3S INSTITUTE
JEE ADVANCED | JEE MAIN | NEET | OLYMPIADS | MHT-CET | FOUNDATION

15-April-2023 (Morning Batch) : JEE Main Paper

MATHEMATICS

Section - A (Single Correct Answer)

1. A

Sol. (1, 1, 1) (3, 3, 3) (5, 5, 5) (8, 8, 8)
(5, 5, 8) (8, 8, 5) (1, 3, 5) (1, 3, 8)

Total number $= 1+1+1+\frac{3!}{2!}+\frac{3!}{2!}+3!+3! = 22$

2. C

Sol. $\mu = 20,\ \sigma = 8$

$$\mu_{\text{Corrected}} = \frac{200-50+40}{10} = 19$$

$$\sigma^2 = \frac{1}{10}\Sigma x_i^2 - 20^2$$

$$(64+400)10 = \Sigma x_i^2$$

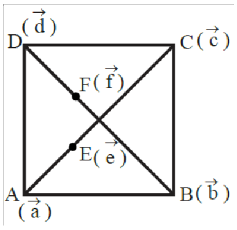
$$\sigma^2{}_{\text{Corrected}} = \frac{1}{10}[(64+400)10-2500+1600]-19^2$$

$= 374 - 361$
$= 13$

3. B

4. C

Sol.



$$\overrightarrow{AB}-\overrightarrow{BC}+\overrightarrow{AB}-\overrightarrow{DC} = k\overrightarrow{FE}$$

$$(\vec{b}-\vec{a})-(\vec{c}-\vec{b})+(\vec{d}-\vec{a})-(\vec{c}-\vec{d}) = k\overrightarrow{FE}$$

$$2(\vec{b}+\vec{d})-2(\vec{a}-\vec{c}) = k\overrightarrow{FE}$$

$$2(2\vec{f})-2(2\vec{e}) = k\overrightarrow{FE}$$

$$4(\vec{f}-\vec{e}) = k\overrightarrow{FE}$$

$$-4\overrightarrow{FE} = k\overrightarrow{FE}$$

$k = -4$

5. C

Sol. $2(y+2)\ln(y+2)dx + (x+4-2\ln(y+2))\,dy = 0$

$$2\ln(y+2)+(x+4-2\ln(y+2))\frac{1}{y+2}\cdot\frac{dy}{dx} = 0$$

let, $\ln(y+2) = t$

$$\frac{1}{y+2}\cdot\frac{dy}{dx} = \frac{dt}{dx}$$

$$2t+(x+4-2t)\cdot\frac{dt}{dx} = 0$$

$$(x+4-2t)\frac{dt}{dx} = -2t$$

$$\frac{dx}{dt} = \frac{2t-4-x}{2t}$$

$$\frac{dx}{dt}+\frac{x}{2t} = \frac{2t-4}{2t}$$

$$x\cdot t^{1/2} = \int\left(t^{1/2}-\frac{2}{t^{1/2}}\right)dt$$

$$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 2\cdot\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$x\cdot t^{\frac{1}{2}} = \frac{2t^{\frac{3}{2}}}{3} - 4t^{\frac{1}{2}} + C$$

$$x = \frac{2}{3}\cdot t - 4 + C\cdot t^{\frac{-1}{2}}$$

$$x = \frac{2}{3}\ln(y+2) - 4 + C\cdot(\ln(y+2))^{\frac{-1}{2}}$$

Put $y = e^4 - 2$, $x = 1$

$$1=\frac{2}{3}\times 4-4+C\times\frac{1}{2}$$

$$\frac{C}{2}=5-\frac{8}{3}=\frac{7}{3}$$

$$C=\frac{14}{3}$$

$$x=\frac{2}{3}\times 9-4+\frac{14}{3}\times\frac{1}{3}$$

$$=2+\frac{14}{9}$$

$$=\frac{32}{9}$$

6. D

Sol. Let $g(x)=1+x+[x]=\begin{cases}1+x; & x\in[0,\ 1)\\ 2+x; & x\in[1,\ 2)\\ 5; & x=2\end{cases}$

$$\lambda(x)=x+2[x]=\begin{cases}x; & x\in[0,\ 1)\\ x+2; & x\in[1,\ 2)\\ 6; & x=2\end{cases}$$

r(x) = 2 + x

$$f(x)=\begin{cases}2+x; & x\in[0,\ 2)\\ 6; & x=2\end{cases}$$

f(x) is discontinuous only at x = 2 $\Rightarrow$ m = 1

f(x) is differentiable in (0, 2) $\Rightarrow$ n = 0

$(m+n)^2 + 2 = 3$

7. B

Sol. x|x| – 5 |x + 2| + 6 = 0

$C-1:\ x\in[0,\ \infty]$

$x^2 - 5x - 4 = 0$

$$x=\frac{5\pm\sqrt{25+16}}{2}=\frac{5+\sqrt{41}}{2}$$

$$x=\frac{5\pm\sqrt{41}}{2}$$

$C–2:\ x\in[-2,\ 0)$

$-x^2 - 5x - 4 = 0$

$x^2 + 5x + 4 = 0$

x = –1, –4

x = –1

$C–3:\ x\in[-\infty,\ -2)$

$-x^2 + 5x + 16 = 0$

$x^2 - 5x - 16 = 0$

$$x=\frac{5\pm\sqrt{25+64}}{2}$$

$$\frac{5\pm\sqrt{89}}{2}$$

$$x=\frac{5-\sqrt{89}}{2}$$

8. B

Sol. $(a+bx+cx^2)^{10}=\sum_{i=0}^{20}p_i x^i$

Coefficient of x^1 = 20

$$20=\frac{10!}{9!1!}\times a^9\times b^1$$

$a^9\cdot b = 2$

a = 1, b = 2

Coeffcient of x^2 = 210

$$210=\frac{10!}{9!1!}\times a^9\times c^1+\frac{0!}{8!2!}\times a^8b^2$$

10c = 30

c = 3

2(a + b = c) = 12

9. A

Sol. |A| = m – n

4m + n = 22

17m + 4n = 93

m = 5, n = 2

|A| = 3

$|2\text{ adj (adj }5A)| = 2^5|5A|^{16}$

$= 2^5\cdot 5^{80}\ |A|^{16}$

$= 2^5\cdot 5^{80}\cdot 3^{16}$

$= 3^{11}\cdot 5^{80}\cdot 6^5$

a + b + c = 96

10. B

Sol. a, A_1, A_2, b are in A.P.

