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29-January-2023 (Morning Batch) : JEE Main Paper

PHYSICS

Section - A (Single Correct Answer)

1. C

Sol. Pressure gradient $= \frac{dp}{dx} = \frac{[ML^{-1}T^{-2}]}{[L]}$

$= [M^1L^{-2}T^{-2}]$

Energy density $= \frac{\text{energy}}{\text{volume}} = \frac{[ML^2T^{-2}]}{[L^3]}$

$= [M^1\ L^{-1}\ T^{-2}]$

Electric field $= \frac{\text{Force}}{\text{charge}} = \frac{[MLT^{-2}]}{[A.T]}$

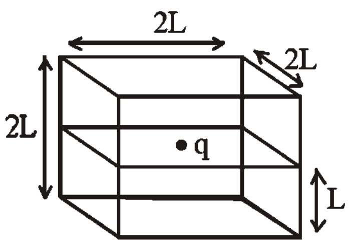
$= [M^1\ L^1\ T^{-3}\ A^{-1}]$

Latent heat $= \frac{\text{heat}}{\text{mass}} = \frac{[ML^2T^{-2}]}{[M]}$

$= [M^0\ L^2\ T^{-2}]$

2. D

Sol. $\phi = \frac{Q/\varepsilon_0}{6}$



Flux passing through shaded face $= \frac{q}{6\varepsilon_0}$

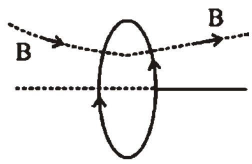
3. A

Sol. $H = \frac{V^2}{R} \times t$

$$\frac{H_1}{H_2} = \frac{\frac{V^2t}{R}}{\frac{V^2t}{3R}} = 3:1$$

4. C

Sol.



$B_y = 0$ in plane of coil

B_y is opposite of each other in –z and +z positions.

5. D

Sol. Magnetic field due to current in BC and ET are outward at point 'O'

$$B_0 = \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i}{4\pi r} = \frac{\mu_0 i}{2\pi r} = \frac{\mu_0 i}{\pi a}$$

6. C

Sol. $\phi = Mi$

$\phi = (BA)$

$$\phi = \pi R^2 \left(4 \frac{\mu_0}{4\pi} \frac{i}{\left(\frac{L}{2}\right)} \sqrt{2} \right)$$

$$\Rightarrow M = \frac{2\sqrt{2}\mu_0 R^2}{L}$$

7. D

Sol. Speed of light does not depend on the motion of source as well as intensity.

8. C

Sol. $A_2P - A_1P = \frac{\lambda}{2}$ (Condition of minima)

$$\sqrt{D^2 + a^2} - D = \frac{\lambda}{2}$$

$$D\left(1+\frac{a^2}{D^2}\right)^{1/2} - D = \frac{\lambda}{2}$$

$$D\left(1+\frac{1}{2}\times\frac{a^2}{D^2}\right) - D = \frac{\lambda}{2}$$

$$\frac{a^2}{2D} = \frac{\lambda}{2} \Rightarrow a = \sqrt{\lambda D}$$

$$= \sqrt{800\times10^{-6}\times50}$$

a = 0.2 mm

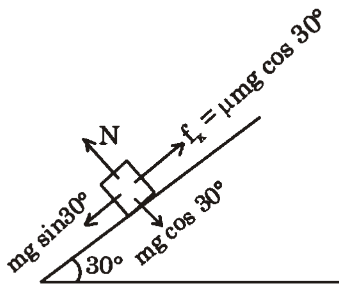
9. D

Sol. $\frac{KE_{POP}}{KE_{top}} = \frac{\frac{1}{2}M(u)^2}{\frac{1}{2}M(u\cos30°)^2} = \frac{4}{3}$

10. B

Sol. Mg sin 30° – μ mg cos 30° = ma

$$\frac{g}{2} - \frac{\sqrt{3}}{2}\cdot\mu g = \frac{g}{4}$$



$$\frac{\sqrt{3}}{2}\mu = \frac{1}{4}$$

$$\mu = \frac{1}{2\sqrt{3}}$$

11. C

Sol. $f_s = \frac{mv^2}{r}$

For maximum speed in safe turning,

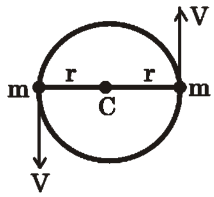
$f_s = f_s \max = \mu mg$

v_{max} for safe turning = $\sqrt{\mu rg}$

$$= \sqrt{0.34\times50\times10} \approx 13 \text{ m/s}$$

12. D

Sol. $\frac{Gm^2}{4r^2} = \frac{mv^2}{r}$



$$v = \sqrt{\frac{Gm}{4r}}$$

13. C

Sol. Surface area of soap bubble = 2 × 4πR^2

Work done = change in surface energy × T_S

$= T_S \times 8\pi \times \left(R_2^2 - R_1^2\right)$

$$= 2\times10^{-2}\times8\times\frac{22}{7}\times49\times\frac{3}{4}\times10^{-4}$$

= 18.48 × 10^{-4} J

14. C

Sol. First law of thermodynamics is based on law of conservation of energy and it can be written as

dQ = dU– dW.

where dW is work done on the system

15. C

Sol. Taking volume constant : $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

$$\Rightarrow P_2 = \frac{P_1}{T_1}\times T_2 = \frac{270\times(309)}{300}$$

= 278kPa

16. B

Sol. $f_1 = 300\left(\frac{330-0}{330-(-30)}\right) = 275$

$$f_2 = 300\left(\frac{330-0}{330-(30)}\right) = 330$$

Δf = 330 – 275 = 55 Hz.

