3S INSTITUTE

JEE ADVANCED | JEE MAIN | NEET | OLYMPIADS | MHT-CET | FOUNDATION

25-January-2023 (Morning Batch) : JEE Main Paper

PHYSICS

Section - A (Single Correct Answer)

1. D

Sol. When electron is accelerated through potential difference V, then

K.E. = eV

$$\Rightarrow \lambda = \frac{h}{\sqrt{2m(KE)}} = \frac{h}{\sqrt{2meV}}$$

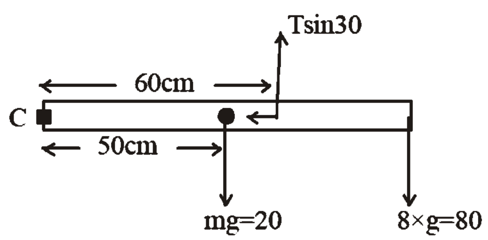
$$\therefore \lambda \propto \frac{1}{\sqrt{V}}$$

$$\therefore \frac{\lambda}{\lambda_0} = \sqrt{\frac{20}{40}}$$

$$\therefore \lambda = \frac{\lambda_0}{\sqrt{2}}$$

2. C

Sol.



Taking torque about point C

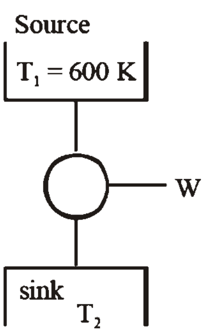
$$\frac{T}{2} \times 60 = 20 \times 50 + 80 \times 100$$

$\Rightarrow$ 3T = 100 + 800

$\Rightarrow$ = 300 N

3. B

Sol.



Initially $\eta = \frac{1}{2}$

But $\eta = 1 - \frac{T_2}{T_1}$

$$\therefore \frac{1}{2} = 1 - \frac{T_2}{600}$$

$$\Rightarrow \frac{T_2}{600} = \frac{1}{2} \Rightarrow T_2 = 300K$$

Now efficiency is increased to 70% and T_2 = 300 K, Let temp of source T_1 = T

$$\Rightarrow \frac{7}{10} = 1 - \frac{300}{T} \Rightarrow \frac{300}{T} = 1 - \frac{7}{10}$$

$$\Rightarrow \frac{300}{T} = \frac{3}{10} \quad \therefore T = 1000\ K$$

4. B

Sol. At surface of earth time period

$$T = 2\pi\sqrt{\frac{l}{g}}$$

At height h = R

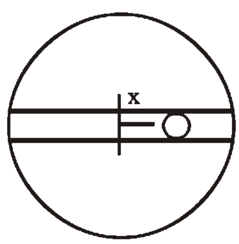
$$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2} = \frac{g}{4}$$

$$\therefore xT = 2\pi\sqrt{\frac{l}{(g+4)}}$$

$$\Rightarrow xT = 2 \times 2\pi\sqrt{\frac{l}{g}} \Rightarrow xT = 2T \Rightarrow x = 2$$

5. B

Sol.



Let at some time particle is at a distance x from centre of Earth, then at that position field

$$E = \frac{GM}{R^3}x$$

$\therefore$ Acceleration of particle

$$\vec{a} = -\frac{GM}{R^3}\vec{x}$$

$$\Rightarrow \omega = \sqrt{\frac{GM}{R^3}} = \sqrt{\frac{g}{R}}$$

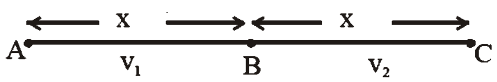
Now $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$

$$\Rightarrow \quad T = 2\times 3.14\times\sqrt{\frac{6400\times 10^3}{10}}$$

= 2 × 3.14 × 800 sec ≈ 1 hour 24 minutes

6. D

Sol.

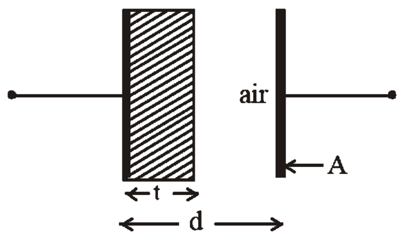


$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

$$= \frac{x+x}{\frac{x}{v_1}+\frac{x}{v_2}} = \frac{2v_1v_2}{v_1+v_2}$$

7. C

Sol.



This can be seen as two capacitors in series combination so

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$= \frac{1}{\frac{K\epsilon_0 A}{t}} + \frac{1}{\frac{\epsilon_0 A}{d-t}}$$

$$= \frac{t}{K\epsilon_0 A} + \frac{d-t}{\epsilon_0 A}$$

$$= \frac{1\times 10^{-3}}{5\epsilon_0\times 40\times 10^{-4}} + \frac{1\times 10^{-3}}{\epsilon_0\, 40\times 10^{-4}}$$

$$\frac{1}{C_{eq}} = \frac{1}{20\epsilon_0} + \frac{1}{4\epsilon_0}$$

$$C_{eq} = \frac{20\times 4\epsilon_0}{24} = \frac{10\epsilon_0}{3}F$$

8. C

Sol. The rms speed of a gas molecule is

$$V_{RMS} = \sqrt{\frac{3RT}{M}}$$

$$V_{RMS} \propto \sqrt{T}$$

9. B

Sol. (A) Surface Tension $= \frac{F}{l} = \frac{MLT^{-2}}{L} = ML^{-1}T^{-2}$

$= Kgs^{-2}(IV)$

(B) Pressure $= \frac{F}{A} = \frac{MLT^{-2}}{L^2}$

$= kgm^{-1}s^{-2}(III)$

(C) Viscosity $= \frac{F}{A\left(\frac{dV}{dz}\right)} = \frac{MLT^{-2}}{L^2\left(\frac{LT^{-1}}{L}\right)}$

$= ML^{-1}T^{-1} = kgm^{-1}s^{-1}(I)$

(D) Impulse $= \int Fdt = MLT^{-2}\times T$

$= MLT^{-1} = Kgms^{-1}(II)$

So A → (IV), B → (III), C → (I), D → (II)

