3S INSTITUTE
JEE ADVANCED | JEE MAIN | NEET | OLYMPIADS | MHT-CET | FOUNDATION

25-January-2023 (Evening Batch) : JEE Main Paper

PHYSICS

Section - A (Single Correct Answer)

1. B

Sol. $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta l / l} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

$$F = 6\pi\eta rv \Rightarrow \eta = \frac{F}{6\pi rv}$$

$$[\eta] = \frac{[MLT^{-2}]}{[L][LT^{-1}]} = [ML^{-1}T^{-1}]$$

$$E = h\nu \Rightarrow h = \frac{E}{\nu} = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$$

Work function has same dimension as that of energy, so $[\phi] = [ML^2T^{-2}]$

2. D

Sol. Diatomic gas molecules have three translational degree of freedom, two rotational degree of freedom & it is given that it has one vibrational mode so there are two additional degree of freedom corresponding to one vibrational mode, so total degree of freedom = 7

$$C_V = \frac{fR}{2} = \frac{7R}{2}$$

3. A

Sol. Plane mirror forms erect, same sized, laterally inverted and virtual image of real object.

4. B

Sol. $\tau = K\theta$

NiAB = $K\theta$

$$A = \frac{K\theta}{NiB} = \frac{4\times10^{-5}\times0.05}{200\times10\times10^{-3}\times0.01}$$

On solving $A = 10^{-4}\ m^2 = 1\ cm^2$

5. B

Sol.

$$\frac{\text{reading on scale} - \text{Lower fixed point}}{\text{upper fixed point} - \text{lower fixed point}} = \text{constant}$$

$$\frac{t_P - 30}{180 - 30} = \frac{t_Q - 0}{100 - 0}$$

6. A

Sol. Gauss's Law of electrostatic

$$\phi = \oint \vec{E}\cdot d\vec{s} = \frac{q}{\epsilon_0}$$

Faraday's law $\oint \vec{E}\cdot d\vec{l} = \frac{-d\phi_B}{dt}$

Gauss's law of magnetism $\oint \vec{B}\cdot d\vec{A} = 0$

Ampere's Maxwell law

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i_C + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$$

Where i_C : Conduction current

$\epsilon_0 \frac{d\phi_E}{dt}$: Displacement current

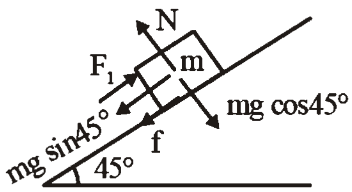
7. D

Sol. Statement - I is correct

When P-N junction is formed an electric field is generated form N-side to P-side due to which barrier potential arises & majority charge carrier can not flow through the junction due to barrier potential so current is zero unless we apply forward bias voltage.

8. A

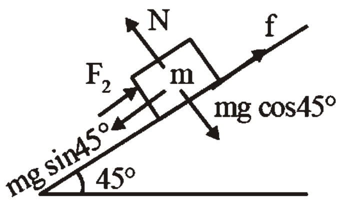
Sol.



$F_1 = mg \sin45° + f = mg \sin45° + \mu N$

$$F_1 = \frac{mg}{\sqrt{2}} + \mu mg \cos 45°$$

$$F_1 = \frac{mg}{\sqrt{2}}(1+\mu)$$



$F_2 = mg \sin 45° – f = mg \sin 45° – \mu N$

$$= \frac{mg}{\sqrt{2}}(1-\mu)$$

$F_1 = 2F_2$

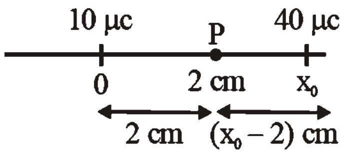
$$\frac{mg}{\sqrt{2}}(1+\mu) = 2\frac{mg}{\sqrt{2}}(1-\mu)$$

$1 + \mu = 2 – 2\mu$

$\mu = 1/3 = 0.33$

9. A

Sol.



$$E_P = \frac{K \times 10}{2^2} - \frac{K \times 40}{(x_0 - 2)^2} = 0$$

$$\frac{1}{2} = \frac{2}{x_0 - 2}$$

$x_0 – 2 = 4$

$x_0 = 6$ cm

10. D

Sol. $\lambda = \dfrac{hc}{\Delta E}$

$\Delta E_A = 2.2$ eV

$\Delta E_B = 5.2$ eV

$\Delta E_C = 3$ eV

$\Delta E_D = 10$ eV

$$\lambda_A = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{2.2 \times 1.6 \times 10^{-19}}$$

$$= \frac{12.41 \times 10^{-7}}{2.2} m$$

$$= \frac{1241}{2.2} nm = 564 nm$$

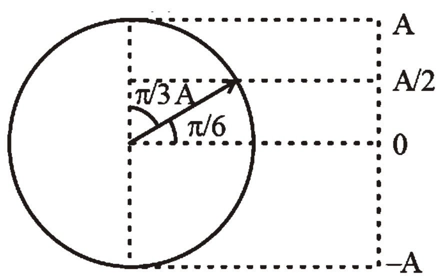
$$\lambda_B = \frac{1241}{5.2} nm = 238.65 nm$$

$$\lambda_C = \frac{1241}{3} nm = 413.66 nm$$

$$\lambda_D = \frac{1241}{10} nm = 124.1 nm$$

11. D

Sol.



Let time from 0 to A/2 is t_1 & from A/2 to A is t_2

then $\omega t_1 = \pi/6$

$\omega t_2 = \pi/3$

$$\frac{t_1}{t_2} = \frac{1}{2}$$

$t_2 = 2t_1 = 2 \times 2 = 4$ sec

12. B

Sol. $\Delta U = nC_V\Delta T$

For isothermal process T is constant

So $\Delta U = 0$

$A \rightarrow II$

Adiabatic process

$\Delta Q = 0$

$\Delta Q = \Delta U + \Delta W$

$\Delta U = – \Delta W$

Work done by gas is positive

So ΔU is negative

$B \rightarrow I$

For Isochoric process $\Delta W = 0$

$C \rightarrow IV$

For Isobaric process

$\Delta W = P\Delta V \neq 0$

$\Delta U = nC_V\Delta T \neq 0$

Heat absorbed goes partly to increase internal energy and partly do work.

13. A

Sol. NCERT fact based

14. C

Sol. $U = \frac{-GM_e m}{r}$

$U_i = \frac{-GM_e m}{R_e}$

$U_f = \frac{-GM_e m}{(R_e + h)} = \frac{-GM_e m}{R_e + 2R_e}$

$\frac{-GM_e m}{3R_e}$

Increase in internal energy $\Delta U = U_f - U_i$

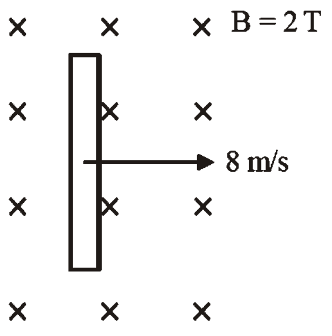
$= \frac{2}{3}\frac{GM_e m}{R_e}$

$\frac{2}{3}\frac{GM_e}{R_e^2} mR_e$

$= \frac{2}{3} mgR_e$

15. D

Sol.