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17. D

Sol. Maximum line of sight distance between two antennas, $d_M = \sqrt{2Rh_T} + \sqrt{2R.h_R}$

$$d_M = 2\times\sqrt{2\times6.4\times10^6\times80} = 64 \text{ km}$$

18. D

Sol. $\lambda < 5500$ Å for photoelectric emission

$\lambda_{uv} < 5500$Å

19. A

Sol. $$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/t_{\frac{1}{2}}} = \left(\frac{1}{2}\right)^{\frac{90}{30}}$$

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

20. C

Sol. LED works in forward biasing and light energy may be slightly less or equal to band gap.

Section - B (Numerical Value)

21. 87

Sol. P = 92 – 2 – 2 + 1 – 1 – 1

P = 92 – 5

P = 87

22. 120

Sol. $$2A\cos\left(\frac{\Delta\phi}{2}\right) = A$$

$$\cos\left(\frac{\Delta\phi}{2}\right) = \frac{1}{2}$$

$$\frac{\Delta\phi}{2} = 60°$$

23. 28

Sol. By average form of Newton's law of cooling

$$\frac{20}{6} = k(50-10) \quad ....(i)$$

$$\frac{40-T}{6} = K\left(\frac{40+T}{2}-10\right) \quad ....(ii)$$

From equation (i) and (ii)

$$\frac{20}{40-T} = \frac{40}{10+T/2}$$

$$10+\frac{T}{2} = 80-2T$$

$$\frac{5T}{2} = 70 \Rightarrow T = 28°C$$

24. 40

Sol. $$KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$2240 = \frac{1}{2}2(v)^2 + \frac{1}{2}\frac{2}{5}(2)R^2\cdot\left(\frac{v}{R}\right)^2$$

$$2240 = v^2 + \frac{2}{5}v^2$$

$\Rightarrow$ v = 40 m/s

25. 300

Sol. Displacement is 8th sec.

$$S_8 = 0 + \frac{1}{2}\times10\times(2\times8-1)$$

S_8 = 5 × 15

ΔU = 0.4 × 10 × 5 × 15

ΔU = 20 × 15 = 300

26. 120

Sol. $v_i = \sqrt{2gh_i}$

$= \sqrt{2\times10\times9.8}\downarrow$

= 14 m/s $\downarrow$

$v_f = \sqrt{2gh_f}$

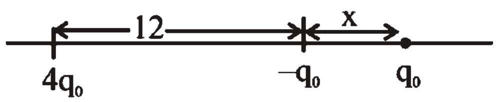
$= \sqrt{2\times10\times5}\uparrow$

= 10 m/s $\uparrow$

$$\left|\vec{a}_{avg}\right| = \left|\frac{\Delta\vec{v}}{\Delta t}\right| = \frac{24}{0.2} = 120 \text{ m/s}^2$$

27. 24

Sol.



$$\frac{q_0}{x^2} = \frac{4q_0}{(x+12)^2}$$

$x + 12 = 2x$

$x = 12$

Distance from origin = x + 12 = 24 cm.

28. 2

Sol. $\dfrac{2}{\left(\dfrac{3x}{3+x}\right)} = \dfrac{40+22.5}{60-22.5} = \dfrac{62.5}{37.5} = \dfrac{5}{3}$

$\dfrac{6}{5} = \dfrac{3x}{3+x}$

$6 + 2x = 5x \Rightarrow x = 2$

29. 10

Sol. $\text{EMF} = \dfrac{d}{dt}\left(B\pi r^2\right)$

$= 2B\pi r\dfrac{dr}{dt} = 2\times\pi\times0.1\times0.8\times2\times10^{-2}$

$= 2\pi \times 1.6 = 10.06$ [round off 10.06 = 10]

30. 24

Sol. By first polaroid P_1 intensity will be halved then P_2 and P_3 will make intensity $\cos^2(60°)$ and $\cos^2(30°)$ times respectively.

Intensity out $= \dfrac{256}{2}\times\dfrac{1}{4}\times\left(\dfrac{\sqrt{3}}{2}\right)^2 = \dfrac{256\times3}{2\times4\times4} = 24$

CHEMISTRY

Section - A (Single Correct Answer)

31. A

Sol. For 1 mole of real gas

PV = ZRT

From graph PV for real gas is less than PV for ideal gas at point A.

Z < 1

$Z = 1 - \dfrac{a}{V_mRT}$

32. B

Sol. For H :

$$\frac{1}{\lambda} = R_H \times 1^2\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) \quad \ldots (1)$$

$$\frac{1}{\lambda_{He^+}} = R_H \times 2^2 \times \left(\frac{1}{4} - \frac{1}{9}\right) \quad \ldots (2)$$

From (1) and (2),

$\dfrac{\lambda_{He^+}}{\lambda} = \dfrac{9}{5}$

$\lambda_{He^+} = \lambda \times \dfrac{9}{5}$

$\lambda_{He^+} = \dfrac{9\lambda}{5}$

33. A

Sol. Formed is negatively charged solution, therefore Al^{3+} has highest coagulating power.

34. A

Sol. Bond energy of F_2 less than Cl_2 due to lone pair – lone pair repulsions.