$$d = \frac{b-a}{3}\ ;\ A_1 = a + \frac{b-a}{3} = \frac{2a+b}{3}$$

$$A_2 = \frac{a+2b}{3}$$

$A_1 + A_2 = a + b$

a, G_1, G_2, G_3, b are in G.P.

$$r = \left(\frac{b}{a}\right)^{\frac{1}{4}}$$

$$G_1 = (a^3b)^{\frac{1}{4}}$$

$$G_2 (a^2b^2)^{\frac{1}{4}}$$

$$G_3 = (ab^3)^{\frac{1}{4}}$$

$$G_1^4 + G_2^4 + G_3^4 + G_1^2G_3^2 =$$

$$a^3b + a^2b^2 + ab^3 (a^3b)^{\frac{1}{2}} \cdot (ab^3)^{\frac{1}{2}}$$

$= a^3b + a^2b^2 + ab^3 + a^2 \cdot b^2$

$= ab(a^2 + 2ab + b^2)$

$= ab(a^2 + 2ab + b^2)$

$= G_1 \cdot G_3 \cdot (A_1 + A_2)^2$

11. D

Sol. Let $z_1 = \left(\frac{z - \bar{z} + z\bar{z}}{2 - 3z + 5\bar{z}}\right)$

Let z = 3 + iy

$\bar{z} = 3 - iy$

$$z_1 = \frac{2iy + (9+y^2)}{2 - 3(3+iy) + 5(3-iy)}$$

$$= \frac{9 + y^2 + i(2y)}{8 - 8iy}$$

$$= \frac{(9+y^2) + i(2y)}{8(1-iy)}$$

$$\text{Re}(z_1) = \frac{(9+y^2) - 2y^2}{8(1+y^2)}$$

$$= \frac{9 - y^2}{8(1+y^2)}$$

$$= \frac{1}{8}\left[\frac{10 - (1+y^2)}{(1+y^2)}\right]$$

$$= \frac{1}{8}\left[\frac{10}{1+y^2} - 1\right]$$

$$1 + y^2 \in [1, \infty]$$

$$\frac{1}{1+x^2} \in (0, 1]$$

$$\frac{10}{1+y^2} \in (-1, 9]$$

$$\text{Re}(z_1) \in \left(\frac{-1}{8}, \frac{9}{8}\right]$$

$$\alpha = \frac{-1}{8},\ \beta = \frac{9}{8}$$

$$24(\beta - \alpha) = 24\left(\frac{9}{8} + \frac{1}{8}\right) = 30$$

12. A

Sol. $C_1\left(9, \frac{15}{2}\right)$ $r_1 = \sqrt{81 + \frac{225}{4} - 131} = \frac{5}{2}$

$C_2(3, 3)$ $r_2 = 5$

$$C_1C_2 = \sqrt{6^2 + \frac{81}{4}} = \frac{15}{2}$$

$$r_1 + r_2 = \frac{15}{2}$$

$C_1C_2 = r_1 + r_2$

Number of common tangents = 3

13. D

Sol. $\sim [p \wedge (q \wedge \sim (p \wedge q))]$

$\sim p \vee (\sim q \vee (p \wedge q))$

$\sim p \vee ((\sim q \vee p) \wedge (\sim q \vee q))$

$\sim p \vee (\sim q \vee p)$

$\sim (p \wedge q) \vee p$

14. D

Sol. $-x + 2y - 9z = 7$ .....(1)

$-x + 3y - 7z = 9$ .....(2)

$-2x + y + 5z = 8$ .....(3)

(2) – (1)

$y + 16z = 2$ .....(4)

(3) – 2 × (1)

$-3y + 23z = -6$ .....(5)

3 × (4) + (5)

$71z = 0 \Rightarrow z = 0$

$y = 2$

$x = -3$

$(-3, 2, 0) \to (\alpha, \beta, \gamma)$

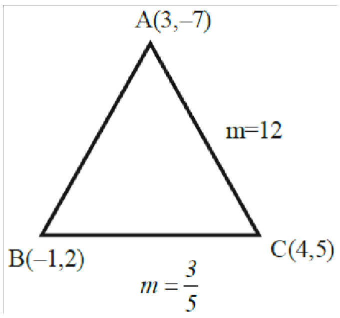
Put in $-3x + y + 13z = \lambda$

$\lambda = 9 + 2 = 11$

$$d = \left|\frac{-6-4-11}{3}\right| = 7$$

15. B

Sol.



Altitude of $BC : y + 7 = \frac{-5}{3}(x - 3)$

$3y + 21 = -5x + 15$

$5x + 3y + 6 = 0$

Altitude of $AC : y - 2 = \frac{-1}{12}(x + 1)$

$12y - 24 = -x - 1$

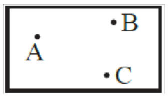
$x + 12y = 23$

$$\alpha = \frac{-47}{19},\ \beta = \frac{121}{57}$$

$9\alpha - 6\beta + 60 = 25$

16. C

Sol. P(3, –2, –9)



Equation of plane through A, B, C

$$\begin{vmatrix} x+1 & y+2 & z+3 \\ 10 & 5 & 7 \\ 10 & 0 & 4 \end{vmatrix} = 0$$

