1 & d_2 we the distance travelled with 10 m/s and 15 m/s respectively in equal time intervals

$$\frac{'t'}{2} \text{ each}$$

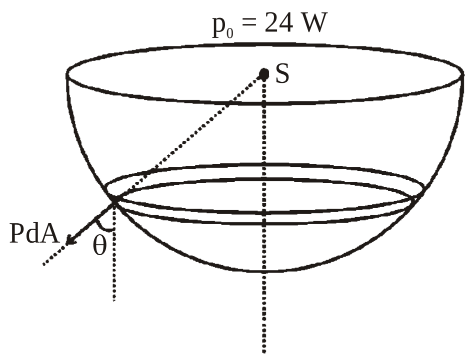
$$d_1 = \frac{10t}{2}, d_2 = \frac{15t}{2}$$

$$d_1 + d_2 = x = \frac{t}{2}(10+15) = \frac{25t}{2}$$

$$< v >= \frac{2x}{\frac{x}{5} + \frac{2x}{25}} = \frac{2 \times 25}{5+2} = \frac{50}{7} \text{ m/s}$$

26. 4

Sol.



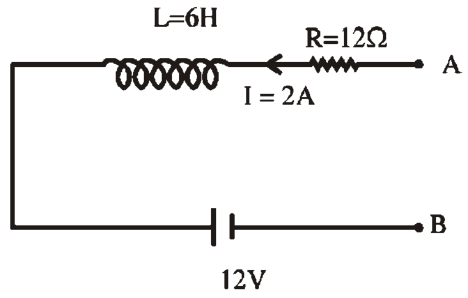
$$\text{Force } = \int PdA\cos\theta$$

$$= \frac{2I}{C}\int dA\cos\theta = \frac{2I}{C}\pi R^2 = 2\frac{p_0}{4\pi R^2} \cdot \frac{\pi R^2}{C}$$

$$=\frac{p_0}{2C}=\frac{24}{2\times3\times10^8}=4\times10^{-8}\,N$$

27. 30

Sol.



$$\frac{dI}{dt}=-1\frac{A}{sec}$$

$$V_A-IR-L\frac{dI}{dt}-12=V_B$$

$V_A - 2 \times 12 - 6(-1) - 12 = V_B$

$V_A - V_B = 36 - 6 = 30$ volt

28. 22

Sol. $\text{Least count}=\frac{\text{Pitch}}{\text{No. of circular divisions}}$

$$=\frac{0.5\text{mm}}{100}$$

Least count = 5×10^{-3} mm

Positive Error = MSR + CSR (LC)

= 0 mm + 6 (5×10^{-3} mm)

Reading of Diameter = MSR + CSR (LC) – Positive zero error

= 4×0.5 mm + $(46(5 \times 10^{-3})) - 6(5 \times 10^{-3})$ mm

= 2 mm + $40 \times 5 \times 10^{-3}$ mm = 2.2 mm

29. 32

Sol. $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\frac{-1}{120}-\frac{1}{40}=\frac{1}{f},\quad f=-30\text{ cm}$$

Now,

$$\frac{-1}{v^2}dv-\frac{1}{u^2}du=-\frac{1}{f^2}df$$

Also $dv=du=\frac{1}{20}\text{cm}$

$$\therefore\frac{\frac{1}{20}}{(120)^2}+\frac{\frac{1}{20}}{(40)^2}=\frac{df}{(30)^2}$$

On solving

$$df=\frac{1}{32}\text{cm}$$

$\therefore$ k = 32

30. 3

Sol. $(KE)_{Rotational}=\frac{1}{2}I\omega^2=E$

$$E=\frac{1}{2}\frac{ml^2}{12}\omega^2$$

$$E=\frac{1}{2}\frac{dAl^3}{12}\omega^2$$

$$E=\frac{dA(2)^3}{24}\omega^2$$

$$\sqrt{\frac{3E}{dA}}=\omega$$

$\alpha = 3$

CHEMISTRY

Section - A (Single Correct Answer)

31. D

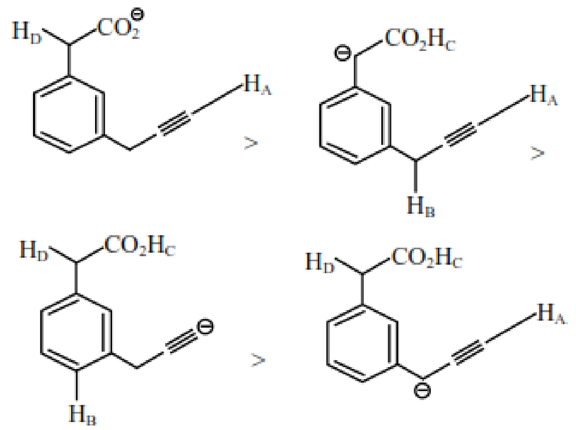
Sol. Aromatic aldehydes do not give Fehling's test.

Both nitrogen and sulfur must be present to obtain blood red colour.

Sodium nitroprusside gives blood red colour with S and N.

32. B

Sol. Acidity of an acid depends upon the stability of its conjugate base.



33. C

Sol. Seliwanoff 's test is a differentiating test for Ketose and aldose. This test relies on the principle that the keto hexose are more rapidly dehydrated to form 5-hydroxy methyl furfural when heated in acidic medium which on condensation with resorcinol, Cherry red or brown red coloured complex is formed rapidly indicating a positive test.

