3S INSTITUTE
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12-April-2023 (Morning Batch) : JEE Main Paper

MATHEMATICS

Section - A (Single Correct Answer)

1. A

Sol. 7 x x x 5
9 x x x 0
9 x x x 5

So Required numbers = $5 \times {}^4P_3 = 120$

2. C

Sol. $\alpha,\ \beta = \frac{-\sqrt{6} \pm \sqrt{6-12}}{2} = \frac{-\sqrt{6} \pm \sqrt{6}i}{2}$

$= \sqrt{3}e^{\pm\frac{3\pi i}{4}}$

Required expression

$$= \frac{(\sqrt{3})^{23}\left(2\cos\frac{69\pi}{4}\right) + (\sqrt{3})^{14}\left(2\cos\frac{42\pi}{4}\right)}{(\sqrt{3})^{15}\left(2\cos\frac{45\pi}{4}\right) + (\sqrt{3})^{10}\left(2\cos\frac{30\pi}{4}\right)}$$

$(\sqrt{3})^8 = 81$

3. B

Sol. $a_n = S_n - S_{n-1} = \frac{n^2+3n}{(n+1)(n+2)} - \frac{(n-1)(n+2)}{n(n+1)}$

$\Rightarrow a_n = \frac{4}{n(n+1)(n+2)}$

$\Rightarrow 28\sum_{k=1}^{10}\frac{1}{a_k} = 28\sum_{k=1}^{10}\frac{k(k+1)(k+2)}{4}$

$= \frac{7}{4}\sum_{k=1}^{10}(k(k+1)(k+2)(k+3) - (k-1)k(k+1)(k+2))$

$= \frac{7}{4}\cdot 10.11.12.13 = 2.3.5.7.11.13$

So m = 6

4. C

Sol. $(3x+2y+z-2) + \mu(x-3y+2z-13) = 0$

$3(3+\mu) + 1\cdot(2-3\mu) - 2(1+2\mu) = 0$

$9 - 4\mu = 0$

$\mu = \frac{9}{4}$

$4(-15-8+\alpha-2) + 9(-5+12+2\alpha-13) = 0$

$-100 + 4\alpha - 54 + 18\alpha = 0$

$\Rightarrow \alpha = 7$

Let $P \equiv (3\lambda-5,\ \lambda-4,\ -2\lambda+7)$

Direction ratio of PQ $(3\lambda-1,\ \lambda-1,\ -2\lambda+5)$

But $PQ \perp \ell_1$

$\Rightarrow 3(3\lambda-1) + 1\cdot(\lambda-1) - 2(-2\lambda+5) = 0$

$\Rightarrow \lambda = 1$

$P(-2, -3, 5) \Rightarrow |a| + |b| + |c| = 10$

5. C

Sol. Let $R(2\cos\theta,\ 3\sin\theta)$

as $OP \perp OR$

so $\frac{3\sin\theta}{2\cos\theta} \times \frac{\frac{6}{\sqrt{7}}}{\frac{2\sqrt{3}}{\sqrt{7}}} = -1$

$\Rightarrow \tan\theta = \frac{-2}{3\sqrt{3}}$

$\Rightarrow R\left(\frac{-6\sqrt{3}}{\sqrt{31}},\ \frac{6}{\sqrt{31}}\right)$ or $R\left(\frac{6\sqrt{3}}{\sqrt{31}},\ \frac{-6}{\sqrt{31}}\right)$

Now $= \frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{1}{4}\left(\frac{1}{(OP)^2} + \frac{1}{(OR)^2}\right)$

$= \frac{1}{4}\left(\frac{1}{\frac{48}{7}} + \frac{1}{\frac{144}{31}}\right) = \frac{1}{4}\left(\frac{7}{48} + \frac{31}{144}\right)$

$$= \frac{13}{144}$$

$$\Rightarrow p + q = 157$$

6. A

Sol. $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

$C_2 \to C_2 - C_1,\ C_3 \to C_3 - C_1$

$$\begin{vmatrix} a & 1-a & 1-a \\ 1 & b-1 & 0 \\ 1 & 0 & c-1 \end{vmatrix} = 0$$

a(b – 1)(c – 1) – (1 – a) (c – 1) + (1 – a)(1 – b) = 0

a(1 – b)(1 – c) + (1 – a) (1 – c) + (1 – a)(1 – b) = 0

$$\frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$$

$$\Rightarrow -1 + \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$$

$$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

7. C

Sol. Let $y = \left(\frac{\sqrt{3e}}{2\sin x}\right)^{\sin^2 x}$

$$\ln y = \sin^2 x \cdot \ln\left(\frac{\sqrt{3e}}{2\sin x}\right)$$

$$\frac{1}{y} y' = \ln\left(\frac{\sqrt{3e}}{2\sin x}\right) 2\sin x\cos x - \sin x\cos x = 0$$

$$\Rightarrow \sin x\cos x\left[2\ln\left(\frac{\sqrt{3e}}{2\sin x}\right) - 1\right] = 0$$

$$\Rightarrow \ln\left(\frac{3e}{4\sin^2 x}\right) = 1 \Rightarrow \frac{3e}{4\sin^2 x} = e \Rightarrow \sin^2 x = \frac{3}{4}$$

$$\Rightarrow \sin x - \frac{\sqrt{3}}{2} \left(\text{as } x \in \left(0, \frac{\pi}{2}\right)\right)$$

$$\Rightarrow \text{local max value } = \left(\frac{\sqrt{3e}}{\sqrt{3}}\right)^{3/4} = e^{3/8} = \frac{k}{e}$$

$$\Rightarrow k^8 = e^{11}$$

$$\Rightarrow \left(\frac{k}{e}\right) + \frac{k^8}{e^5} + k^8 = e^3 + e^6 + e^{11}$$

8. B

Sol. $\frac{6 + 2\log_3 x}{-5x} > 0 \ \&\ x > 0 \ \&\ x \neq \frac{1}{3}$

this gives $x \in \left(0, \frac{1}{27}\right)$ ....(1)

$$-1 \le \log_{3x}\left(\frac{6 + 2\log_3 x}{-5x}\right) \le 1$$

$$3x \le \frac{6 + 2\log_3 x}{-5x} \le \frac{1}{3x}$$

$15x^2 + 6 + 2\log_3 x \ge 0$ $6 + 2\log_3 x + \frac{5}{3} \ge 0$

$x \in \left(0, \frac{1}{27}\right)$ ....(2)

