13-April-2023 (Morning Batch) : JEE Main Paper

MATHEMATICS

Section - A (Single Correct Answer)

1. B

Sol. $1=\int_0^\infty \frac{6}{(e^x+1)(e^x+2)(e^x+3)}dx$

$=6\int_0^\infty \left(\frac{\frac{1}{2}}{e^x+1}+\frac{-1}{e^x+2}+\frac{\frac{1}{2}}{e^x+3}\right)dx$

$=3\int_0^\infty \frac{e^{-x}}{1+e^{-x}}dx-6\int_0^\infty \frac{e^{-x}dx}{1+2e^{-x}}+3\int_0^\infty \frac{e^{-x}}{1+3e^{-x}}dx$

$=3[-\ln(1+e^{-x})]_0^\infty+6\frac{1}{2}[\ln(1+2e^{-x})]_0^\infty$

$-\frac{3}{3}[\ln(1+3e^{-x})]_0^\infty$

= 3 ln 2 – 3 ln 3 + ln 4

$=3\ln\frac{2}{3}+\ln 4$

$=\ln\frac{32}{27}$

2. A

Sol. $f(x)=x-\sin 2x+\frac{1}{3}\sin 3x$

f '(x) = 1 – 2 cos 2x + cos 3x = 0

$x=\frac{5\pi}{6},\ \frac{\pi}{6}$

$\therefore f''(x)=4\sin 2x-3\sin 3x$

$f''\left(\frac{5\pi}{6}\right)<0$

$\Rightarrow \left(\frac{5\pi}{6}\right)$ is point of maxima

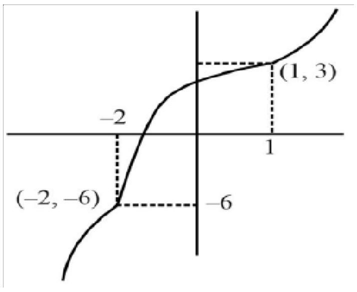
$f\left(\frac{5\pi}{6}\right)=\frac{5\pi}{6}+\frac{\sqrt{3}}{2}+\frac{1}{3}$

3. B

Sol. f(x) = x|x – 1| + |x + 2|

x|x – 1| + |x + 2| + a = 0

x|x – 1| + |x + 2| = –a



All values are increasing

4. A

Sol. $p:((A\wedge(B\vee C))\Rightarrow(A\vee B))\Rightarrow A$

$[\sim(A\wedge(B\vee C))\vee(A\vee B)]\Rightarrow A$

$[(A\wedge(B\vee C))\wedge\sim(A\vee B)]\vee A$

$(f\vee A)=A$

$\sim p\equiv\sim A$

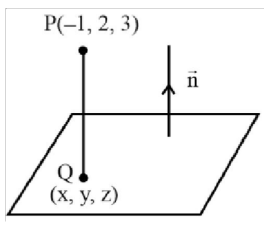
5. C

Sol. Let $L_1:\vec{r}=(\hat{i}-\hat{j})+\lambda(2\hat{i}+\hat{k})$

$L_2:\vec{r}=(2\hat{i}-\hat{j})+\mu(\hat{i}-\hat{j}+\hat{k})$

$\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 0 & 1\\ 1 & -1 & 1\end{vmatrix}$

$\vec{n}=\hat{i}-\hat{j}-2\hat{k}$



Equation of line along shortest distance of L_1 and L_2

$$\frac{x+1}{1}=\frac{y-2}{-1}=\frac{z-3}{-2}=r$$

$\Rightarrow (x,\ y,\ z)\equiv(r-1,\ 2-r,\ 3-2r)$

$\Rightarrow$ (r – 1) –2(2 – r) + 3(3 – 2r) = 10

$\Rightarrow$ r = –2

$\Rightarrow Q(x,\ y,\ z)\equiv(-3,\ 4,\ 7)$

$\Rightarrow PQ=\sqrt{4+4+16}=2\sqrt{6}$

6. A

Sol. $P(H)=\frac{3}{4}$

$P(T)=\frac{1}{4}$

x	1	2	3
P(X)	$\frac{3}{4}$	$\frac{1}{4}\times\frac{3}{4}$	$\left(\frac{1}{4}\right)^3+\left(\frac{1}{4}\right)^2\times\frac{3}{4}$

Mean $\bar{X}=\frac{3}{4}+\frac{3}{8}+3\left(\frac{1}{64}+\frac{3}{64}\right)$

$=\frac{3}{4}+\frac{3}{8}+\frac{3}{16}$

$=3\left(\frac{7}{16}\right)$

$=\frac{21}{16}$

7. A

Sol. $\Delta=\begin{vmatrix}2 & 4 & 2a\\ 1 & 2 & 3\\ 2 & -5 & 2\end{vmatrix}=18(3-a)$

$\Delta_x=\begin{vmatrix}b & 4 & 2a\\ 4 & 2 & 3\\ 8 & -5 & 2\end{vmatrix}=(64+19b-72a)$

For unique solution $\Delta=0$

$\Rightarrow a\neq 3$ and $b\in R$

For infinitely many solution :

$\Delta=\Delta_x=\Delta_y=\Delta_z=0$

$\Rightarrow a=3\ \because \Delta=0$

and b = 8 $\because \Delta_x=0$

8. D

Sol. $\left[3f(x)+2f\left(\frac{1}{x}\right)=\frac{1}{x}-10\right]\times 3$

$\left[2f(x)+3f\left(\frac{1}{x}\right)=x-10\right]\times 2$

$5f(x)=\frac{3}{x}-2x-10$

$f(x)=\frac{1}{5}\left(\frac{3}{x}-2x-10\right)$

$f'(x)=\frac{1}{5}\left(-\frac{3}{x^2}-2\right)$

$\left|f(3)+f'\left(\frac{1}{4}\right)\right|=\left|\frac{1}{5}(1-6-10)+\frac{1}{5}(-48-2)\right|$

|–3 – 10| = 13

9. B

Sol. Given ellipse $\frac{x^2}{36}+\frac{y^2}{4}=1$

$\frac{x}{4\sqrt{3}}+\frac{y}{4}=1$

y = 4

$\frac{x}{4}-\frac{4}{4\sqrt{3}}=\frac{2}{\sqrt{3}}$

y = –8

$x^2 + y^2 + 4y – 32 = 0$

hx + ky + 2(y + k) – 32 = 0

k = –2

hx + 2k – 32 = 0

hx = 36

$\alpha=h=\frac{36}{2\sqrt{5}}$

$\beta = k = -2$

$\alpha^2 - \beta^2 = \frac{304}{5}$

10. D

Sol. $\text{Area} = \left|\int_{-\pi}^{\frac{-3\pi}{4}} \sin x dx\right| + \left|\int_{\frac{-3\pi}{4}}^{\frac{-\pi}{2}} \cos x \ dx\right| + \int_{\frac{-\pi}{2}}^{\frac{\pi}{4}} \cos x \ dx + \int_{\frac{\pi}{4}}^{\pi} \sin x dx$