34. D

Sol. The copper ore contains iron, it is mixed with silica before heating in reverberatory furnace. FeO slags off as $FeSiO_3$.

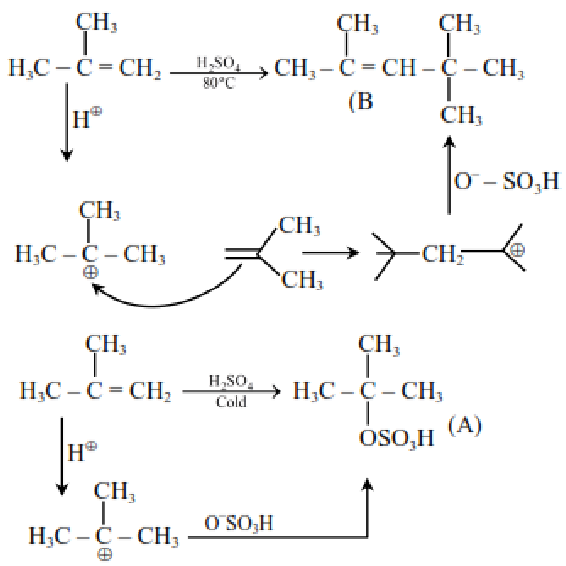
$$FeO + SiO_2 \longrightarrow FeSiO_3$$

35. A

Sol. 1. Ranitidine : Antacid
2. Meprobamate : Tranquilizer
3. Terfenadine : Antihistamine
4. Brompheniramine : Antihistamine

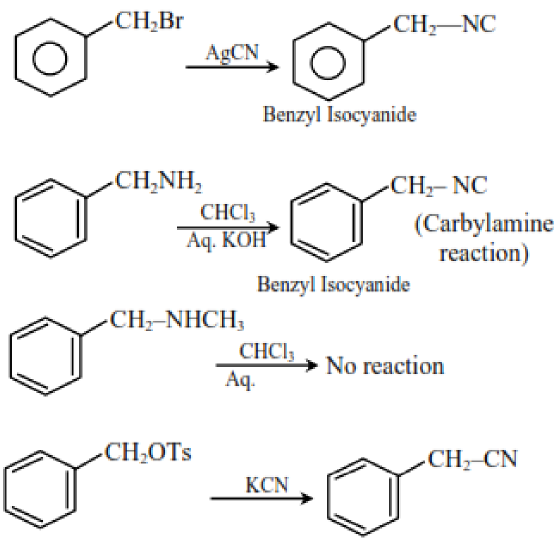
36. A

Sol.



37. C

Sol.

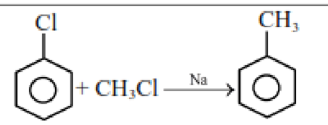
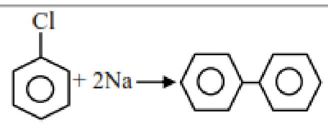
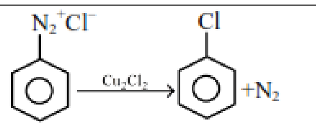


38. C

Sol. Silica gel prevents water corrosion (rusting) and instrument malfunction by adsorbing moisture from the air.

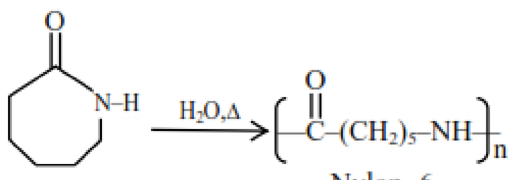
39. D

Sol.

	LIST-I	LIST-II
A.		Wurtz-fitting reaction
B.		Fitting reaction
C.		Sandmeyer reaction
D.	$C_2H_5Cl + NaI \rightarrow C_2H_5I + NaCl$	Finkelstein reaction

40. D

Sol.



Caprolactam Nylon -6

41. C

Sol. Due to high hydration energy Be^{2+} and Mg^{2+}, $BeSO_4$ and $MgSO_4$ are readily soluble in water.

42. B

Sol. The increasing order of field strength of ligands (according to spectrochemical series).

$$S^{2-} < C_2O_4^{2-} < NH_3 < en < CO$$

43. D

Sol. $NO_{2g} \xrightarrow{hv} \underset{(A)}{NO_g} + \underset{(B)}{O_g}$

$$\underset{(B)}{O_g} + O_{2g} \rightleftharpoons \underset{(C)}{O_{3g}}$$

$$\underset{(A)}{NO_g} + \underset{(C)}{O_{3g}} \longrightarrow NO_{2g} + O_{2g}$$

44. C

Sol. $Cr_2O_7^{2-} + SO_3^{2-} \xrightarrow{H^+} \underset{\text{Green}}{Cr^{3+}} + SO_4^{2-}$

45. A

Sol. *cis*–Platin is used in chemotherapy to inhibits the growth of tumors.
(*cis*[Pt$(NH_3)_2Cl_2$])

46. A

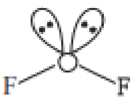
Sol. $Zn^{2+}, Co^{2+}, Ni^{2+}$ = IVth Group
Fe^{3+} = IIIrd Group

47. B

Sol.

IF_7	zero lone pair
ICl_4^-	two lone pair
XeF_6	one lone pair
XeF_2	three lone pair

48. D

Sol. 

