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11-April-2023 (Morning Batch) : JEE Main Paper

MATHEMATICS

Section - A (Single Correct Answer)

1. D

Sol. $I=\int_{-\ln 2}^{\ln 2} e^x\left(\ln\left(e^x+\sqrt{1+e^{2x}}\right)\right)dx$

Put $e^x = t \Rightarrow e^x dx = dt$

$$I=\int_{1/2}^{2}\ln\left(t+\sqrt{1+t^2}\right)dt$$

Applying integration by parts.

$$=\left[t\ln\left(t+\sqrt{1+t^2}\right)\right]_{\frac{1}{2}}^{2}-\int_{1/2}^{2}\frac{t}{t+\sqrt{1+t^2}}\left(1+\frac{2t}{2\sqrt{1+t^2}}\right)dt$$

$$=2\ln(2+\sqrt{5})-\frac{1}{2}\ln\left(\frac{1+\sqrt{5}}{2}\right)-\int_{1/2}^{2}\frac{t}{\sqrt{1+t^2}}dt$$

$$=2\ln(2+\sqrt{5})-\frac{1}{2}\ln\left(\frac{1+\sqrt{5}}{2}\right)-\frac{\sqrt{5}}{2}$$

$$=\ln\left(\frac{(2+\sqrt{5})^2}{\left(\frac{\sqrt{5}+1}{2}\right)^{\frac{1}{2}}}\right)-\frac{\sqrt{5}}{2}$$

2. D

Sol. The equation of plane through (–2, 3, 5) is

$a(x + 2) + b(y – 3) + c(z – 5) = 0$

it is perpendicular to $2x + 4y + 5z = 8$ & $3x – 2y + 3z = 5$

$\therefore\ 2a + 4b + 5c = 0$

$3a – 2b + 3c = 0$

$$\therefore \frac{a}{\begin{vmatrix}4 & 5\\ -2 & 3\end{vmatrix}}=\frac{-b}{\begin{vmatrix}2 & 5\\ 3 & 3\end{vmatrix}}=\frac{c}{\begin{vmatrix}2 & 4\\ 3 & -2\end{vmatrix}}$$

$$\Rightarrow \frac{a}{22}=\frac{b}{9}=\frac{c}{-16}$$

$\therefore$ Equation of Plane is

$22(x + 2) + 9(y – 3) – 16(z – 5) = 0$

$\Rightarrow 22x + 9y – 16z + 97 = 0$

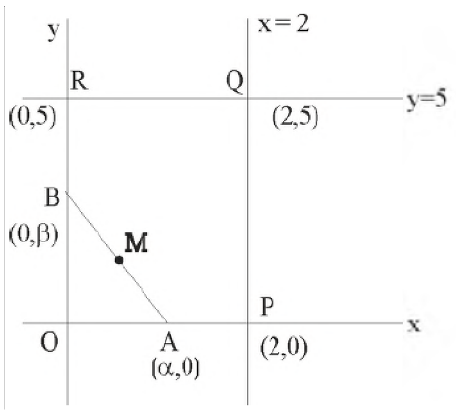
Comparing with $\alpha x+\beta y+\gamma x+97=0$

We get $\alpha+\beta+\gamma=22+9-16=15$

3. B

Sol. $\frac{ar(OPQR)}{or(OAB)}=\frac{4}{1}$

Let M be the mid-point of AB.



$$M(h,\ k)\equiv\left(\frac{\alpha}{2},\ \frac{\beta}{2}\right)$$

$$\Rightarrow \frac{10-\frac{1}{2}\alpha\beta}{\frac{1}{2}\alpha\beta}=4$$

$$\Rightarrow \frac{5}{2}\alpha\beta=10\Rightarrow\alpha\beta=4$$

$\Rightarrow (2h)(2K) = 4$

$\therefore$ Locus of M is $xy = 1$

Which is a hyperbola.

4. B

Sol. ω $A = \{a_1, a_2, a_3, a_4, a_5\}$

$B = \{b_1, b_2, b_3, b_4, b_5\}$

Given, $\sum_{i=1}^{5} ai = 25,\ \sum_{i=1}^{5} bi = 40$

$$\frac{\sum_{i=1}^{5} a_i^2}{5} - \left(\frac{\sum_{i=1}^{5} a_i}{5}\right)^5 = 12,\ \frac{\sum_{i=1}^{5} b_i^2}{5} - \left(\frac{\sum_{i=1}^{5} b_i}{5}\right)^2 = 20$$

$$\Rightarrow \sum_{i=1}^{5} a_i^2 = 185,\ \sum_{i=1}^{5} b_i^2 = 420$$

Now, $C = \{C_1, C_2, \ldots\ldots C_{10}\}$

s.f. $C_i = a_i = 3$ or $b_i + 2$

$$\therefore \text{ Mean of C, } \bar{C} = \frac{(\Sigma a_i - 15) + (\Sigma b_i + 10)}{10}$$

$$\bar{C} = \frac{10 + 50}{10} = 6$$

$$\therefore \sigma^2 = \frac{\sum_{i=1}^{10} C_i^2}{10} (\bar{C})^2$$

$$= \frac{\sum (a_i - 3)^2 + \sum (b_i + 2)^2}{10} - (6)^2$$

$$= \frac{\sum a_i^2 + \sum b_i^2 - 6\sum a_i + 4\sum b_i + 65}{10} - 36$$

$$= \frac{185 + 420 - 150 + 160 + 65}{10} - 36$$

$= 32$

$\therefore$ Mean + Variance $= \bar{C} + \sigma^2 = 6 + 32 = 38$

5. D

Sol. Here $f(x) = [x(x-1)] + \{x\}$

$f(o^+) = -1 + 0 = -1 f(1^+) = 0 + 0 = 0$

$f(0) = 0$ $\qquad$ $f(1) = 0$

$f(1^-) = -1 + 1 = 0$

$\therefore$ f(x) is continous at x = 1, discontinuous at x = 0

6. B

Sol. $x + y + z = 15$

Total no. of solution $= {}^{15+3-1}C_{3-1} = 136$ ....(1)