$= 4$

11. A

Sol. $A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$, a, b, c, d, e, f $\in \{0, 1, 2, ...9\}$

Number of matrices = 10^6

12. A

Sol. $S_1 : \lim_{n\to\infty} \frac{n(n+1)}{n^2} = 1 \Rightarrow \text{True}$

$S_2 : \lim_{n\to\infty} \frac{1}{16}\left(\sum r^{15}\right) = \lim_{n\to\infty} \frac{1}{n} \sum \left(\frac{r}{n}\right)^{15}$

$= \int_0^1 x^{15} dx = \frac{1}{16} \Rightarrow \text{True}$

13. A

Sol. $9\left(t + \frac{1}{t}\right)^2 = 100$

$t = 3$

$\Rightarrow$ P(81, 54) & Q(1, –6)

M(21, 9)

$\Rightarrow$ L is $y - 9(= \frac{-4}{3}(x - 21)$

3y – 27 = –4x + 84

4x + 3y = 111

14. C

Sol. $g(x) = \sqrt{x} + 1$

$fog(x) = x + 3 - \sqrt{x}$

$= (\sqrt{x} + 1)^2 - 3(\sqrt{x} + 1) + 5$

$= g^2(x) - 3g(x) + 5$

$\Rightarrow f(x) = x^2 - 3x + 5$

$\therefore f(0) = 5$

But, if we consider the domain of the composite function fog(x) then in that case f(0) will be not defined as g(x) cannot be equal to zero.

15. B

Sol. $\left\{\frac{4^{2022}}{15}\right\} = \left\{\frac{2^{4044}}{15}\right\}$

$= \left\{\frac{(1+15)^{1011}}{15}\right\}$

$= \frac{1}{15}$

16. A

Sol. $\vec{d} \times \vec{b} = \vec{c} \times \vec{b}$

$\Rightarrow (\vec{d} - \vec{c}) \times \vec{b} = 0$

$\Rightarrow \vec{d} = \vec{c} + \lambda\vec{b}$

Also $\vec{d} \cdot \vec{a} = 24$

$\Rightarrow (\vec{c} + \lambda\vec{b}) \cdot \vec{a} = 24$

$\lambda = \frac{24 - \vec{a}\cdot\vec{c}}{\vec{b}\cdot\vec{a}} = \frac{24 - 6}{9} = 2$

$\Rightarrow \vec{d} = \vec{c} + 2(\vec{b})$

$= 8\hat{i} - 5\hat{j} + 18\hat{k}$

$\Rightarrow \left|\vec{d}\right|^2 = 64 + 25 + 324 = 413$

17. C

Sol. Given, $B = \begin{bmatrix} 1 & 3 & \alpha \\ 1 & 2 & 3 \\ \alpha & \alpha & 4 \end{bmatrix}$

|B| = 4

$1(8 - 3\alpha) - 3(4 - 3\alpha) + \alpha(\alpha - 2\alpha) = 4$

$-\alpha^2 + 6\alpha - 8 = 0$

$\alpha = 2, 4$

Given, $\alpha > 2$

So, $\alpha = 2$ is rejected

$$[4\ -8\ 4]\begin{bmatrix}1 & 3 & 4\\ 1 & 2 & 3\\ 4 & 4 & 4\end{bmatrix}\begin{bmatrix}4\\ -8\\ 4\end{bmatrix}=[-16]_{1\times1}$$

18. D

Sol. $S_k = 6(2k + (11)(2k - 1))$

$S_k = 6(2k + 22k - 11)$

$S_k = 144k - 66$

$$\sum_{1}^{10} S_k = 144\sum_{k=1}^{10} k - 66\times10$$

$$=144\times\frac{10\times11}{2}-660$$

$= 7920 - 660$

$= 7260$

19. B

Sol. $\frac{dy}{dx}=y+7\Rightarrow\frac{dy}{dx}-y=7$

I.F. $= e^{-x}$

$ye^{-x}=\int 7e^{-x}dx$

$\Rightarrow ye^{-x}=-7e^{-x}+c$

$\Rightarrow y=-7+ce^{x}$

$-7 + 7e^x = -7 + 8e^x$

$\Rightarrow e^x = 0$

No solution

20. C

Sol. $(x+2y+az-2)+\lambda(x-y+z-3)=0$

$$\frac{1+\lambda}{5}=\frac{2-\lambda}{-11}=\frac{a+\lambda}{b}=\frac{2+3\lambda}{6a-1}$$

$\lambda=-\frac{7}{2},\ a=3,\ b=1$

$$\frac{2}{\sqrt{a}}=\left|\frac{5a+11c+bc-6a+1}{\sqrt{25+121+1}}\right|$$

$c = -1$

$$\therefore \frac{a+b}{c}=\frac{3+1}{-1}=-4$$

Section - B (Numerical Value)

21. 36

Sol. $T_{k+1} = {}^nC_k(x)^{\frac{n-k}{2}}(-6)^k(x)^{\frac{-3}{2}k}$

$$\frac{n-k}{2}-\frac{3}{2}k=0$$

$n - 4k = 0$

$$(-5)^n-\left({}^nC_k(-6)^{\frac{n}{4}}\right)=640$$

By observation (625 + 24 = 649), we get n = 4

$\because n = 4\ \&\ k = 1$

Required is coefficient of x^{-4} is $\left(\sqrt{4}-\frac{6}{x^{\frac{3}{2}}}\right)^4$

