3S INSTITUTE

JEE ADVANCED | JEE MAIN | NEET | OLYMPIADS | MHT-CET | FOUNDATION

13-April-2023 (Evening Batch) : JEE Main Paper

MATHEMATICS

Section - A (Single Correct Answer)

1. A

Sol. $\begin{vmatrix} 2 & 1 & -1 \\ 2 & -5 & \lambda \\ 1 & 2 & -5 \end{vmatrix} = 0$

$2(25-2\lambda)-(-10-\lambda)-(4+5)=0$

$50-4\lambda+10+\lambda-9=0$

$51=3\lambda \Rightarrow \lambda=17$

$\begin{vmatrix} 2 & 1 & 5 \\ 2 & -5 & \mu \\ 1 & 2 & 7 \end{vmatrix} = 0$

$\Rightarrow 2(-35-2\mu)-(14-\mu)+5(4+5)=0$

$-70-4\mu-14+\mu+45=0$

$-3\mu=39$

$-\mu=13$

$(\lambda+\mu)^2+(\lambda-\mu)^2=2(\lambda^2+\mu^2)$

$= 2(17^2 + 13^2) = 916$

2. C

Sol. $\left(2x^3-\dfrac{1}{3x^2}\right)^5$

$T_{r+1} = {}^5C_r(2x^{35-r})\left(\dfrac{-1}{3x^2}\right)^r = {}^5C_r\dfrac{(2)^{5-r}}{(-3)^r}(x)^{15-5r}$

$\therefore 15-5r=5$

$\therefore r=2$

$T_3=10\left(\dfrac{8}{9}\right)x^5$

So, coefficient is $\dfrac{80}{9}$

3. C

Sol. Points (0, –1, 2) and (–1, 2, 1) parallel to the line of (5, 1, –7) and (1, –1, –1)

Normal vector $= \begin{vmatrix} i & j & k \\ 4 & 2 & -6 \\ -1 & 3 & -1 \end{vmatrix}$

$\vec{n}=16\hat{i}+10\hat{j}+14\hat{k}$

$16x + 10y + 14z = d$

Point (0, –1, 2)

$0 - 10 + 28 = d \Rightarrow d = 18$

$8x + 5y + 7z = 9$ is equation of plane.

4. D

Sol. $x^2-\sqrt{2}x+2=0$

$x=\dfrac{\sqrt{2}\pm\sqrt{2-8}}{2}=\dfrac{\sqrt{2}\pm\sqrt{6}i}{2}$

$\alpha=\dfrac{\sqrt{2}+\sqrt{6}i}{2}=\sqrt{2}e^{\frac{ix}{3}}$ & $\beta=\sqrt{2}e^{\frac{-i\pi}{3}}$

$\alpha^{14}=2^7e^{\frac{i14\pi}{3}}=128\left[e^{\frac{i2\pi}{3}}\right]$

$\beta^{14}=128\left[e^{\frac{-i2\pi}{3}}\right]$

$\alpha^{14}+\beta^{14}=128(2)\cos\left(\dfrac{2\pi}{3}\right)=-128$

5. D

Sol. $ar^5 + ar^7 = 2$

$(ar^2)(ar^4)=\dfrac{1}{9}$

$a^2r^6=\dfrac{1}{9}$

Now, $r > 0$

$ar^5(1 + r^2) = 2$

Now, $ar^3 = \frac{1}{3}$ or $-\frac{1}{3}$(rejected)

$r^2 = 2$

$r = \sqrt{2}$

$a = \frac{1}{6\sqrt{2}}$

Now, $6(a_2 + a_4)(a_4 + a_6)$

$6(ar + ar^3)(ar^3 + ar^5)$

$$6\left(\frac{1}{36.2}\right)(4)(9) = 3$$

6. B

Sol. upon solving we get coordinates as (6, 8), (1, 2) and (5, –7)

So centroid : (α, β) is

$$\alpha = \frac{6+1+5}{3} = 4$$

$$\beta = \frac{8+2-7}{3} = 1$$

$\alpha + 2\beta = 6$

$2\alpha - \beta = 7$

Ans. $x^2 - 13x + 42 = 0$

7. D

Sol. $|\vec{a}| = 2,\ |\vec{b}| = 3$

$$\left|(\vec{a} + 2\vec{b}) \times (2\vec{a} - 3\vec{b})\right|^2$$

$$\left|-3\vec{a} \times \vec{b} + 4\vec{b} \times \vec{a}\right|^2$$

$$\left|-3\vec{a} \times \vec{b} - 4\vec{a} \times \vec{b}\right|^2$$

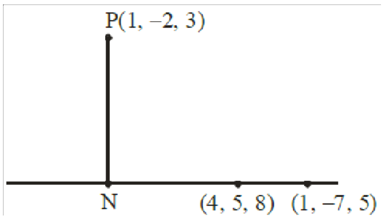
$$\left|-7\vec{a} \times \vec{b}\right|^2$$

$$\left(-7|\vec{a}| \times |\vec{b}| \sin\left(\frac{\pi}{4}\right)\right)^2$$

$$49 \times 4 \times 9 \times \frac{1}{2} = 882$$

8. C

Sol.