$x \ge 3^{-\frac{23}{6}}$ ....(3)

from (1), (2) & (3)

$$x \in \left[3^{-\frac{23}{6}}, \frac{1}{27}\right)$$

$\therefore \alpha$ is small positive quantity

$$\&\ \beta = \frac{1}{27}$$

$\therefore \alpha^2 + \frac{5}{\beta}$ is just greater than 135

9. A

Sol. $(1 + x^2)dy = y(x - y)dx$

$y(0)=1,\ y(2\sqrt{2})=\beta$

$$\frac{dy}{dx}=\frac{yx-y^2}{1+x^2}$$

$$\frac{dy}{dx}+y\left(\frac{-x}{1+x^2}\right)=\left(\frac{-1}{1+x^2}\right)y^2$$

$$\frac{1}{y^2}\frac{dy}{dx}+\frac{1}{y}\left(\frac{-x}{1+x^2}\right)=\frac{-1}{1+x^2}$$

put $\frac{1}{y}=t$ then $\frac{-1}{y^2}\frac{dy}{dx}=\frac{dt}{dx}$

$$\frac{dt}{dx}+t\frac{x}{1+x^2}=\frac{1}{1+x^2}$$

$$I.F=e^{\int\frac{x}{1+x^2}dx}=e^{\frac{1}{2}\ln(1+x^2)}=\sqrt{1+x^2}$$

$$t\sqrt{1+x^2}=\int\frac{1}{\sqrt{1+x^2}}dx$$

$$\frac{\sqrt{1+x^2}}{y}=\ln(x+\sqrt{x^2+1})+c$$

$y(0) = 1 \qquad \Rightarrow c=1$

$$\Rightarrow\sqrt{1+x^2}=y\ln(e(x+\sqrt{x^2+1}))$$

$$\beta=\frac{3}{\ln(e(3+2\sqrt{2}))}\Rightarrow\frac{3}{\beta}=\ln(e(3+2\sqrt{2}))$$

$$e^{\frac{3}{\beta}}=e(3+2\sqrt{2})$$

10. D

Sol. $S_1:(p\to q)\wedge(p\wedge(\sim q))$

p	q	$p\to q$	$p\vee(\sim q)$	S1
T	T	T	F	T
T	F	F	T	F
F	T	T	F	F
F	F	T	F	F

$\Rightarrow S_1$ is Contradiction

S_2

p	q	$p\wedge q$	$(\sim p\wedge q)$	$(p\wedge\sim q)$	$(\sim p)\wedge(\sim q)$	S_2
T	T	T	F	F	F	T
T	F	F	F	T	F	T
F	T	F	T	F	F	T
F	F	F	F	F	T	T

S_2 is tautology

11. B

Sol. $(\vec{a}+\vec{b}+\vec{c})\times\vec{c}=0$

$(\vec{a}+\vec{b})\times\vec{c}=0$

$\vec{c}=\alpha(\vec{a}+\vec{b})=\alpha(\lambda+3)\hat{i}+\alpha\hat{k}$

$\vec{b}\cdot\vec{c}=-20\Rightarrow 3\alpha(\lambda+3)+2\alpha=-20$

$\vec{a}\cdot\vec{c}=-17\Rightarrow\alpha\lambda(\lambda+3)-\alpha=-17$

$\Rightarrow\alpha(3\lambda+9+2)=-20$

$\alpha(\lambda^2+3\lambda-1)=-17$

$17(3\lambda+11)=20(\lambda^2+3\lambda-1)$

$20\lambda^2+9\lambda-207=0$

$\lambda=3 \qquad (\lambda\in Z)$

$\Rightarrow\alpha=-1 \qquad \Rightarrow\vec{c}=-(6\hat{i}+\hat{k})$

$\vec{v}=\vec{c}\times(3\hat{i}+\hat{j}+\hat{k})$

$$=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-6&0&-1\\3&1&1\end{vmatrix}=\hat{i}+3\hat{j}-6\hat{k}$$

$|\vec{v}|^2=(-1)^2+3^2+6^2=46$

12. C

Sol. $(1-x)^{100}=Co-C_1x+C_2x^2-C_3x^3+\ldots.C_{99}x^{99}+C_{100}x^{100}$

$\Rightarrow Co-C_1+C_2+C_3+\ldots..-C_{99}+C_{100}=0$

$2(Co-C_1+C_2+\ldots.-C_9)+C_{50}=0$

$$C_0-C_1+C_2+\ldots.C_{99}=-\frac{1}{2}\,{}^{100}C_{50}$$

$$-\frac{1}{2}\frac{100!}{50!50!}=-\frac{1}{2}\times\frac{100\times99!}{50!50!}=-\,{}^{99}C_{49}$$

13. A

Sol. equation of tangent : y + 1 = 3 (x + 1)

i.e. y = 3x + 2

Point of intersection with curve (2, 8)

So Area $= \int_{-1}^{2}((3x+2)-x^3)dx = \frac{27}{4}$

14. A

Sol. Let $C = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}, D = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

$$DC = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1$$

B = CAD

$B^n DA^n D$ ....(1)

$$A^2 = \begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{2}{51} \\ 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & \frac{3}{51} \\ 0 & 1 \end{bmatrix}$$

similarly $A^n = \begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}$

$$B^n = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{n}{51}+2 \\ -1 & -\frac{n}{51}-1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{n}{51}+1 & \frac{n}{51} \\ -\frac{n}{51} & 1-\frac{n}{51} \end{bmatrix}$$

$$\sum_{n=1}^{50} B^n = \begin{bmatrix} 25+50 & 25 \\ -25 & -25+50 \end{bmatrix} = \begin{bmatrix} 75 & 25 \\ -25 & 25 \end{bmatrix}$$

Sum of the elements = 100

15. D

Sol. Let equation of new position is

$$(4x - y + z - 10) + \lambda(x + y - z - 4) = 0$$

$$4(4+\lambda) - 1\cdot(-1+\lambda) + 1\cdot(1-\lambda) = 0$$

$$\Rightarrow \lambda = -9$$

So equation in new position is

–5x – 10y + 10z + 26 = 0

$$\Rightarrow \alpha = \frac{54}{15}$$

16. B

Sol. $\sum_{r=0}^{n} \frac{{}^nC_r}{r+1} = \frac{1}{n+1}\sum_{r=0}^{n} {}^{n+1}C_{r+1}$

$$= \frac{1}{n+1}(2^{n+1}-1) = \frac{1023}{10}$$

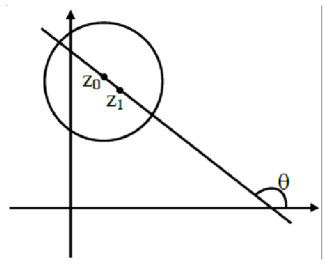
n + 1 = 10 $\Rightarrow$ n = 9

17. B

Sol. $|z_1 - z_0| = \left|\frac{1-i}{2}\right| = \frac{1}{\sqrt{2}}$

$\Rightarrow |z_2 - z_0| = \sqrt{2}$; centre $\left(\frac{1}{2}, \frac{3}{2}\right)$

$z_0\left(\frac{1}{2}, \frac{3}{2}\right)$ and $z_1(1, 1)$



$$\tan\theta = -1 \Rightarrow \theta = 135^\circ$$

$$z_2\left(\frac{1}{2} + \sqrt{2}\cos 135^\circ, \frac{3}{2} + \sqrt{2}\sin 135^\circ\right)$$

or

$$\left(\frac{1}{2} - \sqrt{2}\cos 135^\circ, \frac{3}{2} + \sqrt{2}\sin 135^\circ\right)$$

