3S INSTITUTE

JEE ADVANCED | JEE MAIN | NEET | OLYMPIADS | MHT-CET | FOUNDATION

29-January-2023 (Evening Batch) : JEE Main Paper

PHYSICS

Section - A (Single Correct Answer)

1. A

Sol. $(N_0)A = \frac{320}{16} = 20$ moles

$(N_0)B = \frac{320}{32} = 10$ moles

$$N_A = \frac{(N_0)_A}{(2)^{2/1}} = \frac{20}{4} = 5$$

$$N_B = \frac{(N_0)_B}{(2)^{2/5}} = \frac{20}{2^4} = 0.625$$

Total N = 5.625

No. of atoms = $5.625 \times 6.023 \times 10^{23}$

$= 3.38 \times 10^{24}$

2. B

Sol. $$\sqrt{\frac{3RT}{M}} = \sqrt{\frac{\alpha+5}{\alpha}}\sqrt{\frac{8}{\pi}\frac{RT}{M}}$$

$$3 = \frac{\alpha+5}{\alpha}\frac{8}{\pi}$$

$\alpha = 28$

3. C

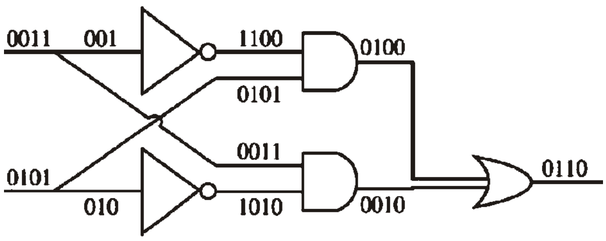
Sol. $$\frac{\lambda_\alpha}{\lambda_p} = \frac{\dfrac{h}{\sqrt{2m_\alpha q_\alpha V}}}{\dfrac{h}{\sqrt{2m_p q_p V}}}$$

$$\frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{1}{8}}$$

m = 8

4. B

Sol.



5. D

Sol. $a_1 = g\sin\theta = g/\sqrt{2}$

$$a_2 = g\sin\theta - Kg\cos\theta = \frac{g}{\sqrt{2}} - \frac{Kg}{\sqrt{2}}$$

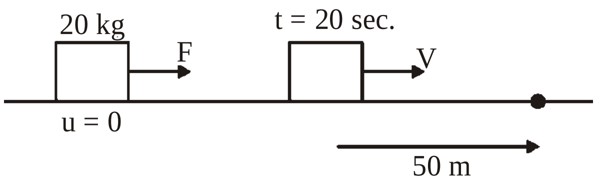
$t_2 = nt_1$ & $a_1 t_1^2 = a_2 t_2^2$

$$\frac{g}{\sqrt{2}}t_1^2 = \left(\frac{g}{\sqrt{2}} - \frac{kg}{\sqrt{2}}\right)n^2 t_1^2$$

$$K = 1 - \frac{1}{n^2}$$

6. B

Sol.



50 = V × 10

V = 5 m/s

V = 0 + a × 20

5 = a × 20

$$a = \frac{1}{4} m/s^2$$

$F = ma = 20 \times \frac{1}{4} = 5N$

7. D

Sol. $P_2A - P_1A = 5.4 \times 10^5 \times g$

$$P_2 - P_1 = \frac{5.4\times10^6}{500} = 5.4\times2\times10^2\times10 = 10.8 \times 10^3$$

$$P_2 + 0 + \frac{1}{2}\rho V_2^2 = P_1 + 0 + \frac{1}{2}\rho V_1^2$$

$$P_2 - P_1 = \frac{1}{2}\rho\left(V_1^2 - V_2^2\right) = \frac{1}{2}\rho\left(V_1 - V_2\right)\left(V_1 + V_2\right)$$

$$10.8\times10^3 = \frac{1}{2}\times1.2\left(V_1 - V_2\right)\times2\times3\times10^2$$

$10.8 \times 10 = 3.6\ (V_1 - V_2)$

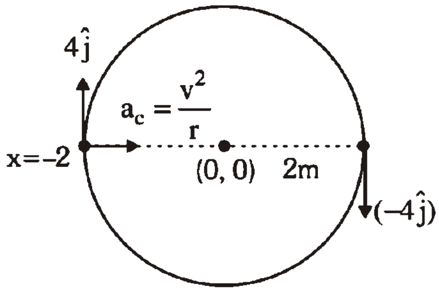
$V_1 - V_2 = 30$

$$\left(\frac{V_1 - V_2}{V}\right)\times100 = \frac{30}{300}\times100 = 10\%$$

8. A

9. B

Sol.



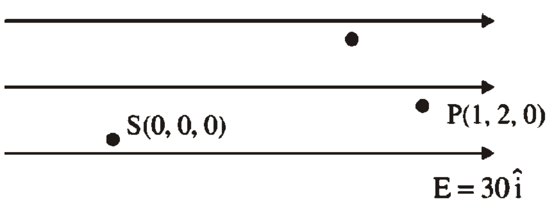
$$a_c = \frac{V^2}{r} = \frac{4^2}{2} = \frac{16}{2} = 8\ m/s^2$$

$$\vec{V} = 4\hat{j}$$

$$\vec{a}_c = 8\hat{i}$$

10. C

Sol.



$$\omega_E = q\vec{E}\cdot\vec{S}$$

$$= 2\times10^{-2}\left[30\hat{i}\cdot\left(-\hat{i}\right)\right]$$

$= 2 \times 10^{-2}\ (-30)$

$= -60 \times 10^{-2}$

$$= -\frac{60}{100} = -0.6\ J$$

$= -600$ mJ

11. B

Sol. μ = modulation index $= \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$

$$= \frac{14-6}{14+6} = 0.4$$

12. C

Sol. $B = \frac{\mu_0 i}{2R}\times4$

$$B' = \frac{\mu_0 i}{2R'}$$

$$R' = 4R$$

$$B' = \frac{\mu_0 i}{8R}$$

$$\frac{B'}{B} = \frac{1}{16}$$

$$B' = 2T$$

13. B

Sol. Sensitivity of potentiometer wire is inversely proportional to potential gradient.

14. B

Sol. $P = \frac{2\mu\sin\theta}{1.22\lambda}$

15. A

Sol. Statement-I is correct as EMW are neutral.