Bond energy order : $Cl_2 > Br_2 > F_2 > I_2$

35. A

Sol. Mond's process uses :

$Ni + 4CO \rightarrow [Ni(CO)_4]$

36. C

Sol. Refer NCERT

37. D

Sol. Hydration enthalpies :

(i) $K^+ > Rb^+ > Cs^+$: (A) > (B) > (D)

(ii) $Mg^{+2} > Ca^{+2}$: (C) > (E)

Option (D)

(C) > (E) > (A) > (B) > (D)

38. D

Sol. $Li_2O \rightarrow O^{2-} \rightarrow$ diamagnetic

$Na_2O_2 \rightarrow O_2^{2-} \rightarrow$ diamagnetic

$KO_2 \rightarrow O_2^{-} \rightarrow$ paramagnetic

39. C

Sol. $4NH_3 + 5O_2 \xrightarrow{\Delta} \underset{(A)}{4NO} + 6H_2O$

$2NO + O_2 \longrightarrow \underset{(B)}{2NO_2}$

40. C

Sol. Metal cation with (–) value of reduction potential (M^{+3}/M^{+2}) or with (+) value of oxidation potential (M^{+2}/M^{+3}) will liberate H_2.

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Therefore they will reduce H^+.

i.e., V^{+2} and Cr^{+2}

41. C

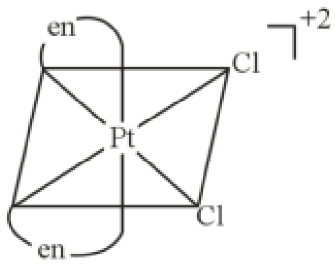
Sol. Photochemical smog has high concentration of oxidising agents.

NO_2 is produced from NO and O_3 in the presence of sunlight.

Classical smog contain smoke, fog and SO_2 and it is known as reducing smog, as chemically it is reducing mixture.

42. A

Sol.



this is chiral complex form

43. B

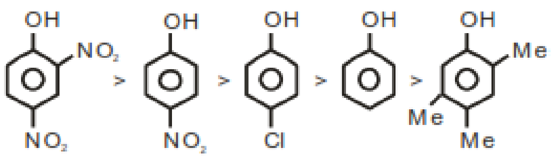
Sol. Boiling point of alkyl halide increases with increase in size, mass of halogen atom and size of alkyl group.

Boiling point of isomeric alkyl halide decreases with increase in branching.

Density increases with increase in atomic mass of halogen atom.

44. B

Sol. Order of acidity for following phenol is



– M and – I increases acidity

+ M and + I decreases acidity

45. C

Sol. Reactions

A. Hoffmann degradation

B. Clemenson reduction

C. Cannizaro reaction

D. Reimer-Tiemann reaction

Reagent used

$Br_2/NaOH$

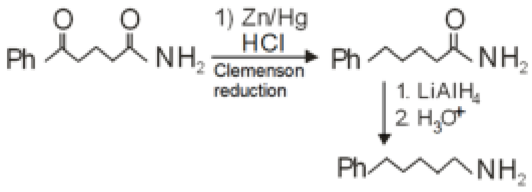
Zn–Hg/HCl

conc. KOH/Δ

$CHCl_3$, $NaOH/H_3O^+$

46. C

Sol.



47. C

Sol. Blue green colour is due to formation of $Cu(BO_2)_2$

$$CuSO_4 \xrightarrow{\Delta} CuO + SO_3$$

$$CuO + B_2O_3 \longrightarrow Cu(BO_2)_2$$

48. A

Sol. (A) Narrow spectrum antibiotic – penicillin-G

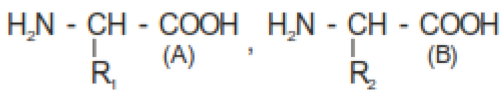
(B) Antiseptic – Furacine

(C) Disinfectants – sulphur dioxide

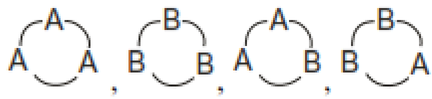
(D) Broad spectrum antisiotics – chloramphenicol

49. D

Sol. Two amino acid are



Tripeptide are formed from three amino acids



50. B

Sol. $CH_3NH_2 \cdot HCl \xrightarrow[\text{fusion}]{\text{Na}} NaCN$ and NaCl

NaCN gives +ve test for nitrogen and

NaCl gives +ve test for halogen

Section - B (Numerical Value)

51. 5

Sol. ∵ pH = 12

∴ $[H^+] = 10^{-12}$ M

∴ $[OH^-] = 10^{-2}$ M

$\therefore$ $[Ca(OH)_2] = 5 \times 10^{-3}$ M

$$5 \times 10^{-3} = \frac{\text{milli moles of } Ca(OH)_2}{100 \text{ mL}}$$

milli moles of $Ca(OH)_2 = 5 \times 10^{-1}$

Ans. = 5

52. 3

Sol. ICl_4^-, BrF_3 and NO_2^+ do not have odd number of e^-.

53. 6

Sol. $\because \Delta G^\circ = -RT \ln K_{eq}$

$$\& \; K_{eq} = \frac{K_f}{K_b}$$

$$\therefore \; K_{eq} = \frac{10^3}{10^2} = 10$$

$$\therefore \; \Delta G = -RT \ln 10$$

$$\Rightarrow -(8.3 \times 300 \times 2.3)$$

$$= -5.7 \text{ kJ mole}^{-1}$$

$$\approx 6 \text{ kJ mole}^{-1}$$

(nearest integer)

= 6

54. 13

Sol. Let a mole $Pb(NO_3)_2$ be added

$$Pb(NO_3)_2 \rightarrow Pb^{2+} + 2NO_3^-$$

a a 2a

$\Delta T_b = 0.15 = 0.5\,[3a] \Rightarrow a = 0.1$

	$Pb^{2+}_{(aq)}$	+	$2Cl^-_{(aq)} \rightarrow PbCl_2(s)$
t = 0	0.1		0.2
t = ∞	(0.1 – x)		(0.2 – 2x)