$2x + 3y - 5z - 7 = 0$

Foot of I^r of P(3, –2, –9) is

$$\frac{x-3}{2} = \frac{y+2}{3} = \frac{z+9}{-5} = -\frac{(\cancel{6} - \cancel{6} + 45 - 7)}{4+9+25}$$

$$\frac{x-3}{2} = \frac{y+2}{3} = \frac{z+9}{-5} = -1$$

$Q(1, -5, -4) \equiv (\alpha, \beta, \gamma)$

$OQ = \sqrt{\alpha^2 + \beta^2 + \gamma^2} = \sqrt{42}$

17. C

Sol. 6 W

4 R

$$\frac{1}{6} \times \left[\frac{{}^6C_1}{{}^{10}C_1} + \frac{{}^6C_2}{{}^{10}C_2} + \frac{{}^6C_3}{{}^{10}C_3} + \frac{{}^6C_4}{{}^{10}C_4} + \frac{{}^6C_5}{{}^{10}C_5} + \frac{{}^6C_6}{{}^{10}C_6}\right]$$

$$= \frac{1}{6}\left(\frac{126+70+35+15+5+1}{210}\right) = \frac{42}{210} = \frac{1}{5}$$

18. A

Sol. $I = \int_0^1 \frac{dx}{(5+2x-2x^2)(1+e^{2-4x})}$ ....(i)

$x \to 1 - x$

$I = \int_0^1 \frac{dx}{(5+2x-2x^2)(1+e^{2-4x})}$ ....(ii)

Add (i) and (ii)

$$2I = \int_0^1 \frac{dx}{5+2x-2x^2} = \int_0^1 \frac{dx}{2\left(\frac{11}{4} - \left(x - \frac{1}{2}\right)^2\right)}$$

$$I=\frac{1}{\sqrt{11}}\ln\left(\frac{\sqrt{11}+1}{\sqrt{10}}\right)\quad \begin{matrix}\alpha=\sqrt{11}\\ \beta=\sqrt{10}\end{matrix}$$

$$\alpha^4-\beta^4=121-100=21$$

19. C

Sol. $\begin{vmatrix}\lambda & -1 & 1\\ 1 & 2 & \mu\\ 3 & -4 & 5\end{vmatrix}=0\ \&\ \lambda-\mu=5$

$$\lambda(10+4\mu)+(5-3\mu)+(-10)=0$$

$$(\mu+5)(4\mu+10)+5-3\mu-10=0$$

$$\mu=-15;\ \lambda=5/4$$

$$\underline{\mu=-3;\ \lambda=2}$$

Hence $\sum\limits_{(\lambda,\ \mu)\in S} 80(\lambda^2+\mu^2)$

$$=80\left(\frac{250}{16}+13\right)$$

$= 1250 + 1040$

$= 2290$

20. B

Sol. $f(x) = \ln(4x^2 + 11x + 6) + \sin^{-1}(4x + 3)$

$$+\cos^{-1}\left(\frac{10x+6}{3}\right)$$

(i) $4x^2 + 11x + 6 > 0$

$4x^2 + 8x + 3x + 6 > 0$

$(4x + 3)(x + 2) > 0$

$$x\in(-\infty,\ -2)\cup\left(-\frac{3}{4},\ \infty\right)$$

(ii) $4x+3\in[-1,\ 1]$

$$x\in[-1,\ -1/2]$$

(iii) $\frac{10x+6}{3}\in[-1,\ 1]$

$$x\in\left[-\frac{9}{10},\ -\frac{3}{10}\right]$$

$$x\in\left(-\frac{3}{4},\ -\frac{1}{2}\right]\ \alpha=-\frac{3}{4},\ \beta=-\frac{1}{2}$$

$$\alpha+\beta=-\frac{5}{4}$$

$$36|\alpha+\beta|=45$$

Section - B (Numerical Value)

21. 7

Sol. $P=\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{2^2}-\frac{1}{2.3}+\frac{1}{3^2}\right)+$

$$\left(\frac{1}{2^3}-\frac{1}{2^2\cdot 3}+\frac{1}{2\cdot 3^2}-\frac{1}{3^3}\right)+\ldots\ldots$$

$$P\left(\frac{1}{2}+\frac{1}{3}\right)=\left(\frac{1}{2^2}-\frac{1}{3^2}\right)+\left(\frac{1}{2^3}+\frac{1}{3^3}\right)+\left(\frac{1}{2^4}-\frac{1}{3^4}\right)+\ldots.$$

$$\frac{5P}{6}=\frac{\frac{1}{4}}{1-\frac{1}{2}}-\frac{\frac{1}{9}}{1+\frac{1}{3}}$$

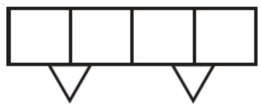
$$\frac{5P}{6}=\frac{1}{2}-\frac{1}{12}=\frac{5}{12}$$

$$\therefore P=\frac{1}{2}=\frac{\alpha}{\beta}\qquad \therefore \alpha=1,\ \beta=2$$

$$\alpha+3\beta=7$$

22. 72

Sol.



Sum of first two digits

Sum of last two digits $=\alpha$

Case-I : $\alpha=7$

$2 \times 12 = 24$ ways.

7	0
0	7

1	6
2	5
3	4
4	3
5	2
6	1

Case-II : $\alpha = 8$

17	26
71	62
	35
	53

2 × 8 ways
= 16 ways

Case-III : $\alpha = 9$

27	36
72	63
	45
	54

2 × 8 ways
= 16 ways

47	56
74	65

2 × 4 ways
= 8 ways

Ans. 24 + 16 + 16 + 8 + 8 = 72

23. 26

Sol. Plane $P \equiv P_1 + \lambda P_2 = 0$

$$(2x + y - z - 3) + \lambda(5x - 3y) + 4z + 9) = 0$$

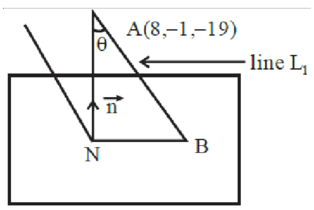
$$(5\lambda + 2)x + (1 - 3\lambda)y + (4\lambda - 1)z + 9\lambda - 3 = 0$$