Induced emf across the ends = Bvl

$= 2 \times 8 \times 1 = 16$ V

16. A

Sol. $x = 4t^2$

$v = \frac{dx}{dt} = 8t$

At t = 5 sec

$v = 8 \times 5 = 40$ m/s.

17. D

Sol. Range $= \frac{u^2 \sin 2\theta}{g}$

Range for projection angle "α"

$R_1 = \frac{u^2 \sin 2\alpha}{g}$

Range for projection angle "β"

$R_2 = \frac{u^2 \sin 2\beta}{g}$

$\alpha + \beta = 90°$ (Given)

$\Rightarrow \boxed{\beta = 90° - \alpha}$

$R_2 = \frac{u^2 \sin 2(90° - \alpha)}{g}$

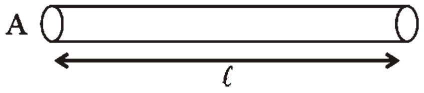
$R_2 = \frac{u^2 \sin(180° - 2\alpha)}{g}$

$R_2 = \frac{u^2 \sin 2\alpha}{g}$

$\Rightarrow \frac{R_1}{R_2} = \frac{\left(\frac{u^2 \sin 2\alpha}{g}\right)}{\left(\frac{u^2 \sin 2\alpha}{g}\right)} = \frac{1}{1}$

18. C

Sol.



$R_{initial} = \frac{\rho l}{A} = 5\Omega$

$\because$ Volume of wire is constant in stretching

$V_i = V_f$

$A_i l_i = A_f l_f$

$Al = A'(5l)$

$$A' = \frac{A}{5}$$

$$R_f = \frac{\rho l_f}{A_f} = \frac{\rho(5l)}{\left(\frac{A}{5}\right)}$$

$$= 25\left(\frac{\rho l}{A}\right)$$

$= 25 \times 5 = 125\ \Omega$

19. D

Sol. Stopping potential $V_S = \frac{KE_{max}}{e}$

$$V_S = \frac{\frac{hC}{\lambda} - \phi}{e}$$

Stopping potential does not depend on intensity or power of light used, it only depends on frequency or wavelength of incident light.

So both statements I and II are correct

20. A

Sol. $F = \frac{Gm_1 m_2}{r^2}$

$$\Rightarrow F \propto \frac{1}{r^2}$$

$\Rightarrow F \propto m_1\ m_2$

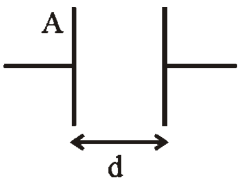
$\Rightarrow$ This force provides centripetal force and acts towards sun

$\Rightarrow T^2 \propto a^3$ (Kepler's third law)

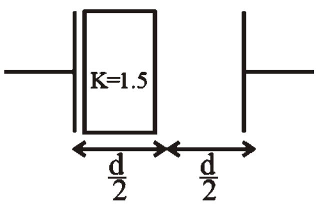
Section - B (Numerical Value)

21. 6

Sol.



$\frac{\in_0 A}{d} = 5\mu F$



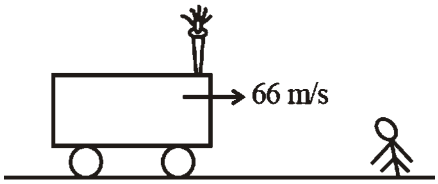
$$C_{new} = \frac{\in_0 A}{\frac{\left(\frac{d}{2}\right)}{1.5} + \frac{\left(\frac{d}{2}\right)}{1}}$$

$$= \frac{\in_0 A}{\left(\frac{d}{3} + \frac{d}{2}\right)} = \frac{6 \in_0 A}{5d}$$

$$= \frac{6}{5} \times 5\mu F = 6\mu F$$

22. 400

Sol.

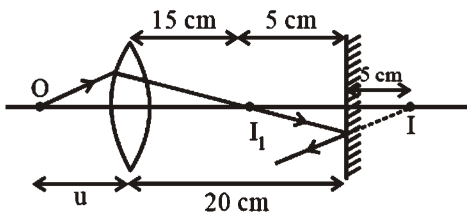


$$f_{app} = f\left(\frac{v}{v - v_S}\right)$$

$$= 320\left(\frac{330}{330 - 66}\right) = 400\ Hz$$

23. 30

Sol.



f = 10 cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

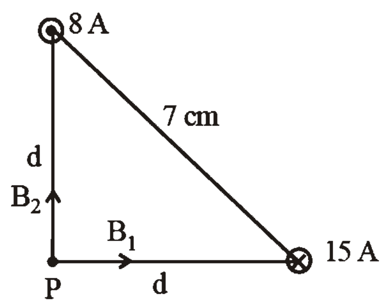
$$\frac{1}{15} - \frac{1}{-u} = \frac{1}{10}$$

$$\Rightarrow \frac{1}{u} = \frac{1}{10} - \frac{1}{15}$$

On solving we get value of u as 30 cm.

24. 68

Sol.



Magnetic fields due to both wires will be perpendicular to each other.

$$B_1 = \frac{\mu_0 i_1}{2\pi d} \quad B_2 = \frac{\mu_0 i_2}{2\pi d}$$

$$B_{net} = \sqrt{B_1^2 + B_2^2} \Rightarrow \frac{\mu_0}{2\pi d}\sqrt{i_1^2 + i_2^2}$$

$$\Rightarrow \frac{4\pi \times 10^{-7}}{2\pi \times \left(7/\sqrt{2}\right) \times 10^{-2}} \times \sqrt{8^2 + 15^2} \left(d = \frac{7}{\sqrt{2}} \text{cm}\right)$$

$\Rightarrow \quad 68 \times 10^{-6}$ T

25. 1

Sol. Surface Tension = T

R : Radius of bigger drop

r : Radius of smaller drop

Volume will remain same

$$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3$$

R = 10 r

$u_i = T \cdot 4\pi R^2$

$u_f = T \cdot 4\pi R^2 \times 1000$

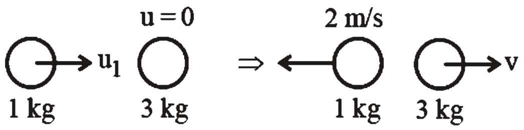
$$\frac{u_f}{u_i} = \frac{1000 r^2}{R^2}$$

$$\frac{u_f}{u_i} = \frac{10}{1}$$

So, x = 1

26. 4

Sol.