10. A

Sol. The resonance frequency of LC oscillations circuit is

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$L \to 2L$

$C \rightarrow 8C$

$$\omega = \frac{1}{\sqrt{2L \times 8C}} = \frac{1}{4\sqrt{LC}}$$

$$\omega = \frac{\omega_0}{4}$$

So $x = \frac{1}{4}$

11. C

Sol. Nuclear density is independent of mass number

As nuclear density $= \frac{Au}{\frac{4}{3}\pi R^3}$

Also, $R = R_0 A^{\frac{1}{3}}$

And $R^3 = R_0^3 A$

$\Rightarrow$ Nuclear density $= \frac{Au}{\frac{4}{3}\pi R_0^3 A}$

Nuclear density $= \frac{3u}{4\pi R_0^3}$

$\Rightarrow$ Nuclear density is independent of A

12. C

Sol. Given

Signal frequency f_m = 5kHz

Carrier wave frequency f_c = 2MHz

f_c = 2000KHz

The resultant signal will have band width of frequency given by

$$\left[(f_c + f_m) - (f_c - f_m)\right]$$

$\Rightarrow$ $\left[(2000+5)-(2000-5)\right]$kHz

$\Rightarrow$ 10 kHz

13. A

Sol. As, poynting vector

$$\vec{S} = \vec{E} \times \vec{H}$$

Given energy transport = negative z direction

Electric field = positive y direction

$$\left(-\hat{k}\right) = \left(+\hat{j}\right) \times \left[\hat{i}\right]$$

Hence according to vector cross product magnetic field should be positive x direction.

14. A

Sol. Given

D = 1m

$\lambda = 600 \times 10^{-9}$

n = 5

As $y_{nth} = \frac{n\lambda D}{d}$

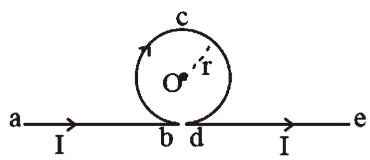
$$\Rightarrow \frac{5 \times 600 \times 10^{-9} \times 1}{d} = 5 \times 10^{-2}$$

$$\Rightarrow d = \frac{5 \times 600 \times 10^{-9} \times 1}{5 \times 10^{-2}} = 60 \times 10^{-6} m$$

$\Rightarrow d = 60\ \mu m$

15. C

Sol.

(A) 

$$B_{ab} = \frac{\mu_0}{4\pi}\frac{1}{r} \text{ (out of the plane)}$$

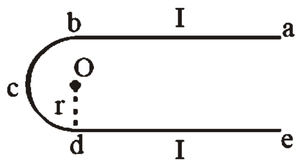
$$B_{bcd} = \frac{\mu_0}{4\pi}\frac{I}{r}(2\pi) \text{ (in the plane)}$$

$$B_{de} = \frac{\mu_0}{4\pi}\frac{I}{r} \text{ (out of the plane)}$$

Hence magnetic field at O is

$$B_0 = -\frac{\mu_0}{4\pi}\frac{I}{r} + \frac{\mu_0}{4\pi}\frac{I}{r}(2\pi) - \frac{\mu_0}{4\pi}\frac{I}{r}$$

$$B_0 = \frac{\mu_0}{2\pi}\frac{I}{r}(\pi - 1) \quad \text{..... (III)}$$

(B) 

$$B_{ab} = \frac{\mu_0}{4\pi}\frac{I}{r} \text{ (out of the plane)}$$

$$B_{bcd} = \frac{\mu_0}{4\pi}\frac{I}{r} (\pi) \text{ (out of the plane)}$$

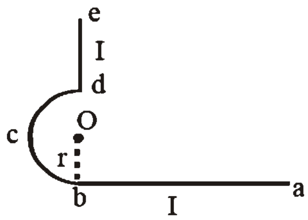
$$B_{de} = \frac{\mu_0}{4\pi}\frac{I}{r} \text{ (out of the plane)}$$

Hence magnetic field at O is

$$B_0 = \frac{\mu_0}{4\pi}\frac{I}{r} + \frac{\mu_0}{4\pi}\frac{I}{r}(\pi) + \frac{\mu_0}{4\pi}\frac{I}{r}$$

$$B_0 = \frac{\mu_0}{4\pi}\frac{I}{r}(\pi + 2) \quad .....(I)$$

(C)



$$B_{ab} = \frac{\mu_0}{4\pi}\frac{I}{r} \quad \text{(in the plane)}$$

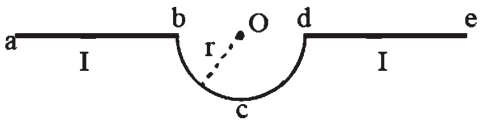
$$B_{bcd} = \frac{\mu_0}{4\pi}\frac{I}{r}(\pi) \text{ (in the plane)}$$

B_{de} = 0 (at the axis)

Hence magnetic field at O is

$$B_0 = \frac{\mu_0}{4\pi}\frac{I}{r}(1 + \pi) \quad .... \text{ (IV)}$$

(D)



B_{ab} = 0 (at the axis)

$$B_{bcd} = \frac{\mu_0}{4\pi}\frac{I}{r}(\pi) \text{ (out of the plane)}$$

B_{de} = 0 (at the axis)

Hence magnetic field at O is

$$B_0 = \frac{\mu_0 I}{4r} \quad .....(II)$$

16. C

Sol. Theory based

Photodiodes are operated in reverse bias condition.

For P-N junction current in forward bias (for $V \geq V_0$) is always greater than current in reverse bias (for $V \leq V_z$)