Let $x = y \neq z$

$2x + z = 15 \Rightarrow z = 15 - 2t$

$\Rightarrow r \in (0, 1, 2, \ldots.7) - \{5\}$

$\therefore$ 7 solutions

$\therefore$ there are 21 solutions in which exactly

Two of $x_1 y_1 z$ are equal ....(2)

There is one solution in which x = y = z ....(3)

Required answer = 136 – 21 – 1 = 114

7. D

Sol. Without loss of generality

Let $|a_1| \leq |a_2| \leq |a_3|$

$$|\vec{a}|^2 = |a_1|^2 + |a_2|^2 + |a_3|^2 \geq (a_3)^2$$

$$\Rightarrow |\vec{a}| \geq |a_3| = \max\{|a_1|,\ |a_2|,\ |a_3|\}$$

A is true

$$|\vec{a}|^2 = |a_1|^2 + |a_2|^2 + |a_3|^2 \leq |a_3|^2 + |a_3|^2 + |a_3|^2$$

$$\Rightarrow |\vec{a}|^2 \leq 3|a_3|^2$$

$$\Rightarrow |\vec{a}| \leq \sqrt{3}|a_3| = \sqrt{3}\max\{|a_1|,\ |a_2|,\ |a_3|\}$$

$$\leq 3\max\{|a_1|,\ |a_2|,\ |a_3|\}$$

(2) is true

8. D

Sol. $W_1 = z_1 i = (5 + 4i)i = -4 + 5i$ ....(1)

$W_2 = z_2(-i) = (3 + 5i)(-i) = 5 - 3i$ ....(2)

$W_1 - W_2 = -9 + 8i$

Principal argument $= \pi - \tan^{-1}\left(\frac{8}{9}\right)$

9. C

Sol. $|A| = 48$

$|B| = 25$

$|C| = 18$

$|A \cup B \cup C| = 60$ [Total]

$|A \cap B \cap C| = 5$

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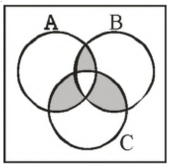


$$|A \cup B \cup C| = \sum|A| - \sum|A \cap B| + |A \cap B \cap C|$$

$$\Rightarrow \sum|A \cap B| = 48 + 25 + 18 + 5 - 60$$

= 36

No. of mean who received exactly 2 medals

$$= \sum|A \cap B| - 3|A \cap B \cap C|$$

= 36 – 15

= 21

10. A

Sol. $M\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where a, b, c, d, $\in$ {0, 1, 2}

n(s) = 3^4 = 81

we first bound $p(\bar{A})$

|m| = 0 $\Rightarrow$ ad = bc

ad = bc = 0 $\Rightarrow$ no. of (a, b, c, d) = $(3^2 - 2^2)^2$ = 25

ad = bc = 1 $\Rightarrow$ no. of (a, b, c, d) = 12 = 1

ad = bc = 2 $\Rightarrow$ no. of (a, b, c, d) = 22 = 4

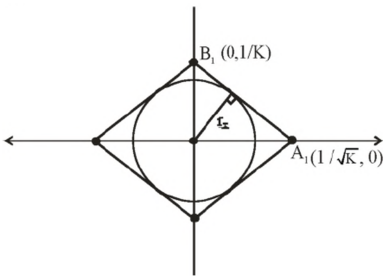
ad = bc = 4 $\Rightarrow$ no. of (a, b, c, d) = 12 = 1

$$: P(\bar{A}) = \frac{31}{81} \Rightarrow p(A) = \frac{50}{81}$$

11. A

Sol. $Kx^2 + K^2y^2 = 1$

$$\frac{x^2}{1/K} + \frac{y^2}{1/K^2} = 1$$



Equation of

$$A_1B_2; \frac{x}{1/\sqrt{K}} + \frac{y}{1/K} = 1 \Rightarrow \sqrt{K}x + Ky = 1$$

$r_K = \perp r$ distance of (0, 0) from line A_1B_1

$$r_K = \left|\frac{(0+0-1)}{\sqrt{K+K^2}}\right| = \frac{1}{\sqrt{K+K^2}}$$

$$\frac{1}{r_K^2} = K + K^2 \Rightarrow \sum_{k=1}^{20}\frac{1}{r_K^2} = \sum_{K=1}^{20}(K+K^2)$$

$$= \sum_{K=1}^{20}K + \sum_{K=1}^{20}K^2$$

$$= \frac{20 \times 21}{2} + \frac{20.21.41}{6}$$

= 210 + 10 × + 7 × 41

= 210 + 2870

= 3080

12. A

Sol. $\log_{x+\frac{7}{2}}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$

Feasible region : $x + \frac{7}{2} > 0 \Rightarrow x > -\frac{7}{2}$

And $x + \frac{7}{2} \neq 1 \Rightarrow x \neq -\frac{5}{2}$

And $\frac{x-7}{2x-3} \neq 0$ and $2x - 3 \neq 0$

$\Downarrow$ $\Downarrow$

$x \neq 7$ $x \neq \frac{3}{2}$

Taking intersection :

$$x \in \left(\frac{-7}{2}, \infty\right) - \left\{-\frac{5}{2}, \frac{3}{2}, 7\right\}$$

