24-January-2023 (Evening Batch) : JEE Main Paper

PHYSICS

Section - A (Single Correct Answer)

1. B

Sol. Potential at centre

$$V=\frac{(\lambda\cdot\pi R_2)}{4\pi\varepsilon_0 R_2}+\frac{(\lambda\cdot\pi R_1)}{4\pi\varepsilon_0 R_1}$$

$$=\frac{\lambda}{2\varepsilon_0}$$

2. C

Sol. For monoatomic gas $\gamma_1=\frac{5}{3}$

For diatomic gas at low temperatures

$$\gamma_2=\frac{7}{5}$$

$$\therefore\frac{\gamma_1}{\gamma_2}=\frac{\frac{5}{3}}{\frac{7}{5}}=\frac{25}{21}$$

3. B

Sol. $\lambda_D=\frac{h}{p}=\frac{h}{\sqrt{2mK}}$

$$\therefore\lambda\propto\frac{1}{\sqrt{m}}$$

$$\because m_\alpha>m_p>m_e$$

$$\lambda_\alpha<\lambda_p<\lambda_e$$

4. C

Sol. $T^2\propto R^3\Rightarrow\left(\frac{T_1}{T_2}\right)^2=\left(\frac{R_1}{R_2}\right)^3$

$$\Rightarrow\left(\frac{1}{2.83}\right)^2=\left(\frac{1.5\times10^6}{R_2}\right)^3$$

$$\Rightarrow R_2=\left[(2.83)^2\times(1.5\times10^6)^3\right]^{\frac{1}{3}}$$

$= 8^{1/3}\times 1.5\times 10^6 = 3\times 10^6$ km

5. A

Sol. AM Broadast $\rightarrow$ 540 – 1600 KHz

FM Broadcast $\rightarrow$ 88 – 108 MHz

Television $\rightarrow$ 54 – 890 MHz

Salellite communication $\rightarrow$ 3.7 – 4.2 GHz

$\therefore$ A-II, B-I, C-IV, D-III

6. B

Sol.

A_1	B_1	Y
0	0	1
0	1	1
1	0	1
1	1	0

$Y=\overline{A_1\odot B_1}$ NAND

7. B

Sol. $B=\mu_0 nI$

$=4\pi\times10^{-7}\times70\times10^2\times2$

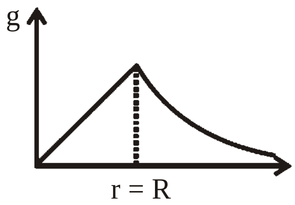
$=56\pi\times10^{-4}$ T

$=176\times10^{-4}$ T

8. C

Sol. $g'=\frac{g}{\left(1+\frac{h}{R}\right)^2}$

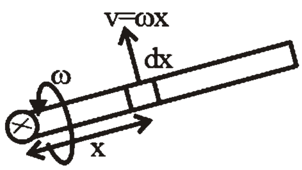
$$g'=g\left\{1-\frac{d}{R}\right\}$$



Statement I is correct & Statement II is incorrect

9. C

Sol.



$$\int d\varepsilon = \int B(\omega x)dx$$

$$\varepsilon = B\omega\int_0^L xdx = \frac{B\omega L^2}{2}$$

10. B

Sol. Based on fact.

11. A

Sol. $\left[T^{-1}\right] = \left[L^1\right]^a \left[M^1L^{-3}\right]^b \left[\frac{MLT^{-2}}{L}\right]^c$

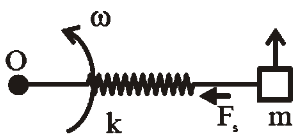
$$\Rightarrow T^{-1} = M^{b+c} \cdot L^{a-3b} \cdot T^{-2c}$$

$$c = \frac{1}{2}, b = -\frac{1}{2}, a - 3b = 0$$

$$a + \frac{3}{2} = 0 \Rightarrow a = -\frac{3}{2}$$

12. C

Sol.



Natural length = L_0

Extension = x

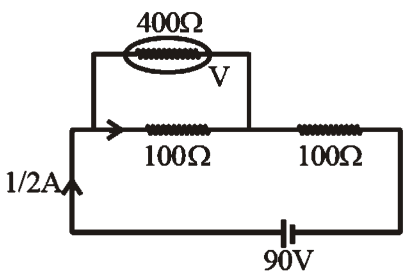
$kx = m(L_0 + x)\,\omega^2$

$$\Rightarrow 12.5x = \frac{1}{2}(L_0 + x)25 \Rightarrow 1.5x = L_0$$

$$\Rightarrow \frac{x}{L_0} = \frac{2}{3}$$

13. A

Sol.



$$R_{eq} = \frac{400\times100}{500} + 100$$

$= 180\Omega$

$$i = \frac{90}{180} = \frac{1}{2}A$$

$$\text{Reading } = \frac{1}{2}\times\frac{400}{500}\times100$$

$= 40\text{volt}$

14. A

Sol. $C = \frac{\omega}{k} = \frac{E_0}{B_0}$

15. D

Sol. Displacement = Σarea = 16 – 8 + 16 – 8 = 16 m

Distance = Σ | area | = 48 m

$$\frac{\text{displacement}}{\text{Distance}} = \frac{1}{3}$$

16. C

Sol. Concept based

17. C

Sol. $T \propto \frac{1}{\sqrt{g}}$

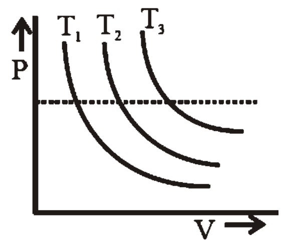
18. A

Sol. $\frac{hc}{\lambda} = \left[1 - \frac{1}{16}\right](13.6\text{ eV})$

So, $\lambda = 94.1$ nm

19. D

Sol. For isothermal process P-V graph is rectangular hyperbola



As dotted line is isobaric line which implies $T_3 > T_2 > T_1$ as volume is increasing.