Hence Assertion if false but Reason is true

17. B

Sol. Magnetic field at centre inside the solenoid is given by

$B = \mu_0 nI$

So magnetic intensity at centre

$$H = \frac{B}{\mu_0} = nI = \left(\frac{1200}{2}\right)(2)$$

$H = 1.2 \times 10^3\ Am^{-1}$

18. B

Sol. I = 2A

$\Delta V = 3.4$ V

Using Ohm's Law

$$R = \frac{3.4}{2} = 1.7\Omega$$

$$1.7 = \frac{\rho L}{A}$$

$$L = \frac{1.7(A)}{\rho}$$

M = (density volume)

$$\text{Volume } = \frac{8.92 \times 10^{-3}}{8.92 \times 10^{3}} = 10^{-6}$$

$$L^2 = \frac{1.7}{\rho}\left(10^{-6}\right) = \frac{1.7}{1.7} \times 10^2$$

L = 10m

19. B

Sol. $$\frac{\Delta Q}{\Delta t} = -K\left(T - T_0\right)$$

$$\frac{\Delta Q}{\Delta t} = -K\left(T_{avg} - T_0\right)$$

(i) $$\frac{ms \times 12}{2} = -K\left(\frac{98+86}{2} - 22\right)$$

$$6 = -\frac{K}{ms}\left[\frac{98+86}{2} - 22\right]$$

$$6 = -\frac{K}{ms}[70] \quad .....(i)$$

(ii) $$\frac{ms \times 6}{\Delta t} = -K\left(\frac{75+69}{2} - 22\right)$$

$$\frac{6}{\Delta t} = -\frac{K}{ms}(50) \quad .....(ii)$$

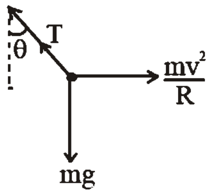
$(ii) \div (i)$

$$\frac{6}{\Delta t(6)} = \frac{50}{70}$$

$$\Delta t = \frac{7}{5} = 1.4 \text{ min}$$

20. C

Sol.



T cos θ = mg

$$T\sin\theta = \frac{mv^2}{R}$$

$$\tan\theta = \frac{v^2}{Rg}$$

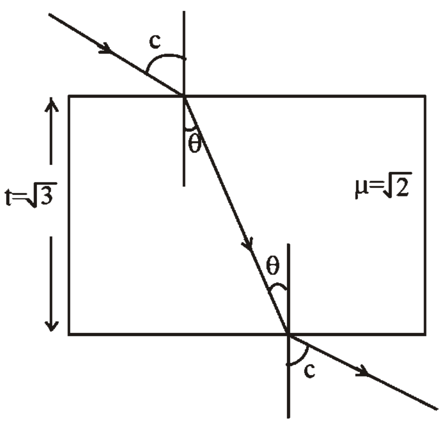
$$\tan\theta = \frac{20^2}{40 \times 10}$$

$$\tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$$

Section - B (Numerical Value)

21. 52

Sol.



$$\sin c = \frac{1}{\sqrt{2}}$$

c = 45°

sin c = μ sin θ

$$\frac{1}{\sqrt{2}} = \sqrt{2}\sin\theta$$

θ = 30°

Lateral displacement:

x = t sin(i – r)sec r

$$x = \sqrt{3}\sin(45° - 30°)\sec 30°$$

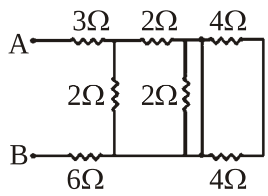
$$x = \sqrt{3}(0.26)\left(\frac{2}{\sqrt{3}}\right)$$

X = 0.52 cm

$x = 52 \times 10^{-2}$ cm

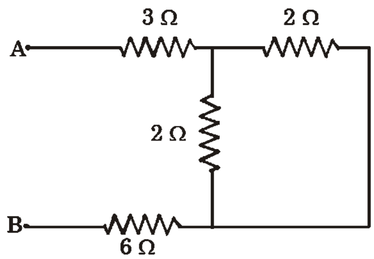
22. 10

Sol.



Both 4Ω resistance gets short.

Remove the resistors that have no current.



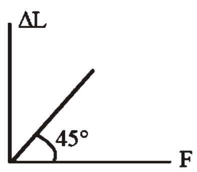
R_{eq} = 3 + (2 || 2) + 6

R_{eq} = 3 + 1 + 6

R_{eq} = 10Ω

23. 5

Sol.



From graph :

F = ΔL

$$Y = \frac{FL}{A\Delta L}$$

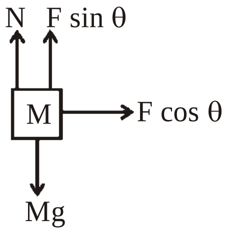
$$Y = \frac{L}{A}$$

$$Y = \frac{62.8\times10^{-2}}{\pi\left(2\times10^{-3}\right)^2}$$

$Y = 5 \times 10^4$ N/m^2

24. 2

Sol.



F cos θ = ma

$$2\cos(kx) = \frac{mvdv}{dx}$$

$$\int_0^v vdv = 2\int_0^x \cos(kx)\,dx$$

$$\frac{mv^2}{2} = \frac{2}{k}\sin kx$$

$$K.E. = \frac{2}{k}\sin\theta$$

n = 2

25. 27

Sol.

______________ n = 4
______________ n = 3
______________ n = 2
______________ n = 1

Second excited state $\rightarrow$ first excited state

$n = 3 \rightarrow n = 2$

$$\frac{hc}{\lambda_0} = 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \quad ....(i)$$

Third excited state $\rightarrow$ second orbit

$n = 4 \rightarrow n = 2$

$$\frac{hc}{(20\lambda_0 / x)} = 13.6\left(\frac{1}{2^2} - \frac{1}{4^2}\right) \quad ....(ii)$$

(ii) ÷ (i)

$$\frac{x}{20} = \frac{\frac{1}{2^2} - \frac{1}{4^2}}{\frac{1}{2^2} - \frac{1}{3^2}}$$

x = 27

26. 17

Sol. $I_{cm} = \frac{mR^2}{2}$

$$I_{AB} = \frac{mR^2}{2} + m\left(\frac{2R}{3}\right)^2 = \frac{17}{18}mR^2$$

$$\frac{I_{AB}}{I_{cm}} = \frac{17}{9} \Rightarrow x = 17$$

27. 18

Sol. $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$

$$\frac{\pi}{3} = \frac{2\pi}{\lambda}(6m) \Rightarrow \quad \lambda = 36m$$

V = fλ = (500 Hz)(36 m)