${}^4C_1(-6)^3$

By calculating we will get $\lambda = 36$

22. 4

Sol. $$\sin^{-1}\left(\frac{(x+1)}{\sqrt{(x+1)^2+1}}\right)-\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right)=\frac{\pi}{4}$$

$$\because \frac{t}{\sqrt{t^2+1}}\in(-1,\ 1)$$

$$\sin^{-1}\left(\frac{(x+1)}{\sqrt{(x+1)^2+1}}\right)=\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right)+\frac{\pi}{4}$$

$$\frac{(x+1)}{\sqrt{(x+1)^2+1}}=\left(\frac{1}{\sqrt{2}}\right)\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right)\right)+\frac{1}{\sqrt{2}}\left(\frac{x}{\sqrt{x^2+1}}\right)$$

$$=\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{x^2+1}}+\frac{x}{\sqrt{x^2+1}}\right)$$

$$\frac{(x+1)}{\sqrt{(x+1)^2+1}}=\frac{1}{\sqrt{2}}\left(\frac{1+x}{\sqrt{x^2+1}}\right)$$

After solving this equation, we get

x = –1 or x = 0

S = {–1, 0}

$$\sum_{x\in R}\left(\sin(x^2+x+5)\frac{\pi}{2}\right)-\cos((x^2+x+5)\pi))$$

$$=\left[\sin\left(\frac{5\pi}{2}\right)-\cos(5\pi)\right]+\left[\sin\left(\frac{5\pi}{2}\right)-\cos(5\pi)\right]$$

= 4

23. 24

Sol. $\omega = z\bar{z}+k_1z+k_2iz+\lambda(1+i)$

$$\text{Re}(w)=x^2+y^2+k_1x-k_2y+\lambda=0$$

$$\text{Centre}\equiv\left(\frac{-k_1}{2},\ \frac{k_2}{2}\right)\equiv(1,\ 2)$$

$\Rightarrow k_1=-2,\ k_2=4$

radius $=1\Rightarrow\lambda=4$

$\text{Im}=k_1y+k_2x+\lambda=0$

$\therefore 2x-y+2=0$

$$d=\frac{2}{\sqrt{5}}$$

$$\frac{1^2}{4}=1-\frac{4}{5}=\frac{1}{5}$$

$\therefore 301^2=24$

24. 18

Sol. Given, $S_k(x)=C_kx+k\int_0^x S_{k-1}(t)dt,$

Put k = 2 and x = 3

$S_2(3)=C_2(3)+2\int_0^3 S_1(t)dt$ ....(1)

Also,

$$S_1(x)=C_1(x)+\int_0^x S_0(t)dt$$

$$=C_1x+\frac{x^2}{2}$$

$$S_3(3)=3C_2+2\int_0^3\left(C_1t+\frac{t^2}{2}\right)dt$$

$= 3C_2 + 9C_1 + 9$

Also,

$$C_1=1-\int_0^1 S_0(x)dx=\frac{1}{2}$$

$$C_2=1-\int_0^1 S_1(x)dx=0$$

$$C_3=1-\int_0^1 S_2(x)dx$$

$$=1-\int_0^1\left(C_2x+C_1x^2+\frac{x^3}{3}\right)dx$$

$$=\frac{3}{4}$$

$$S_2(x)=C_2x+2\int_0^x S_1(t)dt$$

$$=C_2x+C_1x^2+\frac{x^3}{3}$$

$\Rightarrow S_2(3) + 6C_3 = 6C_3 + 3C_2 + 9C_1 + 9$

= 18

25. 1310

Sol. $(2^2 - 3^2 + 4^2 - 5^2 + 20 \text{ terms}) +$

$(2^2 + 4^2 + ... + 10 \text{ terms})$

$-(2 + 3 + 4 + 5 + ...+ 11) + 4[1 + 2^2 + ...10^2]$

$$-\left[\frac{21\times22}{2}-1\right]+4\times\frac{10\times11\times21}{6}$$

$= 1 - 231 + 14 \times 11 \times 10$

$= 1540 + 1 - 231$

= 1310

26. 413

Sol. $x_1+x_2+x_3+x_4+x_5+x_6+x_7=12,\ x_i\in\{1, 2, 3, 4\}$

No. of solutions $= {}^{5+7-1}C_{7-1}-\frac{7!}{6!}-\frac{7!}{5!}=413$

27. 8

Sol. Equation of tangent to the hyperbola $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$

$$y=mx\pm\sqrt{a^2-b^2m^2}$$

passing through (4, 1)

$$1=4m\pm\sqrt{25-16m^2}\Rightarrow 4m^2-m-3=0$$

$$\Rightarrow m=1,\ \frac{-3}{4}$$

Equation of tangent with positive slopes $1\ \&\ \frac{3}{4}$

$$\left.\begin{array}{l}4y=3x-16\\ y=x-3\end{array}\right\}$$ with positive intercept on x-axis.

$$\alpha=\frac{16}{3},\ \beta=3$$

Intersection points :

Q : (–4, –7)

P : (4, 1)

$PQ^2 = 128$

$$\frac{PQ^2}{\alpha\beta}=\frac{128}{16}=8$$

28. 15

Sol. Image of point $\left(\frac{5}{3},\ \frac{5}{3},\ \frac{8}{3}\right)$

$$\frac{x-\frac{5}{3}}{1}=\frac{y-\frac{5}{3}}{-2}=\frac{z-\frac{8}{3}}{1}=\frac{-2\left(1\times\frac{5}{3}+(-2)\times\frac{8}{3}+1\times\frac{8}{3}-2\right)}{1^2+2^2+1^2}$$