- Two lone pair one oxygen
- Molecule is 'v' shaped
- Bond angle is less than 104.5^o(102^o)
- O·S· of 'O' is +2

49. B

Sol. $8LiH + Al_2Cl_6 \longrightarrow 2LiAlH_4 + 5LiCl$

50. D

Sol.

Atomic number	Block
37 (K)	s-block
78 (Pt)	d-block
52 (Te)	p-block
65 (Tb)	f-block

Section - B (Numerical Value)

51. 10

Sol. $Pt_{(s)} | H_2(g, 1\text{ atm}) | H^+(aq, 1\text{ M}) || Fe^{3+}(aq), Fe^{2+}(aq) | Pt(s)$

at anode $H_2 \longrightarrow 2H^+ + 2e^-$

at cathode $Fe^{3+}_{aq} + e^- \longrightarrow Fe^{2+}_{aq}$

$E^o = E^o_{H_2|H^+} + E^o_{Fe^{3+}|Fe^{2+}} = 0.771\text{ V}$

$$E = E^o - \frac{0.06}{1}\log\frac{Fe^{2+}}{Fe^{3+}}$$

$$0.712 = (0 + 0.771) - \frac{0.06}{1}\log\frac{Fe^{2+}}{Fe^{3+}}$$

$$\log\frac{Fe^{2+}}{Fe^{3+}} = \frac{0.059}{0.06} \approx 1$$

$$\boxed{\frac{Fe^{2+}}{Fe^{3+}} = 10}$$

52. 1362

Sol. Mole of CO_2 = 0.2 M × (300 × 10^{-3}) L
= 0.06 Mole
Volume of 0.06 mole CO_2 at S.T.P
= 0.06 × 22.7
= 1.362 L

53. 100

Sol. $\Delta T_b = 373.52 - 373$
$= 0.52$
$\Delta T_b = Kb \cdot m$

54. 623

$A + B \rightarrow P$

Sol.

t = 0	7g
t = t	2g

at constant volume

$$t = \frac{2.303}{K}\log\frac{[A]_0}{[A]_t}$$

$$= \frac{2.303}{2.011\times10^{-3}}\log\frac{7}{2}$$

$$= \frac{2.303\times0.544}{2.011\times10^{-3}}$$

$= 622.989$

≈ 623

55. 798

Sol. For one photon E = hv
For one mole photon,

$$E = 6.023\times10^{23}\times6.626\times10^{-34}\times2\times10^{12}$$

$= 798.16$ J

≈ 798 J

56. 3

Sol. $\overset{+7}{Mn}O_4^- + 4H^+ + 3e^- \longrightarrow \overset{+4}{Mn}O_2 + 2H_2O$

57. 0

Sol. For ideal gas U = f (T)

and for isothermal process, $\Delta U = 0$

58. 186

Sol. Total milimoles of H^+

$= (600 \times 0.01) + (400 \times 0.01 \times 2)$

$= 14$

$[H^+] = \dfrac{14}{1000} = 14 \times 10^{-3}$

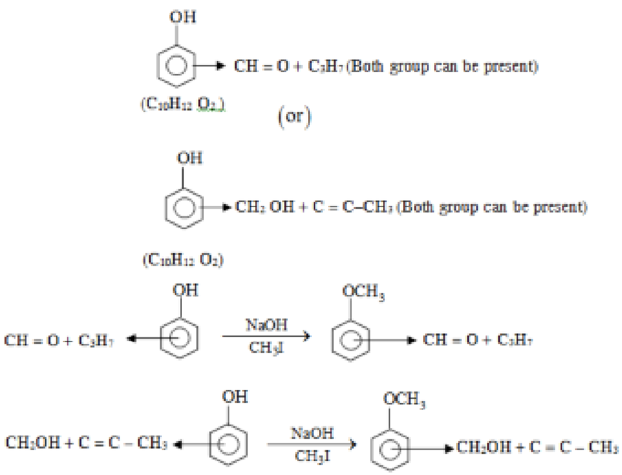
pH $= 3 - \log 14$

$= 1.86$

$= 186 \times 10^{-2}$

59. 4

Sol.



60. 221

Sol. Molarity $= \dfrac{\text{mole}}{\text{volume}}$

$2.6 \times 10^{-3} = \dfrac{x/85}{0.67141}$

$x = 0.148$ g

conc. Fo DCM in ppm

$= \dfrac{0.148}{1.49 \times 671.141} \times 10^6$

$= 148$ ppm

MATHEMATICS

Section - A (Single Correct Answer)

61. A

Sol. $A = \begin{bmatrix} m & n \\ p & q \end{bmatrix}$, $|A - d(\text{adj}A)| = 0$

$\Rightarrow |A - d(\text{adj}A)| = \left| \begin{bmatrix} m & n \\ p & q \end{bmatrix} - d\begin{bmatrix} q & -n \\ -p & m \end{bmatrix} \right|$

$= \begin{vmatrix} m - qd & n(1+d) \\ p(1+d) & q - md \end{vmatrix} = 0$

$\Rightarrow (m - qd)(q - md) - np(1 + d)^2 = 0$

$\Rightarrow mq - m^2d - q^2d + mqd^2 - np(1 + d)^2 = 0$

$\Rightarrow (mq - np) + d^2 (mq - np) - d (m^2 + q^2 + 2np) = 0$

$\Rightarrow d + d^3 - d((m + q)^2 - 2d) = 0$

$\Rightarrow 1 + d^2 = (m + q)^2 - 2d$

$\Rightarrow (1 + d)^2 = (m + q)^2$

$\therefore$ Option (A) is correct.