Statement-II is wrong.

$$E_0 = \sqrt{\frac{1}{\mu_0\varepsilon_0}}B_0$$

16. A

Sol. $\Delta Q = 184 \times 10^3$

m = 0.600 kg at – 12°C

S = 222.3 J/kg/°C

L = 336×10^3 J/kg

$Q_1 = 0.600 \times 2222.3 \times 12 = 16000.56$ J

Remaining heat

$\Delta Q_1 = 184000 - 16000.56 = 167999.44$ J

For meeting at 0°C

$\Delta Q_2 = 0.600 \times 336000 = 201600$ J needed

$\therefore$ 100% ice is not melted

Amount of ice melted

$167999.44 = m \times 336000 = 0.4999$ kg

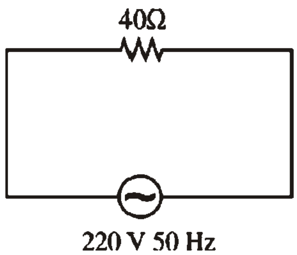
$\therefore$ mass of water = 0.4999 kg

Mass of ice = 0.1001

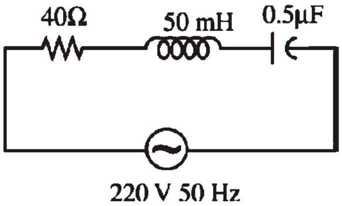
$$\therefore \text{Ratio} = \frac{0.1001}{0.4999} \approx 1:5$$

17. A

Sol.



$$I_{rms} = \frac{220}{40} = 5.5A$$



X_L is not equal to X_C. So rms current in (b) can never be larger than (a).

18. D

Sol. $T^2 \propto R^3$

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R}{\frac{R}{4}}\right)^3$$

$$\therefore \frac{T_1^2}{T_2^2} = 64$$

$$\therefore T_2^2 = \frac{T_1^2}{64}$$

$$\therefore T_2 = \frac{24}{8} = 3$$

19. B

Sol. $$(x - At)^2 + \left(y - \frac{t}{B}\right)^2 = a^2$$

$$[At] = A \times \frac{1}{T} = L$$

$$\therefore [A] = T^1 L^1$$

$\frac{t}{B}$ is in meters

$$\therefore \frac{1}{T[B]} = L$$

$$\therefore [B] = T^{-1} L^{-1}$$

20. B

Sol. l = 50 cm

t = 1 sec

$$\therefore V = \frac{0.05}{1} = 0.05 \text{ m/s}$$

$$i = \frac{40 \times 0.05 \times 0.05}{10} = 0.01 \text{ A}$$

$$\therefore F = B_i l = 40 \times 0.01 \times 0.05$$

F = 0.02 N

$\therefore$ W = 0.02 × l = 0.02 × .05

$\therefore$ W = 1 × 10^{-3} J

Section - B (Numerical Value)

21. 30

Sol. $$\frac{R_1 + R_2}{10} = \frac{60}{40} = \frac{3}{2} \Rightarrow R_1 + R_2 = 15$$

Now $$\frac{R_1 R_2}{(R_1 + R_2) \times 3} = \frac{40}{60} = \frac{2}{3} \Rightarrow R_1 R_2 = 30$$

22. 800

Sol. Use $\Delta L = \int_0^t \tau dt$

$$L_0 = \int_0^2 mg(v_x t) dt$$

$$= mgv_x \frac{t^2}{2} = (0.1)(10)(10\sqrt{2})\frac{2^2}{2}$$

$$= 20\sqrt{2} = \sqrt{800} \text{ kg m}^2/\text{s}$$

23. 41

Sol. $u = \frac{h}{h'} = \frac{5.25}{5.00}$

Least count $= \frac{1}{20}\text{cm} - \frac{49}{50}\cdot\frac{1}{20}\text{cm}$

$= \frac{1}{50}\times\frac{1}{20}\text{cm} = 0.01\text{ mm}$

In u = ln h – ln h′

$\frac{du}{u} = \frac{dh}{h} - \frac{dh'}{h'}$

$du = \left[\frac{0.01}{5.25}+\frac{0.01}{5.00}\right]\frac{5.25}{5.00} = \frac{41}{10}\times10^{-3}$

24. 12

Sol.

$E = -\frac{dV}{dr} = -4ar = \frac{\rho r}{3\varepsilon_0}$ (compare)

Result inside uniformly charged solid sphere.

$\rho = -12\ a\varepsilon_0$

$\lambda = 12$

25. 40

Sol. $v\frac{dv}{dx} = \frac{v^2}{R} \Rightarrow \int_{15}^{v}\frac{dv}{v} = \frac{1}{R}\int_0^x dx$

$v = 15\ e^{x/R}$

$\frac{dx}{dt} = 15e^{x/R}$

$\int_0^{\frac{\pi R}{2}} e^{-x/R}dx = 15\int_0^{t_0} dt$

$t_0 = 40\left(1-e^{-\pi/2}\right)$

26. 3872

Sol. $\frac{1}{2\pi fC} = 2\pi fL$

$C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\times\pi^2\times49\times10^6\times2\times10^{-6}}$

$C = \frac{1}{3872}F$

x = 3872

27. 25

Sol. $|F| = \eta A\frac{\Delta v}{\Delta h} : 0.1 = 5\times10^{-3}\times0.2\times\frac{v}{.25\times10^{-3}}$

$v = 0.025$ m/s or $v = 25\times10^{-3}$ m/s

28. 40

Sol. $\frac{1}{4}a = -24x$;

a = – 100 x

$\omega^2 = 100\ \omega = 10,$

$\omega A = 4$

$A = \frac{4}{10} = 0.4m$

A = 40 cm

29. 7

Sol. $\mu_1 = \sqrt{2.8\times1} = \sqrt{2.8}$

$\mu_2 = \sqrt{6.8\times1} = \sqrt{6.8}$

$\mu_1 \sin i = \mu_2 \cos i$

$\tan i = \frac{\mu_2}{\mu_1} = \sqrt{\frac{6.8}{2.8}}$

$\tan i = \left(\frac{2.8+4}{2.8}\right)^{1/2}$

$i = \tan^{-1}\left(1+\frac{10}{7}\right)^{1/2}$

$\theta = 7$

30. 5

Sol. $\frac{\varepsilon}{r+5}\times5 = 200x$ ....(1)

$\frac{\varepsilon\times15}{r+15} = 300x$ ....(2)

$\Rightarrow$ r = 5 Ω

CHEMISTRY

Section - A (Single Correct Answer)

31. B

Sol. The first ionization energies (as in NCERT) are as follows :

B : 801 kJ/mol

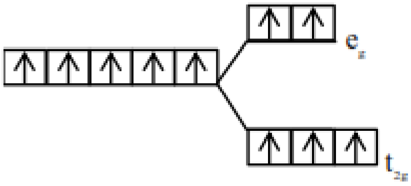
Al : 577 kJ/mol

Ga : 579 kJ/mol

Ga : $[Ar]3d^{10}\ 4s^2\ 4p^1$

32. A

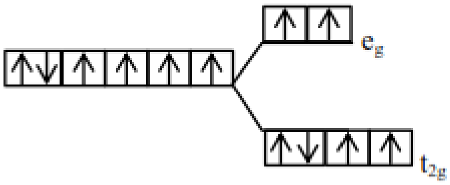
Sol. $[FeF_6]^{3-} : Fe^{3+} = 3d^5\ \Delta_O < P$



Number of unpaired e^- = 5

$\therefore\ \mu = \sqrt{35}$ BM

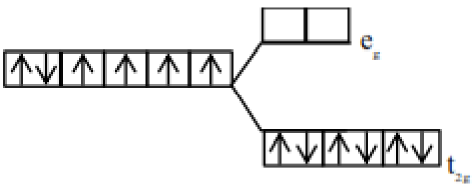
$[CoF_6]^{3-} : Co^{3+} = 3d^6\ (\Delta_O < P)$



Number of unpaired e^- = 4

$\therefore\ \mu = \sqrt{24}$ BM

$[Co(C_2O_4)_3]^{3-} : Co^{3+} = 3d^6\ (\Delta_O > P)$



Number of unpaired e^- = 0

$\therefore\ \mu = 0$ BM

33. B

Sol.