Now $\log_a b \geq 0$ if a > 1 and $b \geq 1$

Or

$a \in (0, 1)$ and $b \in (0, 1)$

$$C-I;\ x+\frac{7}{2}>1 \text{ and } \left(\frac{x-7}{2x-3}\right)^2 \geq 1$$

$$x>-\frac{5}{2}\quad (2x-3)^2-(x-7)^2\leq 0$$

$$(2x-3+n-7)(2x-3-x+7)\leq 0$$

$$(3x-10)(x+4)\leq 0$$

$$x\in\left[-4,\ \frac{10}{3}\right]$$

Intersection : $x\in\left(\frac{-5}{2},\ \frac{10}{3}\right]$

$$C-II\ x+\frac{7}{2}\in(0,\ 1) \text{ and } \left(\frac{x-7}{2x-3}\right)^2\in(0,\ 1)$$

$$0<x+\frac{7}{2}<1\quad \left(\frac{x-7}{2x-3}\right)^2<1$$

$$-\frac{7}{2}<x<\frac{-5}{2}\quad (x-7)^2<(2x-3)^2$$

$$x\in(-\infty,\ -4)\cup\left(\frac{10}{3},\ \infty\right)$$

No common values of x.
Hence intersection with feasible region

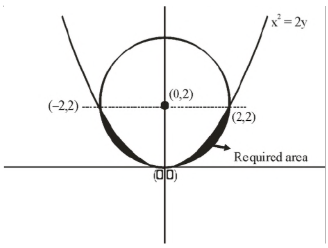
We get $x\in\left(\frac{-5}{2},\ \frac{10}{3}\right]-\left\{\frac{3}{2}\right\}$

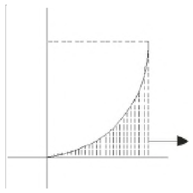
Integral value of x are {–2, –1, 0, 1, 2, 3}
No. of integral values = 6

13. A

Sol. $x^2+(y-2)^2\leq 2^2$ and $x^2\geq 2y$

Solving circle and parabola simultaneously :
$2y + y^2 - 4y + 4 = 4$
$y^2 - 2y = 0$
$y = 0, 2$
Put y = 2 in $x^2 = 2y$ $\rightarrow x=\pm 2$
$\Rightarrow$ (2, 2) and (–2, 2)





$$=2\times 2-\frac{1}{4}\cdot\pi\cdot 2^2=4-\pi$$

Required area $=2\left[\int_0^2 \frac{x^2}{2}dx-(4-\pi)\right]$

$$=2\left[\left.\frac{x^3}{6}\right|_0^2-4+\pi\right]$$

$$=2\left[\frac{4}{3}+\pi-4\right]$$

$$=2\left[\pi-\frac{8}{3}\right]$$

$$=2\pi-\frac{16}{6}$$

14. C

Sol. $x\ell nxf'(x)+\ln x\ f(x)+f(x)\geq I,\ x\in[2,\ 4]$

And $f(2)=\frac{1}{2},\ f(4)=\frac{1}{4}$

Now $x\ln x\frac{dy}{dx}+(\ln+1)y\geq 1$

$$\frac{d}{dx}(y\cdot x\ln x)\geq 1$$

$$\frac{d}{dx}(f(x)\cdot x\ln x)\geq 1$$

$$\Rightarrow \frac{d}{dx}(x\ln x\ f(x)-x)\geq 0,\ x\in[2,\ 4]$$

$\Rightarrow$ The function g(x) = x ln x f(x) – x is increasing in [2.4]
And g(2) = 2 ln 2f(2) – 2 = ln 2 – 2
g(4) = 4 ln 4f(4) – 4 = ln 4 – 4
= 2(ln 2 – 2)

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Now $g(2) \le g(x) \le g(4)$

$$\ln 2 - 2 \le x \ln f(x) - x \le 2(\ln 2 - 2)$$

$$\frac{\ln 2 - 2}{x \ln x} + \frac{1}{\ln x} \le f(x) \le \frac{2(\ln 2 - 2)}{x \ln x} + \frac{1}{\ln x}$$

Now for $x \in [2, 4]$

$$\frac{2(\ell n 2 - 2)}{x \ln x} + \frac{1}{\ell n x} < \frac{2(\ln 2 - 2)}{2 \ln 2} + \frac{1}{\ln 2} = 1 - \frac{1}{\ln 2} < 1$$

$\Rightarrow f(x) \le 1$ for $x \in [2, 4]$

Aslo for $x \in [2, 4]$:

$$\frac{\ln 2 - 2}{x \ln x} + \frac{1}{\ln x} \ge \frac{\ln 2 - 2}{4 \ln 4} + \frac{1}{\ln 4} = \frac{1}{8} + \frac{1}{2\ln 2} > \frac{1}{8}$$

$\Rightarrow f(x) \ge \frac{1}{8}$ for $x \in [2, 4]$

Hence both A and B are true.

LMVT on (yx (ln x)) not satisfied.

Hence no such function exists.

Therefore it should be bonus.

15. D

Sol. $(1 - x^2y^2)dx = y\,dx + x\,dy$, $y(1) = 2$

$y(2) = \propto = ?$

$$dx = \frac{d(xy)}{1 - (xy)^2}$$

$$\int dx = \int \frac{d(xy)}{1 - (xy)^2}$$

$$x = \frac{1}{2} \ln \left| \frac{1 + xy}{1 - xy} \right| + C$$

Put x = 1 and y = 2

$$1 = \frac{1}{2} \ln \left| \frac{1+2}{1-2} \right| + C$$

$$C = 1 - \frac{1}{2} \ln 3$$

Now put x = 2 :

$$2 = \frac{1}{2} \ln \left| \frac{1+2\alpha}{1-2\alpha} \right| + 1 - \frac{1}{2} \ln 3$$

$$1 + \frac{1}{2} \ln 3 = \frac{1}{2} \ln \left| \frac{1+2\alpha}{1-2\alpha} \right|$$

$$2 + \ln 3 = \ln \left(\frac{1+2\alpha}{1-2\alpha} \right)$$

$$\left| \frac{1+2\alpha}{1-2\alpha} \right| = 3e^2$$

$$\frac{1+2\alpha}{1-2\alpha} = 3e^2,\ -3e^2$$

$$\frac{1+2\alpha}{1-2\alpha} = 3e^2 \Rightarrow \alpha = \frac{3e^2+1}{2(3e^2-1)}$$