$1 \times u_1 = -2 + 3v \Rightarrow u_1 = -2 + 3v$ .....(1)

$$1 = \frac{v+2}{u_1} \qquad \Rightarrow v + 2 = u_1 \qquad .....(2)$$

Solving (1) and (2)

$u_1 = 4$ m/s

27. 2

Sol.

$$\frac{v_1}{v_2} = \frac{3}{2}$$

$$m_1 v_1 = m_2 v_2 \Rightarrow \frac{m_1}{m_2} = \frac{2}{3}$$

Since, Nuclear mass density is constant

$$\frac{m_1}{\frac{4}{3}\pi r_1^3} = \frac{m_2}{\frac{4}{3}\pi r_2^3}$$

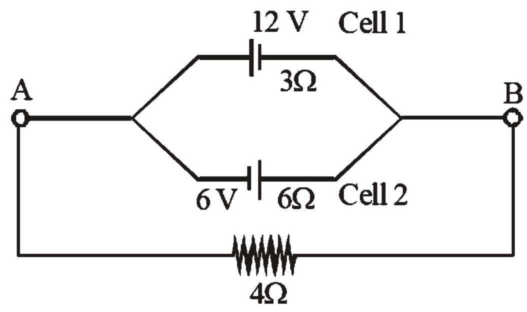
$$\left(\frac{r_1}{r_2}\right)^3 = \frac{m_1}{m_2}$$

$$\frac{r_1}{r_2} = \left(\frac{2}{3}\right)^{\frac{1}{3}}$$

So, x = 2

28. 1

Sol.

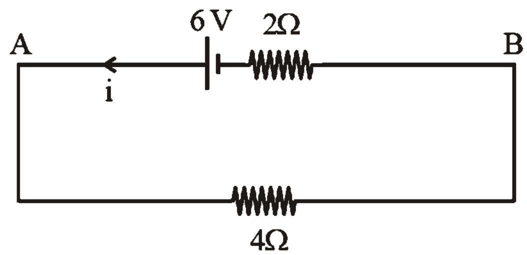


$$E_{eq} = \frac{\frac{12}{3} - \frac{6}{6}}{\frac{1}{3} + \frac{1}{6}}$$

$E_{eq} = 6V$

$r_{eq} = 2\ \Omega$

$R = 4\ \Omega$



$$\text{So, } i = \frac{6}{2+4} = 1A$$

29. 8

Sol. $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_C - X_L)^2}}$

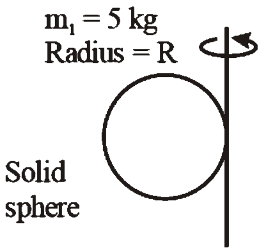
$\cos\phi = \frac{80}{\sqrt{(80)^2 + (60)^2}}$

$\cos\phi = \frac{80}{100} \Rightarrow \frac{8}{10}$

So, x = 8

30. 5

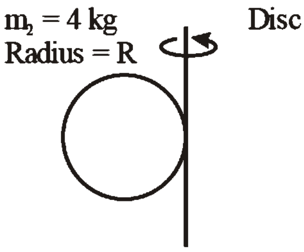
Sol.



$I_1 = \frac{2}{5} m_1 R^2 + m_1 R^2$

$I_1 = m_1 R^2 \left(\frac{7}{5}\right)$

$I_1 = 7R^2$



$I_2 = \frac{m_2 R^2}{4} + m_2 R^2$

$I_2 = \frac{5}{4} m_2 R^2$

$I_2 = 5R^2$

$\frac{I_2}{I_1} = \frac{5}{7}$

x = 5

CHEMISTRY

Section - A (Single Correct Answer)

31. D

Sol. Cobalt catalyst $\rightarrow$ Methanol production

Syn gas $\rightarrow$ Coal gasification

$\left(C_{\text{(Red hot coke)}} + H_2O(g) \rightarrow CO + H_2\right)$

Nickel catalyst $\rightarrow$ Water gas production

Brine solution $\rightarrow$ Production

(aq. NaCl) $\begin{pmatrix} H_2 \rightarrow \text{Cathode} \\ Cl_2 \rightarrow \text{Anode} \end{pmatrix}$

32. B

Sol. In froth floatation method a rotating paddle draws in air and stirs the pulp.

33. A

Sol. Metallic character increases down the group and decreases along the period.

34. C

Sol. The alkali metals and their salts impart characteristic colour to oxidizing flame.

35. C

Sol. Assume : Mass of solvent $\approx$ Mass of solution

Case I :-

$0.25 = \frac{W_1}{62} \times \frac{1000}{500}$

Case II :-

$0.25 = \frac{W_2}{62} \times \frac{1000}{250}$

$\frac{W_1}{W_2} = \frac{2}{1}$

36. D

Sol. Statement II : The corssed arrow symbolises the direction of the shift of electron density in the molecule.

37. A

Sol. Butylated hydroxyl anisole is an antioxidant.

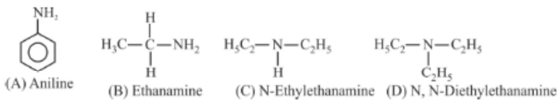
38. D

Sol. B. $\dot{Cl} + O_3 \longrightarrow O_2 + Cl\dot{O}$

D. 'Blue baby' syndrome occurs due to the presence of excess of nitrate ions in water.

39. D

Sol.



Basic strength $\alpha \frac{1}{pK_b}$

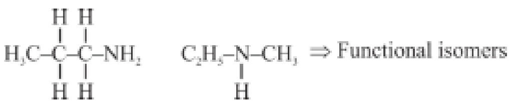
Order for pK_b : A > B > D > C

40. C

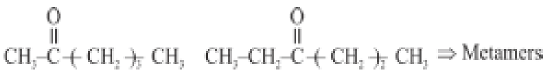
Sol. Sodium have lowest oxidation potential in alkali metals. Hence it is weakest reducing agent among alkali metals.

41. D

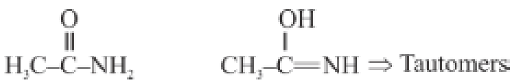
Sol. A. Propanamine N–Methylethanamine



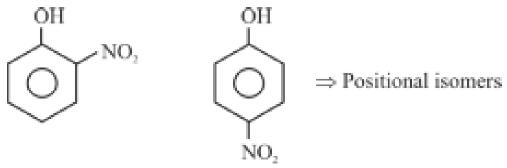
B. Hexan–2–one Hexan–3–one



C. Ethanamide Hydroxyethanimine



D. o–Nitrophenol p–nitrophenol



42. B

43. B

Sol. The oxide which form acid on dissolving in water is acidic oxide.