A.	Osmosis	III
B.	Reverse osmosis	I
C.	Electro osmosis	IV
D.	Electrophoresis	II

34. B

Sol. (i), (ii) and (iv) correct.

Manganese exhibits +7 oxidation state in its oxide. (Mn_2O_7)

Ru and Os from RuO_4 and OsO_4 oxide in +8 oxidation state.

Cr in +6 oxidation act is oxidizing.

Sc does not show +4 oxidation state.

35. C

Sol. Neoprene : Elastomer

Polyester : Fibre

Polystyrene : Thermoplastic

Urea–Formaldhyde Resin : Thermosetting polymer

36. A

Sol. $I^- + H_2O_2 \longrightarrow \underset{(A)}{I_2} + H_2O$

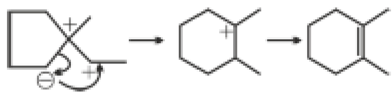
$I_2 + \underset{(indicator)}{Starch} \longrightarrow Blue$

37. D

Sol. Theory based.

38. B

Sol.



39. C

Sol. $CH_3-CH_2-CH(CH_3)-CH_2-Br \longrightarrow$ C–C–C(C)=C (1)

$CH_3-CH(Br)-CH_3 \longrightarrow CH_3CH{=}CH_2$ (1)

$CH_3-CH_2-CH_2-CH(Br)-CH_3 \longrightarrow$

C-C-C-C=C + C-C-C=C-C, cis & trans (3)

C–C–C(CH$_3$)(CH$_3$)–C(Br)–C $\longrightarrow$ C–C–C(C)(C)–C=C (1)

40. A

Sol. $C_xH_y + \left(x+\frac{y}{4}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$

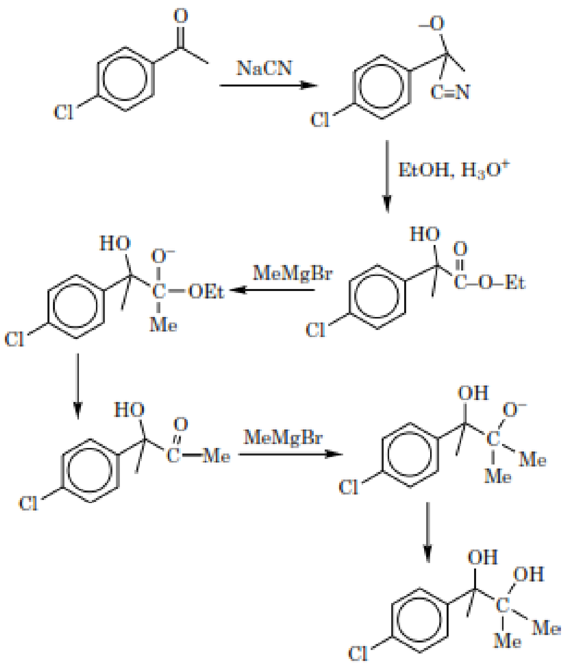
$x+\frac{y}{4}=9.5$

$\frac{y}{2}=3$

$\Rightarrow x=8,\ y=6$

41. B

Sol.



42. A

Sol. Theory based.

43. D

Sol. A solution of CrO_5 in amyl alcohol has a blue colour.

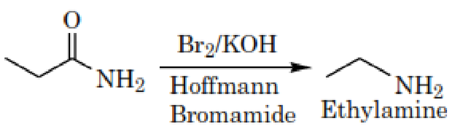
So, option (4) is correct.

44. A

Sol. The growth of fish gets inhibited if the concentration of dissolved Oxygen in water is less than 6 ppm and Biochemical Oxygen demand in clean water should be less than 5 ppm.

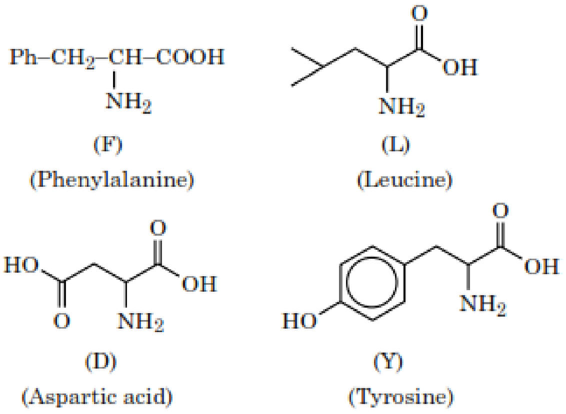
45. D

Sol.



46. B

Sol. Hydrolysis of the given tetrapeptide will give the following :



47. B

Sol. Only (B) and (C) are correct.

(B) $G = H - TS$

At constant T

$\Delta G = \Delta H - T\Delta S$

(A) First law is given by

$\Delta U = Q + W$

If we apply constant P and reversible work.

$\Delta U = Q - P\Delta V$

(C) By definition of entropy change

$$dS = \frac{dq_{rev}}{T}$$

At constant T

$$\Delta S = \frac{q_{rev}}{T}$$

(D) $H = U + PV$

For ideal gas

$H = U + nRT$

At constant T

$\Delta H = \Delta U + \Delta nRT$

48. C

Sol. Calamine : $ZnCO_3$

Siderite : $FeCO_3$

Sphalerite : ZnS

Malachite : $CuCO_3 \cdot Cu(OH)_2$

49. D

Sol. Statement–I is correct.

Ni is used in Hydrogenation of unsaturated fat to make edible fats.

Statements–II is false as hydride of Silicon is electron precise and neither electron deficient nor electron rich.

50. A

Sol. (A) van't Hoff factor, i

$$i = \frac{\text{Normal molar mass}}{\text{Abnormal molar mass}}$$

(B) k_f = Cryoscopic constant

(C) Solutions with same osmotic pressure are known as isotonic solutions.

(D) Solutions with same composition of vapour over them are called Azeotrope.

Section - B (Numerical Value)

51. 3

Sol. $2LiNO_3 \xrightarrow{\Delta} Li_2O + 2NO_2 + \frac{1}{2}O_2$

Hence three products Li_2O, NO_2 and O_2.