$\vec{n} \cdot \vec{b} = 0$ where $\vec{b}(2, 4, 5)$

$$2(5\lambda + 2) + 4(1 - 3\lambda) + 5(4\lambda - 1) = 0$$

$$\lambda = -\frac{1}{6}$$

Plane 7x + 9y – 10z – 27 = 0



Equation of line AB is

$$\frac{x-8}{-3} = \frac{y+1}{4} = \frac{z+19}{12} = \lambda$$

Let $B = (8 - 3\lambda, \; -1 + 4\lambda, \; -19 + 12\lambda)$ lies on plane P

$$\therefore 7(8 - 3\lambda) + 9(4\lambda - 1) - 10(12\lambda - 19) = 27$$

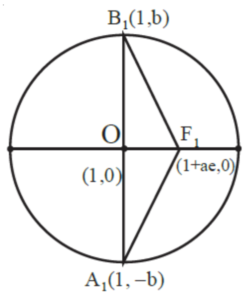
$\lambda = 2$

$\therefore$ Point B = (2, 7, 5)

$$AB = \sqrt{6^2 + 8^2 + 24^2} = 26$$

24. 9

Sol.



$$L.R. = \frac{2b^2}{a} = \frac{1}{2}$$

$4b^2 = a$ ....(i)

Ellipse $\dfrac{(x-1)^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$$m_{B_2F_1} = \frac{1}{\sqrt{3}}$$

$$\frac{b}{ae} = \frac{1}{\sqrt{3}}$$

$3b^2 = a^2e^2 = a^2 - b^2$

$4b^2 = a^2$ ....(ii)

From (i) and (ii)

$a = a^2$

$\therefore a = 1$

$$b^2 = \frac{1}{4}$$

$((2a) + (2b))^2 = 9$

25. 6

Sol. A = {1, 2, 3, 4}

R = {(a, b), (c, d)}

2a + 3b = 4c + 5d = α let

2a = {2, 4, 6, 8}4c = {4, 8, 12, 16}

3b = {3, 6, 9, 12}5d = {5, 10, 15, 20}

$$2a+3b=\begin{Bmatrix}5,\ 8,\ 11,\ 14\\ 7,\ 10,\ 13,\ 16\\ 9,\ 12,\ 15,\ 18\\ 11,\ 14,\ 17,\ 20\end{Bmatrix} 4c+5d\begin{Bmatrix}9,\ 14,\ 19,\ 24\\ 13,\ 18\ ....\\ 17,\ 22.....\\ 21,\ 26......\end{Bmatrix}$$

Possible value of $\alpha = 9,\ 13,\ 14,\ 14,\ 17,\ 18$

Pairs of {(a, b), (c, d)} = 6

26. 15

Sol. $n \in [10,\ 100]$

$3^n - 3$ is multiple of 7

$3^n = 7\lambda + 3$

n = 1, 7, 13, 20,...... 97

Number of possible values of n = 15

27. 5

Sol. $\sin A + \sin B + \sin C = \frac{18}{x}$

$\sin 2A + \sin 2B + \sin 2C = \frac{9}{x}$

∴ sin A + sin B + sin C = 2(sin 2A + sin 2B + sin 2C)

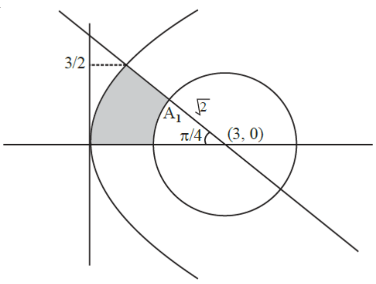
4cos A/2 cosB/2 cosC/2 = 2(4sin A sin B sin C)

16sin A/2 sin B/2 sin C/2 = 1

Hence Ans. 5.

28. 42

Sol.



$y^2 = \frac{3x}{2},\ x + y = 3,\ y = 0$

$2y^2 = 3(3 - y)$

$2y^2 + 3y - 9 = 0$

$2y^2 - 3y + 6y - 9 = 0$

(2y – 3) (y + 2) = 0; y = 3/2

$$\text{Area}\left(\int_0^{\frac{3}{2}} (x_R - x_2)dy\right) - A_1$$

$$= \int_0^{\frac{3}{2}} \left((3-y) - \frac{2y^2}{3}\right)dy - \frac{\pi}{8}(2)$$

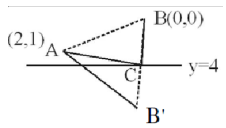
$$A = \left(3y - \frac{y^2}{2} - \frac{2y^3}{9}\right)_0^{\frac{3}{2}} - \frac{\pi}{4}$$

$$4A + \pi = 4\left[\frac{9}{2} - \frac{9}{8} - \frac{3}{4}\right] = \frac{21}{2} = 10.50$$

$\therefore 4(4A + \pi) = 42$

29. 48

Sol. $A(2,\ 1),\ B(0,\ 0),\ C(t,\ 4) : t \in [0,\ 4]$



$B_1(0,\ 8) \equiv$ image of B w.r.t. y = 4

for AC + BC + AB to be minimum

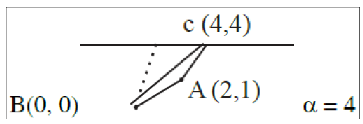
$m_{AB} = \frac{-7}{2}$

line $AB_1 \equiv 7x + 2y = 16$

$C\left(\frac{8}{7},\ 4\right)$

$\beta = \frac{8}{7}$

For max. perimeter



$AB = \sqrt{5} : BC = 4\sqrt{2},\ AC = \sqrt{13}$

$6\alpha + 21\beta = 24 + 24 = 48$

30. 28

Sol. $f(x)=\int\frac{dx}{(3+4x^2)\sqrt{4-3x^2}}$

$x=\frac{1}{t}$

$=\int\frac{\frac{-1}{t^2}dt}{\frac{(3t^2+4)}{t^2}\frac{\sqrt{4t^2-3}}{t}}$

$=\int\frac{-dt\cdot t}{(3t^2+4)\sqrt{4t^2-3}}$: Put $4t^2-3=z^2$

$=-\frac{1}{4}\int\frac{z\,dz}{\left(3\left(\frac{z^2+3}{4}\right)+4\right)z}$

$=\int\frac{-dz}{3z^2+25}=-\frac{1}{3}\int\frac{dz}{z^2+\left(\frac{5}{\sqrt{3}}\right)^2}$