44. B

Sol. $14H^{+} + 6e^{-} + \underset{+6}{Cr_2O_7^{-2}} \longrightarrow \underset{+3}{2Cr^{+3}} + 7H_2O$

45. D

Sol. $\Delta[H^+] = 1000$

$\Delta pH = -\log \Delta[H^+] = -\log 10^3$

$= -3$

46. B

Sol.

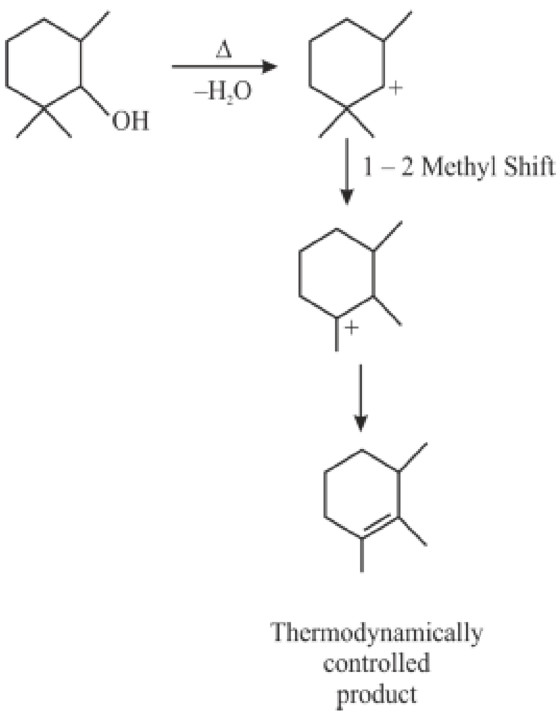
	List I Coordination entity		List II Wavelength of light absorbed in nm
A.	$[CoCl(NH_3)_5]^{2+}$	I.	535
B.	$[Co(NH_3)_6]^{3+}$	II.	475
C.	$[Co(CN)_6]^{3-}$	III.	310
D.	$[Cu(H_2O)_4]^{2+}$	IV.	600

$$E = \frac{hc}{\lambda} \Rightarrow E \propto \frac{1}{\lambda}$$

$$\Rightarrow \Delta(CFSE) \propto \frac{1}{\lambda_{absorb}} \propto \underline{\text{strength of ligand}}$$

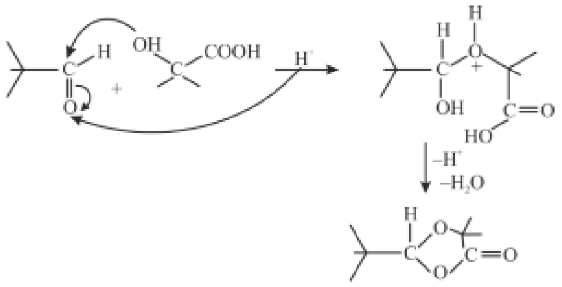
47. A

Sol.



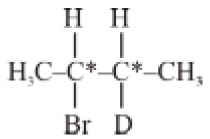
48. B

Sol.



49. C

Sol.



50. C

Sol. $Cl^- + AgNO_3 \longrightarrow \underset{\substack{[A] \\ \text{Curdy white} \\ \text{precipitate}}}{AgCl}$

$AgCl + NH_4OH \longrightarrow \underset{\substack{[B] \\ \text{Soluble complex}}}{[Ag(NH_3)_2]Cl}$

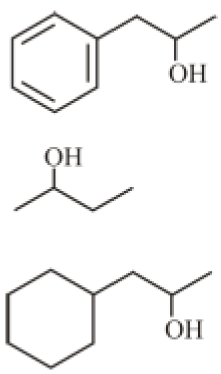
Section - B (Numerical Value)

51. 5

Sol. p_x, p_y, p_z, d_{z^2} and $d_{x^2-y^2}$ are axial orbitals.

52. 3

Sol.



53. 4

Sol. $\pi = iCRT$

$\pi \propto iC$

A, B, D and E have same value of osmatic pressure.

54. 925

Sol. Let, Volume of C_2H_4 is x litre

	C_2H_4 +	$3O_2 \rightarrow$	$2CO_2$ +	$2H_2O$
Initial	x			
Final	–		2x	

	CH_4 +	$2O_2 \rightarrow$	CO_2 +	$2H_2O$
Initial	(16.8 – x)			
Final	–		(16.8 – x)	

Total volume of CO_2 = 2x + 16.8 – x

$\Rightarrow$ 28 = 16.8 + x

x = 11.2 L

$$n_{CH_4} = \frac{PV}{RT} = \frac{1 \times 5.6}{0.082 \times 298} = 0.229 \text{ mole}$$

$$n_{C_2H_2} = \frac{11.2}{0.082 \times 298} = 0.458 \text{ mole}$$

$\therefore$ Heat evolved = 0.229 × 900 + 0.458 × 1400

= 206.1 + 641.2

= 847.3 kJ

55. 5

Sol. $[Co(NH_3)_4Cl_2]Cl \Rightarrow$ Gives 1 mole AgCl

$[Ni(H_2O)_6]Cl_2 \Rightarrow$ Gives 2 moles AgCl

$[Pt(NH_3)_2Cl_2] \Rightarrow$ Gives No AgCl

$[Pd(NH_3)_4]Cl_2 \Rightarrow$ Gives 2 moles AgCl

Total number of moles of AgCl = 5 mole.

56. 12

Sol.

Element	Percentage	Mole	Mole ratio
C	85.8	$\frac{85.8}{12} = 7.15$	1
H	14.2	$\frac{14.2}{1} = 14.2$	2

Empirical formula (CH_2)

14 × n = 84

n = 6

$\therefore$ Molecular formula C_6H_{12}

57. 3

Sol. Overall reaction :–

$$H_{2(g)} + M^{3+}_{(aq)} \longrightarrow M^{+}_{(aq)} + 2H^{+}_{(aq)}$$

$$E_{cell} = E^{o}_{cathode} - E^{o}_{anode} - \frac{0.059}{2}\log\frac{[M^{+}]\times 1^{2}}{[M^{+3}]\times 1}$$

$$0.1115 = 0.2 - \frac{0.059}{2}\log\frac{[M^{+}]}{[M^{+3}]}$$

$$3 = \log\frac{[M^{+}]}{[M^{+3}]}$$

$\therefore$ a = 3

58. 2

Sol. B and D options are correct.