52. 4

Sol. $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g),$

$K_1 = 4 \times 10^5$ ... (i)

$N_2(g) + O_2(g) \rightleftharpoons 2NO(g),$

$K_2 = 1.6 \times 10^{12}$ ... (ii)

$H_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons H_2O(g),$

$K_3 = 1.0 \times 10^{-13}$ ... (iii)

$(ii) + 3 \times (iii) - (i)$

$$2NH_3(g) + \frac{5}{2}O_2(g) \rightleftharpoons 2NO(g) + 3H_2O(g)$$

$$K_{eq} = \frac{k_2 \times k_3^3}{k_1} = \frac{1.6 \times 10^{12} \times (10^{-13})^3}{4 \times 10^5}$$

$$= \frac{1.6}{4} \times 10^{-32} = 4 \times 10^{-33}$$

53. 2

Sol. As unit of rate constant is $(\text{conc.})^{1-n}\ \text{time}^{-1}$

$\Rightarrow (L\ mol^{-1}) \Rightarrow 1 - n = -1$

n = 2

54. 4

Sol. Acidic oxides are N_2O_3, NO_2, Cl_2O_7, SO_2

55. 200

Sol. Let M is the molar mass of the compound (g/mol).

mass of compound = 0.01 M gm

mass of carbon = $0.01\ M \times \frac{60}{100}$

moles of carbon = $\frac{0.01\ M}{12} \times \frac{60}{100}$

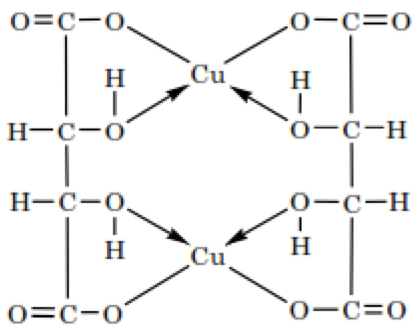
moles of CO_2 from combustion = $\frac{4.4}{44}$ = moles of carbon

$$\frac{0.01\ M}{12} \times \frac{60}{100} = \frac{4.4}{44}$$

$$M = \frac{4.4}{44} \times \frac{100}{60} \times \frac{12}{0.01} = 200\ \text{gm/mol}$$

56. 4

Sol.



Copper tartarate complex

Denticity = 2.

57. 36

Sol. One unit cell of hcp contains = 18 voids

No. of voids in 0.02 mol of hcp

$$= \frac{18}{6} \times 6.02 \times 10^{23} \times 0.02$$

$\approx 3.6 \times 10^{22} \approx 36 \times 10^{21}$

58. 270

Sol. $r \propto \frac{n^2}{Z}$

$$r_{He^+} = r_H \times \frac{n^2}{Z}$$

$$r_{He^+} = 0.6 \times \frac{(3)^2}{2}$$

$$= 2.7\ Å$$

$$r_{He^+} = 270 \text{ pm}$$

59. 17

Sol. $Zn(s) + Sn^{2+}(aq) \rightleftharpoons Zn^{2+}(aq) + Sn(s)$

$$\Delta G^o = -2.303\ RT\ \log_{10} Keq$$

$$-nF(E^o_{cell}) = -2.303\ RT\ \log_{10} Keq$$

$$E^o_{Zn/Zn^{2+}} + E^o_{Sn^{2+}/Sn} = \frac{0.059}{2}\log_{10} Keq$$

$$0.76 + E^o_{Sn^{2+}/Sn} = \frac{0.059}{2}\log_{10} 10^{20}$$

$$0.76 + E^o_{Sn^{2+}/Sn} = \frac{0.059 \times 20}{2}$$

$$E^o_{Sn^{2+}/Sn} = 0.59 - 0.76 = -0.17$$

$$E^o_{Sn/Sn^{2+}} = 17 \times 10^{-2}\ V$$

$= 17$

60. 15

Sol. Mole of Na = $\frac{0.69}{23} = 3 \times 10^{-2}$

$$Na + H_2O \longrightarrow NaOH + \frac{1}{2}H_2$$

By using POAC

Moles of NaOH = 3×10^{-2}

NaOH reacts with HCl

No. of equivalent of NaOH = No. of equivalent of HCl

$$3 \times 10^{-2} \times 1 = \frac{73}{36.5} \times V(\text{in L}) \times 1$$

$V = 1.5 \times 10^{-2}$ L

Volume of HCl = 15 ml.

MATHEMATICS

Section - A (Single Correct Answer)

61. B

Sol.

A	B	~ A	~ A ∨ B	B ⇒ ((~ A) ∨ B)
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

A ⇒ B	~ A ⇒ B	B ⇒ (A ⇒ B)	A ⇒ ((~ A) ⇒ B)	B ⇒ ((~ A) ⇒ B)
T	T	T	T	T
F	T	T	T	T
T	T	T	T	T
T	F	T	T	T

62. B

Sol. $\frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5}$ $\quad \vec{a} = \hat{i} - 8\hat{j} + 4\hat{k}$

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$$

$$\vec{b} = \hat{i} + 2\hat{j} + 6\hat{k}$$

$$\vec{p} = 2\hat{i}7\hat{j} + 5\hat{k},\ \vec{q} = 2\hat{i} + \hat{j} - 3\hat{k}$$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix}$$

$$= \hat{i}(16) - \hat{j}(-16) + \hat{k}(16)$$

$$= 16(\hat{i} + \hat{j} + \hat{k})$$

$$d = \left|\frac{(a-b)\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right| = \left|\frac{(-10\hat{j} - 2\hat{k})\cdot 16(\hat{i}+\hat{j}+\hat{k})}{16\sqrt{3}}\right|$$

$$= \left|\frac{-12}{\sqrt{3}}\right| = 4\sqrt{3}$$

63. A

Sol. $\vec{r}\times\vec{b}-\vec{c}\times\vec{b}=0$

$\Rightarrow(\vec{r}-\vec{c})\times\vec{b}=0$

$\Rightarrow\vec{r}-\vec{c}=\lambda\vec{b}$

$\Rightarrow\vec{r}=\vec{c}+\lambda\vec{b}$

And given that $\vec{r}\cdot\vec{a}=0$

$\Rightarrow(\vec{c}+\lambda\vec{b})\cdot\vec{a}=0$

$\Rightarrow\vec{c}.\vec{a}+\lambda\vec{b}\cdot\vec{a}=0$

$\Rightarrow\lambda=\dfrac{-\vec{c}\cdot\vec{a}}{\vec{b}\cdot\vec{a}}$

Now $\vec{r}\cdot\vec{c}=(\vec{c}+\lambda\vec{b})\cdot\vec{c}$

$$=\left(\vec{c}-\frac{\vec{c}\cdot\vec{a}}{\vec{b}\cdot\vec{a}}\vec{b}\right)\cdot\vec{c}$$

$$=|\vec{c}|-\left(\frac{\vec{c}\cdot\vec{a}}{\vec{b}\cdot\vec{a}}\right)(\vec{b}\cdot\vec{c})$$

$$=74-\left[\frac{15}{3}\right]8$$

$= 74 - 40 = 34$

64. B

Sol. Let $P(w_1)=\lambda$ then

$$P(w_2)=\frac{\lambda}{2}.......P(w_n)=\frac{\lambda}{2^{n-1}}$$

As $\sum_{k=1}^{\infty}P(w_k)=1\Rightarrow\dfrac{\lambda}{1-\frac{1}{2}}=1\Rightarrow\lambda=\dfrac{1}{2}$