$=-\frac{1}{3}\frac{\sqrt{3}}{5}\tan^{-1}\left(\frac{\sqrt{3}z}{5}\right)+C$

$=-\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}z}{5}\right)+C$

$f(x)=-\tan^{-1}\left(\frac{\sqrt{3}}{5}\sqrt{4t^2-3}\right)+C$

$\because f(0)=0\ \because c=\frac{\pi}{10\sqrt{3}}$

$f(1)=-\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}}{5}\right)+\frac{\pi}{10\sqrt{3}}$

$f(1)=\frac{1}{5\sqrt{3}}\cot^{-1}\left(\frac{\sqrt{3}}{5}\right)=\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{5}{\sqrt{3}}\right)$

$\alpha=5:\beta=\sqrt{3}\ \therefore\alpha^2+\beta^2=28$

PHYSICS

Section - A (Single Correct Answer)

31. C

Sol. Electric field due to a dipole at point on its axis

$E=\frac{2kp}{r^3}$

32. A

Sol. $F=-kx$ A true

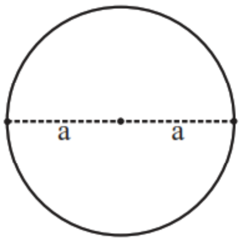
$a=-\omega^2x$ B true

Velocity is maximum at mean position
C true

Acceleration is maximum at extreme points
D false

33. C)

Sol.



$F=m\omega^2r$

$\Rightarrow\frac{Gmm}{(2a)^2}=m\omega^2a\ \Rightarrow\omega=\sqrt{\frac{Gm}{4a^3}}$

34. D

Sol. $d_{max}=\sqrt{2Rh_t}+\sqrt{2Rh_r}$

$=\sqrt{2\times64\times10^5\times180}+\sqrt{2\times64\times10^5\times245}$

$=\{(8\times6\times10^3)+(8\times7\times103)\}$ m = (48 + 56) km = 104 km

35. C

Sol. 15 year = 3 half lives

Number of active nuclei = $N_0/8$

Number of decay = $7N_0/8$

36. C

Sol. $\lambda=\frac{h}{\sqrt{2mE}}$

$\lambda'=\frac{h}{\sqrt{2m\left(\frac{E}{4}\right)}}=\frac{2h}{\sqrt{2mE}}=2\lambda$

37. C

Sol. For voltmeter

$$R = \frac{V}{I_g} - G = \frac{50}{10^{-3}} - 54 \approx 50k\Omega (A)$$

For ammeter

$$S = \frac{I_g G}{I - I_g} = \frac{10^{-3} \times 54}{(10-1)\times 10^{-3}} = 6\Omega (C)$$

38. A

Sol. Average KE per molecule $= \frac{3}{2}kT$

$$\frac{K_{Ar}}{K_H} = \frac{1}{1}$$

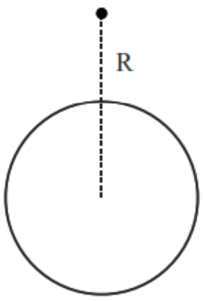
39. B

Sol. $R_{eq} = R_1 + R_2 + R_3$ So St-1 False

Resistivity depends on temperature. St-2 False

40. A

Sol.



By conservation of mechanical energy

$U_i + K_i = U_f + K_i$

$$-\frac{GMm}{2R} + 0 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\frac{GMm}{2R} = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

41. D

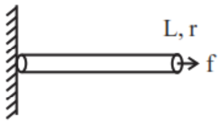
Sol. Induced emf $= -L\frac{dI}{dt}$

$$\Rightarrow 20 = -L\frac{(0-2)}{10^{-3}}$$

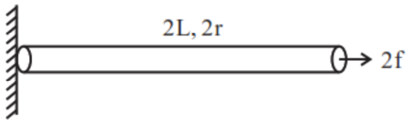
$\Rightarrow$ L = 10 mH

42. B

Sol.



$$\frac{f}{\pi r^2} = Y\frac{\ell}{L}$$



$$\frac{2f}{\pi(2r)^2} = Y\frac{l'}{2L}$$

$$\Rightarrow \frac{2}{1} = \frac{2l'}{l} \Rightarrow l' = l$$

43. C

Sol. $x = 5t^2 - 4t + 5$

v = 10t - 4

At t = 2s v = 16m/s

44. A

Sol. $\vec{r} = 10t\hat{i} + 15t^2\hat{j} + 7\hat{k}$

$\vec{v} = 10\hat{i} + 30t\hat{j}$

$\vec{a} = 30\hat{j}$

So Net force is along +y direction

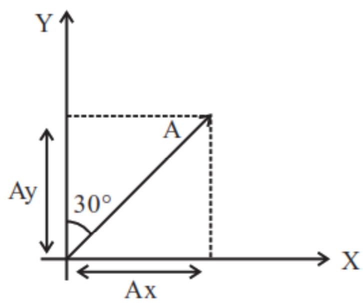
45. B

Sol. Increasing order of wave length

X-ray	1 nm to 10^{-3} nm
Ultra Violet	400 nm to 1 nm
Intra red	1 mm to 700 nm
Micro wave	0.1 m to 1mm

46. D

Sol.



$$A_y = A\cos 30° = 2\sqrt{3}$$

$$\Rightarrow A\frac{\sqrt{3}}{2} = 2\sqrt{3}$$

$$\Rightarrow A = 4$$

Now $A_x = A\sin 30° = 4\ \frac{1}{2} = 2$

47. A

Sol. $\nu = \lambda^a g^b \rho^c$

using dimension formula

$\Rightarrow\ [M^0L^1T^{-1}] = [L^1]^a\ [L^1T^{-2}]^b\ [M^1L^{-3}]^c$

$\Rightarrow\ [M^0L1T^{-1}] = [M^c\ L^{a+b-3c}\ T^{-2b}]$

$\therefore\ c = 0,\ a + b - 3c = 1,\ -2b = -1 \Rightarrow b = \frac{1}{2}$

Now $a + b - 3c = 1$

$$\Rightarrow a + \frac{1}{2} - 0 = 1$$

$$\Rightarrow a = \frac{1}{2}$$

$$\therefore a = \frac{1}{2}, b = \frac{1}{2}, c = 0$$

48. B

Sol. On P-V scale area of loop = work done

$$\Rightarrow W = +\frac{1}{2}(2)\times 300$$

W = 300J

49. B

Sol. As for first minima

$a\sin\theta = \lambda$

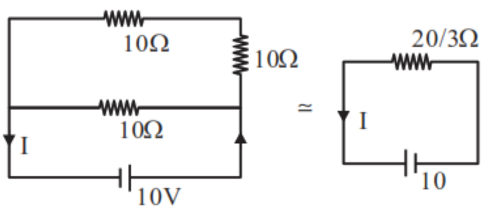
$\Rightarrow\ a\sin 30° = 600 \times 10^{-9}$

$\Rightarrow\ a = 1200 \times 10^{-9}$ m

$\Rightarrow\ a = 1.2\ \mu$m

50. A

Sol. In the circuit D_1 and D_3 are forward biased and D_2 is reverse biased.