Equation of line

$$\frac{x-4}{4-1} = \frac{y-5}{5-(-7)} = \frac{z-8}{8-5}$$

$$\frac{x-4}{3} = \frac{y-5}{12} = \frac{z-8}{3}$$

Let ponit $N(3\lambda + 4,\ 12\lambda + 5,\ 3\lambda + 8)$

$$\overrightarrow{PN} = (3\lambda + 4 - 1)\hat{i} + (12\lambda + 5 - (-2))\hat{j} + (3\lambda + 8 - 3)\hat{k}$$

$$\overrightarrow{PN} = (3\lambda + 3)\hat{i} + (12\lambda + 7)\hat{j} + (3\lambda + 5)\hat{k}$$

And parallel vector to line (say $\vec{a} = 3\hat{i} + 12\hat{j} + 3\hat{k}$)

Now, $\overrightarrow{PN} \cdot \vec{a} = 0$

$(3\lambda + 3)3 + (12\lambda + 7)12 + (3\lambda + 5)3 = 0$

$$162\lambda + 108 = 0 \Rightarrow \lambda = \frac{-108}{162} = \frac{-2}{3}$$

So point N is (2, –3, 6)

Now distance is $= \left|\frac{2(2) - 2(-3) + 6 + 5}{\sqrt{4+4+1}}\right| = 7$

9. C

Sol. $$\lim_{x \to 0} \frac{e^{ax} - \cos(bx) - \frac{cxe^{-cx}}{2}}{\frac{(1 - \cos 2x)}{4x^2} \times 4x^2} = 17$$

On expansion,

$$\lim_{x \to 0} \frac{\left(1 + ax + \frac{a^2x^2}{2}\right) - \left(1 - \frac{b^2x^2}{2}\right) - \frac{cx}{2}(1 - cx)}{2x^2} = 17$$

$$\lim_{x \to 0} \frac{\left(a - \frac{c}{2}\right)x + x^2\left(\frac{a^2}{2} + \frac{b^2}{2} + \frac{c^2}{2}\right)}{2x^2} = 17$$

For limit to be exist $a-\frac{c}{2}=0$

$a=\frac{c}{2}$

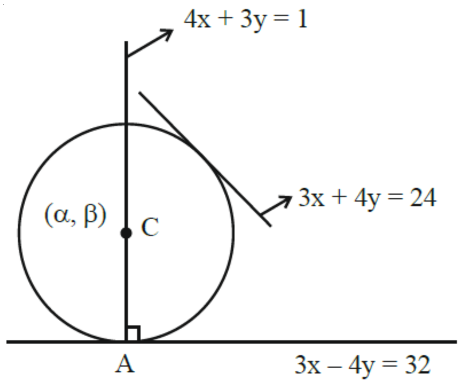
and $\frac{a^2+b^2+c^2}{4}=17$

$a^2 + b^2 + 4a^2 = 17 \times 4$

$5a^2 + b^2 = 68$

10. A

Sol.



First find point A by solving 4x + 3y = 1 and 3x – 4y = 32

After solving, point A is (4, –5)

centre (α, β) lie on 4x + 3y = 1

$4\alpha+3\beta=1 \Rightarrow \beta=\frac{1-4\alpha}{3}$

Now distance from centre to line 3x – 4y – 32 = 0 and 3x + 4y – 24 = 0 are equal.

$$\left|\frac{3\alpha-4\left(\frac{1-4\alpha}{3}\right)-32}{5}\right|=\left|\frac{3\alpha+4\left(\frac{1-4\alpha}{3}\right)-24}{5}\right|$$

after solving $\alpha=1$ and $\alpha=\frac{28}{3}$

For $\alpha=1$, centre (1, –1) $\Rightarrow$ radius = 5

For $\alpha=\frac{28}{3}$, centre $\left(\frac{28}{3}, \frac{-109}{2}\right)$

$\Rightarrow$ radius ≈ 49.78 (rejected)

Hence, $\alpha=1,\ \beta=-1,\ r=5$

$\alpha-\beta+r=7$

11. A

Sol. First arrange in alphabetical order

i.e. ADMNOY

A _ _ _ _ _ = 5!

D _ _ _ _ _ = 5!

M A _ _ _ _ = 4!

M D _ _ _ _ = 4!

M N _ _ _ _ = 4!

M O A _ _ _ = 3!

M O D _ _ _ = 3!

M O N A _ _ = 2

M O N D A Y = 1

= 327

12. B

Sol. $f(x)=4\sin^{-1}\left(\frac{x^2}{x^2+1}\right)$

$\frac{x^2+1-1}{x^2+1}=1-\frac{1}{x^2+1}\Rightarrow[0,\ 1)$

Range of $f(x)=[0,\ 2\pi)$

13. A

Sol. $(p\wedge(\sim q)\vee((\sim p)\wedge q)\vee((\sim p)\wedge(\sim q)))$

$(p\wedge(\sim q))\vee((\sim p)\wedge(q\vee(\sim q)))$

$(p\wedge(\sim q))\vee((\sim p)\wedge t)$

$(p\wedge(\sim q))\vee(\sim p)$

$(\sim p)\vee(p\wedge\sim q)$

$(\sim p\vee p)\wedge(\sim p\vee\sim q)$

$t\wedge(\sim p\vee\sim q)$

$=\sim p\vee\sim q$

14. C

Sol. np – npq = 1

$\Rightarrow np^2=1$

$2\,{}^nC_2p^2q^{n-2} = 3\,{}^nC_1pq^{n-1}$

$\Rightarrow$ np – p = 3q $(\because q=1-p)$

$\Rightarrow p=\frac{1}{2}$

Hence n = 4

P(x > 1) = 1 – (p(x = 0) + p(x = 1))

$=1-\left({}^4C_0\left(\frac{1}{2}\right)^4+{}^4C_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^8\right)=\frac{11}{16}$

15. D

Sol. $A = \begin{bmatrix} 1 & 2 & 3 \\ a & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ $|A| = 2$