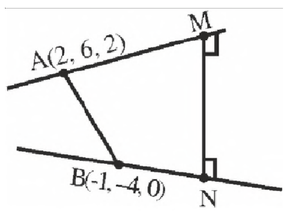
62. D

Sol. Line ℓ_1 is given by

$L_1 : \dfrac{x-2}{2} = \dfrac{y-6}{1} = \dfrac{z-2}{-2}$

Given,

$L_2 : \dfrac{x+1}{2} = \dfrac{y+4}{-3} = \dfrac{z}{2}$



Shortest distance $= \left| \dfrac{\overrightarrow{AB} \cdot \overrightarrow{MN}}{|\overrightarrow{MN}|} \right|$

$\overrightarrow{AB} = 3\hat{i} + 10\hat{j} + 2\hat{k}$

$$\overline{MN} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 2 & -3 & 2 \end{vmatrix} = -4\hat{i} - 8\hat{j} - 8\hat{k}$$

$$= MN = \sqrt{16+64+64} = 12$$

$$\therefore \text{ Shortest distance } = \left|\frac{-12-80-16}{12}\right| = 9$$

$\therefore$ Option (D) is correct.

63. B

Sol. Either all outcomes are positive or any two are negative.

Now, p = P (positive) $= \frac{3}{6} = \frac{1}{2}$

q = p (negative) $= \frac{2}{6} = \frac{1}{3}$

Required probability

$$= {}^5C_2\left(\frac{1}{2}\right)^5 + {}^5C_2\left(\frac{1}{3}\right)^2\left(\frac{1}{2}\right)^3 + {}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{1}{2}\right)^1$$

$$= \frac{521}{2592}$$

$\therefore$ Option (B) is correct

64. D

Sol. $\begin{vmatrix} 1 & 1 & k \\ 2 & 3 & -1 \\ 3 & 4 & 2 \end{vmatrix} = 0$

$\Rightarrow$ 1(10) – 1(7) + k(–1) – 0

$\Rightarrow$ k = 3

For k = 3, 2nd system is

4x + 5y = 7 .....(1)

and 7x + 8y = 10 .....(2)

Clearly, they have a unique solution

(2) – (1) $\Rightarrow$ 3x + 3y = 3

$\Rightarrow$ x + y = 1

65. A

Sol. $\tan 15^\circ = 2 - \sqrt{3}$

$$\frac{1}{\tan 75^\circ} = \cot 75^\circ = 2 - \sqrt{3}$$

$$\frac{1}{\tan 105^\circ} = \cot(105^\circ) = -\cot 75^\circ = \sqrt{3} - 2$$

$$\tan 195^\circ = \tan 15^\circ = 2 - \sqrt{3}$$

$$\therefore 2(2-\sqrt{3}) = 2a \Rightarrow a = 2 - \sqrt{3}$$

$$\Rightarrow a + \frac{1}{a} = 4$$

66. C

Sol. 5f(x + y) = f(x)·f(y)

$5f(0) = f(0)^2 \Rightarrow f(0) = 5$

5f(x + 1) = f(x)·f(1)

$$\Rightarrow \frac{f(x+1)}{f(x)} = \frac{f(1)}{5}$$

$$\Rightarrow \frac{f(1)}{f(0)} \cdot \frac{f(2)}{f(1)} \cdot \frac{f(3)}{f(2)} = \left(\frac{f(1)}{5}\right)^3$$

$$\Rightarrow \frac{320}{5} = \frac{(f(1))^3}{5^3} \Rightarrow f(1) = 20$$

$\therefore$ 5f(x + 1) = 20·f(x) $\Rightarrow$ f(x + 1) = 4f(x)

$$\sum_{n=0}^{5} f(n) = 5 + 5.4 + 5.4^2 + 5.4^3 + 5.4^4 + 5.4^5$$

$$= \frac{5[4^{-6} - 1]}{3} = 6825$$

67. C

Sol. If $a_n = \frac{-2}{4n^2 - 16n + 15}$ then $a_1 + a_2 + \ldots\ldots a_{25}$