59. 2

Sol. $$t_{10\%} = \frac{1}{K}\ln\left(\frac{a}{a-x}\right) = \frac{1}{K}\ln\left(\frac{100}{90}\right)$$

$$t_{10\%} = \frac{2.303}{K}(\log 10 - \log 9)$$

$$t_{10\%} = \frac{2.093}{K}\times(0.04)$$

Similarly,

$$t_{90\%} = \frac{1}{K}\ln\left(\frac{100}{10}\right)$$

$$t_{90\%} = \frac{2.303}{K}$$

$$\frac{t_{90\%}}{t_{10\%}} = \frac{1}{0.04} = 25$$

$$e^{kt} = \frac{a}{a-x}$$

$$\frac{a-x}{a} = e^{-kt}$$

$$1 - \frac{x}{a} = e^{-kt}$$

$$x = a(1-e^{-kt})$$

$$\alpha = \frac{x}{a} = (1-e^{-kt})$$

60. 2

Sol. 'A' water vapours are absorbed by calcium chloride.

C. As the adsorption proceeds, ΔH becomes lessand less negative.

MATHEMATICS

Section - A (Single Correct Answer)

61. C

Sol. $f(x) = 2x^3 + (2p-7)x^2 + 3(2p-9)x - 6$

$f'(x) = 6x^2 + 2(2p-7)x + 3(2p-9)$

$f'(0) < 0$

$\therefore 3(2p-9) < 0$

$$p < \frac{9}{2}$$

$$p \in \left(-\infty, \frac{9}{2}\right)$$

62. D

Sol. $(z-2i)(\overline{z}+2i) = 4(z+i)(\overline{z}-i)$

$$z\overline{z} + 4 + 2i(z-\overline{z}) \setminus 4(z\overline{z} + 1 + i(\overline{z}-z))$$

$$3z\overline{z} - 6i(z-\overline{z}) = 0$$

$x^2 + y^2 - 2i(2iy) = 0$

$x^2 + y^2 + 4y = 0$

63. D

Sol. $$\lim_{x\to\frac{\pi^+}{2}} e^{\frac{\cot 6x}{\cot 4x}} = \lim_{x\to\frac{\pi^+}{2}} e^{\frac{\sin 4x\cdot\cos 6x}{\sin 6x\cdot\cos 4x}} = e^{2/3}$$

$$\Rightarrow \lim_{x\to\frac{\pi^-}{2}} (1+|\cos x|)^{\frac{\lambda}{|\cos x|}} = e^{\lambda}$$

$$\Rightarrow f(\pi/2) = \mu$$

For continuous function $\Rightarrow e^{2/3} = e^{\lambda} = \mu$

$$\lambda = \frac{2}{3},\ \mu = e^{2/3}$$

Now, $9\lambda + 6\log_e \mu + \mu^6 - e^{6\lambda} = 10$

64. B

Sol. $f(x) = 2x^n + \lambda$

$f(4) = 133$

$f(5) = 255$

$133 = 2 \times 4^n + \lambda$ (1)

$255 = 2 \times 5^n + \lambda$ (2)

(2) - (1)

$122 = 2(5^n - 4^n)$

$\Rightarrow 5^n - 4n = 61$

$\therefore n = 3 \;\&\; \lambda = 5$

Now, $f(3) - f(2) = 2(3^3 - 2^3) = 38$

Number of Divisors is 1, 2, 19, 38 ; & their sum is 60

65. A

Sol. Let A : (3, –4, 2) C : (-2, -1, 3)

B : (1, 2, –1) D : (5, –2α, 4)

A, B, C, D are coplanar points, then

$$\Rightarrow \begin{vmatrix} 1-3 & 2+4 & -1-2 \\ -2-3 & -1+4 & 3-2 \\ 5-3 & -2\alpha+4 & 4-2 \end{vmatrix} = 0$$

$$\Rightarrow \alpha = \frac{73}{17}$$

66. D

Sol. $$AA^T = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{-3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$$

$$B^3 = \begin{bmatrix} 1 & -3i \\ 0 & 1 \end{bmatrix}$$

$\vdots$

$$B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$$

$M = A^TBA$

$M^2 = M.M = A^TBA\,A^TBA = A^TB^2A$

$M^3 = M^2.M = A^TB^2AA^TBA = A^TB^3A$

$\vdots$

$M^{2023} = \dots\dots\dots\dots\dots\dots\; A^TB^{2023}A$

$$AM^{2023}A^T = \underline{AA^T}\,B^{2023}\,\underline{AA^T} = B^{2023}$$

$$= \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$$

Inverse of $(AM^{2023}A^T)$ is $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

67. C

Sol. Given $(p \to q)\Delta(p\nabla q)$

Option 1 $\Delta = \wedge,\ \nabla = \vee$

p	q	$(p \to q)$	$(p \vee q)$	$(p \to q) \wedge (p \vee q)$
T	T	T	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

Option 2 $\Delta = \vee,\ \nabla = \wedge$

p	q	$(p \to q)$	$(p \wedge q)$	$(p \to q) \vee (p \wedge q)$
T	T	T	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

Option 3 $\Delta = \vee,\ \nabla = \vee$

p	q	$(p \to q)$	$(p \vee q)$	$(p \to q) \vee (p \wedge q)$
T	T	T	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

Hence, it is tautology.

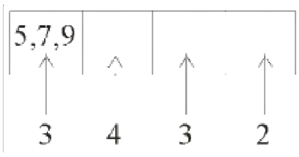
Option 4 $\Delta = \wedge,\ \nabla = \wedge$

p	q	$(p \to q)$	$(p \wedge q)$	$(p \to q) \wedge (p \wedge q)$
T	T	T	T	T
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

68. D

Sol. Numbers between 5000 & 10000

Using digits 1, 3, 5, 7, 9



Total Numbers = 3 × 4 × 3 × 2 = 72

69. D

Sol. $f:\{1,\ 2,\ 3,\ 4\}\to\{a\in\mathbb{Z}:|a|\le 8\}$

$$f(n)+\frac{1}{n}f(n+1)=1,\ \forall n\in\{1,\ 2,\ 3\}$$

f(n + 1) must be divisible by n

f(4) $\Rightarrow$ –6, –3, 0, 3, 6

f(3) $\Rightarrow$ –8, –6, –4, –2, 0, 2, 4, 6, 8

f(2) $\Rightarrow$ –8,...................., 8

f(1) $\Rightarrow$ –8,...................., 8

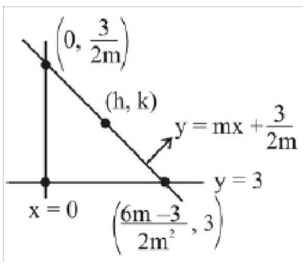
$\frac{f(4)}{3}$ must be odd since f(3) should be even therefore 2 solution possible.

f(4)	f(3)	f(2)	f(1)
-3	2	0	1
3	0	1	0

70. C

Sol. $y^2 = 6x$ & $y^2 = 4ax$

$$\Rightarrow 4a=6\Rightarrow a=\frac{3}{2}$$



$$y=mx+\frac{3}{2m};\ (m\ne 0)$$

$h=\frac{6m-3}{4m^2},\ k=\frac{6m+3}{4m}$, Now eliminating m and we get

$\Rightarrow$ 3h = 2(–2k² + 9k – 9)