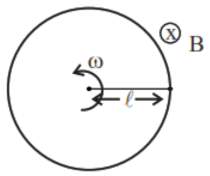


$$\therefore I = \frac{10}{20/3} = \frac{3}{2}A = 1.5A$$

Section - B (Numerical Value)

51. 88

Sol.



Here ω = 210 rpm

$$= 210 \times \frac{2\pi}{60}\ \text{rad/s}$$

$\Rightarrow\ \omega = 7\pi$ rad/s

& $\ l = 0.2$m

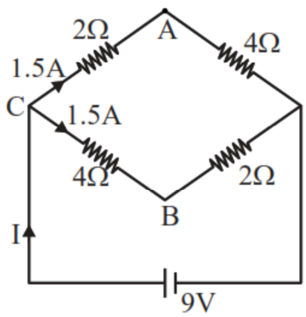
& $\ B = 0.2$T

emf developed across rod is $= \frac{1}{2}B\omega l^2$

$$\frac{1}{2}\times 0.2\times 7\pi\times(0.2)^2 = 88\ \text{mV}$$

52. 3

Sol.



In the circuit $I = \frac{9}{3} = 3A$

$V_C - V_A = 2 \times 1.5 = 3$ ....(I)

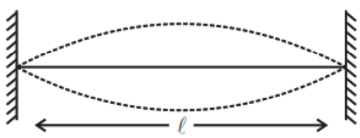
$V_C - V_B = 4 \times 1.5 = 6$ ....(II)

Eqn (II) - Eqn (I)

$V_A - V_B = 6 - 3 = 3$ Volt

53. 90

Sol.



Fundamental frequency = 50 Hz

mass/length = 20g/m

mass = 18g

length of string $= \frac{18}{20}\text{m} = \frac{9}{10}\text{m}$

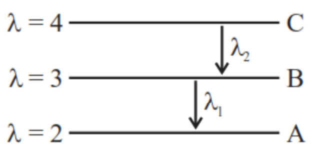
from diagram $\frac{\lambda}{2} = l$

$\Rightarrow \lambda = 2l = \frac{9}{5}\text{m}$

again speed $v = f\lambda = 50 \times \frac{9}{5} = 90$ m/s

54. 5

Sol.



As $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$\frac{1}{\lambda_1} = R(1)^2\left[\frac{1}{(2)^2} - \frac{1}{(3)^2}\right] = R\left(\frac{5}{36}\right)$

.....(i)

& $\frac{1}{\lambda_2} = R(1)^2\left[\frac{1}{(3)^2} - \frac{1}{(4)^2}\right] = R\left(\frac{7}{144}\right)$

....(ii)

(ii) ÷ (i) gives

$\frac{\lambda_1}{\lambda_2} = \frac{7/144}{5/36} = \frac{7}{20} = \frac{7}{4 \times 5}$

∴ n = 5

55. 5

Sol. For solid sphere $\frac{2}{5}mR^2 = mk_{sph}^2$

$k_{sph} = \sqrt{\frac{2}{5}}R$

For solid cylinder $\frac{mR^2}{2} = mk_{cyl}^2$

$\Rightarrow k_{cyl} = \frac{R}{\sqrt{2}}$

$\frac{k_{sph}}{k_{cyl}} = \frac{\sqrt{\frac{2}{5}}}{\frac{1}{\sqrt{2}}} = \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{x}}$

∴ x = 5

56. 30

Sol. As $\mu = \frac{\sin\left(\frac{D_{min} + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$\sqrt{2} = \frac{\frac{\sin(D_{min} + 60)}{2}}{\sin\left(\frac{60}{2}\right)}$

$\Rightarrow \frac{1}{\sqrt{2}} = \sin\left(\frac{D_{min} + 60}{2}\right)$

$\Rightarrow \frac{D_{min} + 60}{2} = 45$

$\Rightarrow$ $D_{min} = 30$

57. 40

Sol. Magnetic field due to moving charge

$B = \frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2}$

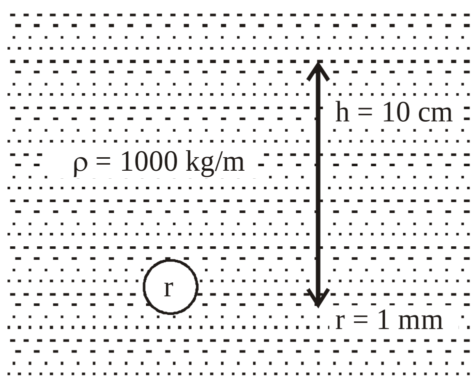
$B = \frac{\mu_0}{4\pi}\frac{ev\sin(\pi/2)}{r^2}$

$B = \frac{10^{-7} \times 1.6 \times 10^{-19} \times 6.76 \times 10^6}{0.52 \times 0.52 \times 10^{-20}}$

B = 40 T

58. 1150

Sol.