= 18000 m/s = 18 km/s

28. 45

Sol. $0.5e = \frac{1}{2}mv_x^2 \Rightarrow v_x = \sqrt{\frac{e}{m}}$

Along x $L = v_x t = \sqrt{\frac{e}{m}}t$

Along y $v_y = \frac{eE}{m}t$

dividing $\frac{v_y}{L} = E\sqrt{\frac{e}{m}} = Ev_x$

$$\Rightarrow \quad Tan\theta = \frac{v_y}{v_x} = E\times L = 10\times0.1 = 1$$

θ = 45°

29. 100

Sol. $f = \frac{1}{2\pi\sqrt{LC}}$

$$2000\text{Hz} = \frac{1}{2\pi\sqrt{L \times 62.5 \times 10^{-9}}}$$

$$L = \frac{1}{4\pi^2 \times 2000^2 \times 62.5 \times 10^{-9}} = 0.1\text{ H} = 100\text{ mH}$$

30. 4

Sol. $\vec{P} \times \vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & \sqrt{3} & 2 \\ 4 & \sqrt{3} & 2.5 \end{vmatrix} = \sqrt{3}\frac{\hat{i}}{2} + \frac{\hat{j}}{2} - \sqrt{3}\hat{k}$

$$\Rightarrow \frac{\vec{P} \times \vec{Q}}{|\vec{P} \times \vec{Q}|} = \frac{1}{2}\left(\sqrt{3}\frac{\hat{i}}{2} + \frac{\hat{j}}{2} - \sqrt{3}\hat{k}\right)$$

$$= \frac{1}{4}\left(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k}\right)$$

x = 4

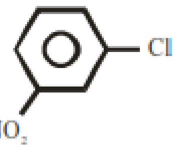
CHEMISTRY

Section - A (Single Correct Answer)

31. D

Sol. Electron withdrawing groups are highly ineffective at meta position in nucleophilic aromatic substitution reactions.

Hence compound



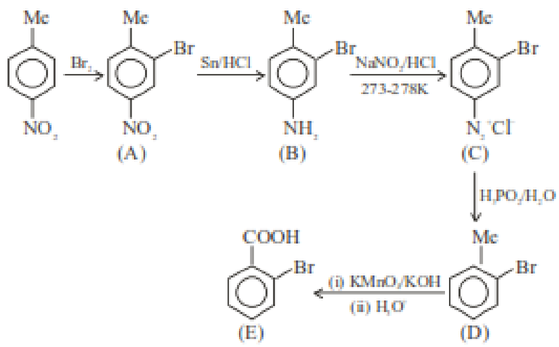
will have lowest rate in nucleophilic aromatic substitution.

32. B

Sol. Fact base.

33. B

Sol.



34. C

Sol.

Element	Colour in flame test
K	Violet
Ca	Brick red
Sr	Crimson red
Ba	Apple green

35. B

Sol. $P_4 + 8SOCl_2 \rightarrow \underset{[A]}{4PCl_3} + 4SO_2 + 2S_2Cl_2$

$PCl_3 + 3H_2O \rightarrow \underset{[B]}{H_3PO_3} + 3HCl$

36. B

Sol. $X_{4\times\frac{1}{8}+1\times1} \quad Y_{6\times\frac{1}{3}\times\frac{1}{2}}$

$\Rightarrow X_{\frac{1}{2}+1} \quad Y_1$

$\Rightarrow X_{\frac{2}{3}} \quad Y_1$

$\Rightarrow X_{1.5} \quad Y_1$

$\Rightarrow X_3 \quad Y_2$

37. C

Sol. Li^{2+}

$r_2 = x = k \times \frac{2^2}{3} = \frac{4k}{2}$

Be^{3+}

$r_3 = y = k \times \frac{3^2}{4}$

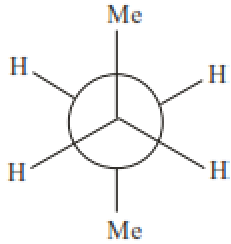
$$\frac{y}{x} = \frac{9}{4} \times \frac{3}{4} = \frac{27}{16}$$

$$y = \frac{27}{16}x$$

38. A

Sol.

Conformation

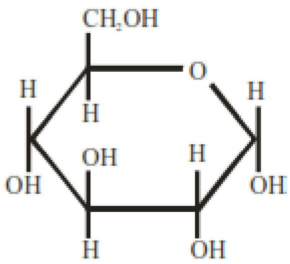


has lowest

vanderwaal and torsional strain. Hence it must be most stable.

39. D

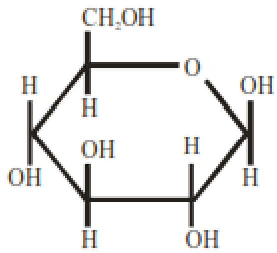
Sol. Structure



[A]

Represents α-D-(+) Glucopyranose

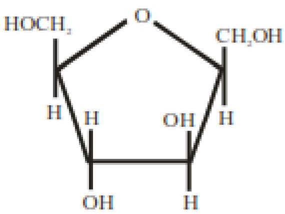
Structure



[B]

Represents β-D-(+) Glucopyranose

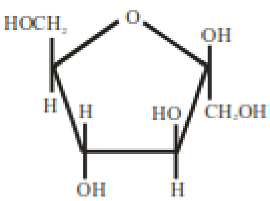
Structure



[C]

Represents β-D-(–) Fructofuranose

Structure



[D]

Represents β-D-(–) Fructofuranose

(from the given options best answer is D)

40. A

Sol. For Assertion :

Acetal and ketals are basically ethers hence they must be stable in basic medium but should break down in acidic medium.

Hence assertion is correct.

For reason :

Alkoxide ion (RO^-) is not considered a good leaving group hence reason must be false.

41. C

Sol.

Element	ΔegH[KJ/mol]
He	+ 48
Ne	+ 116
Kr	+ 96
Xe	+ 77

From NCERT

So, order is : Ne > Kr >Xe > He

42. C

Sol. $CuFeS_2 + O_2 \xrightarrow{\text{Partial roasting}}$

$Cu_2S + FeO + SO_2 + \underset{\text{very small}}{FeS} + \underset{\text{very small}}{Cu_2O}$

$Cu_2S + O_2 \rightarrow Cu_2O + SO_2$

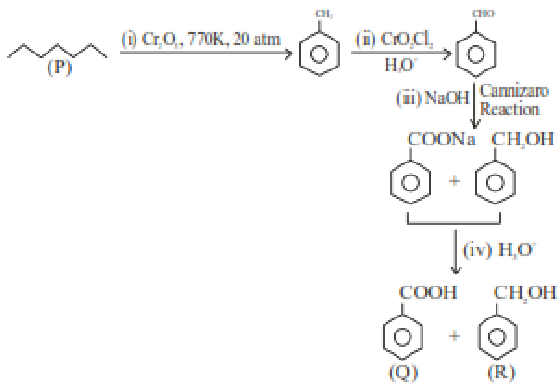
$FeS + O_2 \rightarrow FeO + SO_2$

$FeO + SiO_2 \rightarrow FeSiO_3$

No formation of calcium silicate ($CaSiO_3$) in extraction of Cu.