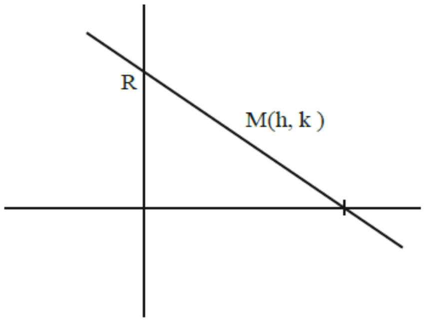
$$\Rightarrow z_2\left(-\frac{1}{2},\ \frac{5}{2}\right) \text{ or } z_2\left(\frac{3}{2},\ \frac{1}{2}\right)$$

$$\Rightarrow |z_2|^2 = \frac{26}{4},\ \frac{5}{2}$$

$$\Rightarrow |z_2|^2_{min} = \frac{5}{2}$$

18. A

Sol. pt $\left(\alpha,\ \frac{7\sqrt{3}}{3}\right)$



$$x\cos\theta + y\sin\theta = 7$$

x-intercept $= \frac{7}{\cos\theta}$

y-intercept $= \frac{7}{\sin\theta}$

$$A:\left(\frac{7}{\cos\theta},\ 0\right)\ B:\left(0,\ \frac{7}{\sin\theta}\right)$$

Locus of mid pt M : (h, k)

$$h = \frac{7}{2\cos\theta},\ k = \frac{7}{2\sin\theta}$$

$$\frac{7}{2\sin\theta} = \frac{7\sqrt{3}}{3} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$\alpha = \frac{7}{2\cos\theta} = 7$$

19. B

Sol.

$\alpha-\beta$	Case	P
5	(6, 1)	1/36
4	(6, 2) (5, 1)	2/36
3	(6, 3) (5, 2) (4, 1)	3/36
2	(6, 4) (5, 3) (4, 3) (3, 1)	4/36
1	(6, 5) (5, 4) (4, 3) (3, 2) (2, 1)	5/36
0	(6, 6), (5, 5) ..... (1, 1)	6/36
–1	....	5/36
–2	....	4/36
–3	....	3/36
–4	(2, 6) (1, 5)	2/36
–5	(1, 6)	1/36

$$\sum(x^2) = \sum x^2P(x) = 2\left[\frac{25}{36}+\frac{32}{36}+\frac{27}{36}+\frac{16}{36}+\frac{5}{36}\right]$$

$$= \frac{105}{18} = \frac{35}{6}$$

$\mu = \sum(x) = 0$ as data is symmetric

$$\sigma^2 = \sum(x^2) = \sum x^2P(x) = \frac{35}{6} \quad P = 35 = 5\times 7$$

Sum of divisors = $(5^0 + 5^1)(7^0 + 7^1) = 6 \times 8 = 48$

20. C

cos A + cos C = 2(1 – cos B)

$$2\cos\frac{A+C}{2}\cos\frac{A-C}{2} = 4\sin^2 B/2$$

$$\text{as } \cos\left(\frac{A+C}{2}\right) = \sin\frac{B}{2}$$

$$\text{so } \cos\frac{A-C}{2} = 2\sin\frac{B}{2}$$

$$2\cos B/2\cos\frac{A-C}{2} = 4\sin B/2\cos B/2$$

$$2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right) = 4\sin B/2\cos B/2$$

sin A + sin C = 2 sin B

a + c = 2b $\Rightarrow$ a = 3, c = 7, b = 5

$$\cos A - \cos C = \frac{b^2 + c^2 - a^2}{2bc} - \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{25 + 49 - 9}{70} - \frac{9 + 25 - 49}{30}$$

$$= \frac{65}{70} + \frac{1}{2} = \frac{20}{14} = \frac{10}{7}$$

Section - B (Numerical Value)

21. 10

Sol. Mean $= 1$, $\frac{n-1}{n} + 2\frac{1}{n}\left(\frac{n-1}{n}\right) + 3\left(\frac{1}{n}\right)^2 + \left(\frac{n-1}{n}\right)$

.....

$$\frac{n}{9} = \left(\frac{n-1}{n}\right)\left(1 + 2\left(\frac{1}{n}\right) + 3\left(\frac{1}{n}\right)^2 ...\right)$$

$$\frac{n}{9} = \left(\frac{n-1}{n}\right)\left(1 - \frac{1}{n}\right)^{-2} = \left(\frac{n-1}{n}\right) \cdot \frac{n^2}{(n-1)^2}$$

$$\frac{n}{9} = \frac{n}{n-1} \Rightarrow n = 10$$

22. 1260

Sol. abc or cba

a b c

\- - - - - -

c b a

$$\frac{{}^7C_1 \times 2 \times 6!}{2!2!2!} = 1260$$

23. 2

Sol. Need to check at doubtful points

discont at $x \in I$ only

at x = –1 $\Rightarrow$ f(–1$^+$) = 1 + 0 = 1

$\Rightarrow$ f(–1$^-$) = 2 + 1 = 3

at x = 0 $\Rightarrow$ f(0$^+$) = 0 + 0 = 0

$\Rightarrow$ f(0$^-$) = 1 + 1 = 2

at x = 1 $\Rightarrow$ f(1$^+$) = 1 + 0 = 1

$\Rightarrow$ f(1$^+$) = 0 + 1 = 1

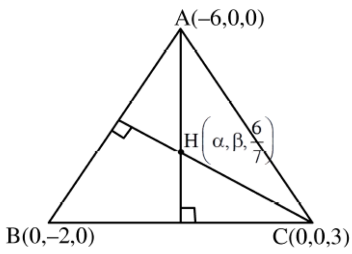
discont at two points

24. 288

Sol. A(–6, 0, 0) B(0, –2, 0) C = (0, 0, 3)

$\overrightarrow{AB} = 6\hat{i} - 2\hat{j}$, $\overrightarrow{BC} = 2\hat{j} + 3\hat{k}$,