20. D

Sol. $\vec{P}\cdot\vec{Q}=0$

$$\left(\hat{i}+2m\hat{j}+m\hat{k}\right)\cdot\left(4\hat{i}-2\hat{j}+m\hat{k}\right)=0$$

$\Rightarrow$ $4-4m+m^2=0$

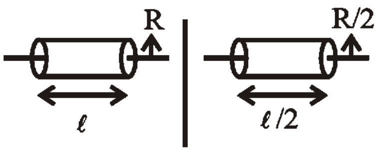
$\Rightarrow$ $(m-2)^2=0$

$\Rightarrow$ $m=2$

Section - B (Numerical Value)

21. 32

Sol.



$$I_1=\frac{m_1R^2}{2}$$

$$I_2=\frac{m_2(R/2)^2}{2}$$

$$\frac{I_1}{I_2}=\frac{4m_1}{m_2}=\frac{4\cdot\rho\pi R^2 l}{\frac{\pi R^2}{4}\times\frac{l}{2}}\Rightarrow\frac{I_1}{I_2}=32$$

22. 1

Sol. $T=2\pi\sqrt{\frac{m}{k}}=1$

$$T'=2\pi\sqrt{\frac{m+3}{k}}=2$$

$$\frac{T}{T'}=\sqrt{\frac{m}{m+3}}=\frac{1}{2}$$

$$\Rightarrow\frac{m}{m+3}=\frac{1}{4}$$

$m=1$

23. 6

Sol. No. of mole $=\frac{120}{240}=\frac{1}{2}$

No. of molecules $=\frac{1}{2}\times N_A$

Energy released $=\frac{1}{2}\times6\times10^{23}\times200$

$=6\times10^{25}$ MeV

24. 105

Sol. $C_0=\frac{\epsilon_0 A}{d}=15pF$

$$C=\frac{K\epsilon_0 A}{2d}=\frac{3.5}{2}\times15pF=\frac{105}{4}pF$$

25. 100

Sol. $\vec{F}=t\hat{i}+3t^2\hat{j}$

$$\frac{md\vec{v}}{dt}=t\hat{i}+3t^2\hat{j}$$

$$m=1kg,\ \int_0^{\vec{v}}dv=\int_0^t tdt\hat{i}+\int_0^t 3t^2dt\hat{j}$$

$$\vec{v}=\frac{t^2}{2}\hat{i}+t^3\hat{j}$$

Power $=\vec{F}\cdot\vec{V}=\frac{t^3}{2}+3t^5$

At t = 2, power $=\frac{8}{2}+3\times32=100$

26. 44

Sol. As volume is constant,

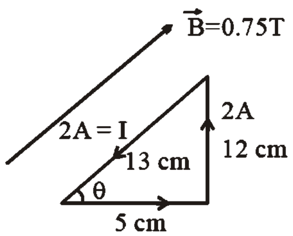
So resistance $\propto$ (length)^2

$\Rightarrow$ % change in resistance

$$=20+20+\frac{400}{100}=44\%$$

27. 9

Sol.



Force on 5 cm side is

$$|\vec{F}|=ILB\sin\theta$$

$$=(2)(5\times10^{-2})\times\frac{3}{4}\times\frac{12}{13}=\frac{9}{130}N$$

So, x = 9

28. 3

Sol. After long time an inductor behaves as a resistance-less path.

So current through cell

$$I=\frac{12}{R/3}=3A \quad \{\because R=12\Omega\}$$

29. 54

Sol. $$\frac{I}{f_{H_2O}}=\left(\frac{\mu_g}{\mu_{H_2O}}-1\right)\left(\frac{2}{R}\right)=\frac{1}{8}\left(\frac{2}{R}\right)=\frac{1}{(4f_{air})}$$

So, $f_{H_2O}=4f_{air}=72$ cm

So change in focal length = 72 – 18= 54 cm

30. 7

Sol. When the ball attain terminal velocity

$F_v = (mg - F_B)\ (\because a=0)$

$=V\sigma_b g - V\rho_l g \ = Vg(\sigma_b-\rho_l)$

$=\frac{4}{3}\pi(10^{-3})^3\times 9.8(10.5-1.5)\times 10^3$

$= 3696 \times 10^{-7}$ N

So, x = 7

CHEMISTRY

Section - A (Single Correct Answer)

31. D

Sol. Ce^{+4} and Tb^{+4} act as oxidising agent.

32. A

Sol. N_2

$$\sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \pi 2p_x^2=\pi 2p_y^2\ \underset{\text{HOMO}}{\sigma 2p_z^2}$$

$$N_2^+ - \sigma 1s^2\ \sigma^* 1s^2\ \sigma^* 2s^2 \sigma^* 2s^2\ \pi 2p_x^2=\pi 2p_y^2\ \underset{\text{HOMO}}{\sigma 2p_z^1}$$

$$O_2 - \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \sigma 2p_z^2$$

$$\pi 2p_x^2=\pi 2p_y^2$$

$$\pi^* 2p_x^1=\pi^* 2p_y^1 (\text{HOMO})$$

$$O_2^+ - \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \sigma^* 2s^2\ \sigma 2p_z^2\ \pi 2p_x^2=\pi 2p_y^2$$

$$\pi^* 2p_x^1=\pi^* 2p_y^0 (\text{HOMO})$$

$N_2 \Rightarrow$ 0 unpaired e^- in HOMO

$N_2^+ \Rightarrow$ 1 unpaired e^- in HOMO

$O_2 \Rightarrow$ 2 unpaired e^- in HOMO

$O_2^+ \Rightarrow$ 1 unpaired e^- in HOMO

33. D

Sol. Crystal field theory introduce spectrochemical series based upon the experimental values of Δ but can't explain it's order. While other three points are explained by CFT. Specially when the CFSE increases thermodynamic stability of the complex increases.

34. B

Sol. $t_{1/2} \propto (P_O)^{1-n}$

$$\frac{(t_{1/2})_1}{(t_{1/2})_2}=\frac{(P_O)^{1-n}}{(P_{O_2})_2^{1-n}}$$

$$\Rightarrow \left(\frac{4}{2}\right)=\left(\frac{50}{100}\right)^{1-n}$$

$$\Rightarrow 2=\left(\frac{1}{2}\right)^{1-n}$$

$$\Rightarrow 2=(2)^{n-1}$$

$$\Rightarrow n-1=1$$

$$\Rightarrow n=2$$

35. D

Sol. Z = 55 [Cs] $\Rightarrow$ [Xe] $6s^1$

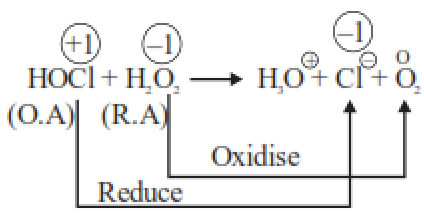
$[Cs^+] \Rightarrow$ [Xe]

i.e., upto 5s count e^- of s-subshell

i.e., 1s, 2s, 3s, 4s, 5s $\Rightarrow$ 10 electrons

36. C

Sol.