In final solution,

$$\Delta T_f = 0.8 = 1.8\left[\frac{0.3 - 3x + 0.2 + 0.2}{1}\right]$$

$$\Rightarrow x = \frac{2.3}{27}$$

$$\Rightarrow K_{sp} = \left(0.1 - \frac{2.3}{27}\right)\left(0.2 - \frac{4.6}{27}\right)^2 = 13 \times 10^{-6}$$

55. 2

Sol. $H_2O(g) \rightleftharpoons H_2(g) + \frac{1}{2}O_2(g)$

$P_0[1-\alpha]$ $\quad P_0\alpha$ $\quad \frac{P_0\alpha}{2}$ $\quad$ partial pr. at eq.

$$P_0\left[1 + \frac{\alpha}{2}\right] = 1$$

... (1)

$$K_p = \frac{(P_{H_2})(P_{O_2})^{1/2}}{P_{H_2O}}$$

$$\frac{(P_0\alpha)\left(\frac{P_0\alpha}{2}\right)^{1/2}}{P_0[1-\alpha]} = 2 \times 10^{-3}$$

since α is negligible w.r.t. 1 so P_0 = 1 and $1 - \alpha \approx 1$

$$\frac{\alpha\sqrt{\alpha}}{\sqrt{2}} = 2 \times 10^{-3}$$

$$\alpha^{3/2} = 2^{3/2} \times 10^{-3}$$

$$\alpha = 2^{3/2 \times 2/3} \times 10^{-3 \times 2/3}$$

$$\alpha = 2 \times 10^{-2}$$

$$\%\alpha = 2\%$$

56. 2

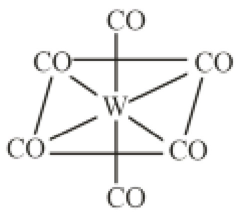
Sol. Statement (A) and Statement (C) are incorrect.

57. 2

Sol. Only option (B) is correct as order cannot be determined.

58. 0

Sol.



$[(CO)_5Mn - Mn(CO)_5]$

59. 6

Sol. $R_f = \dfrac{\text{Distance moved by the substance from base line}}{\text{Distance moved by the solvent from base line}}$

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$$= \frac{3.0 \text{ cm}}{5.0 \text{ cm}} = 0.6 \text{ or } 6 \times 10^{-1}$$

60. 3

Sol. Moles of hydrocarbon = $\frac{17\times10^{-3}}{136} - 1.25\times10^{-4}$

Mole of H_2 gas

$$\Rightarrow 1\times\frac{8.40}{1000} = n\times0.0821\times273$$

$\Rightarrow$ n = 3.75 × 10^{-4}

Hydrogen molecule used for 1 molecule of hydrocarbon is 3.

$$= \frac{3.75\times10^{-4}}{1.25\times10^{-4}} = 3$$

MATHEMATICS

Section - A (Single Correct Answer)

61. B

Sol. $x > 2 > 0 \Rightarrow x > 2$

$x + 1 > 0 \Rightarrow x > -1$

$x+1\neq1 \Rightarrow x\neq0$ and $x > 0$

Denominator

$x^2 - 2x - 3 \neq 0$

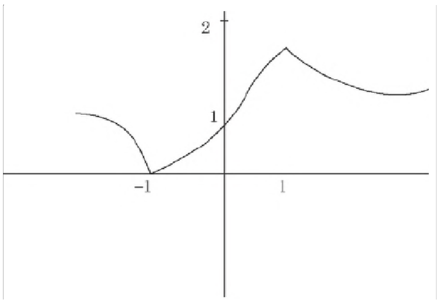
$(x-3)(x+1)\neq0$

$x\neq-1,\ 3$

So Ans. $(2, \infty) - \{3\}$

62. C

Sol.



$$f(x) = \frac{(x+1)^2}{x^2+1} = 1 + \frac{2x}{x^2+1}$$

$$f(x) = 1 + \frac{2}{x+\frac{1}{x}}$$

63. B

Sol. $z_1 = x_1 + iy_1$

$z_2 = x_2 + iy_2$

$Re(z_1z_2) = x_1x_2 - y_1y_2 = 0$

$Re(z_1 + z_2) = x_1 + x_2 = 0$

x_1 & x_2 are of opposite sign

y_1 & y_2 are of opposite sign

64. C

Sol. $14x^2 - 31x + 3\lambda = 0$

$$\alpha+\beta = \frac{31}{14} \quad ....(1)$$

and $$\alpha\beta = \frac{3\lambda}{14} \quad ....(2)$$

$$35x^2 - 53x + 4\lambda = 0$$

$$\alpha+\gamma = \frac{53}{35} \quad ....(3)$$

and $$\alpha\gamma = \frac{4\lambda}{35} \quad ....(4)$$

$$\frac{(2)}{(4)} \Rightarrow \frac{\beta}{\gamma} = \frac{3\times35}{4\times14} = \frac{15}{8} \Rightarrow \beta = \frac{15}{8}\gamma$$

$$(1)-(3) \Rightarrow \beta-\gamma = \frac{31}{14} - \frac{53}{35} = \frac{155-106}{70} = \frac{7}{10}$$

$$\frac{15}{8}\gamma - \gamma = \frac{7}{10} \Rightarrow \gamma = \frac{4}{5}$$

$$\Rightarrow \beta = \frac{15}{8}\times\frac{4}{5} = \frac{3}{2}$$

$$\Rightarrow \alpha = \frac{31}{14} - \beta = \frac{31}{14} - \frac{3}{2} = \frac{5}{7}$$