$1(6-1)-2(2\alpha-1)+3(\alpha-3)=2$

$5-4\alpha+2+3\alpha-9=2$

$-\alpha-4=0$

$\alpha=-4$

$8Adj(2Adj(2A))|$

$8|Adj(2 \times 2^2\ Adj(A))|$

$8|Adj(2^3\ Adj\ A)|$

$2^3(2^6)^3\ |Adj(Adj)|$

$2^3 \cdot 2^{18}\ |A|^4$

$2^{21} \cdot 2^4 = 2^{25} = (2^5)^5 = (32)^5$

$n = 5$

$\alpha=-4$

16. B

Sol. Let $Z = x+iy,\ x \in R,\ y \in R$

$x - iy = i(x^2 - y^2 + (2xy)i + x)$

$x = -2xy$ ....(1)

$-y = -y^2 + x^2 + x$ ....(2)

$\Rightarrow x = 0,\ y = -\frac{1}{2}$ (from (1))

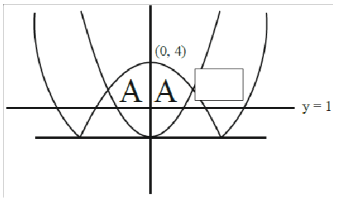
If $x \neq 0$, then $y = 0, 1$

If $y = -\frac{1}{2}$, then $x = \frac{1}{2}, -\frac{3}{2}$

$Z = 0+i0,\ 0+i,\ \frac{1}{2}-\frac{i}{2},\ -\frac{3}{2}-\frac{i}{2}$

17. B

Sol.



Required area

$= 2\left[\int_1^2 \sqrt{y}dy + \int_2^4 \sqrt{4-y}\ dy\right] = \frac{4}{3}[4\sqrt{2}-1]$

18. A

Sol. $\overline{AB}+\overline{BC}+\overline{CA}=\vec{0}$

$\alpha=2,\ \beta=4,\ \gamma-\delta=3$

$\frac{1}{2}\left|\overline{AB}\times\overline{AC}\right| = 5\sqrt{6}$

$(\delta-9)^2+(2\delta+12)^2+100-600$

$\Rightarrow \delta=5,\ \gamma=8$

Hence $\overline{CB}\cdot\overline{CA}=60$

19. B

Sol. Condition of co-planarity

$\begin{vmatrix} x_2-x_1 & a_1 & a_2 \\ y_2-y_1 & b_1 & b_2 \\ z_2-z_1 & c_1 & c_2 \end{vmatrix} = 0$

Where a_1, b_1, c_1 are direction cosine of 1st line and a_2, b_2, c_2 are direction cosine of 2nd line.

Now, solving options

Point (–3, 1, 5) & point (–1, 2, 5)

(1) $\begin{vmatrix} -3 & 1 & 5 \\ 1 & 2 & 5 \\ -2 & -1 & 0 \end{vmatrix}$

= –3(5) – (10) + 5(–1 + 4)

= –15 – 10 + 15 = – 10

(2) Point (–1, 2, 5)

$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 5 \\ -2 & -1 & 0 \end{vmatrix}$

= 3(5) – (10) + 5(1 + 4)

–25 + 25 = 0

(3) Point (–1, 2, 5)

$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 4 \\ -2 & -1 & 0 \end{vmatrix}$

–3(4) – (8) + 5 (1 + 4)

–12 – 8 + 25 = 5

(4) Point (–1, 2, 5)

$$\begin{vmatrix} -3 & 1 & 5 \\ -1 & 2 & 5 \\ 4 & 1 & 0 \end{vmatrix}$$

$-3(-5) - (-20) + 5(-1-8)$

$15 + 20 - 45 = -10$

20. A

Sol. $\int_0^{\pi/4} e^{-x} \tan^{50} x \, dx$

$$[-e^{-y}(\tan x)^{50}]_0^{\pi/4} + \int_0^{\pi/4} e^{-x}(50)(\tan x)^{49} \sec^2 x$$

$$= -e^{-\pi/4} + 0 + 50 \int_0^{\pi/4} e^{-x}(50)(\tan x)^{49} \sec^2 x$$

$$= -e^{-\pi/4} + 50\left(\int_0^{\pi/4} (\tan x)^{51} + (\tan x)^{49}\right) dx$$

Now, $$\frac{-e^{-\pi/4} + \int_0^{\pi/4} e^{-x}(\tan x)^{50} dx}{\int_0^{\pi/4} e^{-x}(\tan^{49} x + \tan^{51} x)\, dx}$$

$$\frac{50\int_0^{\pi/4} e^{-x}((\tan x)^{51} + (\tan x)^{49}) dx}{\int_0^{\pi/4} e^{-x}(\tan^{49} x + \tan^{51})\, dx}$$

Section - B (Numerical Value)

21. 269

Sol. $\bar{x} = 50$

$$\sum x_i = 500$$

$$\sum x_{i_{correct}} = 500 + 20 + 25 - 45 - 50 = 450$$

$$\sigma^2 = 144$$

$$\frac{\sum x_i^2}{10} - (50)^2 = 144$$

$$\sum x_{i\ correct}^2 = (144 + (50)^2) \times 10 - (45)^2 - (50)^2 + (20)^2 + (25)^2$$

$$\text{Correct variance} = \frac{\Sigma(x_{i\ correct})^2}{10} - \left(\frac{\Sigma x_{i\ correct}}{10}\right)^2$$

$= 2294 - (45)^2$

$= 2294 - 2025 = 269$

22. 7

Sol. R = [(–4, 4), (–3, 3), (3, –2), (0, 1), (0, 0), (1, 1), (4, 4), (3, 3)}

For reflexive, add $\Rightarrow$ (–2, –2), (–4, –4), (–3, –3)

For symmetric, add $\Rightarrow$ (4, –4), (3, –3), (–2, 3), (1, 0)