So, $P(w_n)=\dfrac{1}{2^n}$

$A=\{2k+3\ell\ ;\ k,\ \ell\in\mathbb{N}\}=\{5,\ 7,\ 8,\ 9,\ 10......\}$

$B=\{w_n:n\in A\}$

$B = \{w_5, w_7, w_8, w_9, w_{10}, w_n, ...\}$

$A=\mathbb{N}-\{1,\ 2,\ 3,\ 4,\ 6\}$

$\therefore$ $P(B) = 1 - [P(w_1) + P(w_2) + P(w_3) + P(w_4) + P(w_6)]$

$$=1\left[\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{64}\right]$$

$$=1-\frac{32+16+8+4+1}{64}=\frac{3}{64}$$

65. C

Sol. $I=\displaystyle\int_1^2\left(\frac{t^4+1}{t^6+1}\right)dt$

$$=\int_1^2\frac{(t^4+1-t^2)+t^2}{(t^2+1)(t^4-t^2+1)}dt$$

$$=\int_1^2\left(\frac{1}{t^2+1}+\frac{t^2}{t^6+1}\right)dt$$

$$=\int_1^2\left(\frac{1}{t^2+1}+\frac{1}{3}\frac{3t^2}{3(t^3)^2+1}\right)dt$$

$$=\tan^{-1}(t)+\frac{1}{3}\tan^{-1}(t^3)\Big|_1^2$$

$$=(\tan^{-1}(2)-\tan^{-1}(1))+\frac{1}{3}(\tan^{-1}(2)-\tan^{-1}(1^3))$$

$$=\tan^{-1}(2)+\frac{1}{3}\tan^{-1}(8)-\frac{\pi}{3}$$

66. C

Sol. In the expansion of

$(1 + x)^{99} = C_0 + C_1x + C_2x^2 +......+ C_{99}x^{99}$

$K = C_1 + C_3 +....+ C_{99} = 2^{98}$

$a\Rightarrow$ Middle in the expansion of $\left(2+\dfrac{1}{\sqrt{2}}\right)^{200}$

$$T_{\frac{200}{2}+1}={}^{200}C_{100}(2)^{100}\left(\frac{1}{\sqrt{2}}\right)^{100}$$

$$={}^{200}C_{100}\cdot2^{50}$$

So, $\dfrac{{}^{200}C_{99}\times2^{98}}{C_{100}\times2^{50}}=\dfrac{100}{101}\times2^{48}$

So, $\dfrac{25}{101}\times2^{50}=\dfrac{m}{2}2^{\ell}$

$\therefore$ m, n are odd so

(ℓ, n) between (50, 101) Ans.

67. B

Sol. $f''(x) = g''(x) + 6x$ ....(1)

$f'(1) = 4g'(1) - 3 = 9$ ....(2)

$f(2) = 3g(2) = 12$ ....(3)

By integrating (1)

$$f'(x) = g'(x) + 6\frac{x^2}{2} + C$$

At x = 1

$f'(1) = g'(1) + 3 + C$

$\Rightarrow 9 = 4 + 3 + C \Rightarrow C = 3$

$\therefore f'(x) = g'(x) + 3x^2 + 3$

Again by integrating,

$$f(x) = g(x) + \frac{3x^3}{3} + 3x + D$$

At x = 2,

$f(2) = g(2) + 8 + 3(2) + D$

$\Rightarrow 12 = 4 + 8 + 6 + D \Rightarrow D = -6$

So, $f(x) = g(x) + x^3 + 3x - 6$

$\Rightarrow f(x) - g(x) = x^3 + 3x - 6$

At x = –2,

$\Rightarrow g(-2) - f(-2) = 20$ (Option (1) is true)

Now, for $-1 < x, 2$

$h(x) = f(x) - g(x) = x^3 + 3x - 6$

$\Rightarrow h'(x) = 3x^2 + 3$

$\Rightarrow h(x)\uparrow$

So, $h(-1) < h(x) < h(2)$

$\Rightarrow -10 < h(x) < 8$

$\Rightarrow |h(x)| < 10$ (option (2) is NOT true)

Now, $h'(x) = f(x) - g'(x) = 3x^2 + 3$

If $|h'(x)| < 6 \Rightarrow |3x^2 + 3| < 6$

$\Rightarrow 3x^2 + 3 < 6$

$\Rightarrow x^2 < 1$

$\Rightarrow -1 < x < 1$ (option (3) is True)

If $x \in (-1, 1)$ $|f'(x) - g'(x)| < 6$

option (3) is true and now to solve

$f(x) - g(x) = 0$

$\Rightarrow x^3 + 3x - 6 = 0$

$h(x) = x^3 + 3x - 6$

here, $h(1) = -ve$ and $h\left(\frac{3}{2}\right) = +ve$

So there exists $x_0 \in \left(1, \frac{3}{2}\right)$ such that $f(x_0) = g(x_0)$

(option (4) is true)

68. D

Sol. If its invertible, then determinant value $\neq 0$

So,

$$\begin{vmatrix} 0 & -\sin t - \cos t & -3\sin t + \cos t \\ 0 & 2\sin t & -2\cos t \\ 1 & \cos t & \sin t \end{vmatrix} \neq 0$$

By expanding we have,

$e^{-t} \times 1(2\sin t\cos t + 6\cos^2 t + 6\sin^2 t - 2\sin t\cos t) \neq 0$

$\Rightarrow e^{-t} \times 6 \neq 0$

for $\forall t \in \mathbb{R}$

69. D

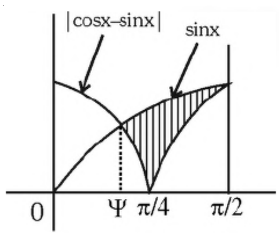
Sol. $|\cos x - \sin x| \le y \le \sin x$

Intersection point of $\cos x - \sin x = \sin x$

$$\Rightarrow \tan x = \frac{1}{2}$$

Let $\psi = \tan^{-1}\frac{1}{2}$

So, $\tan\psi = \frac{1}{2}$, $\sin\psi = \frac{1}{\sqrt{5}}$, $\cos\psi = \frac{2}{\sqrt{5}}$



$$\text{Area} = \int_{\psi}^{\pi/2} (\sin x - |\cos x - \sin x|)dx$$

$$= \int_{\psi}^{\pi/4} (\sin x - (\cos x - \sin x))dx$$

$$+\int_{\pi/4}^{\pi/2}(\sin x(\sin x-\cos x))dx$$

$$=\int_{\psi}^{\pi/4}(2\sin x-\cos x)dx+\int_{\pi/4}^{\pi/2}\cos x dx$$

$$=[-2\cos x-\sin x]_{\psi}^{\pi/4}+[\sin x]_{\pi/4}^{\pi/2}$$

$$=-\sqrt{2}-\frac{1}{\sqrt{2}}+2\cos\psi+\sin\psi+\left(1-\frac{1}{\sqrt{2}}\right)$$

$$=-\sqrt{2}-\frac{1}{\sqrt{2}}+2\left(\frac{2}{\sqrt{5}}\right)+\left(\frac{1}{\sqrt{5}}\right)+1-\frac{1}{\sqrt{2}}$$

$$=\sqrt{5}-2\sqrt{2}+1$$

70. D

Sol. $\lambda=\cos^2 2x-2\sin^4 x-2\cos^2 x$

convert all in to cos x.