37. B

Sol. Ag.

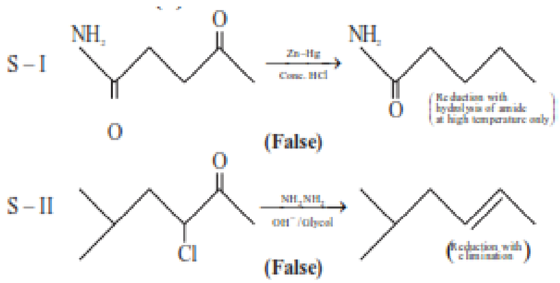
$$4Ag + 8CN^{-} + O_2 + 2H_2O \rightarrow$$

$$4[Ag(CN)_2]^{-} + 4OH^{-}$$

$$2[Ag(CN)_2]^{-1} + Zn \rightarrow 2Ag\downarrow + [Zn(CN)_4]^{-2}$$

38. C

Sol.



39. B

Sol. Be has less negative value compared to other AEM. However it's reducing nature is due to large hydration energy associated with the small size of Be^{2+} ion and relatively large value of the atomization enthalpy of metal.

40. D

Sol. Theoretical, NCERT based.

41. C

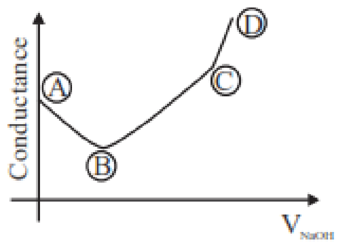
Sol. **Assertion - A :** Benzene is more stable than cyclohexatriene (True)

Reason - R : Delocalised π – e cloud lies B.M.O so more attracted by nuclei of carbon atom.

(True & Correct Explanation)

42. B

Sol. $\underset{(WA)}{C_6H_5COOH} + \underset{(SB)}{NaOH} \rightarrow \underset{(Salt)}{C_6H_5COONa} + H_2O$



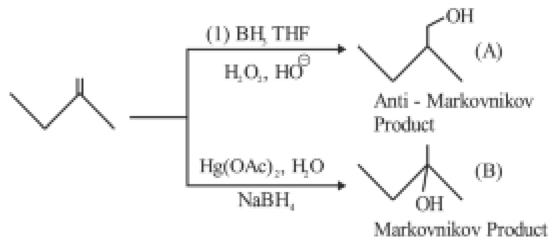
(A) $\rightarrow$(B) Free H^+ ions are replaced by $Na^{\oplus}$ which decreases conductance.

(B) $\rightarrow$(C) Un-dissociated benzoic acid reacts with NaOH and forms salt which increases ions and conductance increases.

(C) $\rightarrow$(D) After equivalence point at (3), NaOH added further increases $Na^{\oplus}$ & $OH^{\ominus}$ ions which further increases the conductance.

43. A

Sol.



44. B

Sol. Theoretical.

45. C

Sol. **Statement – 1 is (True)**

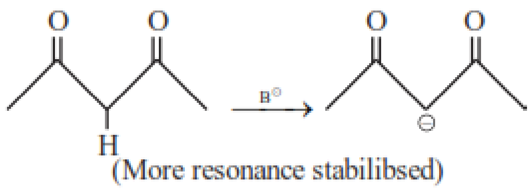
Pure aniline is colourless liquid

Statement – 2 is (False)

Aniline becomes dark brown due to action of air and light [oxidation]

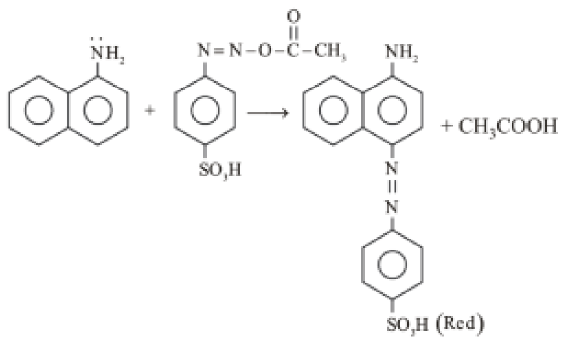
46. A

Sol. Most easily deprotonation



47. C

Sol.



48. C

Sol. (1) Na > Cs > Li – true {If considered with sign}

The low solubility of CsI is due to smaller hydration enthalpy of it's two ions.

Li_2CO_3 is highly stable to heat – false

In conc. NH_3, K formed blue solution – true

All the alkali metal hydrides are ionic solid (True).

49. C

Sol. $[Co(NH_3)_6]^{+3}$ – d^2sp^3, diamagnetic

50. D

Sol. $3SO_2 + Cr_2O_7^{2-} + 2H^+ \rightarrow 3SO_4^{2-} + \underset{(green)}{2Cr^{+3}} + H_2O$

Section - B (Numerical Value)

51. 2

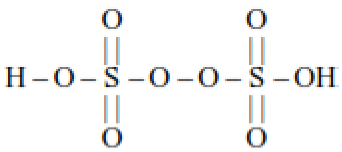
Sol. For physisorptions

(a) Decreases with increase in temperature

(b) No appreciable activation energy is required

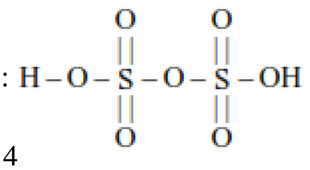
52. 8

Sol. Peroxodisulphuric acid :



No. of π – bonds = 4

Pyro sulphuric acid :

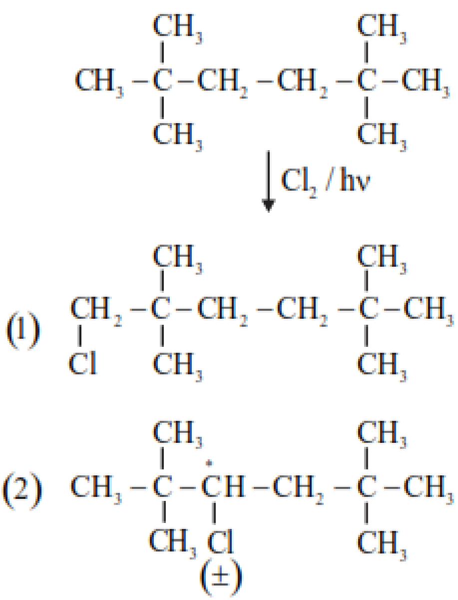


No. of π – bonds = 4

Total π – bonds = 8

53. 3

Sol.