23. 10

Sol. $f(x) = \sum_{k=1}^{10} kx^k$

$f(x) = x + 2x^2 + \ldots\ldots + 10x^{10}$

$f(x) \cdot x = x^2 + 2x^3 + \ldots\ldots + 9x^{10} + 10x^{11}$

$f(x)(1-x) = x + x^2 + x^3 + \ldots\ldots + x^{10} - 10x^{11}$

$$f(x) = \frac{x(1-x^{10})}{(1-x)^2} - \frac{10x^{11}}{(1-x)}$$

$$f(x) = \frac{x - x^{11} - 10x^{11} + 10x^{12}}{(1-x)^2} \Rightarrow \frac{10x^{12} - 11x^{11} + x}{(1-x)^2}$$

Hence $2f(2) + f'(2) = 119.2^{10} + 1$

$\Rightarrow$ So, n = 10

24. 16

Sol. For number to be divisible by '6' unit digit should be even and sum of digit is divisible by 3.

(2, 1, 3), (2, 3, 4), (2, 5, 5), (2, 2, 5), (2, 2, 2), (4, 1, 1), (4, 4, 1), (4, 4, 4), (4, 3, 5)

2, 1, 3 $\Rightarrow$ 312, 132

2, 3, 4 $\Rightarrow$ 342, 432, 234, 324

2, 5, 5 $\Rightarrow$ 552

2, 2, 5 $\Rightarrow$ 252, 522

2, 2, 2 $\Rightarrow$ 222

4, 1, 1 $\Rightarrow$ 114

4, 4, 1 $\Rightarrow$ 414, 144

4, 4, 4 $\Rightarrow$ 444

4, 3, 5 $\Rightarrow$ 354, 534

Total 16 numbers.

25. 825

Sol. $[\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + \ldots.[\sqrt{120}]$

$\Rightarrow$ 1 + 1 + 1 + 2 + 2 + 2 + 2 + 2 + 3 + 3 +......+ 3 = 7 times

+ 4 + 4 +......+ 4 = 9 times + ... 10 + 10 + ...+ 10 = 21 times

$$\Rightarrow \sum_{r=1}^{10}(2r+1)\cdot r$$

$$\Rightarrow 2\sum_{r=1}^{10}(2r+1)\cdot r$$

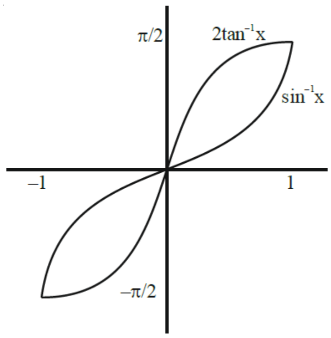
$$\Rightarrow 2\times\frac{10\times11\times21}{6}+\frac{10\times11}{2}$$

$$\Rightarrow 770+55$$

$$\Rightarrow 825$$

26. 2

Sol.



27. 6

Sol. $\frac{dy}{dx}+\frac{4x}{(x^2-1)}y=\frac{x+2}{(x^2-1)^{\frac{5}{2}}}$, $x>1$

$$I.F.=e^{\int\frac{4x}{x^2-1}dx}$$

I.F. = $(x^2 - 1)^2$

$$\Rightarrow d(y\cdot(x^2-1)^2)=\frac{x+2}{(x^2-1)^{\frac{5}{2}}}\cdot(x^2-1)^2$$

$$\Rightarrow \int d(y\cdot(x^2-1)^2)=\int\frac{x+2}{(x^2-1)^{\frac{1}{2}}}dx \quad ....(1)$$

$$y(x^2-1)^2=\sqrt{x^2-1}+2\ln(x+\sqrt{x^2-1})+C$$

$$\Rightarrow C=-\sqrt{3}$$

So $(x^2-1)^2=\sqrt{x^2-1}+2\ln(x+\sqrt{x^2-1})-\sqrt{3}$

$$\Rightarrow \alpha\beta\gamma=6$$

28. 12

Sol. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$$ae=2 \ \& \ e=\frac{3}{2}\Rightarrow a=\frac{4}{3}$$

also $b^2=a^2e^2-a^2\Rightarrow 4-\frac{16}{9}$

$$\Rightarrow b^2=\frac{20}{9}$$

Slope of tangent $=\frac{3}{2}$

So tangent equation will be

$$y=mx\pm\sqrt{a^2m^2-b^2}$$

$$\Rightarrow y=\frac{3x}{2}\pm\sqrt{\frac{16}{9}\cdot\frac{9}{4}-\frac{20}{9}}$$

$$\Rightarrow y=\frac{3x}{2}\pm\frac{4}{3}\Rightarrow|x_{intercept}|=\frac{8}{9}$$

$$|y_{intercept}|=\frac{4}{3}$$

$$\Rightarrow|6a|+|5b|=\frac{48}{9}+\frac{60}{9}=\frac{109}{9}=12$$

29. 41

Sol. $f_n(x)=\int_0^{\frac{\pi}{2}}(1+\sin x+\sin^2 x+\sin^3 x+.....+\sin^{n-1}(x))$