$\overrightarrow{AC} = 6\hat{i} + 3\hat{k}$



$$\overrightarrow{AH} \cdot \overrightarrow{BC} = 0$$

$$\left(\alpha + 6,\ \beta,\ \frac{6}{7}\right) \cdot (0,\ 2,\ 3) = 0$$

$$\beta = \frac{-9}{7}$$

$$\overrightarrow{CH} \cdot \overrightarrow{AB} = 0$$

$$\left(\alpha,\ \beta,\ \frac{-15}{7}\right) \cdot (6,\ -2,\ 0) = 0$$

$$6\alpha - 2\beta = 0$$

$$\alpha = \frac{-3}{7}$$

$$98(\alpha + \beta)^2 = (98)\frac{(144)}{49} = 288$$

25. 64

Sol. $\int \sqrt{\frac{x+7}{x}}\, dx$

Put x = t^2

dx = 2t dt

$$\int 2\sqrt{t^2 + 7}\ dt = 2\int \sqrt{t^2 + \sqrt{7}^2}\, dt$$

$$I(t) = 2\left[\frac{t}{2}\sqrt{t^2 + 7} + \frac{7}{2}\ln\left|t + \sqrt{t^2 + 7}\right|\right] + C$$

$$I(x) = \sqrt{x}\sqrt{x+7} + 7\ln\left|\sqrt{x} + \sqrt{x+7}\right| + C$$

I(9) = 12 + 7 ln 7 = 12 7 (ln (3 + 4)) + C

$\Rightarrow C = 0$

$I(x) = \sqrt{x}\sqrt{x+7} + 7\ln(\sqrt{x} + \sqrt{x+7})$

$I(1) = 1\sqrt{8} + 7\ln(1+\sqrt{8})$

$I(1) = \sqrt{8} + 7\ln(1+2\sqrt{2})$

$\alpha = \sqrt{8}$

$\alpha^4 = (8^{1/2})^4$

$\alpha^4 = 8^2 = 64$

26. 6

Sol. $D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$

$\sum_{k=1}^{n} D_k = 96$

$\Rightarrow \begin{vmatrix} \sum_{k=1}^{n} 1 & \Sigma 2k & \Sigma(2k-1) \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix} = 96$

$\Rightarrow \begin{vmatrix} n & n^2+n & n^2 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix} = 96$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\begin{vmatrix} n & n^2+n & n^2 \\ 0 & 2 & 0 \\ 0 & 0 & n+2 \end{vmatrix} = 96$

$\Rightarrow$ n(2n + 4) = 96 $\Rightarrow$ n(n + 2) = 48 $\Rightarrow$ n = 6

27. 211

Sol. Let $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

Given $\frac{a}{r^2} + \frac{a}{r} + a + ar + ar^2 = 5 \times \frac{31}{10}$ ....(1)

and $\frac{r^2}{a} + \frac{r}{a} + \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} = 5 \times \frac{31}{40}$ ....(2)

(1) + (2) $a^2 = 4 \Rightarrow a = 2 \therefore r + \frac{1}{r} = 5/2$

$(a \neq -2)$

$\Rightarrow r = 2$

$\therefore$ Now $\frac{1}{2}$, 1, 2, 4, 8

$\therefore \sigma^2 = \frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2$

$= \frac{186}{25} = \frac{M}{N} \Rightarrow 211 = m + n$

28. 3.00

Sol. A = {1, 2, 3}

For Reflexive (1, 1) (2, 2), (3, 3) $\in$ R

For transitive : (1, 2) and (2, 3) $\in R \Rightarrow (1, 3) \in R$

Not symmetric : (2, 1) and (3, 2) $\notin$ R

R_1 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}

R_2 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3), (2, 1)}

R_3 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3), (3, 2)}

29. 575

Sol. $\int_{-0.15}^{0.15} |100x^2 - 1| dx = 2\int_{0}^{0.15} |100x^2 - 1| dx$

Now $100x^2 - 1 = 0 \Rightarrow x^2 = \frac{1}{100} \Rightarrow x = 0.1$

$I = 2\left[\int_0^{0.1} (1-100x^2)dx + \int_{0.1}^{0.15} (100x^2-1)dx\right]$

$I = 2\left[x - \frac{100}{3}x^3\right]_0^{0.1} + 2\left[\frac{100x^3}{3} - x\right]_{0.1}^{0.15}$

$= 2\left[0.1 - \frac{0.1}{3}\right] + 2\left[\frac{0.3375}{3} - 0.15 - \frac{0.1}{3} + 0.1\right]$

$= 2\left[0.2 - \frac{0.2}{3} + 0.1125 - 0.15\right]$

$$= 2\left[\frac{5}{100} - \frac{2}{30} + \frac{1125}{10000}\right] = 2\left(\frac{1500 - 2000 + 3375}{30000}\right)$$

$$= \frac{575}{3000} \Rightarrow k = 575$$

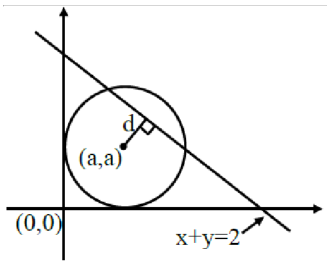
30. 7

Sol. Circle $(x - a)^2 + (y - a)^2 = a^2$

$x^2 + y^2 - 2ax - 2at + a^2 = 0$

intercept = 2

$$\Rightarrow 2\sqrt{a^2 - d^2} = 2$$



Where d = perpendicular distance of centre from line x + y = 2

$$\Rightarrow 2\sqrt{a^2 - \left(\frac{a + a - 2}{\sqrt{2}}\right)^2} = 2$$

$$\Rightarrow a^2 - \frac{(2a - 2)^2}{2} = 1 \Rightarrow 2a^2 - 4a^2 + 8a - 4 = 2$$

$$\Rightarrow 2a^2 - 8a + 6 = 0 \Rightarrow a^2 - 4a + 3 = 0$$

$$\therefore r_1 + r_2 = 4 \text{ and } r_1r_2 = 3$$

$$\therefore r_1^2 + r_2^2 - r_1r_2 = (r_1 + r_2)^2 - 3r_1r_2$$

$= 16 - 9 = 7$

PHYSICS

Section - A (Single Correct Answer)

Sol.

31. B

Sol. $d_{apparent} = \frac{d_{actual}}{\mu_{rel}}$

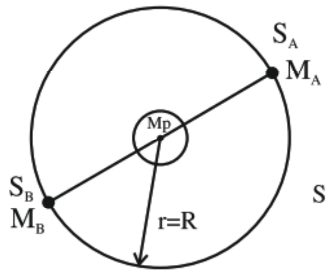
$12 = \frac{x}{\mu}$ .....(1)

$4 = \frac{24 - x}{\mu}$ .....(2)

On solving we get $\mu = 1.5$

32. D

Sol.



$$P.E. = -\frac{GM_PM_A}{R}$$

$$K.E = +\frac{GM_PM_A}{2R}$$

$$T.E = -\frac{GM_PM_A}{2R}$$

$$\text{Speed} = v = \sqrt{\frac{GM_P}{R}}$$

Speed of satellite in Independent of mass of satellite.

33. D

Sol. Equation of Carrier wave

$c(t) = 15 \sin (1000\, \pi t)$

$$f_i = \frac{\omega_c}{2\pi} = \frac{1000\pi}{2\pi} = 500\text{Hz}$$

Equation of modulated wave

$m(t) = 10 \sin (4\, \pi t)$

$$f_m = \frac{\omega_m}{2\pi} = \frac{4\pi}{2\pi} = 2\text{Hz}$$

Frequencies contained in resultant Amplitude modulated wave are (500 – 2)Hz, 500 Hz and (500 + 2) Hz.