43. A

Sol.



44. A

Sol. In aqueous medium basic strength is dependent on electron density on nitrogen as well as solvation of cation formed after accepting H^+.

After considering all these factors overall basic strength order is

$Me_2NH > MeNH_2 > Me_3N > NH_3$

45. B

Sol. Volume = 11.35 × M

Strength

$$M = \frac{25}{11.35} M$$

$$g/L = 25 \times 34/11.35 = 74.889$$

46. C

Sol. An antibiotic should not promote growth or survival of microorganisms. Antibiotics should inhibit growth of microbes.

47. C

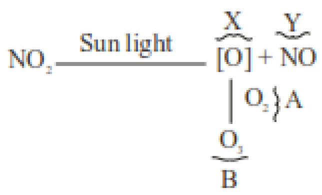
Sol.
$$\underset{(A)}{Ca(OH)_2} + 2NH_4Cl \xrightarrow{\Delta} \underset{(B)}{2NH_3} + CaCl_2 + 2H_2O$$

$$\underset{(B)}{NH_3} + H_2O + \underset{(exc)}{CO_2} \longrightarrow \underset{(C)}{NH_4HCO_3}$$

$$\underset{(C)}{NH_4HCO_3} + NaCl \longrightarrow NaHCO_3 \downarrow + NH_4Cl$$

48. A

Sol.



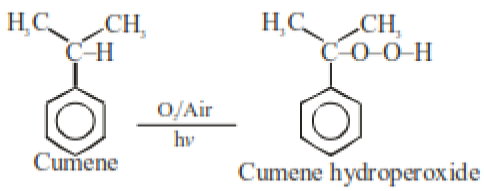
49. D

Sol.

Cations	Group No.	Group reagent
Pb^{+2}, Cu^{+2}	II	$H_2S(g)$ in presence of dilHCl
Al^{+3}, Fe^{+3}	III	NH_4OH in presence of NH_4Cl
CO^{+2}, Ni^{+2}	IV	H_2S in presence of NH_4OH
Ba^{+2}, Ca^{+2}	V	$(NH_4)_2CO_3$ in presence of NH_4OH

50. D

Sol.



Section - B (Numerical Value)

51. 360

Sol. $C_6H_{12}O_6(s) + 6O_2 \rightarrow 6CO_2(g) + 6H_2O(\ell)$

Extra energy used to convert $H_2O(\ell)$ into $H_2O(\ell)$ into $H_2O(g)$.

$$= \frac{1800}{2} = 900 \text{ kJ}$$

$$\Rightarrow \quad 900 = n_{H_2O} \times 45$$

$$n_{H_2O} = \frac{900}{45} = 20 \text{ mole}$$

$$W_{H_2O} = 20 \times 18 = 360 \text{ g}$$

52. 9079

Sol. In resultant solution,

$$n_{NH_3} = 0.1 - 0.02 = 0.08$$

$$n_{NH_4Cl} = n_{NH_4^+} = 0.1 + 0.02 = 0.12$$

$$pOH = pK_b + \log\frac{[NH_4^+]}{[NH_3]}$$

$$= 4.745 + \log\frac{0.12}{0.08}$$

$$= 4.745 + \log\frac{3}{2}$$

$$= 4.745 + 0.477 - 0.301$$

$$pOH = 4.921$$

$$pH = 14 - pH \quad = 9.079$$

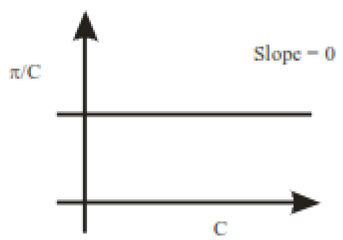
53. 41500

Sol. $\pi = M'RT = \left(\frac{W/M}{V}\right)RT$

$$\Rightarrow \quad \pi = \left(\frac{W}{V}\right)\left(\frac{1}{M}\right)RT = C\left(\frac{RT}{M}\right)$$

$$\Rightarrow \quad \frac{\pi}{C} = \frac{RT}{M} \neq f(c)$$

If we assume graph between $\frac{\pi}{C}$ and C.



Assuming π *vs* C graph.

$$\text{Slope} = \frac{RT}{M} = \frac{0.083\times300}{M} = 6\times10^{-4}$$

$$\therefore\ M = \frac{0.083\times300}{6\times10^{-4}} = \frac{830\times300}{6}$$

$$= 41500\ \text{gm/mole}$$

54. 2

Sol. $\mu_s = \sqrt{n(n+2)}\,BM$ (n=no. of unpaired electrons)

	n
V^{3+} : [Ar] $3d^2 4s^0$	2
Cr^{3+} : [Ar] $3d^3 4s^0$	3
Fe^{2+} : [Ar] $3d^6 4s^0$	4
Ni^{3+} : [Ar] $3d^7 4s^0$	3

Cr^{3+} and Ni^{3+} have same value of μ_s.

55. 12

Sol. millimole of NaOH = 0.24 × 25

$\therefore$ millimole of acid = 0.24 × 25

$\Rightarrow$ mass of acid = 0.24 × 25 × 24.2 mg

for pure acid,

$$V = \frac{w}{d};\ (d = 1.21\ \text{kg/L} = 1.21\ \text{g/ml})$$

$$\therefore\ V = \frac{0.24\times25\times24.2}{1.12}\times10^{-3}$$

$$= 120\times10^{-3}\ \text{ml}$$

$$= 12\times10^{-2}\ \text{ml}$$

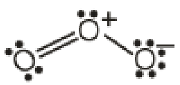
56. 60

Sol. $t_{1/2} = T_{50} = 30$ min

$T_{75} = 2t_{1/2} = 30 \times 2 = 60$ min

57. 6

Sol. Total no. of lone pairs on oxygen atoms = 6.