$$\lambda=(2\cos^2 x-1)^2-2(1-\cos^2 x)^2-2\cos^2 x$$

$$=4\cos^4 x-4\cos^2 x+12(1-2\cos^2 x+\cos^4 x)-$$

$$2\cos^2 x$$

$= 2\cos^4 x - 2\cos^2 x + 1 - 2$

$= 2\cos^4 x - 2\cos^2 x - 1$

$$=2\left[\cos^4 x-\cos^2 x-\frac{1}{2}\right]$$

$$=2\left[\left(\cos^2 x-\frac{1}{2}\right)^2-\frac{3}{4}\right]$$

$$\lambda_{max}=2\left[\frac{1}{4}-\frac{3}{4}\right]=2\times\left(-\frac{2}{4}\right)=-1 \text{ (max Value)}$$

$$\lambda_{min}=2\left[0-\frac{3}{4}\right]=-\frac{3}{2} \text{ (Minimum Value)}$$

So, Range $=\left[-\frac{3}{2},\ -1\right]$

71. A

Sol. Lets arrange the letters of OUGHT in alphabetical order.

G, H, O, T, U

Words starting with

G $\longrightarrow$ 4!

H $\longrightarrow$ 4!

O $\longrightarrow$ 4!

T G $\longrightarrow$ 3!

T H $\longrightarrow$ 3!

T O G $\longrightarrow$ 2!

T O H $\longrightarrow$ 2!

T O U G H $\rightarrow$ 1!

Total = 89

72. A

Sol. A(a, –2, 4), B(2, b, –3)

AC : CB = 2 : 1

$$\Rightarrow C\equiv\left(\frac{a+4}{3},\ \frac{2b-2}{3},\ \frac{-2}{3}\right)$$

C lies on 2x – y + 2 = 4

$$\Rightarrow\frac{2a+8}{3}-\frac{2b-2}{3}-\frac{2}{3}=4$$

$\Rightarrow$ a – b = 2 ....(1)

Also $OC=\sqrt{5}$

$$\Rightarrow\left(\frac{a+4}{3}\right)^2+\left(\frac{2b-2}{3}\right)^2+\frac{4}{9}=5 \quad ....(2)$$

Solving, (1) and (2)

$(b + 6)^2 + (2b – 2)^2 = 41$

$\Rightarrow 5b^2 + 4b – 1 = 0$

$\Rightarrow$ b = –1 or $\frac{1}{5}$

$\Rightarrow$ a = 1 or $\frac{11}{5}$

But ab < 0 $\Rightarrow$ (a, b) = (1, – 1)

$$C\equiv\left(\frac{5}{3},\ \frac{-4}{3},\ \frac{-2}{3}\right),\ P\equiv(2,\ -1,\ -3)$$

$$CP^2=\frac{1}{9}+\frac{1}{9}+\frac{49}{9}=\frac{51}{9}=\frac{17}{3}$$

73. A

Sol. $\vec{a}\times\vec{b} = 15\hat{i} - 20\hat{j} - 25\hat{k}$

Let $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$

$\Rightarrow 15x - 20y - 25z + 25 = 0$

$\Rightarrow 3x - 4y - 5z = -5$

Also $x + y + z = 4$

and $\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|} = 1 \Rightarrow 4x + 3y = 5$

$\Rightarrow \vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}$

Projection of $\vec{c}$ or $\vec{b} = \frac{25}{5\sqrt{2}} = \frac{5}{\sqrt{2}}$

74. B

Sol. Point on $L_1 \equiv (\lambda+1,\ 2\lambda+2,\ \lambda-3)$

Point on $L_2 \equiv (2\mu+a,\ 3\mu-2,\ \mu+3)$

$\lambda - 3 = \mu + 3 \qquad \Rightarrow \lambda = \mu + 6$ ...(1)

$2\lambda + 2 = 3\mu - 2 \qquad \Rightarrow 2\lambda = 3\mu - 4$ ....(2)

Solving, (1) and (2)

$\Rightarrow \lambda = 22\ \&\ \mu = 16$

$\Rightarrow P \equiv (23,\ 46,\ 19)$

$\Rightarrow a = -9$

Distance of P from z = –9 is 28

75. D

Sol. $I = \int_{1/2}^{2} \frac{\tan^{-1}x}{x}dx$ ....(i)

Put $x = \frac{1}{t}dx = -\frac{1}{t^2}dx$

$I = -\int_{2}^{1/2} \frac{\tan^{-1}\frac{1}{t}}{\frac{1}{t}}\cdot\frac{1}{t^2}dt = -\int_{2}^{1/2}\frac{\tan^{-1}\frac{1}{t}}{t}dt$

$I = \int_{1/2}^{2}\frac{\cot^{-1}t}{t}dt = \int_{1/2}^{2}\frac{\cot^{-1}x}{x}dx$ ....(ii)

Add both equation

$2I = \int_{1/2}^{2}\frac{\tan^{-1}x + \cot^{-1}x}{x}dx = \frac{\pi}{2}\int_{1/2}^{2}\frac{dx}{x} = \frac{\pi}{2}(\ell n2)_{1/2}^{2}$

$= \frac{\pi}{2}\left(\ell n2 - \ell n\frac{1}{2}\right) = \pi\ell n2$

$I = \frac{\pi}{2}\ell n2$

76. D

Sol. $y^2 = 3x$

Tangent $P(x_1, y_1)$ is parallel to $x + 2y = 1$

Then slope at $P = -\frac{1}{2}$

$2y\frac{dy}{dx} = 3$

$\Rightarrow \frac{dy}{dx} = \frac{3}{2y} = -\frac{1}{2}$

$\Rightarrow y_1 = -3$

Coordinates of P(3, –3)

Similarly $Q\left(\frac{4}{\sqrt{3}}, \frac{1}{\sqrt{5}}\right)$, $R\left(-\frac{4}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right)$

Area of ΔPQR

$$= \frac{1}{2}\begin{vmatrix} 3 & -3 & 1 \\ \frac{4}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 1 \\ -\frac{4}{\sqrt{5}} & -\frac{1}{\sqrt{5}} & 1 \end{vmatrix}$$

$= \frac{1}{2}\left[3\left(\frac{2}{\sqrt{5}}\right) + 3\left(\frac{8}{\sqrt{5}}\right) + 0\right] = \frac{30}{2\sqrt{5}} = 3\sqrt{5}$

77. A

Sol. $x\log_e x\frac{dy}{dx} + y = x^2\log_e x,\ (x > 1).$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x\ln x} = x$

Linear differential equation

$I.F. = e^{\int\frac{1}{x\ln x}dx} = |\ln x|$

$\therefore$ Solution of differential equation

$$y|\ln x| = \int x|\ln x| dx$$

$$= |\ln x|\frac{x^2}{2} - \int \frac{1}{x}\cdot\frac{x^2}{2}dx$$

$$\Rightarrow y|\ln x| = |\ln x|\left(\frac{x^2}{2}\right) - \frac{x^2}{4} + c$$

For constant

$y(2) = 2 \Rightarrow c = 1$

So, $y(x) = \frac{x^2}{2} - \frac{x^2}{4|\ln x|} + \frac{1}{|\ln x|}$

Hence, $y(e) = \frac{e^2}{2} - \frac{e^2}{4} + 1 = 1 + \frac{e^3}{4}$

78. B

Sol. Total 3 digit number = 900

Divisible by 3 = 300

(Using $\frac{900}{3} = 300$)

Divisible by 4 = 225

(Using $\frac{900}{4} = 225$)

Divisible by 3 & 4 = 108, ...