$$\Rightarrow \sum_{n=1}^{25} a_n = \sum \frac{-2}{4n^2 - 16n + 15}$$

$$= \sum \frac{-2}{4n^2 - 6n - 10n + 15}$$

$$= \sum \frac{-2}{2n(2n-3) - 5(2n-3)}$$

$$= \sum \frac{-2}{(2n-3)(2n-5)}$$

$$= \sum \frac{1}{2n-3} - \frac{1}{2n-5}$$

$=\frac{1}{47}-\frac{1}{(-3)}$

$=\frac{50}{141}$

68. B

Sol. Coefficient of x^{15} in $\left(ax^3+\frac{1}{bx^{1/3}}\right)^{15}$

$T_{r+1} = {}^{15}C_r(ax^3)^{15-r}\left(\frac{1}{bx^{1/3}}\right)^r$

$45-3r-\frac{r}{3}=15$

$30=\frac{10r}{3}$

r = 9

Coefficient of $x^{15} = {}^{15}C_9\, a^6 b^{-9}$

Coefficient of x^{-15} in $\left(ax^{1/3}-\frac{1}{bx^3}\right)^{15}$

$T_{r+1} = {}^{15}C_r(ax^{1/3})^{15-r}\left(-\frac{1}{bx^3}\right)^r$

$5-\frac{r}{3}-3r=-15$

$\frac{10r}{3}=20$

r = 6

Coefficient = ${}^{15}C_6\, a^9 \times b^{-6}$

$\Rightarrow \frac{a^9}{b^6}=\frac{a^6}{b^9}$

$\Rightarrow a^3b^3 = 1 \Rightarrow ab = 1$

69. C

Sol. $\hat{n}\perp\vec{c}$ $\quad\quad$ $\vec{a}=\alpha\vec{b}-\vec{n}$

$\vec{b}\cdot\vec{c}=12$

$\vec{a}\cdot\vec{c}=\alpha(\vec{b}\cdot\vec{c})-\vec{n}\cdot\vec{c}$

$\vec{a}\cdot\vec{c}=\alpha(\vec{b}\cdot\vec{c})$

$|\vec{c}\times(\vec{a}\times\vec{b})|=|(\vec{c}\cdot\vec{b})\vec{a}-(\vec{c}\cdot\vec{a})\vec{b}|$

$=|(\vec{c}\cdot\vec{b})\vec{a}-\alpha(\vec{b}-\vec{c})\vec{b}|$

$=|(\vec{c}\cdot\vec{b})||\vec{a}-\alpha\vec{b}|$

$=12\times(|\vec{n}|)$

= 12 × 1

= 12

70. C

Sol. Normal of line is parallel to line x + 90y + 2 = 0

$m_N=-\frac{1}{90}$

$-\left(\frac{dx}{dy}\right)_{(x_1y_1)}=-\frac{1}{90}\Rightarrow\left(\frac{dy}{dx}\right)_{(x_1y_1)}=90$

Now,

$\frac{dy}{dx}=270x^4-540x^3-210x^2+360x+210=90$

$\Rightarrow x=1,\ 2,\ \frac{-2}{3},\ \frac{-1}{3}$

(4) normals

71. A

Sol. L_1 : y = x + 2, L_2 : 4y = 3x + 6, L_3 : 3y = 4x + 1

Bisector of lines L_2 & L_3

$\frac{4x-3y+1}{5}=\pm\left(\frac{3x-4y+6}{5}\right)$

(+) 4x – 3y + 1 = 3x – 4y + 6 x + y = 5

Centre lies on Bisector of 4x – 3y + 1 = 0 &

(0) 3x - 4y + 6 = 0

$\Rightarrow$ h + k = 5

72. A

Sol. $\frac{dy}{dx}+\left(\frac{-3x^5\tan^{-1}x^3}{(1+x^6)^{3/2}}\right)y=2e^{\left\{\frac{x-\tan x}{\sqrt{1+x^6}}\right\}}$

$I.F.=e^{\int\frac{-3x^5\tan^{-1}x^3}{(1+x^6)^{3/2}}dx}$

$=e^{\frac{\tan^{-1}x^3-x^3}{\sqrt{1+x^6}}}$

Solution of differential equation

$y\cdot e^{\frac{\tan^{-1}x^3-x^3}{\sqrt{1+x^6}}}=\int 2xe^{\left(\frac{x^3-\tan^{-1}x^3}{\sqrt{1+x^6}}\right)}\cdot e^{\left(\frac{\tan^{-1}(x^3)-x^3}{\sqrt{1+x^6}}\right)}dx$

$= \int 2x dx + c$

$y \cdot e^{\frac{\tan^{-1}x^3 - x^3}{\sqrt{1+x^6}}} = x^2 + c$

Also it passes through origin

c = 0

$y(1) \cdot e^{\frac{\tan^{-1}(1)-1}{\sqrt{2}}} = 1$

$y(1) \cdot e^{\frac{\frac{\pi}{4}-1}{\sqrt{2}}} = 1$

$y(1) \cdot e^{\frac{\pi-4}{4\sqrt{2}}} = 1$

$y(1) = \frac{1}{e^{\frac{\pi-4}{4\sqrt{2}}}} = e^{\frac{4-\pi}{4\sqrt{2}}}$

73. A

Sol. Equation of plane :

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 1 & 1 \\ 0 & 3 & 4 \end{vmatrix} = 0$$

$\Rightarrow$ [x – 1] –4 [y – 2] + 3[z – 3] = 0

$\Rightarrow$ x – 4y + 3z = 2

D.R's of normal of plane < 1, –4, 3 >

D.C's of $\left\langle \pm\frac{1}{\sqrt{26}}, \ \mp\frac{4}{\sqrt{26}}, \ \pm\frac{3}{\sqrt{26}} \right\rangle$

$\cos\beta = \frac{4}{\sqrt{26}}$

$\cos\alpha = \frac{-1}{\sqrt{26}} \qquad \frac{\pi}{2} < \alpha < \pi$

$\cos\gamma = \frac{-3}{\sqrt{26}} \qquad \frac{\pi}{2} < \gamma < \pi$

Ans. (A)

74. B

Sol. $\int_1^2 x^2 e^{[x^3]+1} dx$

$x^3 = t$

$3x^2\, dx = dt$

$= \frac{e}{3}\int_1^8 e^{[x]} dt$

$= \frac{e}{3}\left\{\int_1^2 e\, dt + \int_2^3 e^2 dt + ... + \int_7^8 e^7 dt\right\}$