58. 42

Sol. $$\%\text{sulphur} = \frac{32}{233}\times\frac{\text{weight of BaSO}_4\ \text{formed}}{\text{weight of organic compound}}\times100$$

$$= \frac{32}{233}\times\frac{1.4439}{0.471}\times100$$

$$= 42.10$$

Nearest integer 42.

59. 4

Sol. $[Ni(CN)_4]^{2-}$: Ni^{+2} = 3d⁸ [⇅][⇅][⇅][⇅][ ] :

diamagnetic

^{-}CN : strong field ligand

$[Ni(CO)_4]$: Ni = 3d¹⁰ [⇅][⇅][⇅][⇅][⇅] : diamagnetic

$[NiCl_4]^{2-}$: Ni^{2+} = 3d⁸ [⇅][⇅][⇅][↑][↑] : paramagnetic

Cl^- : weak field ligand

$[Fe(CN)_6]^{4-}$: Fe^{2+} 3d⁶ [⇅][⇅][⇅][ ][ ] : diamagnetic

^{-}CN : strong field ligand

$[Cu(NH_3)_4]^{+2}$: Cu^{+2} $\Rightarrow$ one unpaired electron : paramagnetic

$[Fe(CN)_6]^{3-}$: Fe^{+3} : 3d⁵ [⇅][⇅][↑][ ][ ] :
paramagnetic, ^{-}CN : strong field ligand

$[Fe(H_2O)_6]^{2+}$: Fe^{2+} : 3d⁶ [⇅][↑][↑][↑][↑] :
paramagnetic H_2O : Weak field ligand

60. 10

Sol. $$\frac{1}{2}H_2(g) + Fe^{3+}(aq.) \longrightarrow H^+(aq.) + Fe^{2+}(aq.)$$

$$E = E^\circ - \frac{0.059}{1}\log\frac{[Fe^{2+}]}{[Fe^{3+}]}$$

$$\Rightarrow 0.712 = (0.771 - 0) - \frac{0.059}{1}\log\frac{[Fe^{2+}]}{[Fe^{3+}]}$$

$$\Rightarrow \log\frac{[Fe^{2+}]}{[Fe^{3+}]} = \frac{(0.771 - 0712)}{0.059} = 1$$

$$\Rightarrow \frac{[Fe^{2+}]}{[Fe^{3+}]} = 10$$

MATHEMATICS

Section - A (Single Correct Answer)

61. B

Sol. M = 33 × 33

$$x(66 - x) \geq \frac{5}{9} \times 33 \times 33$$

$11 \leq x \leq 55$

A : {12, 15, 18, ... 54}

$$P(A) = \frac{15}{45} = \frac{1}{3}$$

62. D

Sol. $(\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c} = \dfrac{\vec{b} - \vec{c}}{2}$

$$\vec{a}\cdot\vec{c} = \frac{1}{2},\ \vec{a}\cdot\vec{b} = \frac{1}{2}$$

$$\therefore \vec{b}\cdot\vec{d} = \frac{1}{2}$$

$$(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = \vec{a}\cdot(\vec{b}\times(\vec{c}\times\vec{d}))$$

$$= \vec{a}\cdot((\vec{b}\cdot\vec{d})\vec{c} - (\vec{b}\cdot\vec{c})\vec{d})$$

$$= (\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{d}) = \frac{1}{4}$$

63. A

Sol. $\dfrac{dy}{dx} - \dfrac{y}{x} = y^3(1 + \log_e x)$

$$\frac{1}{y^3}\frac{dy}{dx} - \frac{1}{xy^2} = 1 + \log_e x$$

Let $-\dfrac{1}{y^2} = t \Rightarrow \dfrac{2}{y^3}\dfrac{dy}{dx} = \dfrac{dt}{dx}$

$$\therefore \frac{dt}{dx} + \frac{2t}{x} = 2(1 + \log_e x)$$

$$\text{I.F.} = e^{\int \frac{2}{x}dx} = x^2$$

$$\frac{-x^2}{y^2} = \frac{2}{3}\left((1 + \log_e x)x^3 - \frac{x^3}{3}\right) + C$$

y(1) = 3

$$\frac{y^2}{9} = \frac{x^2}{5 - 2x^3(2 + \log_e x^3)}$$

OR

$xdy = ydx + xy^3 (1 + \log_e x)\, dx$

$$\frac{xdy - ydx}{y^3} = x(1 + \log_e x)dx$$

$$-\frac{x}{y}d\left(\frac{x}{y}\right) = x^2(1 + \log_e x)dx$$

$$-\left(\frac{x}{y}\right) = 2\int x^2(1 + \log_e x)dx$$

64. C

Sol. The value of

$$\lim_{n\to\infty}\frac{1 + 2 - 3 + 4 + 5 - 6 + ... + (3n - 2) + (3n - 1) - 3n}{\sqrt{2n^4 + 4n + 3} - \sqrt{n^4 + 5n + 4}}$$

is

(1) $\dfrac{\sqrt{2} + 1}{2}$

(2) $3(\sqrt{2} + 1)$

(3) $\dfrac{3}{2}(\sqrt{2} + 1)$

(4) $\dfrac{3}{2\sqrt{2}}$

65. A

Sol. Only possibility $\alpha = 0, \beta = 1$

$\therefore$ equation of circle $x^2 + y^2 - x - y = 0$

Image of circle in x + y + 2 = 0 is

$x^2 + y^2 + 5x + 5y + 12 = 0$

66. C

Sol. $\sum_{i=1}^{n} x_i = 10n$

$\sum_{i=1}^{n} x_i - 8 + 12 = (10.2)n \qquad \therefore n = 20$

Now $\frac{\sum_{i=1}^{20} x_i^2}{20} - (10)^2 = 4 \Rightarrow \sum_{i=1}^{20} x_i^2 = 2080$