And $\frac{1+2\alpha}{1-2\alpha} = -3e^2 \Rightarrow \alpha = \frac{3e^2+1}{2(3e^2-1)}$

16. C

Sol. $A^T = \alpha A + I$

$$A = \alpha A^T + I$$

$$A = \alpha(\alpha A + I) + I$$

$$A = \alpha^2 A + (\alpha + 1)I$$

$$A(1 - \alpha^2) = (\alpha + 1)I$$

$$A = \frac{I}{1-\alpha} \quad ....(1)$$

$$|A| = \frac{1}{(1-\alpha)^2} \quad ....(2)$$

$|A^2 - A| = |A|\,|A - 1|$ ....(3)

$$A - I = \frac{I}{1-\alpha} - I = \frac{\alpha}{1-\alpha} I$$

$$|A - I| = \left(\frac{\alpha}{1-\alpha} \right)^2 \quad ....(4)$$

Now $|A^2 - A| = 4$

$|A|\,|A - I| = 4$

$$\Rightarrow \frac{1}{(1-\alpha)^2} \frac{\alpha^2}{(1-\alpha)^2} = 4$$

$$\Rightarrow \frac{\alpha}{(1-\alpha)} = \pm 2$$

$\Rightarrow 2(1-\alpha)^2 = \pm\alpha$

$(C_1) 2(1-\alpha)^2 = \alpha \quad (C_2) 2(1-\alpha)^3 = -\alpha$

$2\alpha^2 - 5\alpha + 2 = 0 <^{\alpha_1}_{\alpha_2} \quad 2\alpha^2 - 3\alpha + 2 = 0$

$\alpha \notin R$

$\alpha_1 + \alpha_2 = \frac{5}{2}$

Sum of value of $\alpha = \frac{5}{2}$

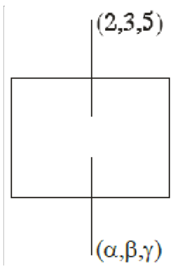
17. A

Sol. $\frac{\alpha-2}{2} = \frac{\beta-3}{1} = \frac{\gamma-5}{-3} = -2\left(\frac{2x^2+3-3\times5-6}{2^2+1^2+1-3^2}\right) = 2$

$\frac{\alpha-2}{2} = 2 \qquad \beta - 3 = 2 \qquad \gamma - 5 = -6$

$\beta = 5 \qquad \gamma = -1$

$\alpha = 6$



$\alpha + \beta + \gamma = 10$

18. D

Sol. $\vec{n}_1$ and $\vec{n}_2$ are normal vector to the plane $\hat{i}+\hat{j},\ \hat{i}+\hat{k}$ and $\hat{i}-\hat{j};\ \hat{j}-\hat{k}$ respectively

$$\vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{j} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \hat{i} - \hat{j} - \hat{k}$$

$$\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{j} \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = \hat{i} + \hat{j} + \hat{k}$$

$\vec{a} = \lambda|\vec{n}_2 \times \vec{n}_2|$

$$= \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = \lambda(-2\hat{j} + 2\hat{k})$$

$\vec{a}\cdot\vec{b} = \lambda|0+4+2| = 6$

$\Rightarrow \lambda = 1$

$\vec{\alpha} = -2\hat{j} + 2\hat{k}$

$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|a||b|}$

$\cos\theta = \frac{6}{2\sqrt{2}\times3} = \frac{1}{\sqrt{2}}$

$\theta = \frac{\pi}{4}$

Now $|\vec{a}\cdot\vec{b}|^2 + |\vec{a}\times\vec{b}|^2 = |a|^2|b|^2$

$36 + |\vec{a}\times b^2| = 8\times9 = 72$

$|\vec{a}\times b|^2 = 36$

$|\vec{a}\times\vec{b}| = 6$

19. C

Sol. $3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0$

$\Rightarrow 3\cos^4\theta - 3\cos^2\theta - 2\cos^2\theta - 2\sin^6\theta + 2 = 0$

$\Rightarrow 3\cos^4\theta - 3\cos^2\theta + 2\sin^2\theta - 2\sin^6\theta = 0$

$\Rightarrow 3\cos^2\theta(\cos^2\theta - 1) + 2\sin^2\theta(\sin^4\theta - 1) = 0$

$\Rightarrow -3\cos^2\theta\sin^2\theta + 2\sin^2\theta(1+\sin^2\theta)\cos^2\theta - 1$

$\Rightarrow \sin^2\theta\cos^2(2 + 2\sin^2\theta - 3) = 0$

$\Rightarrow \sin^2\theta\cos^2\theta(2\sin^2\theta - 1) = 0$

(C1) $\sin^2\theta = 0 \rightarrow 3$ solution ; $\theta = \{0,\ \pi,\ 2\pi\}$

(C2) $\cos^2\theta = 0 \rightarrow 2$ solution ; $\theta = \left\{\frac{\pi}{2},\ \frac{3\pi}{2}\right\}$

(C3) $\sin^2\theta = \frac{1}{2} \rightarrow 4$ solution ; $\theta = \left\{\frac{\pi}{4},\ \frac{3\pi}{4},\ \frac{5\pi}{4},\ \frac{7\pi}{4}\right\}$