(Using $\frac{900}{12} = 75$)

Number divisible by either 3 or 4

= 300 + 2250 - 75 = 450

We have to remove divisible by 48,

144, 192,......, 18 terms

Required number of numbers = 450 – 18 = 432

79. D

Sol. a R a $\Rightarrow$ 5a is multiple it 5

So reflexive

a R b $\Rightarrow$ 2a + 3b = 5α,

Now b R a

$$2b + 3a = 2b + \left(\frac{5\alpha - 3b}{2}\right)\cdot 3$$

$$= \frac{15}{2}\alpha - \frac{5}{2}b = \frac{5}{2}(3\alpha - b)$$

$$= \frac{5}{2}(2a + 2b - 2\alpha)$$

$$= 5(a + b - \alpha)$$

Hence symmetric

a R b $\Rightarrow 2a + 3b = 5\alpha$

b R c $\Rightarrow 2b + 3c = 5\beta$

Now $2a + 5b + 3c = 5(\alpha + \beta)$

$\Rightarrow 2a + 5b + 3c = 5(\alpha + \beta)$

$\Rightarrow 2a + 3c = 5(\alpha + \beta - b)$

$\Rightarrow$ a R c

Hence relation is equivalence relation.

80. D

Sol. Given for $x \geq 2$

f(1) + 2f(2) +.....+ xf(x) = x(x + 1) f(x)

replace x by x + 1

$\Rightarrow$ x(x + 1) f(x) + (x + 1) f(x + 1)

= (x + 1) (x + 2) f(x + 1)

$$\Rightarrow \frac{x}{f(x+1)} + \frac{1}{f(x)} = \frac{(x+2)}{f(x)}$$

$$\Rightarrow xf(x) = (x+1)f(x+1) = \frac{1}{2},\ x \geq 2$$

$$f(2) = \frac{1}{4},\ f(3) = \frac{1}{6}$$

Now $f(2022) = \frac{1}{4044}$

$$f(2028) = \frac{1}{4056}$$

So, $\frac{1}{f(2022)} + \frac{1}{f(2028)} = 4044 + 4056 = 8100$

Section - B (Numerical Value)

81. 3000

Sol. N should be divisible by 2 but not by 3

N = (Numbers divisible by 2) - (Numbers divisible by 6)

$$N=\frac{9000}{2}-\frac{9000}{6}=4500-1500=3000$$

82. 10

Sol. $S_1 : y^2 = 2x \quad S_2 : x^2 + y^2 = 4x$

P(2, 2) is common point on S_1 & S_2

T_1 is tangent to S_1 at P

$\Rightarrow T_1 : y\cdot 2 = x + 2$

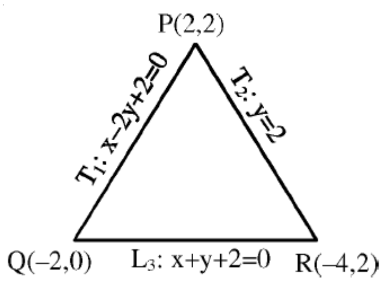
$\Rightarrow T_1 : x - 2y + 2 = 0$

T_2 is tangent to S_2 at P

$\Rightarrow T_2 : x.2 + y\cdot 2 = 2(x + 2)$

$\Rightarrow T_2 : y = 2$

& $L_3 : x + y + 2 = 0$ is third line



$$PQ = a = \sqrt{20}$$

$$QR = b = \sqrt{8}$$

RP = c = 6

$$\text{Area}(\Delta PQR) = \Delta = \frac{1}{2}\times 6\times 2 = 6$$

$$\therefore r = \frac{abc}{4\Delta} = \frac{\sqrt{160}}{4} = \sqrt{10} \Rightarrow r^2 = 10$$

83. 11

Sol. The given line is polar or P(2, β) w.r.t. given circle

$x^2 + y^2 - 4x - 6y - 3 = 0$

Chord or contact

$\alpha x + \beta y - 2(x + \alpha) - 3(y + \beta) - 3 = 0$

$\Rightarrow (\alpha - 2)x + (\beta - 3)y - (2\alpha + 3\beta + 3) = 0$.....(i)

$\therefore$ But the equation of chord of contact is given as : $x + y - 3 = 0$ ....(ii)

comparing the coefficients

$$\frac{\alpha-2}{1}=\frac{\beta-3}{1}=-\left(\frac{2\alpha+3\beta+3}{-3}\right)$$

On solving $\alpha = -6$

$\beta = -5$

Now $4\alpha - 7\beta = 11$

84. 461

Sol. As, $S=\frac{b_1}{2}+\frac{b_2}{2^2}+.....+\frac{b_9}{2^9}+\frac{b_{10}}{2^{10}}$

$$\Rightarrow \frac{S}{2}=\frac{b_1}{2^2}+\frac{b_2}{2^3}+....+\frac{b_9}{2^{10}}+\frac{b_{10}}{2^{11}}$$

subtracting

$$\Rightarrow \frac{S}{2}=\frac{b_1}{2}+\left(\frac{a_1}{2^2}+\frac{a_2}{2^3}.....+\frac{a_9}{2^{10}}\right)-\frac{b_{10}}{2^{11}}$$

$$\Rightarrow S=b_1-\frac{b_{10}}{2^{10}}\left(\frac{a_1}{2}+\frac{a_2}{2^2}......+\frac{a_9}{2^9}\right)$$

$$\Rightarrow \frac{S}{2}=\frac{b_1}{2}-\frac{b_{10}}{2^{11}}+\left(\frac{a_1}{2^2}+\frac{a_2}{2^3}......+\frac{a_9}{2^{10}}\right)$$

subtracting

$$\Rightarrow \frac{S}{2}=\frac{b_1}{2}-\frac{b_{10}}{2^{11}}+\left(\frac{a_1}{2}-\frac{a_9}{2^{10}}\right)+\left(\frac{1}{2^2}+\frac{2}{2^3}+.....+\frac{8}{2^9}\right)$$

$$\Rightarrow \frac{S}{2}=\frac{a_1+b_1}{2}-\frac{(b_{10}+2a_9)}{2^{11}}+\frac{T}{4}$$

$$\Rightarrow 2S=2(a_1+b_1)-\frac{(b_{10}+2a_0)}{2^9}+T$$

$$\Rightarrow 2^7(2S-T)=2^8(a_1+b_1)-\frac{(b_{10}+2a_9)}{4}$$

Given $a_n - a_{n-1} = n - 1$,

$\therefore \quad a_2 - a_1 = 1$

$a_3 - a_2 = 2$

⋮

$a_9 - a_8 = 8$

____

$\Rightarrow \quad a_9 - a_1 = 1 + 2 + ... + 8 = 36$

$\Rightarrow \quad a_9 = 37$ ($a_1 = 1$)