$\Rightarrow$ 4y² – 18y + 3x + 18 = 0

71. A

Sol. Since

$$-\sqrt{2}\le \sin x-\cos x\le\sqrt{2}$$

$$\therefore -2\le\sqrt{2}(\sin x-\cos x)\le 2$$

(Assume $\sqrt{2}$ (sin x – cos x) = k)

$-2\le k\le 2$ ....(i)

$$f(x)=\log_{\sqrt{m}}(k+m-2)$$

Given,

$$0\le f(x)\le 2$$

$$0\le\log_{\sqrt{m}}(k+m-2)\le 2$$

$$1\le k+m-2\le m$$

$-m+3\le k\le 2$ ....(ii)

From eq, (i) & (ii), we get –m + 3 = –2

$\Rightarrow$ m = 5

72. A

Sol. Given, $A^T = A$, $B^T = -B$, $C^T = -C$

Let $M = A^{13}B^{26} - B^{26}A^{13}$

Then, $M^T = (A^{13}B^{26} - B^{26}A^{13})^T$

$= (A^{13}B^{26})^T - (B^{26}A^{13})^T$

$= (B^T)^{26}(A^T)^{13} - (A^T)^{13}(B^T)^{26}$

$= B^{26}A^{13} - A^{13}B^{26} = -M$

Hence, M is skew symmetric

Let, $N = A^{26}C^{13} - C^{13}A^{26}$

then, $N^T = (A^{26}C^{13})^T - (C^{13}A^{26})^T$

$= -(C)^{13}(A)^{26} + A^{26}C_{13} = N$

Hence, N is symmetric,

$\therefore$ Only S_2 is true.

73. A

Sol. $\frac{dy}{dt}+\alpha y=\gamma e^{-\beta t}$

$$\text{I.F.}=e^{\int\alpha dt}=e^{\alpha t}$$

Solution $\Rightarrow y\cdot e^{\alpha t}=\int\gamma e^{-\beta T}\cdot e^{\alpha t}dt$

$$\Rightarrow ye^{\alpha t}=\gamma\frac{e^{(\alpha-\beta)t}}{(\alpha-\beta)}+c$$

$$\Rightarrow y=\frac{\gamma}{e^{\beta t}(\alpha-\beta)}+\frac{c}{e^{\alpha t}}$$

So, $\lim_{t\to\infty} y(t) = \frac{\gamma}{\infty} + \frac{c}{\infty} = 0$

74. C

Sol. $\sum_{k=0}^{6} {}^{51-k}C_3$

$= {}^{51}C_3 + {}^{50}C_3 + ... + {}^{45}C_3$

$= {}^{45}C_3 + {}^{46}C_3 + ... + {}^{51}C_3$

$= {}^{45}C_4 + {}^{45}C_3 + {}^{46}C_3 + ... + {}^{51}C_3 - {}^{45}C_4$

$({}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r)$

$= {}^{52}C_4 - {}^{45}C_4$

75. A

Sol. $\frac{x+1}{1} = \frac{y}{\frac{1}{2}} = \frac{z}{\frac{-1}{12}}$ and $\frac{x}{1} = \frac{y+2}{1} = \frac{z-1}{\frac{1}{6}}$

$\Rightarrow$ Shortest distance $= \frac{(\vec{b}-\vec{a})\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}$

$$\text{S.D.} = (-\hat{i}+2\hat{j}-\hat{k})\cdot\frac{(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}$$

$$\left\{\vec{p}\times\vec{q} \equiv \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \frac{1}{2} & \frac{-1}{2} \\ 1 & 1 & \frac{1}{6} \end{vmatrix} = \frac{1}{6} - \frac{1}{4}\hat{j} + \frac{1}{2}\hat{k} \text{ or } 2\hat{i} - 3\hat{j} + 6\hat{k}\right\}$$

$$\text{S.D.} = \frac{(-\hat{i}+2\hat{j}-\hat{k})\cdot(2\hat{i}-3\hat{j}+6\hat{k})}{\sqrt{2^2+3^2+6^2}} = \left|\frac{-14}{7}\right| = 2$$

76. B

Sol. n(s) =

Given : $N-2, \sqrt{3N}, N+2$ are in G.P.

$3N = (N-2)(N+2)$

$3N = N^2 - 4$

$\Rightarrow N^2 - 3N - 4 = 0$

$(N-4)(N+1) = 0 \Rightarrow \boxed{N = 4}$ or $N = -1$ rejected

(Sum = 4) $\equiv$ {(1, 3), (3, 1), (2, 2)}

n(A) = 3

$P(A) = \frac{3}{36} = \frac{1}{12} = \frac{4}{48} \Rightarrow \boxed{k = 4}$

77. D

Sol. $I = 16\int_1^2 \frac{dx}{x^3(x^2+2)^2}$

$$= 16\int_1^2 \frac{dx}{x^3 x^4\left(1+\frac{2}{x^2}\right)^2}$$

Let, $1 + \frac{2}{x^2} = t \Rightarrow \frac{-4}{x^3}dx = dt$

$$I = -4\int_3^{\frac{3}{2}} \frac{dt}{\left(\frac{2}{t-1}\right)^2 t^2}$$

$$I = -4\int_3^{\frac{3}{2}} \left(\frac{t-1}{2}\right)^2 \frac{dt}{t^2}$$

$$I = -\frac{4}{4}\int_3^{\frac{3}{2}} \left(1 - \frac{2}{t} + \frac{1}{t^2}\right)dt$$

$$I = -1\left[t - 2\ell n|t| - \frac{1}{t}\right]_3^{\frac{3}{2}}$$

$$I = -1\left[\left(\frac{3}{2} - 2\ell n\frac{3}{2} - \frac{2}{3}\right) - \left(3 - 2\ell n 3 - \frac{1}{3}\right)\right]$$