Total numbers of isomer = 03

54. 8

Sol. No. of possible tripeptide :

Val & Pro is 2^3

(1) val – val – val

(2) pro – pro – pro

(3) val – pro – pro

(4) pro – val – pro

(5) val – val – pro

(6) val – pro – val

(7) pro – pro – val

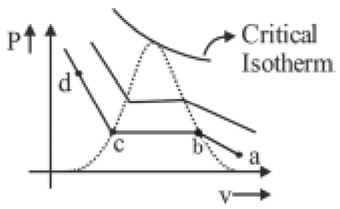
(8) pro – val – val

55. 5

Sol. Mass percent, mole fraction, molarity, ppm, molality are used for measuring concentration terms.

56. 2

Sol.



At

(a) $\rightarrow$ CO_2 exist as gas

(b) $\rightarrow$ liquefaction of CO_2 starts

(c) $\rightarrow$ liquefaction ends

(d) $\rightarrow$ CO_2 exist as liquid.

Between (b) & (c) $\rightarrow$ liquid and gaseous CO_2 co-exist.

As volume changes from (b) to (c) gas decreases and liquid increases.

(A), (C) $\rightarrow$ correct

57. 314

Sol. Let V.P. of pure A be P_A^o

Let V.P of pure B be P_B^o

When $X_A = 0.7$ & $X_B = 0.3$

$P_s = 350$

$\Rightarrow P_A^o \times 0.7 = P_B^o \times 0.3 = 350$ .... (i)

When $X_A = 0.2$ & $X_B = 0.8$

$P_s = 410$

$\Rightarrow P_A^o \times 0.2 + P_B^o \times 0.8 = 410$ .... (ii)

Solving (i) and (ii),

$P_A^o = 314$ mm Hg

$P_B^o = 434$ mm Hg

= (314)

58. 620

Sol. $1 \rightarrow 2 \Rightarrow$ Isobaric process

$2 \rightarrow 3 \Rightarrow$ Isochoric process

$3 \rightarrow 1 \Rightarrow$ Isothermal process

$$W = W_{1\to2} + W_{2\to3} + W_{3\to1}$$

$$= \left(-P(V_2 - V_1) + 0\left[-P_1V_1 \ln\left(\frac{V_2}{V_1}\right)\right]\right)$$

$$= \left[-1\times(40-20) + 0 + \left[-1\times 20\ln\left(\frac{20}{40}\right)\right]\right]$$

$$= -20 + 20\ln 2$$

$$= -20 + 20\times 2.3\times 0.3$$

$$= -6.2 \text{ bar L}$$

$$|W| = 6.2 \text{ bar } 1 = 620 \text{ J}$$

59. 85

Sol. Concentration of calcium lactate = 0.005 M

Concentration of lactate ion = (2×0.005) M

Calcium lactate is a salt of weak acid + strong base

$\therefore$ Salt hydrolysis will take place.

$$pH = 7 + \frac{1}{2}(pKa + \log C)$$

$$= 7 + \frac{1}{2}(5 + \log(2\times 0.005))$$

$$= 7 + \frac{1}{2}[5 - 2\log 10) = 7 + \frac{1}{2}\times 3 = 8.5$$

$$= 85\times 10^{-1}$$

60. 2

Sol. The spectrum of Black body radiation is explained using quantization of energy. With increase in temperature, peak of spectrum shifts to shorter wavelength or higher frequency.

For above graph $\rightarrow T_1 > T_2 > T_3 > T_4$.

MATHEMATICS

Section - A (Single Correct Answer)

61. B

Sol. $a_1 + a_3 = 10 = a_1 + d \Rightarrow 5$

$\mathbf{a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 57}$

$$\Rightarrow \frac{6}{2}[a_1 + a_6] = 57$$

$\Rightarrow a_1 + a_6 = 19$

$\Rightarrow 2a_1 + 5d = 19$ and $a_1 + d = 5$

$\Rightarrow a_1 = 2, d = 3$

Numbers : 2, 5, 8, 11, 14, 17

Variance = σ^2 = mean of squares - square of mean

$$= \frac{2^2 + 5^2 + 8^2 + (11)^2 + (14)^2 + (17)^2}{6} - \left(\frac{19}{2}\right)^2$$

$$= \frac{699}{6} - \frac{361}{4} = \frac{105}{4}$$

$8\sigma^2 = 210$

62. C

Sol. $f(x+y) = f(x)\cdot f(y)\ \forall x, y \in N, f(1) = 3$

$f(2) = f^2(1) = 3^2$

$f(3) = f(1)\, f(2) = 3^3$

$f(4) = 3^4$

$f(k) = 3^k$

$$\sum_{k=1}^{n} f(k) = 3279$$

$f(1) + f(2) + f(3) + \dots\dots\dots + f(k) = 3279$

$3 + 3^2 + 3^3 + \dots\dots\dots\dots 3^k = 3279$

$$\frac{3(3^k - 1)}{3-1} = 3279$$

$$\frac{3^k - 1}{2} = 1093$$

$3^k - 1 = 2186$

$3^k = 2187$

$k = 7$

63. B

Sol. $$3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$$

$$3\left[\left(x+\frac{1}{x}\right)^2-2\right]-2\left(x+\frac{1}{x}\right)+5=0$$

Let $x+\frac{1}{x}=t$

$3t^2 - 2t - 1 = 0$

$3t^2 - 3t + t - 1 = 0$

$3t(t - 1) + 1(t - 1) = 0$

$(t - 1)(3t + 1) = 0$

$t=1,\ -\frac{1}{3}$

$x+\frac{1}{x}=1,\ -\frac{1}{3} \Rightarrow$ No solution.