(1 + 3 sin x + 5 $\sin^2$x + ... + (2n – 1)) $\sin^{n-1}$ x· cos x dx

Multiply & divide by $\sqrt{\sin x}$

$$\int_0^{\frac{\pi}{2}}\left((\sin x)^{\frac{1}{2}}+(\sin x)^{\frac{3}{2}}+(\sin x)^{\frac{5}{2}}+(\sin x)^{\frac{7}{2}}+....(\sin x)^{\frac{2n-1}{2}}\right)$$

$$(1+3\sin x+5\sin^2 x+......+(2n-1)\sin^{n-1}(x))$$

$$\frac{\cos x}{\sqrt{\sin x}}dx$$

Put $(\sin x)^{1/2}$ + $(\sin x)^{3/2}$ + $(\sin x)^{5/2}$ + ... + $(\sin x)^{n-1/2}$ = t

$$\frac{1}{2}\frac{(1+3\sin x+5\sin^2 x+....(2n-1)\sin^{n-1}x)}{\sqrt{\sin x}}\cos x\ dx=dt$$

$$f_n = 2\int_0^n t\,dt$$

$f_n = n^2$

$f_{21} - f_{20} = (21)^2 - (20)^2$

$= 441 - 400$

$= 41$

30. 12

Sol. $7^{103} = 7 \times 7^{102}$

$= 7 \times (49)^{51}$

$= 7 \times (51 - 2)^{51}$

Remainder : $7 \times (-2)^{51}$

$\Rightarrow -7(2^3 \cdot (16)^{12})$

$\Rightarrow -56(17-1)^{12}$

Remainder $= -56 \times (-1)^{12} = -56 + 68 = 12$

PHYSICS

Section - A (Single Correct Answer)

31. D

Sol. Binding energy per nucleon is almost same for nuclei of mass number ranging 30 to 170.

32. A

Sol. Truth table for NAND gate is

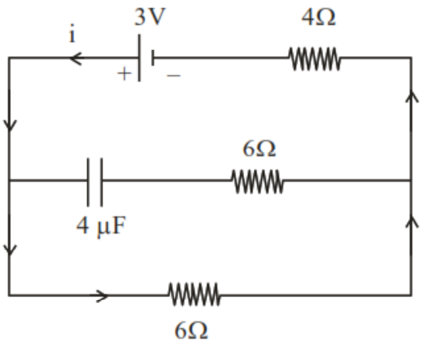
A	B	$Y = \overline{A \cdot B}$
0	0	1
0	1	1
1	0	1
1	1	0

On the basis of given input A and B the truth table is

A	B	Y
1	1	0
0	0	1
0	1	1
1	0	1
1	1	0
0	0	1
0	1	1

33. A

Sol.



No current will flow in capacitor in steady state, current flowing in the circuit in steady state

$$I = \frac{3}{6+4} = \frac{3}{10}$$

Potential difference on 6Ω resistance

$$V = 6 \times \frac{3}{10} = 1.8 \text{ volt}$$

Capacitor will have same potential so charge,

$q = CV = (4\ \mu F) \cdot (1.8 \text{ volt}) = 7.2\ \mu C.$

34. B

Sol. $V_e = \sqrt{\frac{2GM}{R}} \Rightarrow V_e \propto \sqrt{\frac{M}{R}}$

As $\frac{M}{R}$ increases $\Rightarrow V_e$ increases

Statement (I) is correct

Also $V_e \propto \frac{1}{\sqrt{R}}$

As V_e depends upon R

$\Rightarrow$ Statement (B) is incorrect

35. C

Sol. KE = PE

$$\frac{1}{2}M\omega^2\left(A^2 - x^2\right) = \frac{1}{2}M\omega^2 x^2$$

$A^2 - x^2 = x^2 \Rightarrow A^2 = 2 \times 2$

$$\Rightarrow x = \pm\frac{A}{\sqrt{2}}$$

36. D

Sol. Velocity of train A

$$V_A = 90\frac{km}{hr} = 90\times\frac{5}{18} = 25 \text{ m/s}$$

Velocity of train B

$$V_B = 54\frac{km}{hr} = 54\times\frac{5}{18} = 15 \text{ m/s}$$

Velocity of train B w.r.t. train $A = \vec{V}_B - \vec{V}_A$

= 15 – (– 25) m/s = 40 m/s

$$\text{Time of crossing} = \frac{\text{length of train}}{\text{relative velocity}}$$

$$(8) = \frac{l}{40}$$

l = 8 × 40 = 320 meter.

37. B

Sol. As gas is suddenly compressed, the processes is adiabatic.

Equation of gas for adiabatic process is

PV^γ = constant.

$$\Rightarrow P_1V_1^\gamma = P_2V_2^\gamma$$

$$\Rightarrow P_0V_0^\gamma = P_2\left(\frac{V_0}{4}\right)^\gamma$$

$\Rightarrow P_2 = P_0\ (4)^\gamma$

38. D

Sol. Terminal velocity of a spherical body in liquid

$\Rightarrow\quad V_t \propto r^2$

$$\Rightarrow \frac{\Delta V_t}{V_t} = 2\cdot\frac{\Delta r}{r}$$

$$\Rightarrow \frac{\Delta V_t}{V_t}\times 100\% = 2\cdot\frac{(0.1)}{5}\times 100 = 4\%$$

Also $V_t \propto r^2$

Reason R is false

39. B

Sol. Direction of propagation of EM wave will be in the direction of $\vec{E}\times\vec{B}$.

40. D

Sol. Distance (s) = $(2.5)t^2$

$$\text{Speed (v)} \frac{ds}{dt} = \frac{d}{dt}\{(2.5)t^2\}$$

v = 5t

At t = 5, v = 5 × 5 = 25 m/s.

41. A

Sol. $\vec{F} = -e\left(\vec{v}\times\vec{B}\right)$

Force will be along – ve y-axis.

As magnetic force is $\perp$ to velocity, path of electron must be a circle.

42. C

Sol. $g = \frac{GM}{R^2} = \frac{4}{3}\pi G\rho R$

$$\therefore \frac{g_2}{g_1} = \frac{\rho_2}{\rho_1}\times\frac{R_2}{R_1} = \frac{1}{2}\times 1.5 = \frac{3}{4}$$

43. A

Sol. For resonance, ϕ = 0, hence both inductor & capacitor must be present. Also power factor is zero for pure inductor or pure capacitor hence both the component consume zero average power.