____

Also, $b_n - b_{n-1} = a_{n-1}$

$\therefore b_{10} - b_1 = a_1 + a_2 + ... + a_9$

= 1 + 2 + 4 + 7 + 11 + 16 + 22 + 29 + 37

$\Rightarrow b_{10} = 130$ (As $b_1 = 1$)

$\therefore$ 27 (2S – T) = 28 (1 + 1) – (130 + 2 × 37)

$$2^9 - \frac{204}{4} = 461$$

85. 4

Sol. $y = \frac{x-a}{(x+b)(x-2)}$

At point (1, –3),

$$-3 = \frac{1-9}{(1+b)(1-2)}$$

$\Rightarrow$ 1 – a = 3(1 + b) ....(1)

Now, $y = \frac{x-a}{(x+b)(x-2)}$

$$\Rightarrow \frac{dy}{dx} = \frac{(x+b)(x-2)\times(1)-(x-a)(2x+b-2)}{(x+b)^2(x-2)^2}$$

At (1, –3) slope of normal is $\frac{1}{4}$ hence $\frac{dy}{dx} = -4$,

So, $-4 = \frac{(1+b)(-1)(1-a)b}{(1+b)^2(-1)^2}$

Using equation (1)

$$\Rightarrow -4 = \frac{(1+b)(-1)-3(b+1)b}{(1+b)^2(1)^2}$$

$$\Rightarrow -4 = \frac{(-1)-3b}{(1+b)}(b \neq -1)$$

$\Rightarrow$ b = –3

So, a = 7

Hence, a + b = 7 – 3 = 4

86. 5

Sol. $\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix}\begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$

Now ac – b^2 = 2 and 2a + b = 1

and 2b + c = 2

solving all these above equations we get

$$\frac{1-b}{2}\times\left(\frac{2-2b}{1}\right) - b^2 = 2$$

$\Rightarrow (1-b)^2 - b^2 = 2$

$\Rightarrow$ 1 – 2b = 2

$\Rightarrow b = -\frac{1}{2}$ and $a = \frac{3}{4}$ and c = 3

Hence $\alpha = 3a + \frac{3b}{2} = \frac{9}{4} - \frac{3}{4} = \frac{3}{2}$

and $\beta = 3b + \frac{3c}{2} = -\frac{3}{2} + \frac{9}{2} = 3$

also $s = a + c = \frac{15}{4}$

$$\therefore \frac{\beta s}{\alpha^2} = \frac{3\times 15}{4\times\frac{9}{4}} = 5$$

87. 9

Sol. Given that

$c_k = a_k + b_k$ and $a_1 = b_1 = 4$

also $a_2 = 4r_1$ $a_3 = 4r_1^2$

$b_2 = 4r_2$ $b_3 = 4r_2^2$

Now $c_2 = a_2 + b_2 = 5$ and $c_3 = a_3 + b_3 = \frac{13}{4}$

$\Rightarrow r_1 + r_2 = \frac{5}{4}$ and $r_1^2 + r_2^2 = \frac{13}{16}$

Hence $r_1 r_2 = \frac{3}{8}$ which gives $r_1 = \frac{1}{2}$ & $r_2 = \frac{3}{4}$

$$\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$$

$$= \frac{4}{1-r_1} + \frac{4}{1-r_2} - \left(\frac{48}{32} + \frac{27}{2}\right)$$

= 24 – 15 = 9

88. 603

Sol. $\bar{x} = \frac{\sum_{i=11}^{41} i}{31} = \frac{11+41}{2} = 26$ (31 elements)

$$\bar{y} = \frac{\sum_{j=61}^{91} j}{31} = \frac{61+91}{2} = 76 \text{ (31 elements)}$$

Combined mean, $\mu = \dfrac{31\times 26 + 31\times 76}{31+31}$

$= \dfrac{26+76}{2} = 51$

$\sigma^2 = \dfrac{1}{62}\times\left(\sum_{i=1}^{31}(x_i-\mu)^2 + \sum_{i=1}^{31}(y_i-\mu)^2\right) = 705$

Since, $x_i \in X$ are in A.P. with 31 elements & common difference 1, same is $y_i \in y$, when written in increasing order.

$\therefore \sum_{i=1}^{31}(x_i-\mu)^2 = \sum_{i=1}^{31}(y_i-\mu)^2$

$= 10^2 + 11^2 + ... + 40^2$

$= \dfrac{40\times 41\times 81}{6} = \dfrac{9\times 10\times 19}{6} = 21855$

$\therefore \left|\bar{x}+\bar{y}-\sigma^2\right| = |26+76-705| = 603$

89. 14

Sol. $\alpha = 8 - 14i$

$z = x + iy$

$az = (8x + 14y) + i(-14x + 8y)$

$z+\bar{z} = 2x$ $\qquad$ $z-\bar{z} = 2iy$

Set A : $\dfrac{2i(-14x+8y)}{i(4xy-112)} = 1$

$(x - 4)(y + 7) = 0$

$x = 4$ or $y = -7$

Set B : $x^2 + (y + 3)^2 = 16$

when $x = 4$ $\qquad$ $y = -3$

when $y = -7$ $\qquad$ $x = 0$

$\therefore A\cap B = \{4-3i,\ 0-7i\}$

So, $\sum_{z\in A\cap B}(\operatorname{Re} z - \operatorname{Im} z) = 4-(-3)+(0-(-7)) = 14$

90. 9

Sol. Given equation can be rearranged as

$x(x^6 + 3x^4 - 13x^2 - 15) = 0$

clearly $x = 0$ is one of the root and other part can be observed by replacing $x^2 = t$ from which we have t3 + 3t2 - 13t - 15 = 0

$\Rightarrow (t - 3)(t^2 + 6t + 5) = 0$

So, $t = 3, t = -1, t = -5$

Now we are getting $x^2 = 3$, $x^2 = -1$, $x^2 = -5$

$\Rightarrow x = \pm\sqrt{3},\ x = \pm i,\ x - \pm\sqrt{5}i$

From the given condition $|\alpha_1| \ge |\alpha_2| \ge ...... \ge |\alpha_7|$

We can clearly say that $|\alpha_7| = 0$ and

$|\alpha_6| = \sqrt{5} = |\alpha_5|$ and $|\alpha_4| = \sqrt{3} = |\alpha_3|$ and $|\alpha_2| = 1 = |\alpha_1|$

So we can have, $\alpha_1 = \sqrt{5}i,\ \alpha_2 = -\sqrt{5}i,\ \alpha_3 = \sqrt{3}i,$

$\alpha_4 = -\sqrt{3},\ \alpha_5 = i,\ \alpha_6 = -i$

Hence

$\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6$

$= 1 - (-3) + 5 = 9$

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