No. of solution = 9

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20. C

Sol. Mean = 200

$$\Rightarrow \frac{\frac{100}{2}(2\times+99d)}{100}=200$$

$\Rightarrow$ 4 + 99d = 400

$\Rightarrow$ d = 4

$y_i = (xi - i)$

$= i(2 + (i - 1)4 - i) = 3i^2 - 2i$

$$\text{Mean}=\frac{\sum y_i}{100}$$

$$=\frac{1}{100}\sum_{i=1}^{100}3i^2-2i$$

$$=\frac{1}{100}\left\{\frac{3\times100\times101\times201}{6}-\frac{2\times100\times101}{2}\right\}$$

$$=101\left\{\frac{201}{2}-1\right\}=101\times99.5$$

$= 10049\cdot50$

Section - B (Numerical Value)

21. 2736

Sol. Coefficient of x = $^9C_1 2^8$

Of x^2 = $^9C_2 2^7$

Of x^7 = $^9C_7\cdot 2^2$

$$\text{Mean}=\frac{^9C_1\cdot2^8+\,^9C_2\cdot2^7......+\,^9C_7\cdot2^2}{7}$$

$$=\frac{(1+2)^9-\,^9C_0\cdot2^9-\,^9C_8\cdot2^1-\,^{-9}C_9}{7}$$

$$=\frac{3^9-2^9-18-1}{7}$$

$$=\frac{19152}{7}=2736$$

22. 2175

Sol. $S=109+\frac{108}{5}+\frac{107}{5^2}.....+\frac{1}{5^{108}}$

$$\frac{S}{4S}=\frac{109}{5}+\frac{108}{5^2}......+\frac{2}{5^{108}}+\frac{1}{5^{109}}$$

$$\frac{4S}{5}=109-\frac{1}{5}-\frac{1}{5^2}....-\frac{1}{5^{108}}-\frac{1}{5^{109}}$$

$$=109-\left(\frac{1}{5}\frac{\left(1-\frac{1}{5^{109}}\right)}{\left(1-\frac{1}{5}\right)}\right)$$

$$=109-\frac{1}{4}\left(1-\frac{1}{5^{109}}\right)$$

$$=109-\frac{1}{4}+\frac{1}{4}\times\frac{1}{5^{109}}$$

$$s=\frac{5}{4}\left(109-\frac{1}{4}+\frac{1}{4.5^{109}}\right)$$

$$16S=20\times109-5+\frac{1}{5^{108}}$$

$16S - (25)^{-54} = 2180 - 5 = 2175$

23. 32

Sol. $\alpha(m,\ n)=\int_0^2 t^m(1+3t)^n dt$

If $11\alpha(10,\ 6)+18\alpha(11,\ 5)=p(14)^6$ then P

$$=11\int_0^2 \underset{II}{t^{10}}\,\underset{I}{(1+3t)^6}+10\int^2 t^{11}(3t)^5 dt$$

$$=11\left[(t+3t)^6\cdot\frac{t^{11}}{11}-\int 6(1+3t)^3\cdot3\frac{t^{11}}{11}\right]_0^2+t^{11}(1+3t)^5 dt$$

$$=(t^{11}(1+3t)^6)_0^2$$

$= 2^{11}\ (7)^6$

$= 2^5\ (14)^6$

$= 32\ (14)^6$

24. 44

Sol. Derangement of 5 students

$$D_5=5!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right)$$

$$=120\left(\frac{1}{2}-\frac{1}{6}+\frac{1}{24}-\frac{1}{120}\right)$$

= 60 – 20 + 5 – 1

= 40 + 4

= 44

25. 5

Sol. Let $\ell = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \gamma(a\hat{i} + b\hat{j} + c\hat{k})$

$= \gamma(a\hat{i} + b\hat{j} + c\hat{k})$

$$a\hat{i} + b\hat{j} + c\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 2 & 1 \end{vmatrix}$$

$= \hat{i}(2-6) - \hat{j}(1-6) + \hat{k}(2-4)$

$= -4\hat{i} - 5\hat{j} - 2\hat{k}$

$\ell = \gamma(-4\hat{i} + 5\hat{j} - 2\hat{k})$

P is intersection of ℓ and ℓ_1

$-4\gamma = 1 + \lambda,\ 5\gamma = -11 + 2\lambda,\ -2\gamma = -7 + 3\lambda$

By solving there equation $\gamma = -1,\ P(4,\ -5,\ 2)$

Let $Q(-1 + 2\mu,\ 2\mu,\ 1 + \mu)$

$\overrightarrow{PQ} \cdot (2\hat{i} + 2\hat{j} + \hat{k}) = 0$

$-2 + 4\mu + 4\mu + 1 + \mu = 0$

$9\mu = 1$

$\mu = \frac{1}{9}$

$Q\left(\frac{-7}{9}, \frac{2}{9}, \frac{10}{9}\right)$

$9(\alpha + \beta + \gamma) = 9\left(\frac{-7}{9} + \frac{2}{9} + \frac{10}{9}\right)$

$= 5$

26. 171

Sol. The number of integral term in the expression of

$\left(3^{\frac{1}{2}} + 5^{\frac{1}{4}}\right)^{680}$ is equal to

General term $= {}^{680}C_r \left(3^{\frac{1}{2}}\right)^{680-r} \left(5^{\frac{1}{4}}\right)^r$

$= {}^{680}C_r 3^{\frac{680-r}{2}} 5^{\frac{r}{4}}$

Value's of r, where $\frac{r}{4}$ goes to integer

All value of r are accepted for $\frac{680-r}{2}$ as well

so No of integeral terms = 171.