Correct ans is (D)

34. A

Sol. Current gain in common emitter transistor

$$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{16mA - 5mA}{200\mu A - 100\mu A} = \frac{11mA}{100\mu A} = 110$$

35. C

Sol. $V_{rms} = \sqrt{\frac{3RT}{M}}$ $\quad \frac{\upsilon_{Ar}}{\upsilon_{Cl}} = \sqrt{\frac{M_{Cl}}{M_{Ar}}}$

$\Rightarrow \upsilon_{Ar} = 1.33 \times 490 = 651.7$ m/s

36. A

Sol. $\lambda = \frac{h}{m\upsilon} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mq\Delta V}}$

$$\frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{m_p V_p q_p}{m_\alpha V_\alpha q_\alpha}}$$

$$\Rightarrow \frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{1\times2\times1}{4\times4\times2}} = \frac{1}{4}$$

$$\Rightarrow \lambda_p : \lambda_\alpha = 4:1$$

37. A

Sol. Conceptual

38. A

Sol. Volume = Constant

$A_1L_1 = A_2L_2$

$$A_1L = A_2\frac{L}{4}$$

$$\boxed{4A_1 = A_2}$$

$$R_1 = \frac{\rho L_1}{A_1} \qquad R_2 = \frac{\rho L_2}{A_2}$$

$$\frac{R_2}{R_1} = \frac{L_2A_1}{A_2L_1} = \frac{L}{4}\frac{A_1}{4A_1L}$$

$$R_2 = \frac{1}{16}R_1 = 10\Omega$$

39. A

Sol. Spring Constant

$$[K] = \frac{[F]}{[x]} = \frac{MLT^{-2}}{L} = MT^{-2}$$

$$[\omega] = \frac{[\theta]}{[t]} = \frac{1}{T} = T^{-1}$$

40. A

Sol. Resultant of $\vec{F}_2$ and $\vec{F}_3$ should be opposite to $\vec{F}_1$

$$a = \frac{10}{5} = 2\ m/s^2$$

41. A

Sol. Rate of cooling α Temperature difference

$$\frac{80-60}{5} = k\{70-20\}$$

.....(1)

$$\frac{60-40}{t} = k\{50-20\}$$

.....(2)

$$\frac{4t}{20} = \frac{50}{30}$$

$$t = \frac{25}{3}\min = 500\sec$$

$\Rightarrow$ t = 500 seconds

42. C

Sol. $\eta = \left(1-\frac{273}{373}\right)\times100 = 26.8\%$

43. A

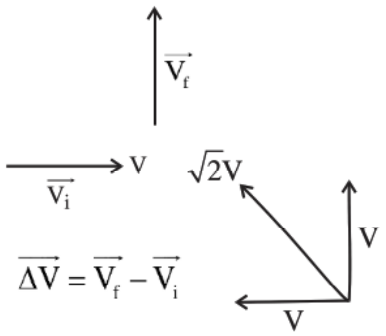
Sol. Work done = ΔKE

Work done = – FS = 0 – K

$$S = \frac{K}{F}$$

Statement 1 $\rightarrow$ correct

Statement 2 $\rightarrow$ incorrect



Velocity is changing $\Rightarrow \vec{a} \neq 0$

44. D

Sol. $x = \frac{A}{2}, \quad P.E. = \frac{1}{2}kx^2$

$$K.E. = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

$$\frac{P.E.}{K.E.}=\frac{x^2}{A^2-x^2}=\frac{A^2}{4\left(\frac{3A^2}{4}\right)}=\frac{1}{3}$$

45. B

Sol. $\vec{v}=\vec{u}+\vec{a}t$

V = 150 – 10 t

V(3) = 150 – 30 = 120

V(5) = 150 – 50 = 100

$$\frac{120}{100}=\frac{x+1}{x}=\frac{6}{5}\Rightarrow x=5$$

46. A

Sol. $\vec{P}=30\times10^{-5}\,Cm$

Using Gauss law

$$\phi=\frac{Q_{in}}{\varepsilon_0}\text{ and } Q_{in}=0$$

$$\Rightarrow\phi=0$$

Statement 1 and Statement 2 are correct.

47. A

Sol. Optical communication is performed in the frequency range of 1THz to 1000 THz.

(Microwave to UV)

So, EM waves used for optical communication have shorter wavelength than that of microwaves used in RADAR.

Also, $\upsilon_{INFRARED}>\upsilon_{MICROWAVE}$

$\therefore$ Infrared EM waves are more energetic than microwave

48. C

Sol. According to Bohr's postulates, an electron makes jump to higher energy orbital if it absorbs a photon of energy equal to difference between the energies of an excited state and the ground state. Assuming that collided electron takes energy equal to 10.2 eV or 12.09 eV from incoming electron beam (some part lost due to collision). The maximum excited state is n = 3. So, number of spectral lines is $\frac{3(3-1)}{2}=3$

49. B

Sol. $V_{escape}=\sqrt{\frac{2GM}{R}}$

$$\therefore V_{escape}\text{ for planet}=\sqrt{\frac{2G(16M_E)}{(4R_E)}}=2\sqrt{\frac{2GM_E}{R_E}}$$

$= 2(V_{escape}$ for Earth)

50. D

Sol. Both statements are correct. Theory based.

Section - B (Numerical Value)

51. 1

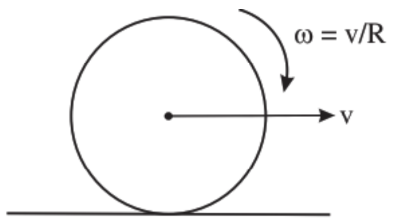
Sol. For 5th harmonic in closed organ pipe,

$$f_5=\frac{5V}{4l}\Rightarrow 405=\frac{5\times324}{4l}$$

$$\Rightarrow l=1\text{ m}$$

52. 2

Sol.



$$\frac{K_{rot}}{K_{Total}}=\frac{\frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{V}{R}\right)^2}{\frac{1}{2}mv^2+\frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{V}{R}\right)^2}$$

$$\Rightarrow\frac{x}{5}=\frac{2}{5}\Rightarrow x=2$$

53. 243

Sol. Period of oscillation $\alpha\frac{1}{\sqrt{B_H}}$

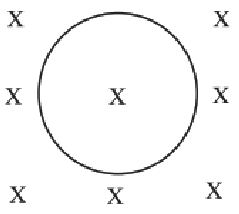
$$T\alpha\frac{1}{\sqrt{B\cos\theta}}\Rightarrow\frac{T_1}{T_2}=\sqrt{\frac{B_2\cos\theta_2}{B_1\cos\theta_1}}$$

$$\Rightarrow\frac{60/20}{60/30}=\sqrt{\frac{B_2\cos60^\circ}{B_1\cos30^\circ}}\Rightarrow\frac{3}{2}=\sqrt{\frac{B_2}{\sqrt{3}B_1}}$$

$$\Rightarrow\frac{9}{4}=\frac{B_2}{\sqrt{3}B_1}\Rightarrow\frac{B_1}{B_2}=\frac{4}{9\sqrt{3}}=\frac{4}{\sqrt{243}}$$