$$\Rightarrow \lambda = \frac{14}{3}\alpha\beta = \frac{14}{3}\times\frac{5}{7}\times\frac{3}{2} = 5$$

so, sum of roots $\frac{3\alpha}{\beta} + \frac{4\alpha}{\gamma} = \left(\frac{3\alpha\gamma + 4\alpha\beta}{\beta\gamma}\right)$

$$= \frac{\left(3\times\frac{4\lambda}{35} + 4\times\frac{3\lambda}{14}\right)}{\beta\gamma} = \frac{12\lambda(14+35)}{14\times35\beta\gamma}$$

$$= \frac{49 \times 12 \times 5}{490 \times \frac{3}{2} \times \frac{4}{5}} = 5$$

Product of roots

$$= \frac{3\alpha}{\beta} \times \frac{4\alpha}{\gamma} = \frac{12\alpha^2}{\beta\gamma} = \frac{12 \times \frac{25}{49}}{\frac{3}{2} \times \frac{4}{5}} = \frac{250}{49}$$

So, required equation is $x^2 5x + \frac{250}{49} = 0$

$\Rightarrow 49x^2 - 245x + 250 = 0$

65. B

Sol. $D = \begin{vmatrix} \alpha & 2 & 1 \\ 2\alpha & 3 & 1 \\ 3 & \alpha & 2 \end{vmatrix} = 0 \Rightarrow \alpha = -1, 3$

$$D_x = \begin{vmatrix} 2 & 1 & 1 \\ 3 & 1 & 1 \\ \alpha & 2 & \beta \end{vmatrix} = 0 \Rightarrow \beta = 2$$

$$D_y = \begin{vmatrix} \alpha & 1 & 1 \\ 2\alpha & 1 & 1 \\ 3 & 2 & \beta \end{vmatrix} = 0$$

$$D_z = \begin{vmatrix} \alpha & 2 & 1 \\ 2\alpha & 3 & 1 \\ 3 & \alpha & \beta \end{vmatrix} = 0$$

$\beta = 2, \ \alpha = -1$

$\alpha = -1, \ \beta = 2$ Infinite solution.

66. D

Sol. $A^2 = 3A + \alpha I$

$A^3 = 3A^2 + \alpha A$

$A^3 = 3\ (3A + \alpha I) + \alpha A$

$A^3 = 9A + \alpha A + 3\alpha I$

$A^4 = (9 + \alpha)\ A^2 + 3\alpha A$

$= (9 + \alpha)\ (3A + \alpha I) + 3\alpha A$

$= A\ (27 + 6\alpha) + \alpha(\ 9 + \alpha)$

$\Rightarrow 27 + 6\alpha = 21 \ \Rightarrow \ \alpha = -1$

$\Rightarrow \ \beta = \alpha(\ 9 + \alpha) = -8$

67. C

Sol. $\lim_{x \to 2p^+} \left(\frac{1 - \cos(x^2 - 4px + q^2 + 8q + 16)}{(x^2 - 4px + q^2 + 8q + 16)^2} \right)$

$$\left(\frac{(x^2 4px + q^2 + 8q + 16)}{(x - 2p)^2} \right)$$

$$\lim_{h \to 0} \frac{1}{2} \left(\frac{(2p + h)^2 - 4p(2p + h) + q^2 + 82 + 16}{h^2} \right)^2 = \frac{1}{2}$$

Using L'Hospital's

$\lim_{x \to 2p^+} [f(x)] = 0$

68. B

Sol. $f(x) = x + \int_0^{\pi/2} (\sin x \cos y + \cos x \sin y) f(y) dx$

$$f(x) = x + \int_0^{\pi/2} ((\cos y f(y) dy) \sin x + (\sin y f(y) dy) \cos x)$$

....(1)

On comparing with

$$f(x) = x + \frac{a}{\pi^2 - 4} \sin x + \frac{b}{\pi^2 - 4} \cos x, \ x \in \mathbb{R}$$

then

$$\Rightarrow \frac{a}{\pi^2 - 4} = \int_0^{\pi/2} \cos y f(y) dy \quad ....(2)$$

$$\Rightarrow \frac{b}{\pi^2 - 4} = \int_0^{\pi/2} \sin y f(y) dy \quad ....(3)$$

Add (2) and (3)

$$\frac{a + b}{\pi^2 - 4} = \int_0^{\pi/2} (\sin y + \cos y) f(y) dy \quad ....(4)$$

$$\frac{a + b}{\pi^2 - 4} = \int_0^{\pi/2} (\sin y + \cos y) f\left(\frac{\pi}{2} - y\right) dy \quad ......(5)$$

Add (4) and (5)

$$\frac{2(a + b)}{\pi^2 4} = \int_0^{\pi/2} (\sin y + \cos y) \left(\frac{\pi}{2} + \frac{(a + b)}{\pi^2 - 4} (\sin y + \cos y) \right) dy$$

$$= \pi + \frac{a + b}{\pi^2 - 4} \left(\frac{\pi}{2} + 1 \right)$$

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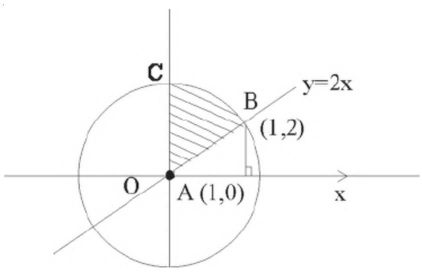
$(a+b)=-2\pi(\pi+2)$