27. 7

Sol.

p	q	r	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$	$q \vee r$	$(p \vee q) \wedge (p \vee r) \to q \vee r$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	F	T	F	T	T
F	F	F	F	F	F	F	T

Hence total no of ordered triplets are 7

28. 306

Sol. $Hn \Rightarrow \frac{x^2}{1+n} - \frac{y^2}{3+n} = 1$

$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{3+n}{1+n}} = \sqrt{\frac{2n+4}{n+1}}$

$e = \sqrt{\frac{2n+4}{n+1}}$

n = 48 (smallest even values for which $e \in Q$)

$e = \frac{10}{7}$

$a^2 = n + 1,\quad b^2 = n + 3$

$= 49 \qquad\qquad = 51$

1 = length of $LR = \frac{2b^2}{a}$

$L = 2 \cdot \frac{51}{7}$

$$1=\frac{102}{7}$$

$$21\ell=306$$

29. 51

Sol. $x^2-7x-1=0<_b^a$

By newton's theorem

$S_{n+2}-7S_{n+1}-S_n=0$

$S_{21}-7S_{20}-S_{19}=0$

$S_{20}-7S_{19}-S_{18}=0$

$S_{19}-7S_{18}-S_{17}=0$

$$\frac{S_{21}+S_{17}}{S_{19}}=\frac{S_{21}+(S_{19}-7S_{18})}{S_{19}}$$

$$=\frac{S_{21}+S_{19}-7(S_{20}-7S_{19})}{S_{19}}$$

$$=\frac{50S_{19}+(S_{20}-7S_{19})}{S_{19}}$$

$$=51\cdot\frac{S_{19}}{S_{19}}=51$$

30. 2

Sol. $A=\begin{bmatrix}0&1&2\\a&0&3\\1&c&0\end{bmatrix}$

$A^3=A$

$$A^2=\begin{bmatrix}0&1&2\\a&0&3\\1&c&0\end{bmatrix}\begin{bmatrix}0&1&2\\a&0&3\\1&c&0\end{bmatrix}$$

$$A^2=\begin{bmatrix}a+2&2c&3\\3&a+3c&2a\\ac&1&2+3c\end{bmatrix}$$

$$A^3=\begin{bmatrix}a+2&2c&3\\3&a+3c&2a\\ac&a&2+3c\end{bmatrix}\begin{bmatrix}0&1&2\\a&0&3\\1&c&0\end{bmatrix}$$

$$A^3=\begin{bmatrix}2ac+3&a+2+3c&2a+4+6c\\a(a+3c)+2a&3+2ac&6+3a+9c\\a+2+3c&ac+c(2+3c)&2ac+3\end{bmatrix}$$

Given $A^3=A$

$2ac+3=0$ .....(1) and $a+2+3c=1$

$a+1+3c=0$

$$a+1-\frac{9}{2a}=0$$

$2a^2+2a-9=0$

$f(1)<0$, $f(2)>0$

$a\in(1,\ 2]$ $\qquad$ $n=2$

PHYSICS

Section - A (Single Correct Answer)

31. C

Sol. $\vec{E}=20\sin\omega\left(t-\frac{x}{c}\right)\vec{j}\,N/C$

Average energy density of an em wave

$$=\frac{1}{2}\epsilon_0 E_0^2$$

Energy stored $=\left(\frac{1}{2}\epsilon_0 E_0^2\right)(\text{volume})$

$$=\frac{1}{2}\times8.85\times10^{-12}\times(20)^2\times\left(5\times10^{-4}\right)J$$

$=8.85\times10^{-13}$ J

32. C

Sol. Area under the graph from t = 0 to t = 20 sec = 200 m Area under the graph from t = 20 to t = 25 sec = 50 m So distance covered = (200 + 50)m = 250 m Displacement = (200 – 50)m = 150 m

$$\frac{250}{150}=\frac{5}{3}$$

33. C

Sol. $g=\frac{GM}{R^2}=\frac{G}{R^2}\times\rho\times\frac{4\pi}{3}R^3=\left(\frac{4\pi}{3}G\right)\rho R$

$$\frac{g_A}{g_B}=\frac{R\times\rho}{4R\times\frac{\rho}{3}}=\frac{3}{4}$$

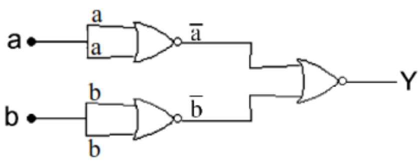
34. D

Sol. $f_{s\,max}=\mu\,mg=m\,\omega^2R\Rightarrow R=\frac{\mu g}{\omega^2}$

So if ω becomes $\frac{\omega}{2}$, R will become 4R.

So distance from the center will be 4 cm.

35. A

Sol. 

$Y = \overline{\bar{a} + \bar{b}} = a \cdot b$

The truth table for the given circuit will be

a	b	output
0	0	0
0	1	0
1	0	0
1	1	1

Hence it will be equivalent to AND gate.

36. B

Sol. Initially

$Q_1 = CV = (2)\ V$

$E_1 = 1/2\ CV^2 = 1/2\ (2)V^2 = V^2$

Finally

Charge on each capacitor, $Q_2 = \frac{Q_1}{2} = \frac{2V}{2} = V$

$E_2 = 2\left(\frac{1}{2}\frac{Q_2^2}{C}\right) = \frac{V^2}{2} \qquad \therefore \frac{E_2}{E_1} = \frac{1}{2}$