$\therefore$ x = 2, y = 1, z = 3

$$13^2=(6-2)^2+(-2-1)^2+(\alpha-3)^2$$

$$\Rightarrow(\alpha-3)^2=144\Rightarrow\alpha=15\ (\because\alpha>0)$$

29. 66

Sol. $|\vec{a}|=\sqrt{11},\ |\vec{c}|=\sqrt{22}$

$$|\vec{a}|=|\vec{b}\times\vec{c}|=|\vec{b}||\vec{c}|\sin\theta$$

$$\sqrt{11}=\sqrt{50}\sqrt{22}\sin\theta$$

$$\Rightarrow\sin\theta=\frac{1}{10}$$

$$|\vec{b}+\vec{c}|^2=|\vec{b}|^2+|\vec{c}|^2+2\vec{b}\cdot\vec{c}$$

$$=|\vec{b}|^2+|\vec{c}|^2+2|\vec{b}||\vec{c}|\cos\theta$$

$$=50+22+2\times\sqrt{50}\times\sqrt{22}\times\frac{\sqrt{99}}{10}$$

= 72 + 66

$$\left|72-|\vec{b}+\vec{c}|^2\right|=66$$

30. 8

Sol. $5=\bar{x}=\frac{\sum x_i f_i}{\sum f_i}=\frac{4+72+140+7\alpha+72}{64+\alpha}$

$$\Rightarrow 320+5\alpha=288+7\alpha\Rightarrow 2\alpha=32\Rightarrow\alpha=16$$

$$\text{M.D.}(\bar{x})=\frac{\sum f_i|x_i-\bar{x}|}{\sum f_i}\ \text{where}\ \sum f_i=64+16=80$$

$$\text{M.D.}(\bar{x})=\frac{4\times4+24+2+28\times0+16\times2+8\times4}{80}$$

$$=\frac{8}{5}$$

$$\text{Variance}=\frac{\sum f_i(x_i-\bar{x})^2}{\sum f_i}$$

$$=\frac{4\times16+24\times4+0+16\times4+8\times16}{80}=\frac{352}{80}$$

$$\therefore\frac{3\alpha}{m+\sigma^2}=\frac{3\times16}{\frac{128}{80}+\frac{352}{80}}=8$$

PHYSICS

Section - A (Single Correct Answer)

31. C

Sol. Based on equations of Maxwell

32. D

Sol. $P = I^2 R$

$$R_1=\frac{3R}{2},R_2=\frac{2R}{3},R_3=\frac{R}{3},R_4=3R$$

Since i is same, hence P α R so options (D) is correct

33. B

Sol. Formula used $d_{app}=\frac{d_1}{n_1}+\frac{d_2}{n_2}$

$$d_{app}=\frac{d}{2}\left[\frac{n_1+n_2}{n_1n_2}\right]$$

34. A

Sol. Source of time varying magnetic field may be

→ accelerated or retarded charge which produces varying electric and magnetic fields.

→ An electric field varying linearly with time will not produce variable magnetic field as current will be constant

35. C

Sol. $\frac{60l+4l}{20}-\frac{61l}{30}=35$

$\Rightarrow l=\frac{1050}{35}$

$\Rightarrow L=60l=\frac{1050}{35}\times 60=1800\text{ m}$

36. D

Sol. Average Power $=\frac{\text{total work done}}{\text{total time}}$

So $P=\frac{mgh}{t}$

$\frac{P_1}{P_2}=\frac{\frac{m_1gh}{t_1}}{\frac{m_2gh}{t_2}}=\frac{m_1}{t_1}\frac{t_2}{m_2}$

$\frac{P_1}{P_2}=\frac{300\times 2}{5\times 50}=\frac{12}{5}=\frac{3\sqrt{x}}{\sqrt{x}+1}$

$12\sqrt{x}+12=15\sqrt{x}$

$3\sqrt{x}=12$

x = 16

37. B

Sol. $V_p=\sqrt{\frac{2GM_P}{R_p}}V_E=\sqrt{\frac{2GM_E}{R_E}}$

$\frac{V_P}{V_E}=\frac{\sqrt{\frac{2GM_P}{R_P}}}{\sqrt{\frac{2GM_E}{R_E}}}=\sqrt{\frac{R_E}{R_P}\times\frac{M_P}{M_E}}$

$V_P=\sqrt{\frac{1}{4}\times 9}\times V_E=\frac{3}{2}V_E$

$V_P=\frac{3}{2}\times 11.2\text{ km/sec}$

= 16.8 km/ sec

38. B

Sol. $\lambda_A=\left(\frac{1242}{9}\right)=138\text{nm}$

$\lambda_B=\left(\frac{1242}{4.5}\right)=276\text{nm}$

$\lambda_B-\lambda_A=138\text{ nm}$

39. B

Sol. By momentum conservation

0 = 3(– v) + 0.01(600 – v)

$v\simeq 2$ m/s

Impulse on gun = 3 × 2 = 6 Ns

40. B

Sol. $|\vec{P}|=qd$

$=0.01\times 0.4\times 10^{-3}=4\times 10^{-6}$

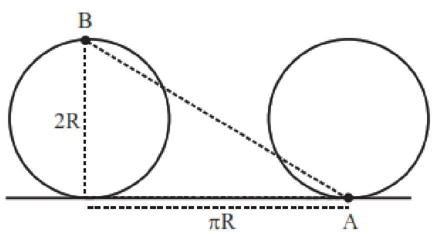
$|\vec{\tau}|=PE\sin\theta$

$=4\times 10^{-6}\times 10\times 10^{-5}\times\sin 30$

$=4\times 10^{-6-5+1}\times\frac{1}{2}=2\times 10^{-10}$

41. A

Sol.



Displacement $=\sqrt{(2R)^2+(\pi R)^2}=R\sqrt{4+\pi^2}$

42. D

Sol. Based on Theory

43. A

Sol. $\sqrt{\frac{3RT}{M}}=\left(1+\frac{5}{x}\right)^{\frac{1}{2}}\sqrt{\frac{8RT}{\pi M}}$

$\Rightarrow\frac{3\times 22}{7\times 8}=1+\frac{5}{x}$

$\Rightarrow\quad x=28$

44. A

Sol. $k=\frac{1}{2}mv^2$

$$k=\frac{1}{2}\times 5\times 400=5\times 200=1000J$$

$$\frac{\Delta k}{2k}=\frac{\Delta m}{m}+\frac{2\Delta v}{v}=\frac{0.5}{5}+\frac{2\times 0.4}{20}$$

$$\Delta k=1000\left(\frac{1}{10}+\frac{4}{100}\right)=1000\left(\frac{10+4}{100}\right)=140J$$

45. D

Sol. $\frac{K_1}{K_2}=\frac{p_1^2}{2m_1}\times\frac{2m_2}{p_2^2}=\frac{m_2}{m_1}=\frac{16}{9}$