44. A

Sol. $F_c = m\omega^2 r = 200 \times (0.2)^2 \times 70 = 560$ N

45. B

Sol. Minimum length of antenna should be $\lambda/4$

46. A

Sol. UV rays have maximum frequency hence are most effective for emission of electrons from a metallic surface.

$KE_{max.} = hf - hf_0$

47. B

Sol. Divide q = 10 μC into two parts x & q – x.

$$F = \frac{Kx(q-x)}{r^2}$$

For F to be maximum

$$\frac{dF}{dx} = \frac{K}{r^2}(q-2x) = 0$$

$$x = \frac{q}{2}$$

48. A

Sol. X and $\frac{a}{Y^2}$ have same dimensions

Y and b have same dimensions

$\therefore$ [a] = $[ML^5T^{-2}]$

[b] = $[L^3]$

$\frac{[a]}{[b]} = [ML^2T^{-2}]$ has dimensions of energy

49. B

Sol. Given that $\frac{A_1}{A_2} = \frac{2}{1}$

$$\frac{I_{max}}{I_{min}} = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2 = \frac{9}{1} = 9:1$$

50. B

Sol. Mean free path

$$\lambda = \frac{RT}{\sqrt{2}\pi d^2 N_A P}$$

$\lambda \propto T$

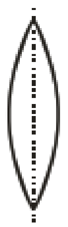
$$\frac{1500d}{\lambda} = \frac{273}{373}$$

λ = 2049 d

Section - B (Numerical Value)

51. 5

Sol.



Let power of each part is P_1, then

$P_1 + P_1 = P = 1/f$

$2P_1 = 1/0.1 = 10$

$P_1 = 5D$

52. 4125

Sol. $E = E_1 - E_2 = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$

$$= 6.6\times10^{-34} \times 3\times10^8\left(\frac{1}{500\times10^{-9}} - \frac{1}{600\times10^{-9}}\right)$$

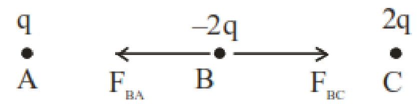
$= 6.6 \times 10^{-20}$ J

$$= \frac{6.6\times10^{-20}}{1.6\times10^{-19}} eV = 4.125\times10^{-1} eV$$

$= 4125 \times 10^{-4}$ eV

53. 5440

Sol.



$$F_{BA} = \frac{Kq(2q)}{\left(\frac{3}{4}R\right)^2} = \frac{32Kq^2}{9R^2}$$

$$F_{BC} = \frac{K(2q)(2q)}{\left(\frac{R}{4}\right)^2} = \frac{64Kq^2}{R^2}$$

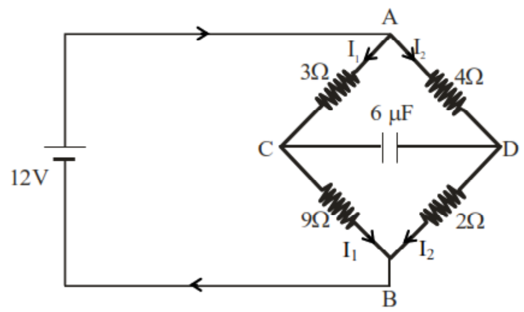
$$F_B = F_{BC} - f_{BA} = \frac{544Kq^2}{9R^2}$$

$$= \frac{544\times9\times10^9\times(2\times10^{-6})^2}{9\times(2\times10^{-2})^2} = 5440N$$

54. 75

Sol. $I_1 = \frac{12}{3+9} = 1A$

$I_2 = \frac{12}{4+2} = 2A$



$V_A - V_C = 3I_1 = 3V$ ....(1)

$V_A - V_D = 2 \times 4 = 8V$ ....(2)

Subtracting eq. (1) from eq. (2)

$V_C - V_D = 5V \Rightarrow V = 5V$

$U = \frac{1}{2}CV^2 = \frac{1}{2}\times 6\times 5^2 = 75\mu J$

55. 60

Sol. $\Delta Q = -\frac{\Delta\phi}{R} = -\left(\frac{\phi_2-\phi_1}{R}\right)$

$\phi_1 = NBA$

$\phi_2 = -NBA$

$\therefore \Delta Q = \frac{2NBA}{R} = \frac{2\times100\times1.5\times24\times10^{-4}}{12}$

$= 6 \times 10^{-2}$ C = 60 mC

56. 500

Sol. $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$ (T : Tension)

$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}}$

$\left(\frac{50}{30}\right)^2 = \frac{mg}{180g} \Rightarrow m = \frac{25}{9}\times180 = 500$ gram

57. 15

Sol. $\tau = I\alpha$

$\Rightarrow$ $FR = mR^2\alpha$

$\alpha = \frac{F}{mR} = \frac{52.5}{5\times0.7} = 15$ rad s^{-2}

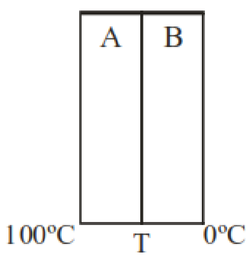
58. 3

Sol. $E_1 = \frac{1}{2}mu^2 - 0 = \frac{1}{2}mu^2 = E$

$E_2 = \frac{1}{2}m(2u)^2 - \frac{1}{2}mu^2 = \frac{3}{2}mu^2 = 3E$

59. 40

Sol.