Pressure inside the bubble

$$P = P_0 + h\rho g + \frac{2T}{r}$$

$$P - P0 = h\rho g + \frac{2T}{r}$$

$$= 0.1 \times 1000 \times 10 + \frac{2 \times .075}{10^{-3}}$$

$= 1000 + (0.15)(1000)$

$= 1150$ Pa

59. 30

Sol. Work done $= \int F dx$

$$\int_2^4 5x dx = 5\left[\frac{x^2}{2}\right]_2^4$$

$$= \frac{5}{2}[16 - 4] = 30 \text{ J}$$

60. 5

Sol. Charge on C_1 is

$Q_1 = 2 \times 10 = 20\ \mu C$ .....(i)

Charge on C_2 is

$Q_2 = x \times 10 = 10\ x\mu C$ .....(ii)

Charge on C_3 is

$Q_3 = 3 \times 10 = 30\ \mu C$ ....(iii)

Total charge $20 + 10x + 30 = 100$

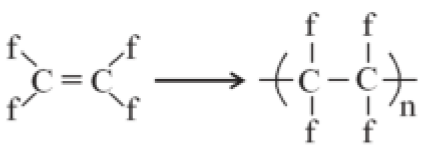
$\Rightarrow \quad x = 5$

CHEMISTRY

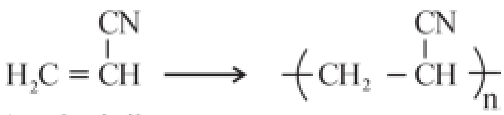
Section - A (Single Correct Answer)

61. A

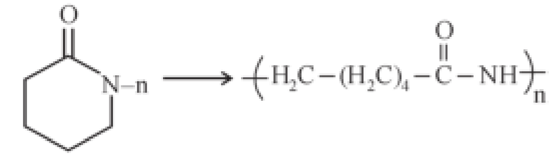
Sol.



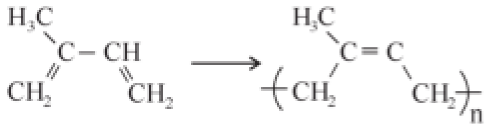
Tetra Fluoroethene Teflon



Acrylonitrile Orlon



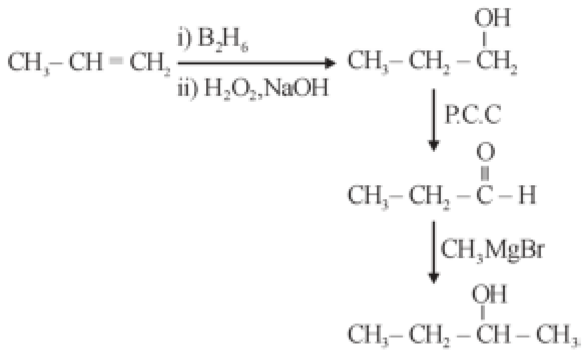
Caprolactam Nylon - 6



Isoprene Natural rubber

62. A

Sol.



63. D

Sol. Photochemical smog occurs in warm, dry and sunny climate.

64. D

Sol. $2\ PbS + 3\ O_2(g) \xrightarrow{\Delta} 2\ PbO + 2\ SO_2(g)$

It is a roasting reaction.

65. D

Sol. (A) NF_3 has trigonal pyramidal shape.

(B) Bond order $\Rightarrow N_2 > O_2$

Bond length $\Rightarrow N_2 < O_2$

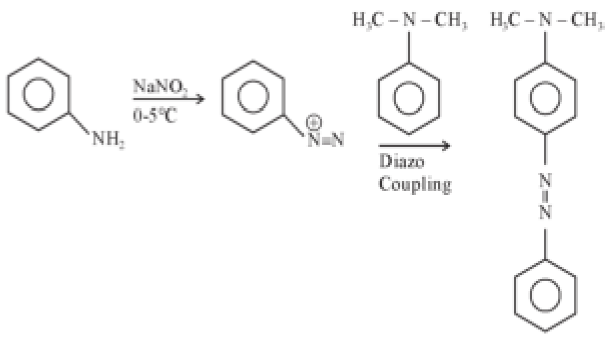
$\Rightarrow$ (C)

(D) Dipole moment $H_2O > H_2S$

Due to Electronegativity difference.

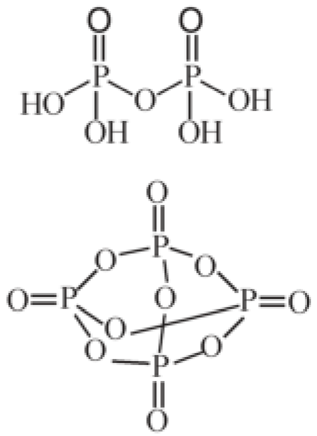
66. B

Sol.



67. A

Sol.



Molecule	Number of P-O-P Bond
$H_4P_2O_7$	1
$(HPO_3)_3$	3
P_4O_{10}	6

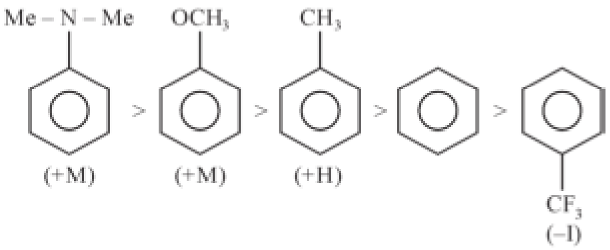
68. A

Sol. According to Bohr's model the angular momentum is quantised and equal to $\frac{nh}{2\pi}$.

Heisenberg uncertainty principle explains orbital concept, which is based on probability of finding electron.

69. B

Sol. Higher the electron density on Benzene Ring, Higher its reactivity towards electrophilic substitution reaction.



70. D

Sol.

(A) pH of 10^{-8} M HCl is in acidic range (6.98).

(B) Conjugate Base of $H_2PO_4^-$ is HPO_4^{2-}

(C) K_W increases with increasing Temperature, as the temperature increases, the dissociation of water increases.

(D) At half neutralisation point, half of the acid is present in the form of salt.