$$I = -1\left[2\ell n 2 - \frac{11}{6}\right]$$

$$I = \frac{11}{6} - \ell n 4$$

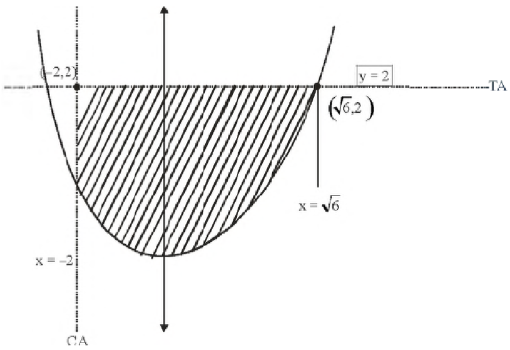
78. B

Sol. $16(x^2 + 4x) - (y^2 - 4y) + 44 = 0$

$16(x + 2)^2 - 64 - (y - 2)^2 + 4 + 44 = 0$

$16(x + 2)^2 - (y - 2)^2 = 16$

$$\frac{(x+2)^2}{1} - \frac{(y-2)^2}{16} = 1$$



$$A=\int_{-2}^{\sqrt{6}}(2-(x^2-4))dx$$

$$A=\int_{2}^{\sqrt{6}}(6-x^2)dx=\left(6x-\frac{x^3}{3}\right)_{-2}^{\sqrt{6}}$$

$$A=\left(6\sqrt{6}-\frac{6\sqrt{6}}{3}\right)-\left(-12+\frac{8}{3}\right)$$

$$A=\frac{12\sqrt{6}}{3}+\frac{28}{3}$$

$$A=4\sqrt{6}+\frac{28}{3}$$

79. B

Sol. $\vec{a}\times\vec{b}=(\hat{i}-\hat{j})$

Taking cross product with $\vec{a}$

$\Rightarrow \vec{a}\times(\vec{a}\times\vec{b})=\vec{a}\times(\hat{i}-\hat{j})$

$\Rightarrow (\vec{a}\cdot\vec{b})\vec{a}-(\vec{a}\cdot\vec{a})\vec{b}=\hat{i}+\hat{j}+2\hat{k}$

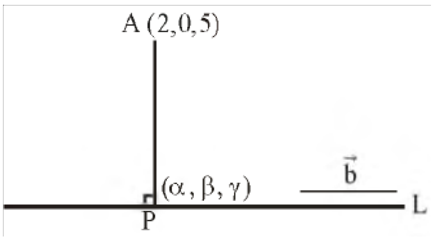
$\Rightarrow \vec{a}-3\vec{b}=\hat{i}+\hat{j}+2\hat{k}$

$\Rightarrow 2\vec{a}-6\vec{b}=2\hat{i}+2\hat{j}+4\hat{k}$

$\Rightarrow \vec{a}-6\vec{b}=3\hat{i}+3\hat{j}+3\hat{k}$

80. C

Sol. $L:\frac{x+1}{2}=\frac{y-1}{5}=\frac{z+1}{1}=\lambda$ (let)



Let foot of perpendicular is

$P(2\lambda-1,\ 5\lambda+1,\ -\lambda-1)$

$\overrightarrow{PA}=(3-2\lambda)\hat{i}-(5\lambda+1)\hat{j}+(6+\lambda)\hat{k}$

Direction ratio of line $\Rightarrow \vec{b}=2\hat{i}+5\hat{j}-\hat{k}$

Now, $\Rightarrow \overrightarrow{PA}\bullet\vec{b}=0$

$\Rightarrow 2(3-2\lambda)-5(5\lambda+1)-(6+\lambda)=0$

$\Rightarrow \lambda=\frac{-1}{6}$

$P(2\lambda-1,\ 5\lambda+1,\ -\lambda-1)\equiv P(\alpha,\ \beta,\ \gamma)$

$\Rightarrow \alpha=2\left(-\frac{1}{6}\right)-1=-\frac{4}{3}\Rightarrow \alpha=-\frac{4}{3}$

$\Rightarrow \beta=5\left(-\frac{1}{6}\right)+1=\frac{1}{6}\Rightarrow \beta=\frac{1}{6}$

$\Rightarrow \gamma=-\lambda-1=\frac{1}{6}-1\Rightarrow \gamma=-\frac{5}{6}$

Section - B (Numerical Value)

81. 3

Sol. $a,\ b,\ \frac{1}{18}\rightarrow GP$

$\frac{a}{18}=b^2$ ....(i)

$\frac{1}{a},\ 10,\ \frac{1}{b}\rightarrow AP$

$\frac{1}{a}+\frac{1}{b}=20$

$\Rightarrow$ a + b = 20ab, from eq. (i) ; we get

$\Rightarrow 18b^2+b=360b^3$

$\Rightarrow 360b^2-180b-1=0\ \{\because b\neq 0\}$

$\Rightarrow b=\frac{18+\sqrt{1764}}{720}$ $\{\because b>0\}$

$\Rightarrow b=\frac{1}{12}$

$\Rightarrow a=18\times\frac{1}{144}=\frac{1}{8}$

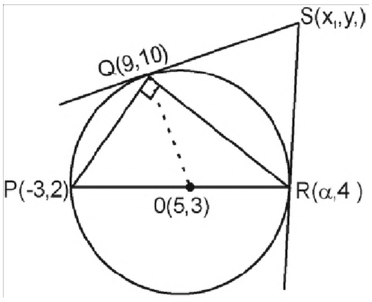
Now, $16a+12b=16\times\frac{1}{8}+12\times\frac{1}{12}=3$

82. 3

Sol. $m_{PQ} \cdot m_{QR} = -1$

$$\Rightarrow \frac{10-2}{9+3} \times \frac{10-4}{9-\alpha} = -1 \Rightarrow \alpha = 13$$

$$m_{OP} \cdot m_{QS} = -1 \Rightarrow m_{QS} = -\frac{4}{7}$$



Equation of QS

$$y - 10 = -\frac{4}{7}(x-9)$$

$$\Rightarrow 4x + 7y = 106 \quad .....(1)$$

$$m_{OR} \cdot m_{RS} = -1 \Rightarrow m_{RS} = -8$$

Equation of RS

$y - 4 = -8(x - 13)$

$\Rightarrow 8x + y = 108 \quad ....(2)$

Solving eq. (1) & (2)

$$x_1 = \frac{25}{2} y_1 = 8$$

$S(x_1, y_1)$ lies on $2x - ky = 1$

$25 - 8k = 1$

$\Rightarrow 8k = 24$

$\Rightarrow k = 3$

83. 45

Sol. $x^2 + 60^{\frac{1}{4}} x + a = 0$ $\begin{cases} \alpha \\ \beta \end{cases}$

$$\alpha + \beta = -60^{\frac{1}{4}} \ \& \ \alpha\beta = a$$

Given $\alpha^4 + \beta^4 = -30$

$$\Rightarrow (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 = -30$$

$$\Rightarrow \{(\alpha+\beta)^2 - 2\alpha\beta\}^2 - 2a^2 = -30$$

$$\Rightarrow \{60^{\frac{1}{2}} - 2a\}^2 - 2a^2 = -30$$

$$\Rightarrow 60 + 4a^2 - 4a \times 60^{\frac{1}{2}} - 2a^2 = -30$$

$$\Rightarrow 2a^2 - 4.60^{\frac{1}{2}} a + 90 = 0$$

$$\text{Product } = \frac{90}{2} = 45$$

84. 6860

Sol. 7 Red apple(RA), 5 white apple(WA), 8 oranges (O) 5 fruits to be selected (Note:- fruits taken different)