46. A

Sol. From continuity theorem $A_1V_1 = A_2V_2$

$1.5 \times V_1 = 25 \times 10^{-2} \times 60$

$$V_1=\frac{25\times 60\times 10^{-2}\times 10}{1.5}$$

V_1 = 10 cm/s

By Bernoulli's theorem

$$P_1+\frac{1}{2}\times 10000\times(0.1)^2=P_2+\frac{1}{2}\times 1000\times(0.6)^2$$

$$P_1+5=P_2+\frac{1}{2}\times 1000\times 36\times 10^{-2}$$

$P_1 + 5 = P_2 + 180$

$P_1 - P_2 = 175$ Pa

47. B

Sol. $B=-\frac{dP}{dv/v}$

$Pv^3 = a$

Differentiating w.r.t to pressure

$$v^3+P3v^2\frac{dv}{dP}=0$$

$$v=-3\frac{Pdv}{dP}=0$$

$$\frac{dP\cdot v}{dv}=-3P$$

$$B=-\left(\frac{dPv}{dv}\right)=-(-3P)=3P$$

48. D

Sol. $Y=\overline{\overline{A}+B}=A+\overline{B}$

49. D

Sol. T.E. – P.E. = K.E.

$$K.E.=\frac{1}{2}m\omega^2\left(A^2-x^2\right)$$

Which is the equation of downward parabola.

50. D

Sol. $Q = (m_A - m_B - m_D) \times 931.5$ MeV

$= (238.05079 - 234.04363 - 4.00260) \times 931.5$

$\Rightarrow$ 4.25Mev

Section - B (Numerical Value)

51. 40

Sol. Energy per unit volume = $\frac{1}{2}$ stress × strain

Energy = $\frac{1}{2}$ stress × strain × volume

$$80=\frac{1}{2}\times Y\times strain^2 A\times l$$

$$80=\frac{1}{2}\times 2\times 10^{11}\times\frac{\left(2\times 10^{-2}\right)^2}{400}\times A\times 20$$

$$20=\frac{10^{+7}}{20}\times A$$

40×10^{-6} m^2 = A

A = 40 mm

52. 16

Sol. $P=VI=I^2R=\frac{V^2}{R}$

Now $R=\frac{\rho l}{A}$

If wire is cut in two equal half

$$R'=\frac{R}{2}$$

Initial $P_1 = \frac{V_0^2}{R}$

After $P_2 = \frac{V_0^2}{R'} \times 2 \Rightarrow \frac{V_0^2}{R} \times 4$

$\frac{P_2}{P_1} = 4 = \frac{\sqrt{x}}{1}$

x = 16

53. D

Sol. General equation for displacement is given by

x = Asin(ωt + ϕ)

at given time

$\Rightarrow \quad \omega t + \phi = 30°$

$\Rightarrow x = 40 \times \frac{\sqrt{3}}{2} \Rightarrow 20\sqrt{3}$ cm

$\Rightarrow \quad A = 40$ cm

$\Rightarrow K.E. = \frac{1}{2}k\left(A^2 - x^2\right) = 200$

$200 = \frac{1}{2}k\left(\frac{1600-1200}{100 \times 100}\right)$

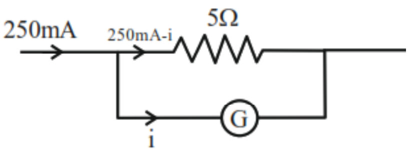
400 × 100 × 100 = k × 400

$k = 10^4$

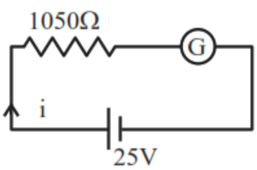
x = 4

54. 50

Sol.



$\frac{250mA \times 5}{5 + R_G} = i \quad$ .....(i)



$i = \frac{25}{1050 + R_G} \quad$ .....(ii)

From (i) and (ii)

$\frac{25}{1050 + R_G} = \frac{5}{4(5 + R_G)}$

$100(5 + R_G) = 1050 \times 5 \times R_G \times 5$

$95 R_G = 4750$

$R_G = 50\ \Omega$

55. 2

Sol.

$r \propto \frac{n^2}{z}$

$\frac{r_{He^+}}{r_{Be^{3+}}} = \frac{2^2 \times 4}{2 \times 4 \times 4} = \frac{1}{2}$

56. 6

Sol. $E_A = \frac{\lambda}{2\pi\varepsilon_0 r_A} - \frac{\sigma}{2\varepsilon_0}\left\{r_A = \frac{3}{\pi}\right\}$

$= \frac{1}{2\varepsilon_0}\left[\frac{\lambda}{3} - \sigma\right]$

$E_B = \frac{\lambda}{2\pi\varepsilon_0 r_A} - \frac{\sigma}{2\varepsilon_0}\left\{r_B = \frac{4}{\pi}\right\}$

$= \frac{1}{2\varepsilon_0}\left[\frac{\lambda}{4} - \sigma\right]$

$\frac{E_A}{E_B} = \frac{4}{3}\left(\frac{\lambda - 3\sigma}{\lambda - 4\sigma}\right)$

$= \frac{4}{3}\left[\frac{2\sigma - 3\sigma}{2\sigma - 4\sigma}\right] = \frac{4}{3}\left[\frac{-\sigma}{-2\sigma}\right] = \frac{4}{6}$

57. 25

Sol. $E = \frac{1}{2}LI^2$

Rate of energy storing $= \frac{dE}{dt} = LI\frac{dI}{dt}$

Now we Know for R – L circuit

$I = \frac{E}{R}\left(1 - e^{-t\frac{R}{L}}\right)$

So $\frac{dI}{dt} = \frac{E}{L}e^{-\frac{tR}{L}}$

$$\frac{dE}{dt}=\frac{E^2}{R}\left(1-e^{-\frac{tR}{L}}\right)\left(e^{-t\frac{R}{L}}\right)$$

Time at which rate of power storing will be max,

$$t=\frac{L}{R\ln 2}$$

So $\frac{dE}{dt}=\frac{E^2}{R}\left(1-\frac{1}{2}\right)\times\frac{1}{2}$

a = 2, b = 50

So $\frac{b}{a}=25$

58. 3

Sol. $\frac{V_{b/f}}{\frac{4}{3}}=\frac{-8}{\frac{4}{3}}+\frac{(-v)}{1}$

$$\Rightarrow \frac{-12}{\frac{4}{3}}=\frac{-8}{\frac{4}{3}}+\frac{(-v)}{1}$$

$\Rightarrow$ v = 3m/ s

59. D

Sol. $L=(I_{com})(\omega)$ and $K=\frac{1}{2}(I_{com})(\omega^2)+\frac{1}{2}MV_{com}^2$

$$L=\frac{2}{5}MR^2\frac{V_{com}}{R}\quad K=\frac{1}{2}\left(\frac{2}{5}MR^2\right)\frac{V_{com}^2}{R^2}+\frac{1}{2}MV_{com}^2$$

$$L=\frac{2MRV_{com}}{5}\quad K=\frac{7}{10}MV_{com}^2$$

Ratio $\frac{L}{K}=\frac{4}{7}\frac{R}{V_{com}}=\frac{\pi}{22}\Rightarrow\omega=\frac{4}{7}\times\frac{22}{22}\times7=4$