$\frac{\sum_{i=1}^{20} x_i^2 - 8^2 + 12^2}{20} - (10.2)^2$

67. C

Sol. $y = \frac{1-x^{32}}{1-x} \Rightarrow y - xy = 1 - x^{32}$

$y' - xy' - y = -32x^{31}$

$y'' - xy'' - y' - y' = -(32)(31)x^{30}$

at $x = -1 \Rightarrow y' - y'' = 496$

68. A

Sol. $\vec{b} = \lambda \vec{a} \times (\vec{a} \times \hat{j})$

$\Rightarrow \vec{b} = \lambda(-2\hat{i} - 2\hat{j} + 2\hat{k})$

$|\vec{b}| = |\vec{a}| \qquad \therefore \sqrt{6} = \sqrt{12}|\lambda| \Rightarrow \lambda = \pm\frac{1}{\sqrt{2}}$

($\lambda = \frac{1}{\sqrt{2}}$ rejected $\because \vec{b}$ makes acute angle with y axis)

$\vec{b} = -\sqrt{2}(-\hat{i} - \hat{j} + \hat{k})$

$\frac{(3\vec{a} + \sqrt{2}\vec{b}) \cdot \vec{c}}{|\vec{c}|} = 3\sqrt{2}$

69. A

Sol. For $x \le 0$

$f(x) = \int_0^2 e^{t-x} dt = e^{-x}(e^2 - 1)$

For $0 < x < 2$

$f(x) = \int_0^x e^{x-t} dt + \int_x^2 e^{t-x} dt = e^x + e^{2-x} - 2$

For $x \ge 2$

$f(x) = \int_0^2 e^{x-t} dt = e^{x-2}(e^2 - 1)$

For $x \le 0$, $f(x)$ is $\downarrow$ and $x \ge 2$, $f(x)$ is $\uparrow$

$\therefore$ Minimum value of f(x) lies in $x \in (0, 2)$

Applying A.M. $\ge$ G.M.

minimum value of f(x) is 2(e – 1)

70. A

Sol. Let $P(2\lambda + 1, \lambda + 3, 2\lambda + 2)$

Let $Q = (\mu + 2, 2\mu + 2, 3\mu + 3)$

$\Rightarrow \lambda = \mu = 3 \Rightarrow P(7, 6, 8)$ and $Q(5, 8, 12)$

$PQ = 2\sqrt{6}$

71. A

Sol. $f'(x) = 8x^3 - 36x + 8 = 4(2x^3 - 9x + 2)$

$f'(x) = 0$

$\therefore x = \frac{\sqrt{6} - 2}{2}$

Now

$f(x) = \left(x^2 - 2x - \frac{9}{2}\right)(2x^2 + 4x - 1) + 24x + 7.5$

$\therefore f\left(\frac{\sqrt{6} - 2}{2}\right) = M = 12\sqrt{6} - \frac{33}{2}$

72. A

Sol. $((x - 2)^2 + (y - 3)^2) - ((x - 3)^2 - (y - 4)^2) = 1 + 1$

$\Rightarrow x + y = 7$

73. B

Sol. For

$y^2 = \frac{x}{2}$, $T : y = mx + \frac{1}{8m}$

For tangent to $y^2 + 1 = x$

$\Rightarrow \left(mx + \frac{1}{8m}\right)^2 + 1 = x$

$\therefore T : x - 2\sqrt{2}y + 1 = 0$

$d = \left|\frac{6 + 8 + 1}{\sqrt{9}}\right| = 5$

74. D

Sol. $\Delta = \begin{vmatrix} a & 2a & -3a \\ 2a+1 & 2a+3 & a+1 \\ 3a+5 & a+5 & a+2 \end{vmatrix}$

$= a(15a^2 + 31a + 36) = 0 \Rightarrow a = 0$

$\Delta \neq 0$ for all $a \in R - \{0\}$

Hence $S_1 = R - \{0\}$ $\quad S_2 = \Phi$

75. A

Sol. Put $x^2 = t$

$$\int \frac{dt}{(t+1)(t+3)} = \frac{1}{2}\int\left(\frac{1}{t+1} - \frac{1}{t+3}\right)dt$$

$$f(x) = \frac{1}{2}\ln\left(\frac{x^2+1}{x^2+3}\right) + C$$

$$f(3) = \frac{1}{2}(\ln 10 - \ln 12) + C$$

$$\Rightarrow C = 0$$

$$f(4) = \frac{1}{2}\ln\left(\frac{17}{19}\right)$$

76. B

Sol. $(p \wedge \sim q) \to (p \to \sim q)$

$\equiv (\sim (p \wedge \sim q)) \vee (\sim p \vee \sim q)$

$\equiv (\sim p \vee q) \vee (\sim p \vee \sim q)$

$\equiv \sim p \vee t \equiv t$

77. D

Sol. $g(x) = f(-x) - f(x) = \dfrac{1+e^x}{1-e^x}$

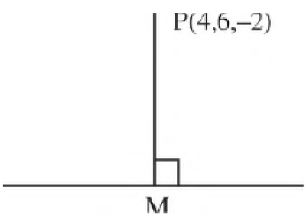
$$\Rightarrow g'(x) = \frac{2e^x}{(1-e^x)^2} > 0$$

$\Rightarrow$ g is increasing in (0, 1)

$\Rightarrow$ g is one-one in (0, 1)

78. D

Sol.



Equation of line is $\dfrac{x+3}{3} = \dfrac{y-2}{3} = \dfrac{z-3}{-1} = \lambda$

$M(3\lambda - 3,\ 3\lambda + 2,\ 3 - \lambda)$

D.R of $PM(3\lambda - 7,\ 3\lambda - 4) - 1(5 - \lambda) = 0$

$\Rightarrow \lambda = 2$

$\Rightarrow M(3,\ 8,\ 1) \Rightarrow PM = \sqrt{14}$

79. B

Sol. $|A| = \dfrac{1}{\log x \cdot \log y \cdot \log z}\begin{vmatrix} \log x & \log y & \log z \\ \log x & 2\log y & \log z \\ \log x & \log y & 3\log z \end{vmatrix}$

$\Rightarrow |adj(adj A^2)| = |A^2|^4 = 2^8$

80. B

Sol. $a_r = {}^{10}C_{10-r} = {}^{10}C_r$

$$= \sum_{r=1}^{10} r^3 r^3 \left(\frac{{}^{10}C_r}{{}^{10}C_{r-1}}\right)^2 = \sum_{r=1}^{10} r^3\left(\frac{11-r}{r}\right)^2 = \sum_{r=1}^{10} r(11-r)^2$$

$$= \sum_{r=1}^{10}(121r + r^3 - 22r^2) = 1210$$

Section - B (Numerical Value)