64. D

Sol. $f(x)=\frac{4^x}{4^x+2}$

$f(x)+f(1-x)=\frac{4^x}{4^x+2}+\frac{4^{1-x}}{4^{1-x}+2}$

$=\frac{4^x}{4^x+2}+\frac{4}{4+2(4^x)}$

$=\frac{4^x}{4^x+2}+\frac{2}{2+4^x}$

$= 1$

$\Rightarrow f(x) + f(1 - x) = 1$

Now $f\left(\frac{1}{2023}\right)+f\left(\frac{2}{2023}\right)+f\left(\frac{3}{2023}\right)+.....+$

$.....+f\left(1-\frac{3}{2023}\right)+f\left(1-\frac{2}{2023}\right)+f\left(1-\frac{1}{2023}\right)$

Now sum of terms equidistant from beginning and end is 1

Sum = 1 + 1 + 1 +............+ 1 (1011 times)

= 1011

65. C

Sol. $f(x) - x^3 - x^2 f'(1) + xf''(2) - f'''(3),\ x \in R$

Let $f'(1) = a,\ f''(2) = b,\ f'''(3) = c$

$\mathbf{f(x) = x^3 - ax^2 + bx - c}$

$f'(x) = 3x^2 - 2ax + b$

$f''(x) = 6x - 2a$

$f'''(x) = 6$

$c = 6,\ a = 3,\ b = 6$

$f(x) = x^3 - 3x^2 + 6x - 6$

$f(1) = -2,\ f(2) = 2,\ f(3) = 12,\ f(0) = -6$

$2f(0) - f(1) + f(3) = 2 = f(2)$

66. B

Sol. Four digit numbers greater than 7000

$= 2 \times 4 \times 3 \times 2 = 48$ Five digit number $= 5! = 120$

Total number greater than 7000

$= 120 + 48 = 168$

67. C

Sol. $x + 2y + 3z = 3$ ......(i)

$4x + 3y - 4z = 4$ .....(ii)

$8x + 4y - \lambda z = 9 + m$ .....(iii)

(i) $\times 4 -$ (ii) $\Rightarrow 5y + 16z = 8$ .....(iv)

(ii) $\times 2 -$ (iii) $\Rightarrow 2y + (\lambda - 8)z = -1 - \mu$ .....(v)

(iv) $\times 2 -$ (iii) $\times 5$ ^ $(32 - 5(\lambda - 8))z = 16 - 5(-1 - \mu)$

For infinite solutions $\Rightarrow 72-5\lambda=0 \Rightarrow \lambda=\frac{72}{5}$

$21+5\mu=0 \Rightarrow \mu=\frac{-21}{5}$

$\Rightarrow (\lambda,\ \mu) \equiv \left(\frac{72}{5},\ \frac{-21}{5}\right)$

68. C

Sol. Let $\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}=z$

$$\left(\frac{1+z}{1+\bar{z}}\right)^3=\left(\frac{1+z}{1+\frac{1}{z}}\right)^3=z^3$$

$$\Rightarrow \left(i\left(\cos\frac{2\pi}{9}-i\sin\frac{2\pi}{9}\right)\right)^3$$

$$\Rightarrow -i\left(\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}\right)=-i\left(\frac{-1}{2}-i\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \frac{-1}{2}(\sqrt{3}-i).$$

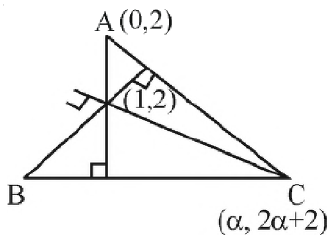
69. B

Sol. AB : $(\lambda + 1)x + \lambda y = 4$

AC : $\lambda x + (1 - \lambda) y + \lambda = 0$

Vertex A is on y-axis

$\Rightarrow x = 0$



So $y = \frac{4}{\lambda}, \; y = \frac{\lambda}{\lambda - 1}$

$\Rightarrow \frac{4}{\lambda} = \frac{\lambda}{\lambda - 1}$

$\Rightarrow \lambda = 2$

AB : $3x + 2y = 4$

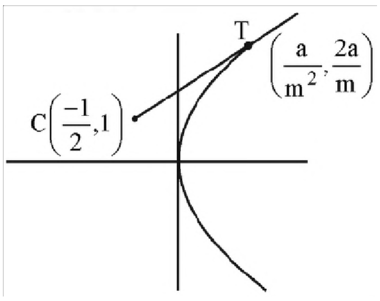
AC : $2x - y + 2 = 0$

$\Rightarrow$ A(0, 2) Let C $(\alpha, 2\alpha + 2)$

Now (Slope of Altitude through C) $\left(-\frac{3}{2}\right) = -1$

$\left(\frac{2\alpha}{\alpha - 1}\right)\left(-\frac{3}{2}\right) = -1 \Rightarrow \alpha = -\frac{1}{2}$

So $C\left(-\frac{1}{2}, 1\right)$



Let Equation of tangent be $y = mx + \frac{3}{2m}$

$m^2 + 2m - 3 = 0$

$\Rightarrow m = 1, -3$

So tangent which touches in first, quadrant at T is

$T \equiv \left(\frac{a}{m^2}, \frac{2a}{m}\right)$

$\equiv \left(\frac{3}{2}, 3\right)$

$\Rightarrow CT = \sqrt{4 + 4} = 2\sqrt{2}$

70. A

Sol. $\lim_{x \to a}([x - 5] - 2[2x + 2]) = 0$

$\lim_{x \to a}([x] - 5 - [2x] - 2) = 0$

$\lim_{x \to a}([x] - [2x]) = 7$

$[a] - [2a] = 7$

$a \in I, \; a = -7$

$a \notin I, \; a = I + f$

Now, $[a] - [2a] = 7$

$-I - [2f] = 7$

Case - I : $f \in \left(0, \frac{1}{2}\right)$

$2f \in (1, 2)$

$-I - 1 = 7$

$I = -8 \Rightarrow a \in (-7.5, -7)$

Hence, $a \in (-7.5, -6.5)$

71. C

Sol. $S = 0.({}^{30}C_0)^2 + 1.({}^{30}C_1)^2 + 2.({}^{30}C_2)^2 + \ldots\ldots + 30.({}^{30}C_{30})^2$

$S = 30.({}^{30}C_0)^2 + 29.({}^{30}C_1)^2 + 28.({}^{30}C_2)^2 + \ldots\ldots + 0.({}^{30}C_0)$

$2S = 30.({}^{30}C_0^{\,2} + {}^{30}C_1^{\,2} + \ldots\ldots + {}^{30}C_{30}^{\,2})$