54. 50

Sol.



$$\frac{dr}{dt} = 10^{-3} \text{ m/s}$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\varepsilon = \left|\frac{-d\phi}{dt}\right| = \left|\frac{BdA}{dt}\right|$$

$= 0.4 \times 2 \times \pi \times 2 \times 10^{-2} \times 10^{-3}$ V

$= 16\pi\mu V = 50.24\ \mu V$

55. 784

Sol. For constant speed, WD by engine + WD by friction = 0 [by WET]

$WD_{engine} = - WD_{friction} = -[-\mu mgx]$

$= 0.04 \times 500 \times 9.8 \times 4 \times 10^3 = 784$ KJ

56. 4

Sol. Energy released = $(\Delta m)_{amu} \times 931.5$ MeV

$= (m_u - m_{Th} - m_{He})_{amu} \times 931.5$ MeV

$= 0.0044 \times 931.5$ MeV $= 4.0986$ MeV

57. 15

Sol. $i_o R_o = i_{100} R_{100}$ [For same source]

$\Rightarrow \quad 2 R_o = 1.2 R_o [1 + 100\alpha]$ ......(1)

$$\Rightarrow 1 + 100\alpha = \frac{5}{3} \Rightarrow 100\alpha = \frac{2}{3}$$

$$\Rightarrow 50\alpha = \frac{1}{3}$$

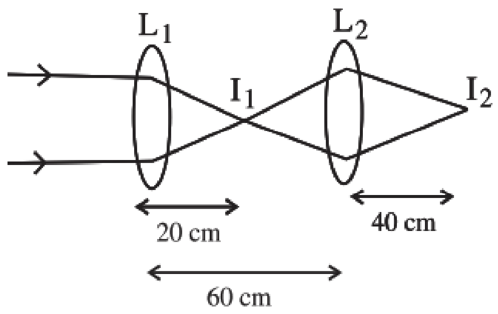
$$\therefore i_{50} R_{50} = i_0 R_0$$

$$\Rightarrow i_{50} = \frac{i_0 R_0}{R_{50}} = \frac{2 \times R_0}{R_0(1+50\alpha)} = \frac{2}{1+\frac{1}{3}} = 1.5 \text{ A}$$

$= 15 \times 10^2$ mA

58. 100

Sol. **$f_1 = 20$ cm** **$f_2 = 20$ cm**



1^{st} refraction in $L_1(I_1)$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{\infty} = \frac{1}{f}$$

$\therefore \quad v = f$

2^{nd} refraction in L_2

$I_1 \to$ object

$I_2 \to$ image

$u = -40$ cm $\qquad f = 20$ cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{(-40)} = \frac{1}{20}$$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{40} = \frac{6-3}{120}$$

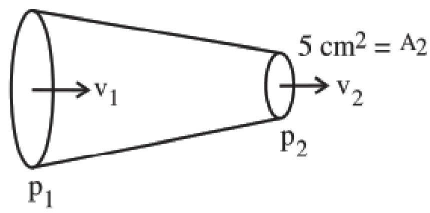
$$\frac{1}{v} = \frac{3}{120} = \frac{1}{40}$$

$\therefore$ v = 40 cm

Correct Answer is 100.

59. 4

Sol. $A_1 = 10\ cm^2$



$\Delta P = P_1 - P_2 = 3\ N/m^2$ (given)

By continuity eq^n

$A_1v_1 = A_2v_2$

$\therefore v_1 = \frac{A_2}{A_1}v_2$

....(1)

By Bernoulli's eq^n

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$\Delta P = \frac{1}{2}\rho\left(v_2^2 - \frac{A_2^2}{A_1^2}v_2^2\right)$$

$$\Delta P = \frac{1}{2}\rho\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]v_2^2$$

$$3 = \frac{1}{2}\times 1.25\times 10^3\left[1 - \left(\frac{5}{10}\right)^2\right]v_2^2$$

$$3 = \frac{1}{2}\times 1.25\times 10^3\left[1 - \frac{1}{4}\right]v_2^2$$

$$3 = \frac{1}{2}\times 1.25\times 10^3 \times \frac{3}{4}v_2^2$$

$\therefore \quad v_2 = 8 \times 10^{-2}$ m/s

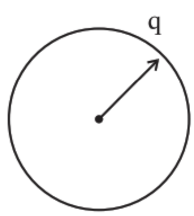
So discharge rate $= A_2\ v_2$

$= 5 \times 10^{-4} \times 8 \times 10^{-2} = 4 \times 10^{-5}\ m^3/s$

Correct ans is x = 4

60. 160

Sol.



Let q = charge on each drop

$V = \frac{Kq}{r}$ .....(1)

Now for combination of 64 drop

$$64 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

R = 4r

And Q = 64 q

Potential of bigger drop

$$= \frac{KQ}{R} = \frac{K64q}{4r} = 16\frac{Kq}{r}$$

= 16 × 10 mV = 160 mV.

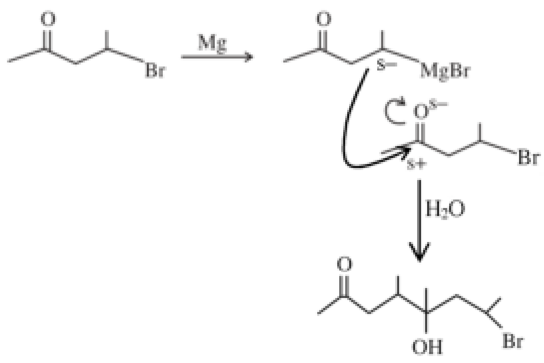
Correct answer is 160.

CHEMISTRY

Section - A (Single Correct Answer)

61. D

Sol.



62. D

Sol. Extent of adsorption a critical temp.

63. B

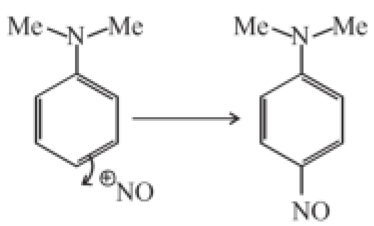
Sol. 5f orbital not buried as 4f orbitals so e^- present in 5f orbital experience less nuclear attraction than e^- present in 4f orbital. Hence electrons of 5f orbital can take part in bonding to a far greater extent.