Let the temperature of contact surface is T, then

$H_A = H_B$

$\frac{K_A A(T_A - T)}{L} = \frac{K_B A(T - T_B)}{L}$

84(100 – T) = 126 (T – 0)

2(100 – T) = 3T

200 – 2T = 3T

T = 40°C

60. 2

Sol. For equilibrium

Mg = I*l*B

$I = \frac{mg}{lB} = \frac{40\times10^{-3}\times10}{50\times10^{-2}\times0.4} = 2A$

CHEMISTRY

Section - A (Single Correct Answer)

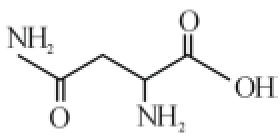
61. B

Sol. In wet testing, $(NH_4)_2CO_3$ is used as group reagent for 5th group cations (Ba^{2+}, Ca^{2+}, Sr^{2+}).

$Ba^{+2} + (NH_4)_2CO_3 \rightarrow \underset{\text{(white precipitate)}}{BaCO_3\downarrow} + NH_4^{\oplus}$

62. C

Sol. Asparagine has only one basic functional group in its chemical structure.



Others are basic amino acid with more than one basic functional group.

63. D

Sol. Statement I is correct, Ellingham diagram can be constructed for formation of oxides, sulphides and halides of metals. (Ref : NCERT)

Statement II is incorrect because Ellingham diagram consists of $\Delta_f G°$ vs T for formation of oxides of elements.

64. C

Sol. Tyndall effect is observed only when the following two conditions are satisfied

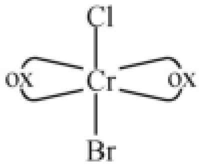
(a) The diameter of the dispersed particle is not much smaller than the wave length of light used.

(b) Refractive indices of dispersed phase and dispersion medium differ greatly in magnitude.

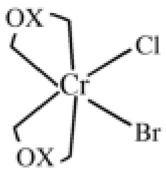
65. B

Sol. $[Cr(Ox)_2ClBr]^{-3}$

- No. of isomers -



- This structure has plane of symmetry, So no optical isomerism will be shown.



- This structure does not contain plane of symmetry, So two forms d as well as 1 will be shown.

66. A

Sol. As per NCERT (s block), the better method of preparation of BeF_2 is heating $(NH_4)_2BeF_4$.

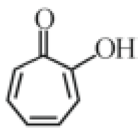
$$(NH_4)_2BeF_4 \xrightarrow{\Delta} BeF_2 + NH_4F$$

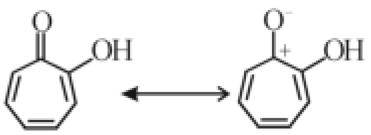
67. B

Sol. Source NCERT

Since the isotopes have the same electronic configuration, they have almost same chemical properties. The only difference is in their rates of reactions, mainly due to their different enthalpy of bond dissociation.

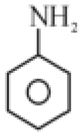
68. B

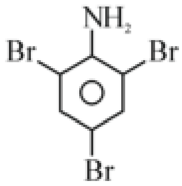
Sol.  Tropolone is an aromatic compound and has 8π electrons ($6\pi e^-$ are endocyclic and $2\pi e^-$ are exocyclic) and π electrons of $>C=O$ group in tropolone is not involved in aromaticity.

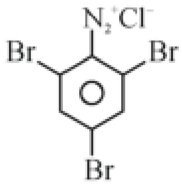


aromatic compound ($6\pi e^-$)

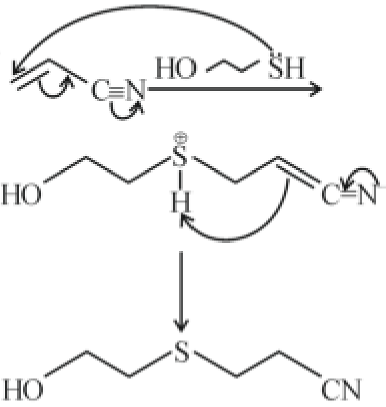
69. D

Sol. A =  .

B = 

C = 

70. A

Sol. 

71. D

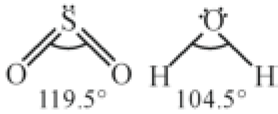
Sol. Green house gases are CO_2, CH_4, water vapour, nitrous oxide, CFC_s and ozone.

72. B

Sol.

- Hexamethylenediamine on reaction with adipic acid forms Nylon 6, 6 which shows H-bonding due to presence of amide group.
- $AlEt_3 + TiCl_4$ is Ziegler-Natta catalyst used to prepare high density polyethylene.
- 2-chloro-1, 3-butadiene (chloroprene) is monomer of neoprene which is a rubber (an elastomer)
- Phenol - formaldehyde forms Bakelite which is heavily branched (cross-linked) polymer

73. B

Sol. 

Both are bent in shape.

Bond angle of SO_2 (sp^2) is greater than that of H_2O (sp^3) due to higher repulsion of multiple bonds.

74. D

Sol. Only I^- among halides can be oxidised to iodine by oxygen in acidic medium.

$$4I^-_{(aq)} + 4H^+_{(aq)} + O_{2(g)} \rightarrow 2I_{2(S)} + 2H_2O_{(\ell)}$$

75. A

Sol. Adiabatic boundary does not allow heat exchange thus heat generated in container can't escape out thereby increasing the temperature.

In case of Diathermic container, heat flow can occur to maintain the constant temperature.

76. C

Sol. Acidic strength α – I effect

$$\alpha \frac{1}{+I} \text{ effect}$$

F, Cl exerts –I effect, Methyl exerts +I effect, C is least acidic.

Among A and B; since inductive effect is distance dependent, Extent of –I effect is higher in A followed by B even though F is stronger electron withdrawing group than Cl. Thus, A is more acidic than B.

77. A

Sol. For a given metal $\Delta_f H°$ always becomes less negative from fluoride to iodide.

78. B

Sol.