$S = 15 \cdot {}^{60}C_{30} = 15 \cdot \frac{60!}{(30!)^2}$

$\frac{15 \cdot 10!}{(30!)^2} = \frac{\alpha \cdot 60!}{(30!)^2}$

$\Rightarrow \alpha = 15$

72. D

Sol. Equation of plane passing through point of intersection of P1 and P2

P = P1 + kP2

(x + (λ + 4)y + z -1) + k (2x + y + z – 2) = 0

Passing through (0, 1, 0) and (1, 0, 1)

(λ + 4 – 1) + k(1 – 2) = 0

(λ + 3) – k = 0 ....(1)

Also passing (1, 0, 1)

(1 + 1 – 1) + k(2 + 1 – 2) = 0

1 + k = 0

k = – 1

put in (1)

λ + 3 + 1 = 0

λ = – 4

Then point (2λ, λ, –λ)

(–8, –4, 4)

$$d = \left|\frac{-16-4+4-2}{\sqrt{6}}\right|$$

$$d = \frac{18}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = 3\sqrt{6}$$

73. C

Sol. Let $\vec{\beta}_1 = \lambda\vec{\alpha}$

Now $\vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1$

$$= (\hat{i} + 2\hat{j} - 4\hat{k}) - \lambda(4\hat{i} + 3\hat{j} + 5\hat{k})$$

$$= (1-4\lambda)\hat{i} + (2-3\lambda)\hat{j} - (5\lambda+4)\hat{k}$$

$$\vec{\beta}_2 \cdot \vec{\alpha} = 0$$

$$\Rightarrow 4(1-4\lambda) + 3(2-3\lambda) - 5(5\lambda+4) = 0$$

$$\Rightarrow 4 - 16\alpha + 6 - 9\lambda - 25\lambda - 20 = 0$$

$$\Rightarrow 50\lambda = -10$$

$$\Rightarrow \lambda = \frac{-1}{5}$$

$$\vec{\beta}_2 = \left(1 + \frac{4}{5}\right)\hat{i} + \left(2 + \frac{3}{5}\right)\hat{j} - (-1+4)\hat{k}$$

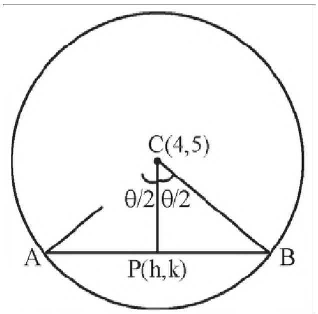
$$\vec{\beta}_2 = \frac{9}{5}\hat{i} + \frac{13}{5}\hat{j} - 3\hat{k}$$

$$5\vec{\beta}_2 = 9\hat{i} + 13\hat{j} - 15\hat{k}$$

$$5\vec{\beta}_2 \cdot (\hat{i} + \hat{j} + \hat{k}) = 9 + 13 - 15 = 7$$

74. D

Sol. In ΔCPB



$$\cos\frac{\theta}{2} = \frac{PC}{2} \Rightarrow PC = 2\cos\frac{\theta}{2}$$

$$\Rightarrow (h-4)^2 + (k-5)^2 = 4\cos^2\frac{\theta}{2}$$

$$\text{Now } (x-4)^2 + (y-5)^2 = \left(2\cos\frac{\theta}{2}\right)^2$$

$$\Rightarrow r_1 = 2\cos\frac{\pi}{6} = \sqrt{3}$$

$$r_2 = 2\cos\frac{\theta_2}{2}$$

$$r_3 = 2\cos\frac{\pi}{3} = 1$$

$$\Rightarrow 3 = 4\cos^2\frac{\theta_2}{2} + 1$$

$$\Rightarrow 4\cos^2\frac{\theta_2}{2} = 2$$

$$\Rightarrow \cos^2\frac{\theta_2}{2} = \frac{1}{2}$$

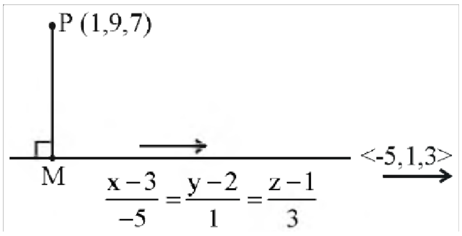
$$\Rightarrow \theta_2 = \frac{\pi}{2}$$

75. D

Sol. Direction ratio of line $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{vmatrix}$

$$= \hat{i}(-5) - \hat{j}(-1) + \hat{k}(3)$$

$= -5\hat{i} + \hat{j} + 3\hat{k}$



$M(-5\lambda + 3,\ \lambda + 2,\ 3\lambda + 1)$

$\overrightarrow{PM} \perp (-5\hat{i} + \hat{j} + 3\hat{k})$

$-5(-5\lambda + 2) + (\lambda - 7) + 3(3\lambda - 6) = 0$

$\Rightarrow 25\lambda + \lambda + 9\lambda - 10 - 7 - 18 = 0$

$\Rightarrow \lambda = 1$

Point M = $(-2, 3, 4) = (\alpha, \beta, \gamma)$

$\alpha + \beta + \gamma = 5$

76. C

Sol. $(x^2 - 3y^2)\,dx + 3xy\,dy = 0$

$$\frac{dy}{dx} = \frac{3y^2 - x^2}{3xy} \Rightarrow \frac{dy}{dx} = \frac{y}{x} - \frac{1}{3}\frac{x}{y} \quad (1)$$

Put y = vx

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$(1) \Rightarrow v + x\frac{dv}{dx} = v - \frac{1}{3}\frac{1}{v}$$

$$\Rightarrow vdv = \frac{-1}{3x}$$

Integrating both side

$$\frac{v^2}{2} = \frac{-1}{3}\ln x + c$$

$$\Rightarrow \frac{y^2}{2x^2} = \frac{-1}{3}\ln x + c$$

y(1) = 1

$$\Rightarrow \frac{1}{2} = c$$

$$\Rightarrow \frac{y^2}{x^2} = \frac{-1}{3}\ln x + \frac{1}{2}$$

$$\Rightarrow y^2 = -\frac{2}{3}x^2 \ln x + x^2$$

$$y^2(e) = -\frac{2}{3}e^2 + e^2 = \frac{e^2}{3}$$

$$\Rightarrow 6y^2(e) = 2e^2$$

77. C

Sol. $\sim(p \wedge (p \to \sim q))\ \sim(p \wedge (p \to \sim q))$

$\equiv \sim p \vee \sim(\sim p \vee \sim q)$

$\equiv \sim p \vee (p \wedge q)$

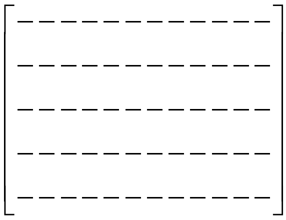
$\equiv (\sim p \vee p) \wedge (\sim p \vee q)$

$\equiv t \wedge (\sim p \vee q)$

$\equiv \sim p \vee q$

78. B

Sol.