37. A

Sol. Parallel combination

$$H_p = \left[\frac{V^2}{\left(\frac{R}{2}\right)}\right]t = \frac{2V^2t}{R}$$

Series combination

$H_s = \left(\frac{V^2}{2R}\right)t \qquad \therefore \frac{H_p}{H_s} = 4$

38. A

Sol. $d_r = \sqrt{2h_rR} \qquad \therefore h_r = \frac{d_r^2}{2R}$

$$= \frac{(4km)^2}{2(6400km)} = \left(\frac{1}{800}\right)km = 1.25m$$

39. C

Sol. $X_C = \frac{1}{\omega C} = \frac{1}{(2\pi f)C}$

$\therefore X_C \propto \frac{1}{f}$

$\therefore$ Curve A

$X_L = \omega L = (2\pi f)\ L$

$\therefore$ $X_L \propto f$

$\therefore$ Curve B

40. A

Sol. $\Delta Q = \Delta U + \Delta W$

$\therefore$ $\Delta U = \Delta Q - \Delta W$

$= mL_V - P\Delta V$

$= (1Kg)\ (2257 \times 10^3\ J/kg) - (1 \times 10^5\ Pa)\ (1.67\ m^3 - 1 \times 10^{-3}\ m^3)$

= 2090 KJ

41. B

Sol. i_C = Critical angle

$\frac{v}{C} = \frac{1}{\mu} = \sin i_C = \sin 45° = \frac{1}{\sqrt{2}}$

$\Rightarrow v = \frac{C}{\sqrt{2}} = \frac{3\times10^8}{\sqrt{2}}\ m/s = 2.12\times10^8 m/s$

42. D

Sol. From the equation of photoelectric effect

$eV_o = \frac{hc}{\lambda} - \phi_0 = \frac{hc}{\lambda} - \frac{hc}{\lambda_o}$

$\& \frac{eV_o}{4} = \frac{hc}{2\lambda} = \frac{hc}{\lambda_o}$

$\Rightarrow \frac{1}{4}\left(\frac{hc}{\lambda} - \frac{hc}{\lambda_0}\right) = \frac{hc}{2\lambda} - \frac{hc}{\lambda_0}$

$\frac{1}{\lambda_0} - \frac{1}{4\lambda_0} = \frac{1}{2\lambda} - \frac{1}{4\lambda}$

$\frac{3}{4\lambda_0} = \frac{1}{4\lambda}$

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$\Rightarrow \lambda_0 = 3\lambda$

43. A

Sol. As $X_m = 2 \times 10^{-2}$

$\mu_r = 1 + X_m = 1.02$

$\Rightarrow \quad B = \mu_r B_0 = 1.02 B_0$

So percentage increase in magnetic field

$$= \frac{B - B_0}{B_0} \times no\% = 2\%$$

44. A

Sol. $I_S = \frac{NBA}{C}$ & $V_S = \frac{NBA}{CG}$

$\Rightarrow V_S = \frac{I_S}{G}$, If G (galvanometer resistance) is constant, then $V_S \propto I_S$

so percentage change in V_S is also 25%.

45. D

Sol. For a particle executing SHM

$$KE = \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$$

When x = 0, KE is maximum & when x = A, KE is zero and KE V/S x graph is parabola.

46. D

Sol. $\frac{X - X_{freez}}{X_{boil} - X_{freez}} = \frac{t - 32}{212 - 32}$

$$\frac{-95 - (-15)}{65 - (-15)} = \frac{t - 32}{180}$$

$$\frac{-80}{80} = \frac{t - 32}{180}$$

t = – 180 + 32

t = – 148°f

47. B

Sol. 1 AU = 1.496×10^{11} m

1 par sec = 3.08×10^{16} m

1 light year = 9.46×10^{15} m

So, Au < ly < Per sec

48. B

Sol. $v_{rms}(mono) = \sqrt{\frac{3RT}{4 \times 10^{-3}}}$

$$v_{rms}(dia) = \sqrt{\frac{3RT}{71 \times 10^{-3}}}$$

$$v_{rms}(ply) = \sqrt{\frac{3RT}{146 \times 10^{-3}}}$$

So correct relation is

v_{rms} (mono) > v_{rms} (dia) > v_{rms} (poly)

49. B

Sol. F = n m v

where n = number of bullets fired per second

$$n = \frac{f}{mv} = \frac{125}{10 \times 10^{-3} \times 250} = 50$$

50 C

Sol. $T_{1/2}$ (A) = T_{av} (B)

$$\frac{ln 2}{\lambda_A} = \frac{1}{\lambda_B}$$

$$\lambda_A = \lambda_B l2$$

Section - B (Numerical Value)

51. 3

Sol. $6 = {}^4C_2 \Rightarrow n_2 = 4$

$h\nu = E_4 - E_1$

$$\therefore \nu = 13.6\left(\frac{1}{1^2} - \frac{1}{4^2}\right) \times \frac{1}{4.25 \times 10^{-25}} = 3 \times 10^{15} \text{ Hz}$$

52. 4

Sol. $P = (1.8 - 1)\left(\frac{1}{20} + \frac{1}{20}\right)$ by lens maker's formula

$$P' = \left(\frac{1.8}{1.5} - 1\right)\left(\frac{1}{20} + \frac{1}{20}\right)$$

Dividing $\frac{P}{P'} = \frac{0.8}{1.2 - 1} = 4$

53. 1152

Sol. $V = \frac{\omega}{k} = \frac{2\pi \times 60}{2\pi \times 0.5} = \frac{160}{0.5}$ m/s

$$= \frac{160}{0.5} \times \frac{18}{5} \text{ km/h}$$

= 1152 km

54. 32

Sol. $W = \int_0^4 (2+3x)\,dx$

$= \left[2x + \frac{3x^2}{2}\right]_0^4$

$= 8 + 3 \times 8$

$= 32$ J

55. 300

Sol. For equilibrium along the plane

$$mg\sin\theta = \frac{1}{4\pi\epsilon_0} \times \frac{q_0^2}{(h\ \text{cosec}\ 30^\circ)^2}$$