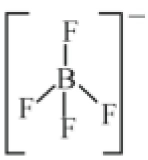
$$CH_3-CH_2-CH_2-Br + KOH(Alc) \rightarrow \underset{(Alkene)}{CH_3-CH=CH_2}$$

$$CH_3-CH_2-CH_2-Br + KCN\ (Alc) \rightarrow \underset{(Nitrile)}{CH_3-CH_2-CH_2-CN}$$

$$CH_3-CH_2-CH_2-Br + AgNO_2 \rightarrow \underset{(Nitroalkane)}{CH_3-CH_2-CH_2-NO_2} + AgBr\downarrow$$

$$CH_3-CH_2-CH_2-Br + CH_3-COOAg \rightarrow \underset{(Ester)}{CH_3-COO-CH_2-CH_2-CH_3} + AgBr\downarrow$$

79. A

Sol. 

Number of covalent bond formed by Boron is 4

Oxidation number of fluorine is –1,

Oxidation number of B + 4 × (–1) = –1,

Thus, Oxidation number of B = +3

80. B

Sol. Complex with maximum number of unpaired electron will exhibit maximum attraction to an applied magnetic field.

$[Zn(H_2O)_6]^{2+} \rightarrow d^{10}$ system $\rightarrow t_{2g}^6\ eg^4$, 0 unpaired e^-

$[Co(H_2O)_6]^{2+} \rightarrow d^7$ system $\rightarrow t_{2g}^5\ eg^2$, 3 unpaired e^-

$[Co(en)_3]^{3+} \rightarrow d^6$ system $\rightarrow t_{2g}^6\ eg^0$, 0 unpaired e^-

$[Ni(H_2O_6]^{2+} \rightarrow d^8$ system $\rightarrow t_{2g}^6\ eg^2$, 2 unpaired e^-

Section - B (Numerical Value)

81. 40

Sol. Mole of AgBr = $\frac{0.376}{188}$

Mole of Br^- = Mole of AgBr = $\frac{0.376}{188}$

Mass of Br^- = $\frac{0.376}{188} \times 80$

% of Br^- = $\frac{0.376 \times 80}{188 \times 0.4} \times 100 = 40\%$

82. 100

Sol. $\underset{1gm}{M_2CO_3} + \underset{Excess}{2HCl} \rightarrow \underset{0.02\ mole}{2MCl} + H_2O + \underset{0.01\ mole}{CO_2}$

From principle of atomic conservation of carbon atom,

Mole of M_2CO_3 × 1 = Mole of CO_2 × 1

$$\frac{1\ gm}{\text{Molar mass of } M_2CO_3} = 0.01 \times 1$$

$\therefore$ Molar mass of M_2CO_3 = 100 gm/mole

83. 23

Sol.

$$Cr_2O_7^{2-} + 14H^+ + 6Fe^{2+} \rightarrow 6Fe^{3+} + 2Cr^{3+} + 7H_2O$$

x = 14

y = 2

z = 7

Hence (x + y + z) = 14 + 2 + 7 = 23

84. 17

Sol. Formula of borax is $Na_2B_4O_5(OH)_4 \cdot 8H_2O$

85. 25

Sol. $Cu(OH)_2(s) \rightleftharpoons Cu^{2+}(aq) + 2OH^-(aq)$

$K_{sp} = [Cu^{2+}][OH^-]^2$

$pH = 14;\ pOH = 0;\ [OH^-] = 1\ M$

$\therefore\ [Cu^{2+}] = \frac{K_{sp}}{[1]^2} = 10^{-20} M$

$Cu^{2+}(aq) + 2e^- \rightarrow Cu(s)$

$E = E^0 - \frac{0.059}{2}\log_{10}\frac{1}{[Cu^{2+}]}$

$= 0.34 - \frac{0.059}{2}\log_{10}\frac{1}{10^{-20}}$

$= -0.25 = -25 \times 10^{-2}$

86. 458

Sol. $CH_3COOH + NaOH \rightarrow CH_3COONa + H_2O$

	CH_3COOH	$NaOH$	CH_3COONa	H_2O
Initially	5mmol	2mmol	0	0
after Rxn	3mmol	0	2 mmole	2 mmole

$pH = pKa + \log_{10}\frac{[salt]}{[acid]}$

$pH = 4.76 + \log_{10}\frac{2}{3}$

$pH = 4.58 = 458 \times 10^{-2}$

87. 2200

Sol. $t_{\frac{1}{2}} = 10$ minutes

$(P_A)_{30\,min.} = (P_A)_0\left(\frac{1}{2}\right)^{30/10}$

$(P_A)_{30\,min.} = 100$ mm Hg

	$A(g) \rightarrow$	$2B(g)$ +	$C(g)$
at t = 0	800mm	0	0
at t = 30	100mm	1400mm	700mm

Total pressure after 30 minutes = 2200 mm Hg.

88. 0

Sol. Orbital angular momentum = $\sqrt{1(\ell+1)}\frac{h}{2\pi}$

Value of ℓ for s = 0

89. 17

Sol. $\sqrt{3}a = 4r$

$\sqrt{3} \times 4 = 4r$

$r = 1.732$ Å

$= 17.32 \times 10^{-1}$

90. 116

Sol. Amount of solvent= 100 – (29.25 + 19)

= 51.75 g

$\Delta T_b = \left[\frac{2 \times 29.25 \times 1000}{58.5 \times 51.75} + \frac{3 \times 19 \times 1000}{95 \times 51.75}\right] \times 0.52$

$\Delta T_b = 16.075$

$\Delta T_b = (T_b)_{solution} - (T_b)_{solvent}$

$(T_b)_{solution} = 100 + 16.07$

$= 116.07^0 C$

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