In each row and each column exactly one is to be placed -

$\therefore$ No. of such materials = 5 × 4 × 3 × 2 × 1 = 120

Alternate :

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \longrightarrow 5 \text{ ways} \\ \longrightarrow 4 \text{ ways} \\ \longrightarrow 3 \text{ ways} \\ \longrightarrow 2 \text{ ways} \\ \longrightarrow 1 \text{ ways} \end{matrix}$$

Step - 1 : Select any 1 place for 1's in row 1. Automatically some column will get filled with 0's.

Step - 2 : From next now select 1 place for 1's.

Automatically some column will get filled with 0's.

$\Rightarrow$ Each time one less place will be available for putting 1's.

Repeat step-2 till last row.

Req. ways = 5 × 4 × 3 × 2 × 1 = 120

79. D

Sol. $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}dx$

We have $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a}+C$

Hence $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}}dx = \frac{48}{2}\times\left[\sin^{-1}\frac{2x}{3}\right]_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}}$

$= 24\times\left[\sin^{-1}\left(\frac{2}{3}\times\frac{3\sqrt{3}}{4}\right)-\sin^{-1}\left(\frac{2}{3}\times\frac{3\sqrt{2}}{4}\right)\right]$

$= 24\times\left[\sin^{-1}\frac{\sqrt{3}}{2}-\sin^{-1}\frac{1}{\sqrt{2}}\right]$

$= 24\times\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = 24\times\frac{\pi}{12} = 2\pi$

80. A

Sol. Given $|adj\ (adj\ (adj\ .A))| = 12^4$

$\Rightarrow |A|^{(n-1)^3} = 12^4$

Given n = 3

$\Rightarrow |A|^8 = 12^4 \Rightarrow |A|^2 = 12$

$|A| = 2\sqrt{3}$

We are asked

$|A^{-1}\cdot adj\ A|$

$= |A^{-1}|\cdot|adj\ A| = \frac{1}{|A|}\cdot|A|^{3-1} = |A| = 2\sqrt{3}$

Section - B (Numerical Value)

81. 432

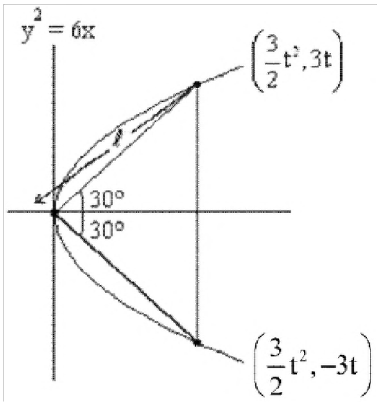
Sol.

Urn A		Urn B		Urn C	
Red	Black	Red	Black	Red	Black
4	6	5	5	λ	4

$$P\left(\frac{C}{R}\right) = \frac{P(C)P\left(\frac{R}{C}\right)}{P(A)P\left(\frac{R}{A}\right)+P(B)P\left(\frac{R}{B}\right)+P(C)P\left(\frac{R}{C}\right)}$$

$$0.4 = \frac{\frac{1}{3}\times\frac{\lambda}{(\lambda+4)}}{\frac{1}{3}\times\frac{4}{10}+\frac{1}{3}\times\frac{5}{10}+\frac{1}{3}\frac{\lambda}{(\lambda+4)}}$$

$\Rightarrow \lambda = 6$



$\tan 30° = 3t = \frac{3}{2}t^2$

$\frac{1}{\sqrt{3}} = \frac{2}{t}$

$t = 2\sqrt{3}$

$\left(\frac{3}{2}t^2,\ 3t\right) = (18,\ 6\sqrt{3})$

$\ell^2 = 18^2 + (6\sqrt{3})^2$

$= 324 + 108$

$= 432$

82. 36

Sol. $y^2 - 2y = -x$

$\Rightarrow y^2 - 2y + 1 = -x + 1$

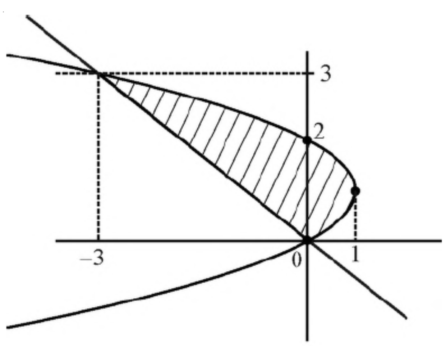
$(y-1)^2 = -(x-1)\ y = -x$

Points of intersection

$x^2 + 2x = -x$

$x^2 + 3x = 0$

$x = 0, -3$



$$A = \int_0^3 (-y^2 + 2y + y)dy$$

$$= \frac{3y^2}{2} - \frac{y^3}{3}\Big|_0^3 = \frac{9}{2}$$

8A = 36

83. 5

Sol. $1^3 + 2^3 + 3^3 ...... + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

1·3 + 2·5 + 3·7 + ... + n terms =

$$\sum_{r=1}^{n} r(2r+1) = \sum_{r=1}^{n}(2r^2 + r)$$

$$= \frac{2 \cdot n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{6}(2(2n+1)+3)$$

$$= \frac{n(n+1)}{2} \times \frac{(4n+5)}{3}$$

$$= \frac{\frac{n^2(n+1)^2}{4}}{\frac{n(n+1)}{2} \times \frac{(4n+5)}{3}} = \frac{9}{5}$$

$$\Rightarrow \frac{5n(n+1)}{2} = \frac{9(4n+5)}{3}$$

$\Rightarrow$ 15n (n + 1) = 18 (4n + 5)