69. A

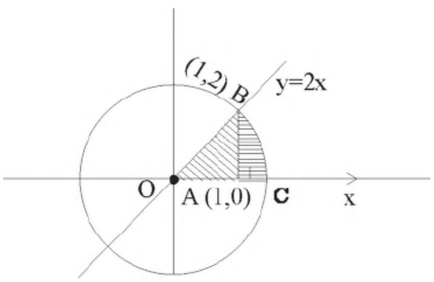
Sol. $y^2 + (x-1)^2 = 4$



shaded portion = circular (OABC)

$$=\frac{\pi(4)}{4}-\frac{1}{2}(2)(1)$$

$$A=(\pi-1)$$



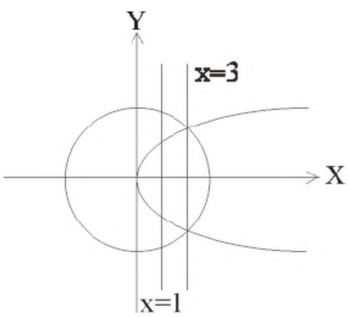
Area $B=Ar(\Delta AOB)+$ Aare of arc of circle (ABC)

$$=\frac{1}{2}(1)(2)+\frac{\pi(2)^2}{4}=\pi+1$$

$$\frac{A}{B}=\frac{\pi-1}{\pi+1}$$

70. D

Sol.



Area $2\int_1^3 2\sqrt{x}dx+2\int_3^{\sqrt{21}}\sqrt{21-x^2dx}$

$$\Delta=\frac{8}{3}(3\sqrt{3}-1)+21\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)-6\sqrt{3}$$

$$\frac{1}{2}\left(\Delta-21\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)\right)=\frac{2\sqrt{3}\frac{8}{3}}{2}$$

$$=\sqrt{3}-\frac{4}{3}$$

71. B

Sol. Slope of reflected ray $=\tan 60^\circ=\sqrt{3}$

Line $y=\frac{x}{\sqrt{3}}$ intersect $y + x = 1$ at

$$\left(\frac{\sqrt{3}}{\sqrt{3}+1},\frac{1}{\sqrt{3}+1}\right)$$

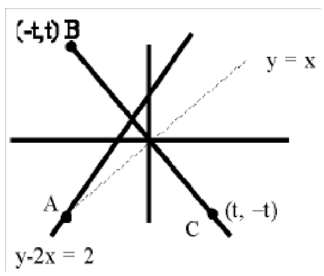
Equation of reflected ray is

$$y-\frac{1}{\sqrt{3}+1}=\sqrt{3}\left(x-\frac{\sqrt{3}}{\sqrt{3}+1}\right)$$

Put $y=0\Rightarrow x=\frac{2}{3+\sqrt{3}}$

72. C

Sol.



At A x = y

Y – 2x = 2

(–2, –2)

Height from line x + y = 0

$$h=\frac{4}{\sqrt{2}}$$

Area of $\Delta = \frac{\sqrt{3}}{4}\frac{h^2}{\sin^2 60} = \frac{8}{\sqrt{3}}$

73. D

Sol. Equation of tangent at A(4, –11) on circle is

$\Rightarrow 4x11y - 3\left(\frac{x+4}{2}\right) + 10\left(\frac{y-1}{2}\right) - 15 = 0$

$\Rightarrow 5x - 12y - 152 = 0$ ....(1)

Equation of tangent at B(8, –5) on circle is

$\Rightarrow 8x - 5y - 3\left(\frac{x+8}{2}\right) + 10\left(\frac{y-5}{2}\right) - 15 = 0$

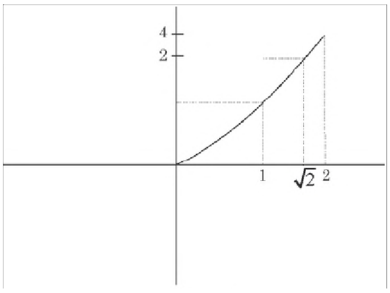
$\Rightarrow 13x - 104 = 0 \Rightarrow x = 8$

put in (1) $\Rightarrow y = \frac{28}{3}$

$r = \left|\frac{3.5 + \frac{2.28}{3} - 34}{\sqrt{13}}\right| = \frac{2\sqrt{13}}{3}$

74. A

Sol.



$A = \int_0^1 1\cdot dx + \int_1^{\sqrt{2}} 2dx + \int_{\sqrt{2}}^{2} x^2 dx$

$= 1 + 2\sqrt{2}2 + \frac{8}{3}\frac{2\sqrt{2}}{3}$

$= \frac{5}{3} + \frac{4\sqrt{2}}{3}$

75. C

Sol. $\begin{vmatrix} \lambda & \mu & 4 \\ 2 & 4 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 0$

$\lambda(10) = \mu(3) + 4(-14) = 0$

$10\lambda - 2\mu = 56$

$5\lambda - \mu = 28$ ....(1)

$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \sqrt{54}$

$\frac{-2\lambda + 4\mu - 8}{\sqrt{24}} = \sqrt{54}$

$-2\lambda + 4\mu - 8 = \sqrt{54\times 24}$ ....(2)

By solving equation (1) & (2)