$= \frac{e}{3}(e + e^2 + ... + e^7)$

$= \frac{e^2}{3}(1 + e + ... + e^6) = \frac{e^2}{3}\frac{(e^7-1)}{(e-1)}$

$\frac{3(e-1)}{e}\int_1^2 x^2 \times e^{|x|+|x^3|} dx = \frac{3}{e}(e-1) \times \frac{e^2}{3}\frac{(e^7-1)}{(e-1)}$

$= e\,(e^7 - 1)$

$= e^8 - e$

Ans : (B)

75. D

Sol. Equation of normal

$y = -tx + 2\,at + at^3$

$y = -tx + \frac{2}{16}t + \frac{1}{16}t^3$

It passes through (0, 33)

$33 = \frac{1}{8} + \frac{t^3}{16}$

$t^3 + 2t - 528 = 0$

$(t - 8)(t^2 + 8t + 66) = 0$

t = 8

$P(at^2, \ 2at) = \left(\frac{1}{16} \times 64, \ 2 \times \frac{1}{16} \times 8\right) = (4, \ 1)$

Parabola :

$y^2 = 4(x + y)$

$\Rightarrow y^2 - 4y = 4x$

$\Rightarrow (y - 2)^2 = 4(x + 1)$

Equation of directix :-

x + 1 = –1

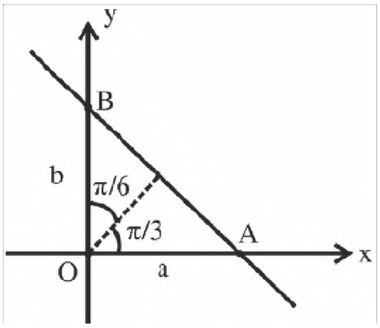
x = –2

Distance of point = 6

Ans. : (D)

76. A

Sol.



Equation of straight line : $\frac{x}{a}+\frac{y}{b}=1$

Or $x\cos\frac{\pi}{3}+y\sin\frac{\pi}{3}=p$

$\frac{x}{2}+\frac{y^3}{2}=p$

$\frac{x}{3p}+\frac{y}{2p}=1$

Comparing both : $a=2p,\ b=\frac{2p}{\sqrt{3}}$

Now are of $\Delta OAB=\frac{1}{2}\cdot ab=\frac{98}{3}\cdot\sqrt{3}$

$\frac{1}{2}\cdot 2p\cdot\frac{2p}{\sqrt{3}}=\frac{98}{3}\cdot\sqrt{3}$

$p^2 = 49$

$a^2-b^2=4p^2-\frac{4p^2}{3}=\frac{2}{3}4p^2$

$=\frac{8}{3}\cdot 49=\frac{392}{3}$

77. D

Sol. $(1 + x)^{500} + x(1 + x)^{499} + x^2(1 + x)^{498} + \ldots\ldots + x^{500}$

$=(1+x)^{500}\cdot\frac{((1+x)^{501}-x^{501})}{(1+x)^{501}}\cdot(1+x)$

$= (1 + x)^{501} - x^{501}$

Coefficient of x^{301} in $(1 + x)^{501} - x^{501}$ is given by ${}^{501}C_{301} = {}^{501}C_{200}$

78. B

Sol. $S_1\equiv((p\vee q)\Rightarrow r)\Leftrightarrow(p\Rightarrow r)$

p	q	r	$p\vee q$	$(p\vee q)\Rightarrow r$	$p\Rightarrow r$	$((p\vee q)\Rightarrow r)\Leftrightarrow(p\Rightarrow r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	T	T	T	T
F	T	T	T	T	T	T
T	F	F	T	F	F	T
F	T	F	T	F	T	T
F	F	F	T	T	T	T
F	F	F	F	T	T	T

$S_2\equiv(p\vee q)\Rightarrow r\Leftrightarrow((p\Rightarrow r)\vee(q\Rightarrow r))$

p	q	r	$(p\vee q)\Rightarrow r$	$p\Rightarrow r$	$q\Rightarrow r$	$(p\Rightarrow r\vee(q\Rightarrow r))$	S2
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	T
T	F	T	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	F	T	T	T	F
F	T	F	F	F	F	T	F
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

S2 $\rightarrow$ not a tautology

79. B

Sol. For Symmetric (a, b), (b, c) $\in$ R

$\Rightarrow (b,\ a),\ (c,\ b)\in R$

For Transitive $(a,\ b),\ (b,\ c)\in R$

$\Rightarrow (a,\ c)\in R$

Now

1. Symmetric

$\therefore (a,\ c)\in R\Rightarrow(c,\ a)\in R$

2. Transitive

$\therefore (a,\ b),\ (b,\ a)\in R$

$\Rightarrow (a,\ a)\in R\ \&\ (b,\ c),\ (c,\ b)\in R$

$\Rightarrow (b,\ b)\ \&\ (c,\ c)\in R$

$\therefore$ Element to be added

{(b, a), (c, b), (a, c), (c, a), (a, a), (b, b), (c, c)}

Number of elements to be added = 7

80. D

Sol. $\log_{\cos x}\cot x + 4\log_{\sin x}\tan x = 1$

$\Rightarrow\frac{\ln\cos-\ln\sin x}{\ln\cos x}+4\frac{\ln\sin x-\ln\cos x}{\ln\sin x}=1$