$\Rightarrow$ $15n^2 + 15n = 72n + 90$

$\Rightarrow$ $15n^2 - 57n - 90 = 0 \Rightarrow 5n^2 - 19n - 30 = 0$

$\Rightarrow$ (n – 5) (5n + 6) = 0

$$\Rightarrow n = \frac{-6}{5} \text{ or } 5$$

$$\Rightarrow n = 5.$$

84. 27

Sol. $f(x) + \int_0^x f(t)\sqrt{1 - (\log_e f(t))^2}\,dt = e$

$\Rightarrow$ f(0) = e

$$f'(x) + f(x)\sqrt{1 - (\ln f(x))^2} = 0$$

f(x) = y

$$\frac{dy}{dx} = -y\sqrt{1 - (\ln y)^2}$$

$$\int \frac{dy}{y\sqrt{1 - (\ln)^2}} = -\int dx$$

Put ln y = t

$$\int \frac{dt}{\sqrt{1 - t^2}} = -x + C$$

$\sin^{-1}t = -x + C \Rightarrow \sin^{-1}(\ln y) = -x + C$

$\sin^{-1}(\ln f(x)) = -x + C$

f(0) = e

$$\Rightarrow \frac{\pi}{2} = C$$

$$\Rightarrow \sin^{-1}(\ln f(x)) = -x + \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}\left(\ln f\left(\frac{\pi}{6}\right)\right) = \frac{-\pi}{6} + \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}\left(\ln f\left(\frac{\pi}{6}\right)\right) = \frac{\pi}{3}$$

$$\Rightarrow \ln f\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}, \text{ we need } \left(6 \times \frac{\sqrt{3}}{2}\right)^2 = 27.$$

85. 13

Sol. Given R = {(a, b), (b, c), (b, d)}

In order to make it equivalence relation as per given set, R must be

{(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (b, c), (c, b), (b, d), (d, b), (a, c), (a, d), (c, d), (d, c), (c, a), (d, a)}

There already given so 13 more to be added.

86. 8

Sol. $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k},\ \vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k},\ \vec{a} \cdot \vec{c} = 7$

$\vec{a} \times \vec{c} - \vec{b} \times \vec{c} = \vec{0},$

$(\vec{a} - \vec{b}) \times \vec{c} = 0 \Rightarrow (\vec{a} - \vec{b})$ is parallel to $\vec{c}$

$\vec{a} - \vec{b} = \mu\vec{c}$, where μ is a scalar

$-2\hat{i} + 7\hat{j} + 2\lambda\hat{k} = \mu \cdot \vec{c}$

Now $\vec{a}\cdot\vec{c}=7$ gives $2\lambda^2+12=7\mu$

And $\vec{b}\cdot\vec{c}=-\frac{43}{2}$ gives $4\lambda^2+82=43\mu$

$\mu=2$ and $\lambda^2=1$

$|\vec{a}\cdot\vec{b}|=8$

87. 405

Sol. Given Binomilal $\left(x-\frac{3}{x^2}\right)^n$, $x\neq 0,\ n\in N,$

Sum of coefficients of first three terms

${}^nC_0 - {}^nC_1\cdot 3 + {}^nC_2 3^2 = 376$

$\Rightarrow 3n^2 - 5n - 250 = 0$

$\Rightarrow (n-10)(3n+25) = 0$

$\Rightarrow n = 10$

Now general term ${}^{10}C_r x^{10-r}\left(\frac{-3}{x^2}\right)^r$

$= {}^{10}C_r x^{10}\ (-3)^r\cdot x^{-2r}$

$= {}^{10}C_r(-3)\cdot\ x^{10-3r}$

Coefficient of $x^4 \Rightarrow 10-3r = 4$

$\Rightarrow r = 2$

${}^{10}C_2(-3)^2 = 405$

88. 384

Sol. Shortest distance between the lines

$$\frac{x+\sqrt{6}}{2}=\frac{y-\sqrt{6}}{3}=\frac{z-\sqrt{6}}{4}$$

$$\frac{x-\lambda}{3}=\frac{y-2\sqrt{6}}{4}=\frac{2+2\sqrt{6}}{5}\ \text{is } 6$$

Vector along line of shortest distance

$=\begin{vmatrix} i & j & k\\ 2 & 3 & 4\\ 3 & 4 & 5\end{vmatrix}$, $\Rightarrow -\hat{i}+2\hat{j}-\hat{k}$ (its magnitude is $\sqrt{6}$)

Now $\frac{1}{\sqrt{6}}\begin{vmatrix}\sqrt{6}+\lambda & \sqrt{6} & -3\sqrt{6}\\ 2 & 3 & 4\\ 3 & 4 & 5\end{vmatrix}=\pm 6$

$\Rightarrow \lambda=-2\sqrt{6},\ 10\sqrt{6}$

So, square of sum of these values is 384.

89. 2

Sol. $\tan(\pi\cos\theta)+\tan(\pi\sin\theta)=0$

$\tan(\pi\cos\theta)=-\tan(\pi\sin\theta)$

$\tan(\pi\cos\theta)=\tan(-\pi\sin\theta)$

$\pi\cos\theta=n\pi-\pi\sin\theta$

$\sin\theta+\cos\theta=n$ where $n\in I$

possible values are n = 0, 1 and –1 because

$-\sqrt{2}\le\sin\theta+\cos\theta\le\sqrt{2}$

Now it gives $\theta\in\left\{0,\ \frac{\pi}{2},\ \frac{3\pi}{4},\ \frac{7\pi}{4},\ \frac{3\pi}{2},\ \pi\right\}$

So $\sum_{\theta\in S}\sin^2\left(\theta+\frac{\pi}{4}\right)=2(0)+4\left(\frac{1}{2}\right)=2$

90. 122

Sol. Assume $B(\alpha,\ -2\alpha)$ and $C(\beta+3,\ \beta)$

$\frac{\alpha+\beta+3+1}{3}=2$ also $\frac{-2\alpha-2+\beta}{3}=a$

$\Rightarrow\alpha+\beta=2$ $\quad -2\alpha-2+\beta=3a$

$\Rightarrow\beta=2-\alpha$ $\quad -2\alpha-\cancel{2}+\cancel{2}-\alpha=3a\Rightarrow\alpha=-a$

Now both B and C lies as given line

$\alpha-p\cdot 2\alpha=21a$

$\alpha(1-2p)=21a$ ....(1)

$-\alpha(1-2p)=21a\Rightarrow p=11$

$\beta+3+p\beta=21a$

$\beta+3+11\beta=21a$

$21\alpha+12\beta+3=0$

Also $\beta=2-\alpha$

$21\alpha+12(2-\alpha)+3=0$

$21\alpha+24-12\alpha+3=0$

$9\alpha+27=0$ $\quad \alpha=-3,\ \beta=5$

So $BC=\sqrt{122}$ and $(BC)^2 = 122$

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