Possible selections :- (2O, 1RA, 2WA) or (2O, 2RA, 1WA) or (3O, 1RA, 1WA)

$\Rightarrow {}^8C_2\ {}^7C_1\ {}^5C_2 + {}^8C_2\ {}^7C_2\ {}^5C_1 + {}^8C_3\ {}^7C_1\ {}^5C_1$

$\Rightarrow 1960 + 2940 + 1960$

$\Rightarrow 6860$

85. 25

Sol. $\cos 2\theta \cdot \cos\frac{\theta}{2} = \cos 3\theta \cdot \cos\frac{9\theta}{2}$

$$\Rightarrow 2\cos 2\theta \cdot \cos\frac{\theta}{2} = 2\cos\frac{9\theta}{2} \cdot \cos 3\theta$$

$$\Rightarrow \cos\frac{5\theta}{2} + \cos\frac{3\theta}{2} = \cos\frac{15\theta}{2} + \cos\frac{3\theta}{2}$$

$$\Rightarrow \cos\frac{15\theta}{2} = \cos\frac{5\theta}{2}$$

$$\Rightarrow \frac{15\theta}{2} = 2k\pi \pm \frac{5\theta}{2}$$

$$5\theta = 2k\pi \text{ or } 10\theta = 2k\pi$$

$$\theta = \frac{2k\pi}{5} \qquad \theta = \frac{k\pi}{5}$$

$$\therefore \theta = \left\{-\pi, \frac{-4\pi}{5}, \frac{-3\pi}{5}, \frac{-2\pi}{5}, \frac{-\pi}{5}, 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi\right\}$$

$m = 5, n = 5$

$\therefore m \cdot n = 25$

86. 20

Sol. $\int_{\frac{1}{3}}^{3} |\ell nx| dx = \int_{\frac{1}{3}}^{1} (-\ell nx) dx + \int_{1}^{3} (\ell nx) dx$

$= -[x\ell nx - x]_{1/3}^{1} + [x\ell nx - x]_1^3$

$= -\left[-1-\left(\frac{1}{3}\ell n\frac{1}{3}-\frac{1}{3}\right)\right]+[3\ell n3-3-(-1)]$

$= \left[-\frac{2}{3}-\frac{1}{3}\ell n\frac{1}{3}\right]+[3\ell n3-2]$

$= -\frac{4}{3}+\frac{8}{3}\ell n3$

$= \frac{4}{3}(2\ell n3-1)$

$= \frac{4}{3}\left(\ell n\frac{9}{e}\right)$

$\therefore$ m = 4, n = 3

Now, $m^2 + n^2 - 5 = 16 + 9 - 5 = 20$

87. 7

Sol. $(2023)^{2023}$

$= (2030 - 7)^{2023}$

$= (35\,K - 7)^{2023}$

$= {}^{2023}C_0(35\,K)^{2023}(-7)^0 + {}^{2023}C_1\,(35\,K)^{2022}\,(-7)$ +.....+

...........+ ${}^{2023}C_{2023}\,(-7)^{2023}$

$= 35\,N - 7^{2023}$.

Now, $-7^{2023} = -7 \times 7^{2022} = -7\,(7^2)^{1011}$

$= -7\,(50 - 1)^{1011}$

$= -7({}^{1011}C_0\;50^{1011} - {}^{1011}C_1(50)^{1010} + \ldots\ldots {}^{1011}C_{1011})$

$= -7\,(5\lambda - 1)$

$= -35\lambda + 7$

$\therefore$ when $(2023)^{2023}$ is divided by 35 remainder is 7

88. 18

Sol. $\vec{r} = (\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}+\hat{j}+\hat{k})$ $\quad \vec{r} = \vec{a}+\lambda\vec{p}$

$\vec{r} = (+\hat{i}-\hat{j}+2\hat{k})+\mu(2\hat{i}-\hat{j})$ $\quad \vec{r} = \vec{b}+\mu\vec{q}$

$$\vec{p}\times\vec{q} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 2 & -1 & 0\end{vmatrix} = \hat{i}+2\hat{j}-3\hat{k}$$

$$d = \left|\frac{(\vec{b}-\vec{a})\cdot(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right|$$

$$d = \left|\frac{(-3\hat{j}-\hat{k})\cdot(\hat{i}+2\hat{j}-3\hat{k})}{\sqrt{14}}\right|$$

$$= \left|\frac{-6+3}{\sqrt{14}}\right| = \frac{3}{\sqrt{14}}$$

$$\alpha = \frac{3}{\sqrt{14}}$$

Now, $28\alpha^2 = \overset{2}{\cancel{28}} \times \frac{9}{\cancel{14}} = 18$

89. 9

E_1 : Smokers

$P(E_1) = \frac{1}{4}$

E_2 : non-smokers

$P(E_2) = \frac{3}{4}$

E : diagnosed with lung cancer

$P(E/E_1) = \frac{27}{28}$

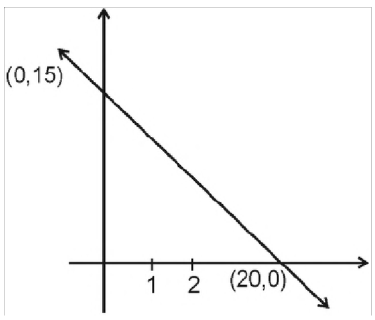
$P(E/E_2) = \frac{1}{28}$

$P(E_1/E) = \frac{(E_1)P(E/E_1)}{P(E)}$

$$= \frac{\frac{1}{2}\times\frac{27}{28}}{\frac{1}{4}\times\frac{27}{28}+\frac{3}{4}\times\frac{1}{28}} = \frac{\cancel{27}^{9}}{\cancel{30}_{10}} = \frac{9}{10}$$

90. 31

If $x = 1,\ y = \frac{57}{4} = 14.25$



(1, 1) (1, 2) – (1, 14) $\Rightarrow$ 14 pts.

If x = 2, $y = \frac{27}{2} = 13.5$

(2, 2) (2, 4) ...... (2, 12) $\Rightarrow$ 6 pts.

If x = 3, $y = \frac{51}{4} = 12.75$

(3, 3) (3, 6) – (3, 12) $\Rightarrow$ 2 pts.

If $x = 5 \cdot y = \frac{45}{4} = 11.25$

(5, 5), (5, 10) $\Rightarrow$ 2 pts.

If $x = 6,\ y = \frac{21}{2} = 10.5$

(6, 6) $\Rightarrow$ 1 pt.

If $x = 7,\ y = \frac{39}{4} = 9.75$

(7, 7) $\Rightarrow$ 1 pt.

If x = 8, x = 9

(8, 8) $\Rightarrow$ 1 pt.

If $x = 9,\ y = \frac{33}{4} = 8.25$ $\Rightarrow$ no pt.

Total = 31 pts.

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