60. 3

Sol. Power gain

$$\Rightarrow A_v\cdot A_1=B\frac{R_C}{R_B}\cdot B=B^2\frac{R_C}{R_B}$$

$$=\left(\frac{(20-10)\times10^{-3}}{(200-100)\times10^{-6}}\right)\times\frac{1\times10^3}{10\times10^3}=10^3$$

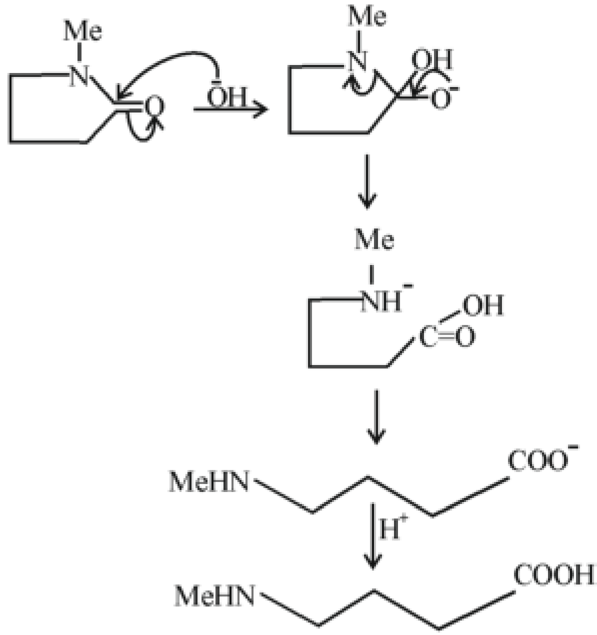
Hence x = 3

CHEMISTRY

Section - A (Single Correct Answer)

61. A

Sol.



62. D

Sol. Nowadays hard water is softened by using synthetic ion exchangers. This method is more efficient than zeolite process/Permutit process.

63. A

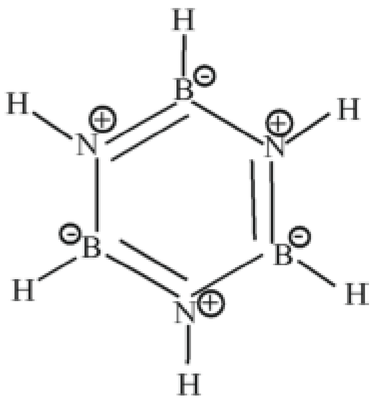
Sol. Mg is present in chlorophyll and in black and white photography the developed film is fixed by washing with hypo solution which dissolves the undecomposed AgBr to form a complex ion $[Ag(S_2O_3)_2]^{3-}$.

64. B

Sol. NO is paramagnetic with BO = 2.5, NO^+ is diamagnetic with BO = 3.

65. C

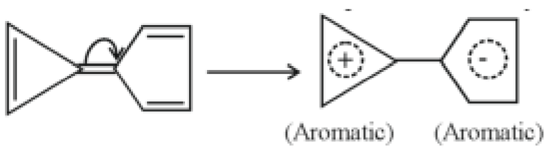
Sol. Borazine is $B_3N_3H_6$



$B_3N_3H_6 + 9H_2O \rightarrow 3NH_3 + 3H_3BO_3 + 3H_2$

66. A

Sol. Among the given compounds, the following compound has the highest dipole moment because both the +ve and –ve ends acquire aromaticity.



67. D

Sol. Nylon-6 – Caprolactum (Monomer)

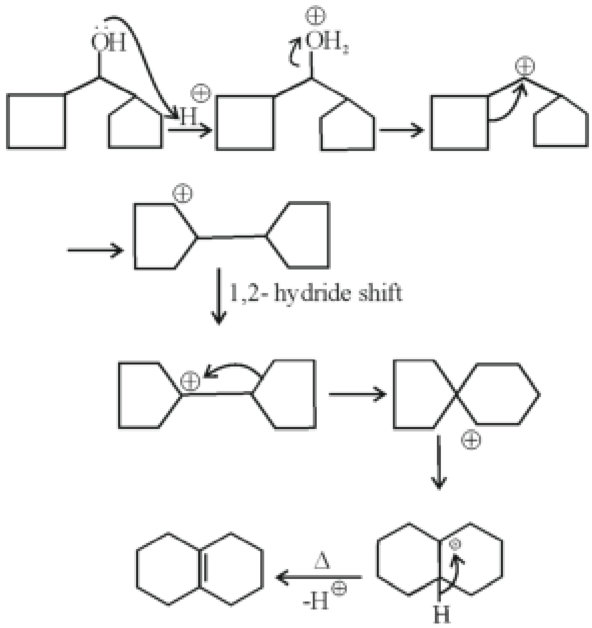
Natural rubber – Isoprene (Monomer)

Vulcanized rubber – Sulphur containing rubber

Neoprene – Chloroprene (Monomer)

68. A

Sol.