$\Rightarrow (\ln\sin x)^2 - 4(\ln\sin x)(\ln\cos x)$

$+ 4 (\ln \cos x)^2 = 1$

$\Rightarrow \ln \sin x = 2 \ln \cos x$

$\Rightarrow \sin^2 x + \sin x - 1 = 0 \Rightarrow \sin x = \dfrac{-1+\sqrt{5}}{2}$

$\therefore \alpha + \beta = 4$

Correct option (D)

Section - B (Numerical Value)

81. 3240

Sol. Let S ={1, 2, 3, 4, 5, 6}, then the number of one-one functions, $f : S \to P(S)$, where P(S) denotes the power set of S, such that $f(n) \subset f(m)$ where $n < m$ is

(S) = 6

$$P(S) = \begin{Bmatrix} \phi, \{1\},......\{6\}, \{1, 2\},.... \\ \{5, 6\},........, \{1, 2, 3, 4, 5, 6\} \end{Bmatrix}$$

– 64 elements

Case - 1

f (6) = S i.e. 1 option,

f (5) = any 5 element subset A of S i.e. 6 options,
f (4) = any 4 element subset B of A i.e. 5 options,
f (3) = any 3 element subset C of B i.e. 4 options,
f (2) = any 2 element subset D of C i.e. 3 options,
f (1) = any 1 element subset E of D or empty subset i.e. 3 options,

Total functions = 1080 Case - 2

f (6) = any 5 element subset A of S i.e. 6 options,
f (5) = any 4 element subset B of A i.e. 5 options,
f (4) = any 3 element subset C of B i.e. 4 options,
f (3) = any 2 element subset D of C i.e. 3 options,
f (2) = any 1 element subset E of D i.e. 2 options,
f (1) = empty subset i.e. 1 option

Total functions = 720

Case - 3

f(6)= S

f(5) = any 4 element subset A of S i.e. 15 options,
f(4) = any 3 element subset B of A i.e. 4 options,
f(3) = any 2 element subset C of B i.e. 3 options,
f(2) = any 1 element subset D of C i.e. 2 options,
f(1) = empty subset i.e. 1 option

Total functions = 360

Case - 4 f(6) = S

f(5) = any 5 element subset A of S i.e. 6 options,
f(4) = any 3 element subset B of A i.e. 10 options,
f(3) = any 2 element subset C of B i.e. 3 options,
f(2) = any 1 element subset D of C i.e. 2 options,
f(1) = empty subset i.e. 1 option

Total functions = 360

Case - 5 f(6) = S

f(5) = any 5 element subset A of S i.e. 6 options,
f(4) = any 4 element subset B of A i.e. 5 options,
f(3) = any 2 element subset C of B i.e. 6 options,
f(2) = any 1 element subset D of C i.e. 2 options,
f(1) = empty subset i.e. 1 option

Total functions = 360

Case - 6 f(6) = S

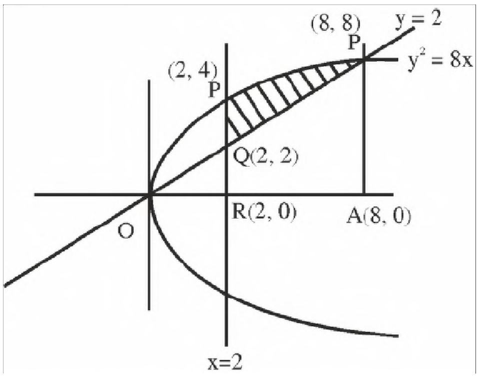
f(5) = any 5 element subset A of S i.e. 6 options,
f(4) = any 4 element subset B of A i.e. 5 options,
f(3) = any 3 element subset C of B i.e. 4 options,
f(2) = any 1 element subset D of C i.e. 3 options,
f(1) = empty subset i.e. 1 option

Total functions = 360

$\therefore$ Number of such functions = 3240

82. 22

Sol.



y = x

& $y^2 = 8x$

Solving it

$x^2 = 8x$

$\therefore x = 0, 8$

$\therefore y = 0, 8$

x = 2 will intersect occur at

$y^2 = 16 \quad \Rightarrow y = \pm 4$

$\therefore$ Area of shaded

$$= \int_2^8 (\sqrt{8x} - x)dx = \int_2^8 (2\sqrt{2}\sqrt{x} - x)dx$$

$$= \left[2\sqrt{2}\cdot\frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^8$$

$$= \left(\frac{4\sqrt{2}}{3}\cdot 2^{9/2} - 32\right) - \left(\frac{4\sqrt{2}}{3}\cdot 2^{9/2} - 2\right)$$

$$= \frac{128}{3}323\frac{16}{3} + 2 = \frac{112-90}{3} = \frac{22}{3} = A$$

$$\therefore 3A = 22$$

83. 315

Sol. $P_1 = \vec{r}\cdot(3\hat{i} - 5\hat{j} + \hat{k}) = 7$

$$P_2 = \vec{r}\cdot(\lambda\hat{i} + \hat{j} - 3\hat{k}) = 9$$

$$\theta = \sin^{-1}\left(\frac{2\sqrt{6}}{5}\right)$$

$$\Rightarrow \sin\theta = \frac{2\sqrt{6}}{5}$$

$$\therefore \cos\theta = \frac{1}{5}.$$

$$\cos\theta = \frac{\vec{r}\cdot\vec{r}}{|\vec{r}_1||\vec{r}_2|}$$

$$= \frac{(3i - 5j + K)(\lambda i + \hat{j} - 3K)}{\sqrt{35}\cdot\sqrt{\lambda^2 + 10}}$$