64. B

Sol. $NaNO_2 + HX \rightarrow HNO_2 + NaX$

$$H - O - N = O \xrightarrow[-H_2O]{H^{\oplus}} NO^{\oplus}$$

(Nitrosonium ion)



P – Nitroso product

65. A

Sol. CFSE = $(-0.4\, nt_{2g} + 0.6\, n_2)\,\Delta_0$

nt_{2g} = number of electrons in t_{2g}

orbital n_{eg} = number of electrons in eg orbital

Complex	No.of at electrons	CFSE(Δ_0)
$[Cu(NH_3)_6]^{+2}$	d^9 (S.L.) $t_{2g}^{2,2,2} eg^{2,1}$	–0.6
$[Ti(H_2O)_6]^{+3}$	d^1 (W.L.) $t_{2g}^{1,0,0} eg^{0,0}$	–0.4
$[Fe(CN)_6]^{3-}$	d^5 (S.L.) $t_{2g}^{2,2,1} eg^{0,0}$	–2.0
$[NiF_6]^{4-}$	d^8 (W.L.) $t_{2g}^{2,2,2} eg^{1,1}$	–1.2

66. D

Sol. FACT

67. B

Sol. In general moving down the group, mass increases more prominently as compared to volume (size) hence density increases for Group I metal. Due to empty 3d subshell in K increase in size is more prominent as compare to mass.

Li < K < Na < Rb < Cs

68. A

Sol. Statement I : Is correct according to Fajan's rule Sb^{+5} more polarising power than Sb^{+3}.

Statement II : Stability of higher oxides of halogen is primarily due to

a) Higher oxidation state

b) More EN halogen

c) Resonance stabilization

69. A

Sol. Molecular, weight of metal chloride

$$=\frac{0.57}{100}\times 22700$$

$$=129.39$$

weight of Cl = 129.39×0.55

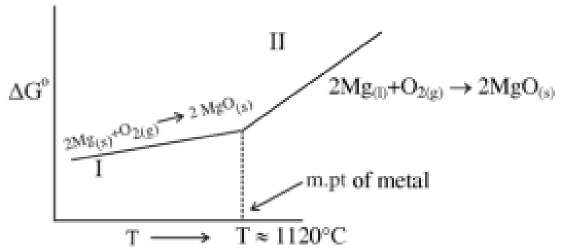
= 71.1645

$$\therefore \text{ Mole of Cl} = \frac{71.1645}{35.5} \cong 2$$

Hence MCl_2.

70. B

Sol.



For line II, ΔS is more –ve than line I. hence higher slope.

For I : $\Delta S_I = (S_{solid}) - (S_{solid} + S_{gas})$

For II : $\Delta S_{II} = (S_{solid}) - (S_{liq.} + S_{gas})$

Hence ΔS_{II} more –ve than ΔS_I

71. A

Sol.

i. $4NO_2(g) + O_2(g) + 2H_2O(\ell) \rightarrow 4HNO_3(aq)$

SO_2 & NO_2 have major contribution in acid rain.

ii. CO_2, CH_4, O_3, CFC are responsible for global warming.

iii. $H_2O(\ell) + CO_2(g) \rightleftharpoons H_2CO_3(aq.)$

$H_2CO_3(aq.) \rightleftharpoons H^+(aq.) + HCo_3^-(aq.)$

Rain water has pH of 5.6 due to the Presence of H^+ ions formed by the reaction of rain water with CO_2.

iv. Phosphates present in fertilizers contribution for Eutrophication (Process in which nutrient enriched water bodies support a dense plant population, which kills animal life by depriving it of oxygen and results in subsequent loss of biodiversity is known as Eutrophication.)

72. A

Lead storage battery consists of lead anode and a grid of lead packed with lead oxide (PbO_2) as cathode, a 38% solution of H_2SO_4 is used as an electrolyte.

On charging the battery the reaction is reversed and $PbSO_4$(s) on anode and cathode is converted into Pb and PbO_2 respectively.

73. A

Sol. 

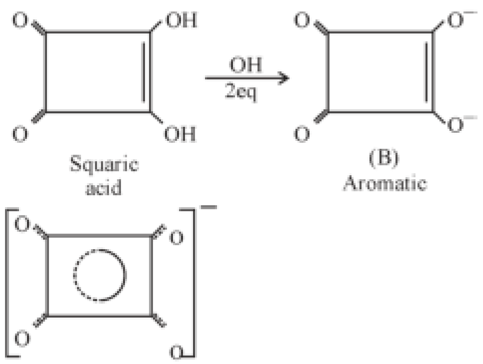
$\xrightarrow[\text{ii) } H_2O]{\text{i) } O_3} CH_3COOH + CH_3CH_2CH_2COOH$

Acetic acid   Butanoic acid

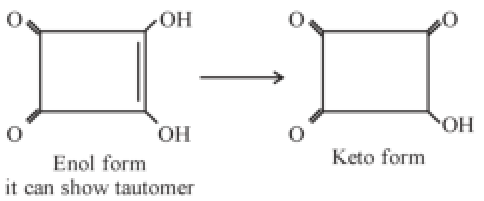
It is oxidative ozonolysis.

74. D

Sol.



Resonance hybrid of B showing all C-C bond length same.



75. A

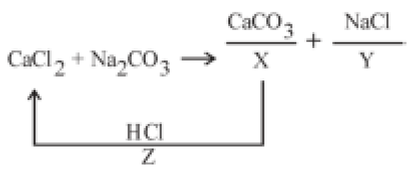
Sol. Acetylide ion $\rightarrow C_2^{2-} (\bar{C} \equiv \bar{C})$

Bond order = 3 & Diamagnetic

NO^+ $14e^-$ $\rightarrow$ Bond order = 3 & Diamagnetic

76. C

Sol.



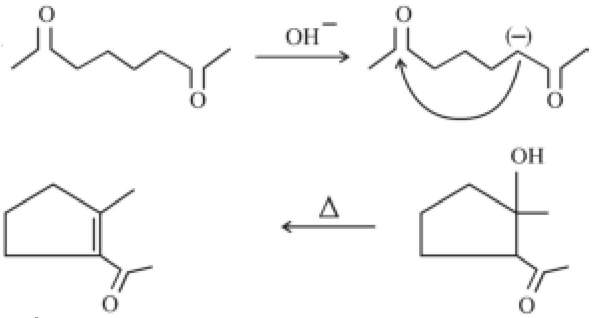
77. B

Sol. Boron is non- metallic in nature. It is extremely hard and black coloured solid. It exists in many allotropic forms. Due to very strong crystalline lattice, boron has unusually high melting point and boiling point.

	Element				
	B	Al	Ga	In	Tl
Melting point/K	2453	933	303	430	576
Boiling point/K	3923	2740	2676	2353	1730

78. C

Sol.



79. A

Sol. $B_2H_6 \Rightarrow e^-$ deficient hydride

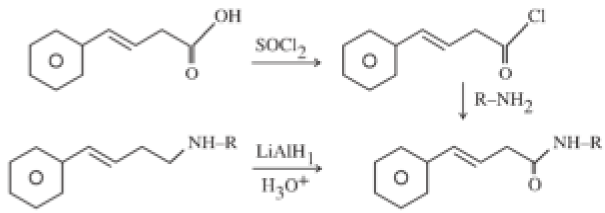
$HF \Rightarrow e^-$ rich hydride

$CH_4 \Rightarrow e^-$ Precise hydride

$MgH_2 \Rightarrow$ Saline hydride

80. D

Sol.