$\Rightarrow \lambda + \mu = 24$

76. C

Sol. Required probability

$= 1 - \frac{D_{(15)} + {}^{15}C_1\cdot D_{(14)} + {}^{15}C_2 D_{(13)}}{15!}$

Taking $D_{(15)}$ as $\frac{15!}{e}$

$D_{(14)}$ as $\frac{14!}{e}$

$D_{(13)}$ as $\frac{13!}{e}$

We get, $1 - \left(\frac{\frac{15!}{e} + 15\cdot\frac{14!}{e} + \frac{15\times 14}{2}\times\frac{13!}{6}}{15!}\right)$

$= 1\left(\frac{1}{e} + \frac{1}{e} + \frac{1}{2e}\right) = 1 - \frac{5}{2e} \approx .08$

77. B

Sol. $f(\theta) = 3\left(\sin^4\left(\frac{3\pi}{2} - \theta\right) + \sin^4(3x + \theta)\right) - 2(1 - \sin^2 2\theta)$

$S = \left\{\theta \in [0, \pi] : f'(\theta) = -\frac{\sqrt{3}}{2}\right\}$

$\Rightarrow f(\theta) = 3(\cos^4\theta + \sin^4\theta) - 2\cos^2 2\theta$

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$$\Rightarrow f(\theta) = 3\left(1\frac{1}{2}\sin^2 2\theta\right) - 2\cos^2 2\theta$$

$$\Rightarrow f(\theta) = 3\frac{3}{2}\sin^2 2\theta - 2\cos^2\theta$$

$$= \frac{3}{2} - \frac{1}{2}\cos^2 2\theta = \frac{3}{2} - \frac{1}{2}\left(\frac{1+\cos 4\theta}{2}\right)$$

$$f(\theta) = \frac{5}{4} - \frac{\cos 4\theta}{4}$$

$$f'(\theta) = \sin 4\theta$$

$$\Rightarrow f'(\theta) = \sin 4\theta = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow 4\theta = n\pi + (-1)^n \frac{\pi}{3}$$

$$\Rightarrow \theta = \frac{n\pi}{4} + (-1)^n \frac{\pi}{12}$$

$$\Rightarrow \theta = \frac{\pi}{12}, \left(\frac{\pi}{4} - \frac{\pi}{12}\right), \left(\frac{\pi}{2} + \frac{\pi}{12}\right), \left(\frac{3\pi}{4} - \frac{\pi}{12}\right)$$

$$\Rightarrow 4\beta = \frac{\pi}{4} + \frac{\pi}{2} + \frac{3\pi}{4} = \frac{3\pi}{2}$$

$$\Rightarrow \beta = \frac{3\pi}{8} \Rightarrow f(\beta) = \frac{5}{4} - \frac{\cos\frac{3\pi}{2}}{4} = \frac{5}{4}$$

78. C

Sol.

	p	q	r	$(p \vee q) \wedge ((\sim p) \vee r)$	$\sim q \vee r$
(1)	T	F	T	T	T
(2)	T	T	F	F	F
(3)	F	T	F	T	F
(4)	F	F	F	F	T

Option (3) $(p \vee q) \wedge (\sim q \vee r) \rightarrow (\sim p \vee r)$ will be False.

79. A

Sol.

X	P(x)	XP(X)	$X^2P(X)$
0	1/6	0	0
1	1/2	1/2	1/2
2	3/10	6/10	12/10
3	1/30	1/10	9/30

$$\sum xP(x) = \frac{6}{2} = \mu$$

$$\sigma^2 = \sum x^2 P(x) - \mu^2$$

$$\sigma^2 + \mu^2 = 0 + \frac{1}{2} + \frac{12}{10} + \frac{9}{30} = 2$$

$$10(\sigma^2 + \mu^2) = 20 \text{ Ans.}$$

80. A

Sol. $\frac{x+1}{x^2}dx = \frac{dy}{y}$

$$\ln x - \frac{1}{x} = \ln y + c$$

(1, c)

c = –2

$$\ln x - \frac{1}{x} = \ln y - 2$$

$$y = e^{\ln x} - \frac{1}{x} + 2$$

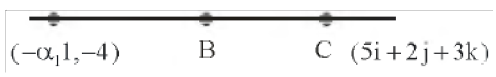
$$\lim_{x \to 0^+} e^{\ln x - 1} - \frac{1}{x} + 2$$

$$= e^{-\infty} = 0$$

Section - B (Numerical Value)

81. 9

Sol. A.$(O_1 2, \alpha)$



$$\left|\frac{1}{2} \cdot 2\sqrt{21} \cdot \begin{vmatrix} i & j & k \\ \alpha & 1 & \alpha+4 \\ 5 & 2 & 3 \end{vmatrix} \frac{1}{\sqrt{25+4+9}}\right| = 21\sqrt{21}$$

$$\sqrt{(2\alpha+5)^2 + (2\alpha+20)^2 + (2\alpha-5)^2} = \sqrt{21}\sqrt{38}$$

$$\Rightarrow 12\alpha^2 + 80\alpha + 450 = 798$$

$$\Rightarrow 12\alpha^2 + 80\alpha - 348 = 0$$

$$\Rightarrow 12\alpha^2 + 80\alpha - 348 = 0$$

$$\Rightarrow \alpha = 3 \Rightarrow \alpha^2 = 9$$

82. 355

Sol. The line $\frac{x+10}{1}=\frac{y-8}{-2}=\frac{z}{1}$ have a point

(–10, 8, 0) with d.r. (1, –2, 1)

$\because$ the plane ax + by + 3z = 2 (a + b)