69. C

Sol. $O_2(g) \xrightarrow{UV} O(g) + O(g)$

$O_2(g) + O(g) \longrightarrow O_3(g)$

$CF_2Cl_2(g) \xrightarrow{UV} \dot{C}l(g) + \dot{C}F_2Cl(g)$

$\dot{C}l(g) + O_3(g) \longrightarrow Cl\dot{O}(g) + O_2(g)$

$Cl\dot{O}(g) + O(g) \longrightarrow \dot{C}l(g) + O_2(g)$

70. C

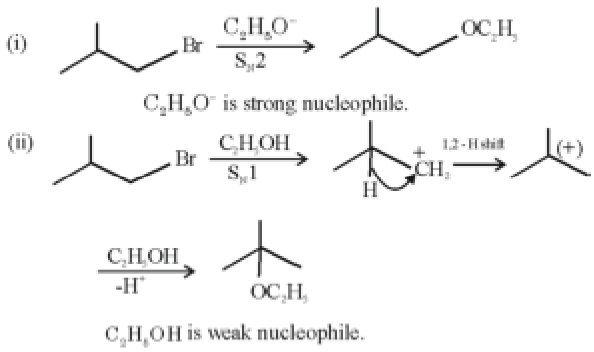
Sol. n-alkanes on heating in this presence of anhydrous $AlCl_3$ and hydrogen chloride gas isomerise to branched chain alkanes. The major product has one methyl side chain.

$$CH_3-(CH_2)_4-CH_3 \xrightarrow[HCl,\Delta]{Anhy.AlCl_3} CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-(CH_2)_2-CH_3$$

2 – methylpentane (major)

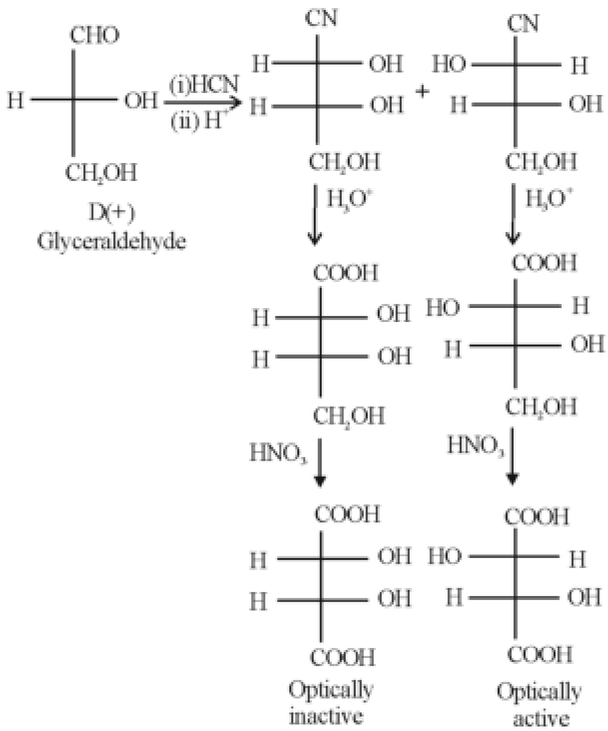
71. A

Sol.



72. B

Sol.

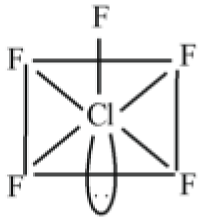


73. A

Sol. Zinc is refined by distillation method, which is used for metals having low boiling point.

74. C

Sol.



ClF_6 is colourless liquid.

75. D

Sol. $Be(OH)_2$ is amphoteric in nature.

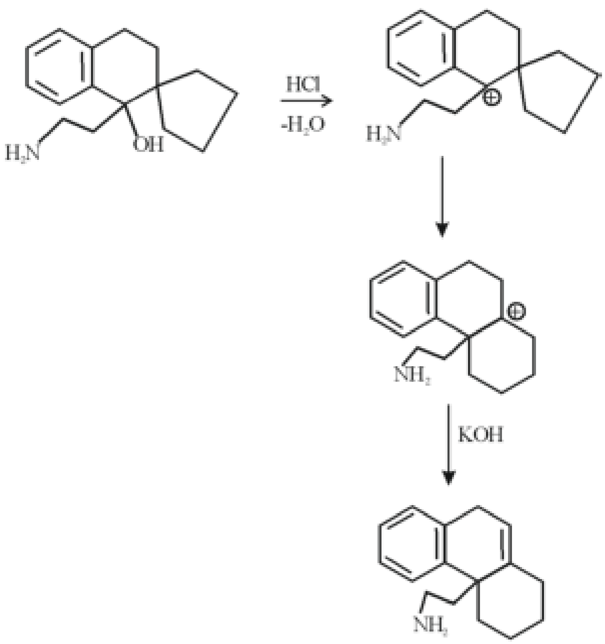
$Sr(OH)_2$ is basic in nature.

These two undergo acid - base reaction to form a salt.

$$Be(OH)_2 + Sr(OH)_2 \rightarrow \underset{\text{(salt)}}{Sr[Be(OH)_4]}$$

76. C

Sol.



77. B

Sol. Electronegativity of an element depends on the atom with which it is attached.

NO = neutral oxide

Al_2O_3 = amphoteric oxide

78. D

Sol. $$E_n = \frac{-2.18\times10^{-18}\ Z^2}{n^2}$$

i.e., $E_n \propto \frac{1}{n^2}$

79. D

Sol. Lyophilic sol is used as protective action for lyophobic **Sol.** It fomis a layer / film around the lyophobic **Sol.**

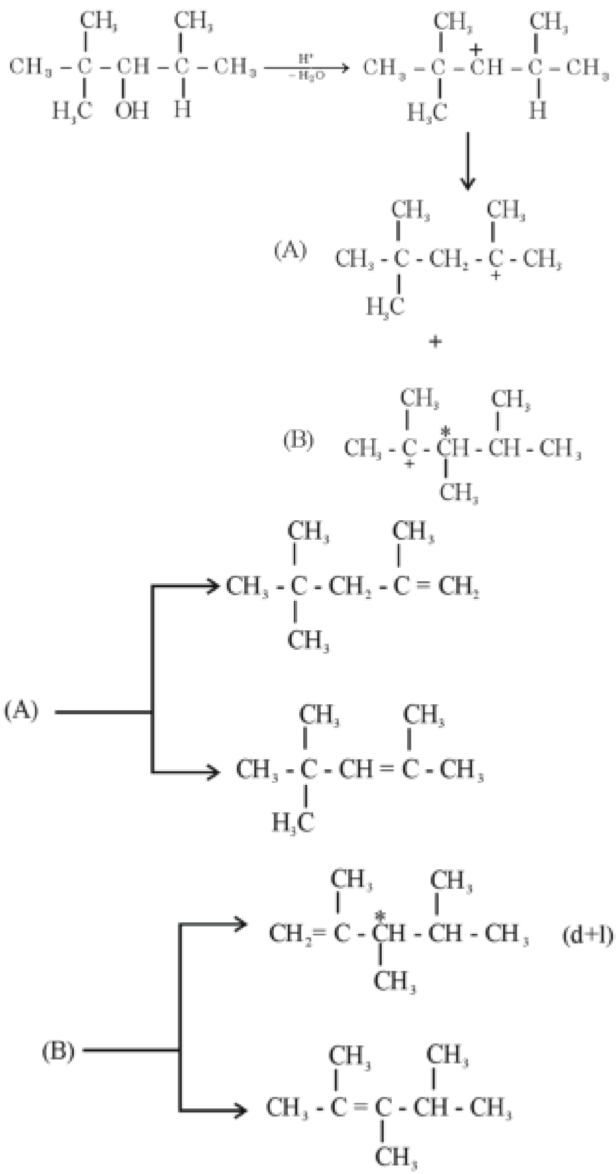
80. B

Sol. $\left.\begin{array}{l} Eu^{+2}:[Xe]4f^{7} \\ Yb^{+2}:[Xe]4f^{14} \end{array}\right\}$ High IE due to half filled & fully filled configurations