Section - B (Numerical Value)

81. 3

Sol. $\Delta S_{system} = nR\ell n\left(\frac{V_2}{V_1}\right)$

$= 1 \times 8.314\ \ell n\left(\frac{3}{2}\right)$

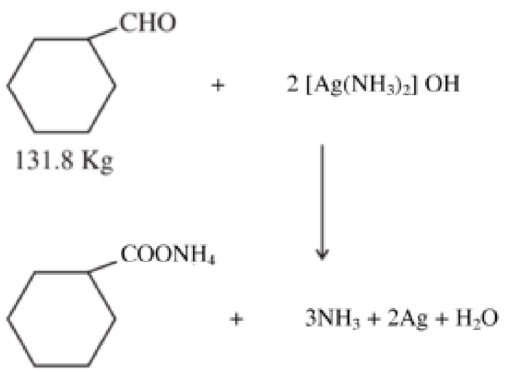
$\Delta S_{system} = 3.37$

$\Delta S_{surr.} = 3.37$

Correct Ans : 3

82. 60

Sol.

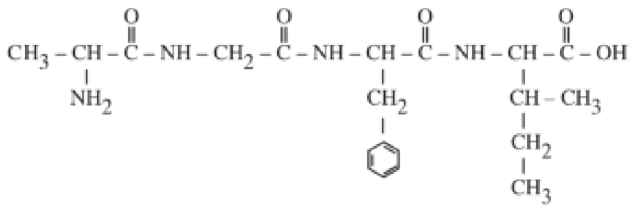


$W_{NH_3} = \frac{131.8 \times 1000}{112} \times 3 \times 17$

$= 60$ Kg

83. 10

Sol.



84. 9000

Sol. $\frac{(M_1 \times V_1)}{10^{-1} \times 1} = \frac{(M_2 \times V_2)}{10^{-2} \times V_2}$

$V_2 = 10$ L

Water added = 10 – 1

= 9 Litre

= 9000 mL

85. 25

Sol. More R_f, less its polarity

$$R_f = \frac{\text{Distance travelled by compound 'X'}}{\text{Distance travelled by solvent 'Y'}}$$

$$= \frac{2}{8} = 0.25 = 25 \times 10^{-2}$$

86. 48

Sol. $MgCl_2 \rightarrow Mg^{+2} + 2Cl^-$

$1-\alpha \quad \alpha \quad 2\alpha$

$i = 1 + 2\alpha \ (\alpha = 0.8)$

$i = 2.6$

$\frac{\Delta p}{p^o} = \frac{i \times n_2}{n_1}$

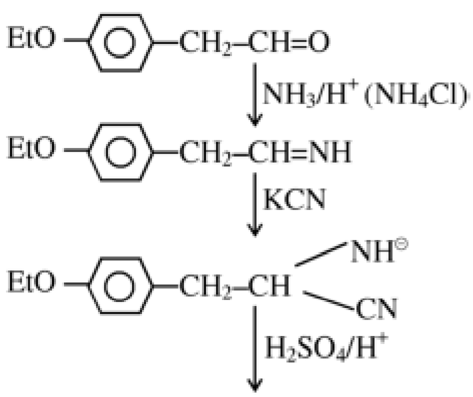
$\Delta p = 2.34$

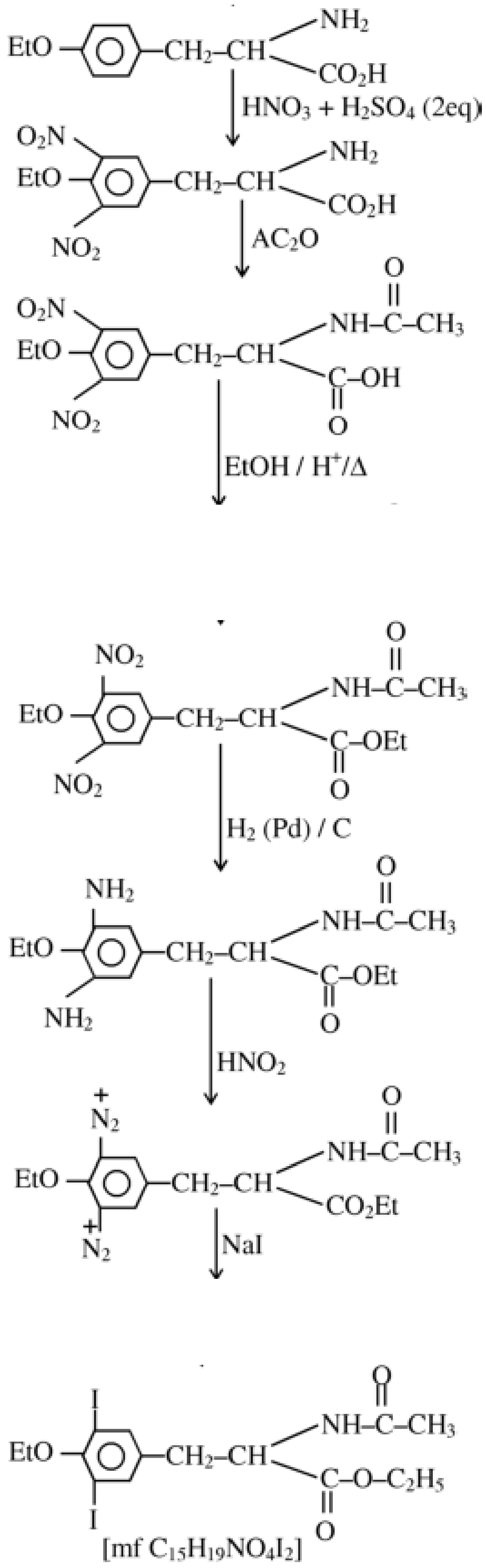
$p_s = 47.66$

$p_s \cong 48$

87. 15

Sol.





88. 4

Sol. $(U_{rms})_{X,600} = (U_{mp})_{Y,90}$

$$\sqrt{\frac{2\times R\times 600}{40}} = \sqrt{\frac{2\times R\times 90}{M}}$$

$M = 4$

89. 3

Sol. $RDS: NOBr_2 + NO \rightarrow 2NOBr$

$r = K[NOBr_2][NO]$ ... (i)

$K_{eq} = \frac{[NOBr_2]}{[NO][Br_2]}$ ... (ii)

From (i) & (ii),

$r = K.\ Keq.\ [NO]\ [Br_2]\ [NO]$

$r = K'\ [NO]^2\ [Br_2]$

Overall order = 3

Ans. 3

90. 3

Sol. $E(ev) = \frac{1240}{400} = 3.1\ ev$

Mg, Cu, Ag

Ans. 3

❒ ❒ ❒