81. 43

Sol. Elements of the type 3k = 3

Elements of the type 3k + 1 = 1, 7, 9

Elements of the type 3k + 2 = 2, 5, 11

Subsets containing one element $S_1 = 1$

Subsets containing two elements

$S_2 = {}^3C_1 \times {}^3C_1 = 9$

Subsets containing three elements

$S_3 = {}^3C_1 \times {}^3C_1 + 1 + 1 = 11$

Subsets containing four elements

$S_4 = {}^3C_3 + {}^3C_3 + {}^3C_2 \times {}^3C_2 = 11$

Subsets containing five elements

$S_5 = {}^3C_2 \times {}^3C_2 \times 1 = 9$

Subsets containing six elements $S_6 = 1$

Subsets containing seven elements $S_7 = 1$

$\Rightarrow$ sum = 43

82. 2039

Sol. Let fog(x) = h(x)

$$\Rightarrow h^{-1}(x) = \left(\frac{x-7}{2}\right)^{\frac{1}{3}}$$

$\Rightarrow$ h(x) = fog(x) = $2x^3 + 7$

fog(x) = $a(x^b + c) - 3$

$\Rightarrow$ a = 2, b = 3, c = 5

$\Rightarrow$ fog(ac) = fog(10) = 2007

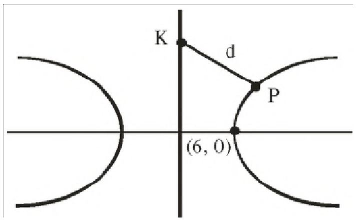
g(f(x) = (2x – 3)3 + 5

$\Rightarrow$ gof(b) = gof(3) = 32

$\Rightarrow$ sum = 2039

83. 216

Sol.



$H : \frac{x^2}{36} - \frac{y^2}{9} = 1$

equation of normal is $6x\cos\theta + 3y\cot\theta = 45$

slope $= -2\sin\theta = -\sqrt{2}$

$\Rightarrow \theta = \frac{\pi}{4}$

Equation of normal is $\sqrt{2}x + y = 15$

$P : (a\sec\theta,\ b\tan\theta)$

$\Rightarrow P(6\sqrt{2},\ 3)$ and K(0, 15)

$d^2 = 216$

84. 25

Sol. $\log_2(9^{2\alpha-4} + 13) - \log_2\left(\frac{5}{2}\cdot 3^{2\alpha-4} + 1\right) = 2$

$\Rightarrow \frac{9^{2\alpha-4} + 13}{\frac{5}{2}3^{2\alpha-4} + 1}$

$\Rightarrow \alpha = 2$ or 3

$\sum_{\alpha\in S}\alpha = 5$ and $\sum_{\alpha\in S}(\alpha+1)^2 = 25$

$\Rightarrow x^2 - 50x + 25\beta = 0$ has real roots

$\Rightarrow \beta \le 25$

$\Rightarrow \beta_{max} = 25$

85. 1080

Sol. General term is $\sum \frac{5!(2x)^{n_1}(x^{-7})^{n_2}(3x^2)^{n_3}}{n_1!n_2!n_3!}$

For constant term,

$n_1 + 2n_3 = 7n_2$

& $n_1 + n_2 + n_3 = 5$

Only possibility $n_1 = 1$, $n_2 = 1$, $n_3 = 3$

$\Rightarrow$ constant term = 1080

86. 495

Sol. $\begin{vmatrix} A+6d & 7 & 1 \\ 2(A+1+8d) & 17 & 1 \\ A+2+16d & 17 & 1 \end{vmatrix} + 70 = 0$

$\Rightarrow$ A = –7 and d = 6

$\therefore$ c – a - b = 20

$S_{20} = 495$

87. 2

Sol. **Case I :** x > 0

$\tan^{-1}\frac{2x}{1-x^2} + \tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3}$

$x = 2 - \sqrt{3}$

Case II : x < 0

$\tan^{-1}\frac{2x}{1-x^2} + \tan^{-1}\frac{2x}{1-x^2} + \pi = \frac{\pi}{3}$

$x = \frac{-1}{\sqrt{3}} \Rightarrow \alpha = 2$

88. 9

Sol. Equation of plane is

(x – 2y – z – 5) + b(x + y + 3z – 5) = 0

$\begin{vmatrix} 1+b & -2+b & -1+3b \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix} = 0$

$\Rightarrow$ b = 12

$\therefore$ plane is 13x + 10y + 35z = 65

Distance from given point to plane = 9

89. 120

Sol. $x + y = 5\lambda$

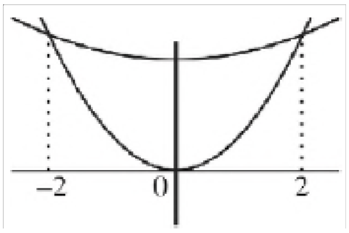
Cases :

x	y	Number of ways
5λ	5λ	20
$5\lambda+1$	$5\lambda+4$	25
$5\lambda+2$	$5\lambda+3$	25
$5\lambda+3$	$5\lambda+2$	25
$5\lambda+4$	$5\lambda+1$	25

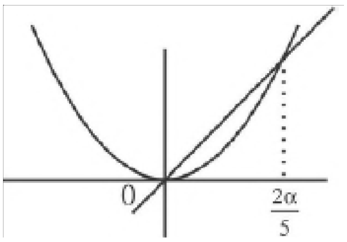
$\text{Total} = 120$

90. 600

Sol.



Abscissa of point of intersection of $2y - 5x^2$ and $y - x^2 + 6$ is ± 2



$$\text{Area} = 2\int_0^2 \left(x^2 + 6 - \frac{5x^2}{2}\right)dx = \int_0^{\frac{2\alpha}{5}} \left(\alpha x - \frac{5x^2}{2}\right)dx$$

$$\Rightarrow \int_0^{\frac{2\alpha}{5}} \left(\alpha x - \frac{5x^2}{2}\right)dx = 16$$

$$\Rightarrow \alpha^3 = 600$$

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