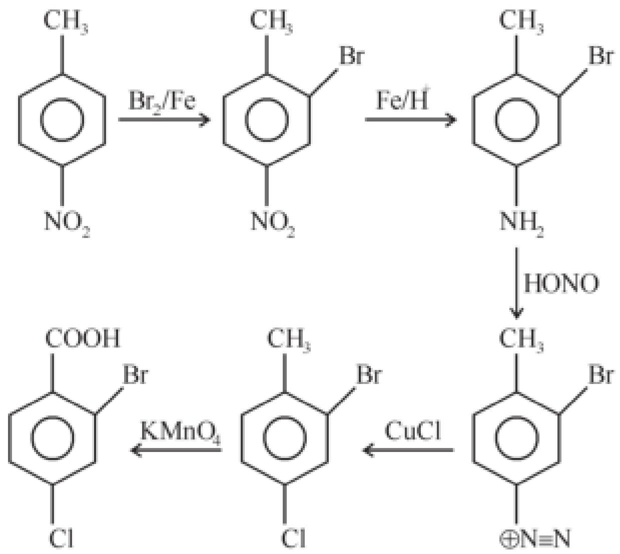
$$pH = Pk_a + \log\frac{1}{1} = pK_a$$

71. B

Sol. Be, Mg do not give colour to flame due to high excitation energy.

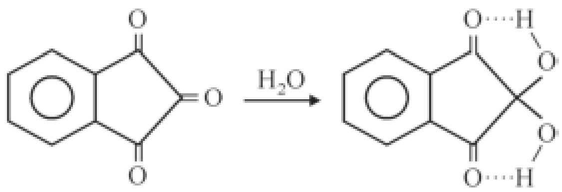
72. C

Sol.



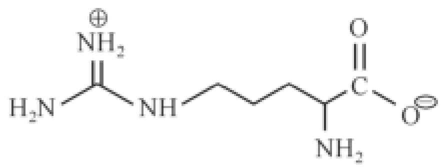
73. D

Sol.



74. D

Sol.



Arginine exist is zwitterion, so solid in nature and soluble in polar solvent.

75. A

Sol. Water gas shift reaction

$$\underbrace{[CO(g) + H_2]}_{\text{water gas}} + \underset{\text{Steam}}{H_2O} \xrightarrow[\text{Catalyst}]{\text{Iron Chromate}} CO_2(g) + 2H_2(g)$$

76. A

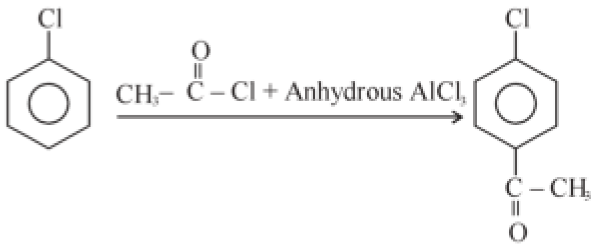
Sol. For good quality cement, the ratio of silica (SlO_2) to Alumina (Al_2O_3) should be between 2.5 to 4.

77. B

Sol. In paper chromatography, a special quality paper known as chromatography paper is used. Paper contains water trapped in it, which acts as the stationary phase.

78. A

Sol.



Chlorine is ortho/para directing, para is major.

79. A

Sol. Ti^{+3} = 67 pm radius

Cr^{3+} = 62 pm radius

Mn^{+3} = 65 pm radius

Fe^{+3} = 65 pm radius

So, Cr^{3+} has highest tendency to attract ligand.

80. C

Sol. For CsCl, $Cs^{\oplus}$ is present at Body centre and Cl^{-} at all corner.

$$\frac{\sqrt{3}a}{2} = r_{cs^{\oplus}} + r_{Cl^{-}}$$

Section - B (Numerical Value)

81. 1

Sol. Co^{2+} : $3d^7$ configuration

$t_{2g}^{221}\ e_g^{11}$

82. 4

Sol. $[Cr(H_2O)_5Cl]\ Cl_2 + 2AgNO_3 \rightarrow$

0.01 M, 20 mL   0.1 M

For 0.2 milimole $AgNO_3$ required = 0.4 milimole

0.4 = 0.1 × V(ml)

V = 4 mL

83. 5

Sol. $KMnO_4 \rightarrow Mn^{2+}$

$\downarrow$ +7 $\downarrow$ +2

Change in oxidation state of Mn = 5.

84. 6

Sol. $CrO_2Cl_2 x - 4 - 2 = 0$

Oxidation State = +6

85. 5

Sol. Isoelectronic species

$O^{2\ominus}, F^{\ominus}, Mg^{2+}, Na^{\oplus}, Al^{3+}$

86. 23

Sol. $$\frac{24 - P_s}{P_s} = \frac{m \times 18}{1000}$$

wt. of solute = 30 gm

volume of solution =100 mL

wt. of solution = 1.2 × 100 = 120 gm

wt. of solvent = 120 – 30 = 90 gm

$$m = \frac{30 \times 1000}{180 \times 90} = 1.85$$

$$\frac{24 - P_s}{P_s} = \frac{1.85 \times 18}{1000}$$

$24 - P_s = 0.0333\ P_s$

$P_s\ (1.033) = 24$

$P_s = 23.22$

87. 50

Sol. Coagulating value is required milimole of electrolyte needed to coagulate 1 L sol in 2 hours.

$$\text{Coagulating value} = \frac{20 \times 0.5}{200} \times 1000 = 50$$

88. 130

Sol. $E_a = 300\ kJ\ mol^{-1}$

$$\frac{E_a}{T} = \frac{E'_a}{T'}$$

(Since rate of catalysed and uncatalysed reaction is same)

$$\frac{300}{600} = \frac{E'_{a,f}}{300}$$

$E'_{a,f} = 150$

$20 = 150 - E'_{a,b}$

$E'_{a,b} = 130$

89. 3

Sol.

(A) Conductivity decreases with dilution for strong electrolyte as well as weak electrolyte.

(B) On dilution, The number of ions per unit volume that carry current in a solution decreases.

(C) Molar conductivity increases with dilution.

(D) Molar conductivity of strong electrolyte follows DHO equation but it is not applicable for weak electrolyte.

(E) On dilution degree of dissociation of weak electrolyte increases.

So answer is (A), (C) & (D).

90. 1070

Sol. $\Delta S = \frac{\Delta H}{T_{mp}}$

$$28.4 = \frac{30.4 \times 1000}{T_{mp}}$$

$$T_{mp} = 1070.422 \text{ K}$$

❒ ❒ ❒