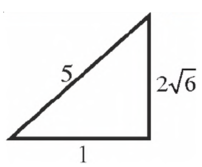
$$\frac{1}{5} = \left|\frac{3\lambda - 8}{\sqrt{35}\cdot\sqrt{\lambda^2+10}}\right|$$

Square $\Rightarrow \frac{1}{25} = \frac{9\lambda^2 + 64 - 48\lambda}{35(\lambda^2+10)}$

$$\Rightarrow 19\lambda^2 120\lambda + 125 = 0$$

$$\Rightarrow 19\lambda^2 - 95\lambda - 25\lambda + 125 = 0$$

$$\Rightarrow x = 5,\ \frac{25}{19}$$



Perpendicular distance of point

$(38\lambda_1,\ 10\lambda_2,\ 2) \equiv (50,\ 50,\ 2)$ from plane P_1

$$= \frac{|3\times 50 - 5\times 50 + 27|}{\sqrt{35}} = \frac{105}{\sqrt{35}}$$

Square $= \frac{105\times 105}{35} = 315$

84. 9

Sol. $z = 1 + i$

$$z_1 = \frac{1 + i\bar{z}}{\bar{z}(1-z) + \frac{1}{z}}$$

$$z_1 = \frac{1 + i(1-i)}{(1-i)(1-1i) + \frac{1}{1+i}}$$

$$= \frac{1 + i - i^2}{(1-i)(-i) + \frac{1-i}{2}}$$

$$= \frac{2+i}{\frac{3i-1}{2}} = \frac{4+2i}{3i-1}$$

$$= \frac{-(4+2i)(3i-1)}{(3i)^2 - (1)^2}$$

$$\text{Arg}(z_1) = \frac{3\pi}{4}$$

$$\therefore \frac{12}{\pi}\arg(z_1) = \frac{12}{\pi}\times\frac{3\pi}{4} = 9$$

85. 12

Sol. $48\lim_{x\to 0}\frac{\int_0^x \frac{t^3}{t^6}dt}{x^4}\left(\frac{0}{0}\right)$

Applying L'Hospitals Rule

$$48\lim_{x\to 0}\frac{x^3}{x^6+1}\times\frac{1}{4x^3}$$

$= 12$

86. 37

Sol. $\frac{x_1 + x_2 + ... + x_7}{7} = 8$

$\frac{x_1 + x_2 + x_3 .... + x_6 + 14}{7} = 8$

$\Rightarrow x_1 + x_2 + ... + x_6 = 42$

$\therefore \frac{x_1 + x_2 ..... + x_6}{6} = \frac{42}{6} = 7 = a$

$\frac{\Sigma x_i^2}{7} - 8^2 = 16$

$\Sigma xi^2 - 8^2 = 560$

$\Rightarrow x_1^2 + x_2^2 + ... + x_6^2 = 364$

$b = \frac{x_1^2 + x_2^2 + ... + x_6^2}{6} 7^2$

$= \frac{364}{6} - 49$

$b = \frac{70}{6}$

$a + 3b - 5 = 7 + 3 \times \frac{70}{6} - 5$

$= 37$

87. 15

Sol. $x - 3y + 2z - 1 = 0$

$4x - y + z = 0$

$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 4 & -1 & 1 \end{vmatrix}$

$= -\hat{i} + 7\hat{j} + 11\hat{k}$

Dr^s of normal to the plane is –1, 7, 11

Equation of plane :

$-1(x - 1) + 7(y - 1) + 11(z - 2) = 0$

$-x + 7y + 11z = 28$

$\frac{-1}{28}x + \frac{7y}{28} + \frac{11z}{28} = 1$

$Ax + By + Cz = 1$

$140(C - B + A) = 140\left(\frac{11}{28} - \frac{7}{28} - \frac{1}{28}\right)$

$= 140 \times \frac{3}{28} = 15$

88. 26

Sol. $\sum_{n=0}^{\infty} \frac{n^3((2n)!) + (2n-1)(n!)}{(n!)((2n)!)}$

$= \sum_{n=0}^{\infty} \frac{1}{(n-3)!} + \sum_{n=0}^{\infty} \frac{3}{(n-2)!}$

$+ \sum_{n=0}^{\infty} \frac{1}{(n-1)!} + \sum_{n=0}^{\infty} \frac{1}{(2n-1)!} - \sum_{n=0}^{\infty} \frac{1}{(2n)!}$

$= e + 3e + e + \frac{1}{2}\left(e - \frac{1}{e}\right) - \frac{1}{2}\left(e + \frac{1}{e}\right)$

$= 5e - \frac{1}{e}$

$\therefore a^2 - b + c = 26$

89. 21

Sol. For number to be divisible by 15, last digit should be 5 and sum of digits must be divisible by 3.

Possible combinations are

1	2	1	5

Number = 3

2	2	3	5

Number = 3

3	3	1	5

Number = 3

1	1	5	5

Number = 3

2	3	5	5

Number = 6

3	5	5	5

Number = 3

Total Numbers = 21

90. 3125

Sol. $f^1(x) = \frac{3x+2}{2x+3}$

$\Rightarrow f^2(x) = \frac{13x+12}{12x+13}$

$\Rightarrow f^3(x) = \frac{63x+62}{62x+63}$

$\therefore f^5(x) = \frac{1563x+1562}{1562x+1563}$

a + b = 3125

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