$\Rightarrow b = 2a$

& do product of d.r.'s is zero

$\therefore a-2b+3=0$

$\therefore$ a = 1 & b = 2

Distance from (1, 27, 7) is

$$c=\frac{1+54+21-6}{\sqrt{14}}=\frac{70}{\sqrt{14}}=5\sqrt{14}$$

$\therefore a^2+b^2+c^2=1+4+350$

$=355$

83. 10

Sol. $\because f(1)=\frac{1}{5}\therefore f(2)=f(1)+f(1)=\frac{2}{5}$

$$f(2)=\frac{2}{5}f(3)=f(2)+f(1)=\frac{3}{5}$$

$$f(3)=\frac{3}{5}$$

$$\therefore\sum_{n=1}^{m}\left(\frac{1}{n+1}\frac{1}{n+2}\right)$$

$$=\frac{1}{5}\sum_{n=1}^{m}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$$

$$=\frac{1}{5}\left(\frac{1}{2}-\frac{1}{m+2}\right)=\frac{m}{10(m+2)}=\frac{1}{12}$$

$\therefore m=10$

84. 60

Sol. $a_4\cdot a_6=9\Rightarrow (a_5)^2=9\Rightarrow a_5=3$

& $a_5+a_7=24\Rightarrow a_5+a_5r^2=24\Rightarrow (1+r^2)=8$

$\Rightarrow r=\sqrt{7}$

$$\Rightarrow a=\frac{3}{49}$$

$\Rightarrow a_1a_9 + a_2a_4a_9 + a_5 + a_7 = 9 + 27 + 3 + 21 = 60$

85. 2

Sol. $\overline{AB}=(\lambda-1)\bar{a}-2\bar{b}+3\bar{c}$

$\overline{AC}=2\bar{a}+3\bar{b}-4\bar{c}$

$\overline{AD}=\bar{a}-3\bar{b}+5\bar{c}$

$$\begin{vmatrix}\lambda-1 & -2 & 3\\ 2 & 3 & -4\\ 1 & -3 & 5\end{vmatrix}=0$$

$\Rightarrow(\lambda-1)(1512)+2(-10+4)+3(6-3)=0$

$\Rightarrow(\lambda-1)=1\Rightarrow\lambda=2$

86. 32

Sol.

1	2					$= {}^7C_4 = 35$
1	3					$= {}^6C_4 = 15$
1	4					$= {}^5C_4 = 5$
1	5					$= {}^4C_4 = 1$
2	3					$= {}^6C_4 = 15$

71 words

2 4 5 6 7 8 $\rightarrow$ 72th word

2 + 4 + 5 + 6 + 7 + 8 = 32

87. 3

Sol. f(x + y) = f(x) + f(y) – 1

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(y)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{f(h)-f(0)}{h}=f'(0)=2$$

f '(x) = 2 $\Rightarrow$ dy = 2dx

y = 2x + C

x = 0, y = 1, c = 1 y = 2x + 1

|f (–2)| = |–4 + 1| = |–3| = 3

88. 1

Sol. Coefficient of x^9 in $\left(\alpha x^3+\frac{1}{\beta x}\right) = {}^{11}C_6\cdot\frac{\alpha^3}{\beta^6}$

$\because$ Both are equal

$$\therefore\frac{11}{C_6}\cdot\frac{\alpha^5}{\beta^6}=-\frac{11}{C_5}\cdot\frac{\alpha^6}{\beta^5}$$

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$$\Rightarrow \frac{1}{\beta} = -\alpha$$

$$\Rightarrow \alpha\beta = -1$$

$$\Rightarrow (\alpha\beta)^2 = 1$$

89. 1120

Sol. $t_{r+1} = {}^nC_r\,(2x)^r$

$$\Rightarrow \frac{{}^nC_{r1}(2)^{r-1}}{{}^nC_r(2)^r} = \frac{2}{5}$$

$$\Rightarrow \frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!(2)}{r!(n-r)!}} = \frac{2}{5}$$

$$\Rightarrow \frac{r}{nr+1} = \frac{4}{5} \Rightarrow 5r = 4n - 4r + 4$$

$$\Rightarrow 9r = 4(n+1) \qquad ....(1)$$

$$\Rightarrow \frac{{}^nC_r(2)^r}{{}^nC_{r+1}(2)^{r+1}} = \frac{5}{8}$$

$$\Rightarrow \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{5}{4} \Rightarrow \frac{r+1}{n-r} = \frac{5}{4}$$

$$\Rightarrow 5r + 4 = 5n - 5r \Rightarrow 5n - 4 = 9r \qquad ....(2)$$

From (1) and (2)

$\Rightarrow$ 4n + 4 = 5n – 4 $\Rightarrow$ n = 8

(1) $\Rightarrow$ r = 4

so, coefficient of middle term is

$$^8C_4 2^4 = 16 \times \frac{8\times7\times6\times5}{4\times3\times2\times1} = 16\times70 = 1120$$

90. 1436

Sol. No. of 5 digit numbers starting with digit 1

= 5 × 5 × 5 × 5 = 625

No. of 5 digit numbers starting with digit 2

= 5 × 5 × 5 × 5 = 625

No. of 5 digit numbers starting with 31

= 5 × 5 × 5 = 125

No. of 5 digit numbers starting with 32

= 5 × 5 × 5 = 125

No. of 5 digit numbers starting with 33

= 5 × 5 × 5 = 125

No. of 5 digit numbers starting with 351

= 5 × 5 = 25

No. of 5 digit numbers starting with 352

= 5 × 5 = 25

No. of 5 digit numbers starting with 3531

= 5

No. of 5 digit numbers starting with 3532

= 5

Before 35337 will be 4 numbers,

So rank of 35337 will be 1690

So, in descending order serial number will be

3125 – 1690 + 1 = 1436

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