Section - B (Numerical Value)

81. 4

Sol.



82. 240

Sol. For Isotonic solutions

$\pi_1 = \pi_2$

$\Rightarrow C_1 = C_2$

$\frac{12}{x} = 0.05$ [x $\rightarrow$ Molar mass of A]

$X = 240$

83. 5

Sol. $[Ba^{+2}] = \frac{25 \times 0.05}{50} = 0.025$ M

$[F^-] = \frac{25 \times 0.2}{50} = 0.01$ M

$[Ba^{+2}][F^-]^2 = 25 \times 10^{-7}$

$K_{sp} = 5 \times 10^{-7}$ (given)

$\text{Ratio} = \frac{[Ba^{+2}][F^-]^2}{K_{sp}} = 5$

84. 400

Sol. $A_2 + B_2 \rightarrow 2AB$; $\Delta H_f^0 = -200$ kJ mol^{-1}

$\Rightarrow \Delta H_f^0$ (AB) = –200 kJ mol^{-1}

$\therefore \Delta H_R^0$ for reaction

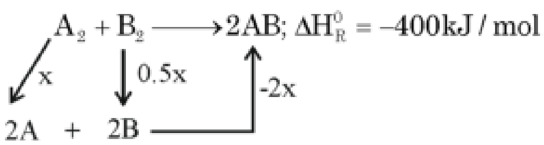
$A_2 + B_2 \rightarrow 2AB$ is –400 kJ mol^{-1}

Given : Bond Enthalpy of A_2, B_2 and AB is 1 : 0.5 : 1

Assuming bond enthalpy of A_2 be x kJ mol^{-1}

$\therefore$ Bond enthalpy B_2 = 0.5 x kJ mol^{-1}

$\therefore$ Bond enthalpy AB = (x) kJ mol^{-1}



$-400 = x + 0.5x - 2x$

$-400 = -0.5x$

$\therefore$ x = 800 kJ/mol

85. 56

Sol. Moles of $CO_2 = \frac{0.22}{44} = \frac{1}{200}$

$\therefore$ Moles of carbon

$= (\text{Moles of } CO_2) \times 1$

$= \frac{1}{200}$

$\therefore$ wt. of C = $\frac{1}{200} \times 12 = 0.06$

(W = wt. of Organic Compound)

W = 0.25

Moles of $H_2O = \frac{0.126}{18} = 0.007$

$\therefore$ Moles of H atom = 2 × 0.007 = 0.014

% of Hydrogen $= \frac{0.014 \times 1}{W} \times 100$

$= \frac{0.014 \times 1}{0.25} \times 100$

$= 5.6$

$= 56 \times 10^{-1}$

86. 1

Sol. Reaction with HCl

$Ca(OH)_2 + 2HCl \rightarrow CaCl_2 + 2H_2O$

Volume of $Ca(OH)_2$ = 10 ml

Volume of HCl = 20 ml

Concentration of HCl = 0.5 M.

No. of milli moles of HCl = 10

No. of milli moles of $Ca(OH)_2$ = 5

i.e., $M_{Ca(OH)_2} = \frac{\text{no. of milli moles}}{V(ml)} = \frac{5}{10}$

= 0.5 M.

Reaction with H_2SO_4

$Ca(OH)_2 + H_2SO_4 \rightarrow CaSO_4 + 2H_2O$

No. of milli moles of $Ca(OH)_2$ = 20 × 0.5 = 10

i.e., no. of milli moles of H_2SO_4 = 10

$\Rightarrow M_{H_2SO_4} = \frac{\text{no. of millimoles}}{V(ml)}$

$= \frac{10}{10}$

$= 1$ M

87. 392

Sol. $z = \frac{PV}{nRT}$; $n = \frac{PV}{ZRT}$

$Z_1 = 1.07, P_1 = 100$ atm, $V_1 = 0.15$ L, $T_1 = 500K$

$Z_2 = 1.4, P_2 = 300$ atm, $T_2 = 300$ K, $V_2 = ?$

$$\frac{P_1V_1}{Z_1RT_1} = \frac{P_2V_2}{Z_2RT_2} = n$$

$$V_2 = \frac{1.4}{1.07} \times .03$$

$$= 392 \times 10^{-4}\,dm^3$$

88. 3

Sol. $A_t = A_0 \times \frac{12.5}{100} = \frac{A_0}{8}$ [87.5% complete]

$$A_0 \xrightarrow{t_{1/2}} \frac{A_0}{2} \xrightarrow{t_{1/2}} \frac{A_0}{4} \xrightarrow{t_{1/2}} \frac{A_0}{8}$$

$t_{87.5} = 3t_{1/2}$

$\therefore$ x = 3

89. 68

Sol. By balancing the redox reaction we get,

$$10\left[FeSO_4.(NH_4)_2SO_4.6H_2O\right] + 2KMnO_4 + 8H_2SO_4$$

$$\downarrow$$

$$5Fe_2(SO_4)_3 + 2MnSO_4 + 10(NH_4)_2SO_4 + K_2SO_4 + 68H_2O$$

90. 161

Sol. $W = z \times i \times t$

$$\text{Density} \times \text{Volume} = \frac{E \times i \times t}{96500}$$

$$10 \times 100 \times 0.0001 = \frac{\left(\frac{\text{at. wt.}}{\text{v.f.}}\right) \times 2 \times x}{96500} \text{(v.f.} = 2)$$

$\therefore$ x = 161 sec.

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