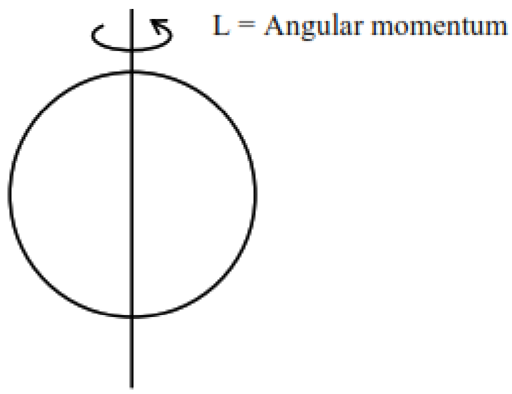
$$\therefore h^2 = \frac{1}{4\pi\epsilon_0} \times \frac{q_0^2}{mg\ \text{cosec}\ 30^\circ}$$

$$= 9\times10^9 \times \frac{(2\times10^{-6})^2}{0.02\times10\times2}$$

$$\therefore h = 3\times10^4 \times \frac{2\times10^{-6}}{0.2} = 0.3\text{m} = 300\ \text{mm}$$

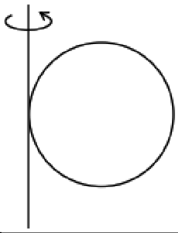
56. 35

Sol.



m = 500 g = 0.5 kg
R = 5cm
ω = 10 rad/sec

moment of inertia about tangent = I_T



$I_t = x \times 10^{-2}$ L

$$\frac{7}{5}mR^2 = x\times10^{-2}\,\frac{2}{5}mR^2\omega$$

$$\frac{7}{2\omega} = x\times10^{-2} = \frac{7}{2\times10}$$

57. 2

Sol. Let the original length be 'l_0'

When T_1 = 100 N, Extension = $l_1 - l_0$

When T_2 = 120 N, Extension = $l_2 - l_0$

Then 100 = K $(l_1 - l_0)$
......(1)

And 120 = K $(l_2 - l_0)$
......(2)

$$\frac{1}{2} \Rightarrow \frac{5}{6} = \frac{l_1 - l_0}{l_2 - l_0}$$

$$5l_2 - 5l_1 = 6l_1 - 6l_0$$

$$l_0 = 6l_1 - 5l_2$$

$$l_0 = 6l_1 - 5\left(\frac{11l_1}{10}\right)$$

$$l_0 = 6l_1 - \frac{11l_1}{2}$$

$$l_0 = \frac{l_1}{2}$$

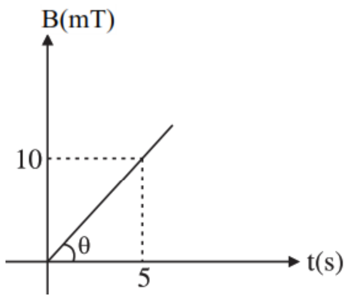
$\therefore$ x = 2

58. 8

Sol. $m = \tan\theta = \frac{10}{5} = 2$

B = mt

B = 2t



$$\varepsilon = \left|\frac{d\phi}{dt}\right| = \frac{d(BA)}{dt} = \frac{AdB}{dt}$$

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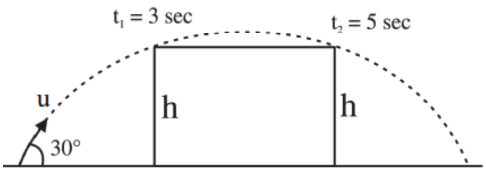
$$\varepsilon = \frac{4d(2t)}{dt} = 4 \times 2 = 8\text{mVolt}$$

59. 80

Sol. Time of flight $t_1 + t_2 = 3 + 5 = 8$ sec



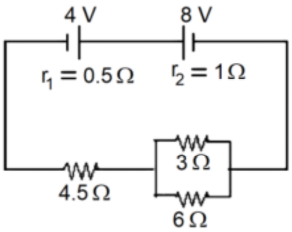
$$T = \frac{2u\sin 30^\circ}{g}$$

$$8 = \frac{2u\sin(30^\circ)}{10}$$

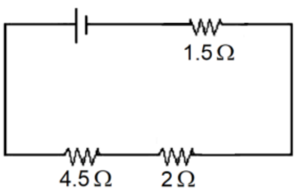
u = 80m/s

60. 1

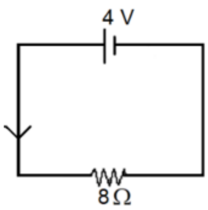
Sol.



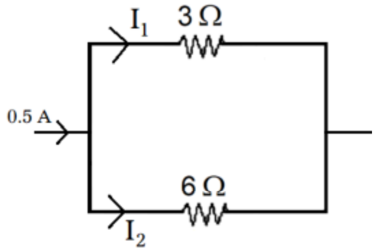
$E_2 - E_1 = 8 - 4 = 4V$



$$\frac{1}{3} + \frac{1}{6} = \frac{1}{2} = \frac{1}{R}$$



$$I = \frac{4}{8} = 0.5A$$



$$I_1 = \left(\frac{6}{3+6}\right) \times 0.5$$

$$I_1 = \frac{2}{3} \times 0.5 = \frac{1}{3}A$$

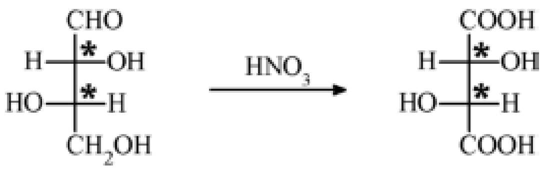
$$I_1 = \frac{x}{3} = \frac{1}{3} \qquad \therefore x = 1$$

CHEMISTRY

Section - A (Single Correct Answer)

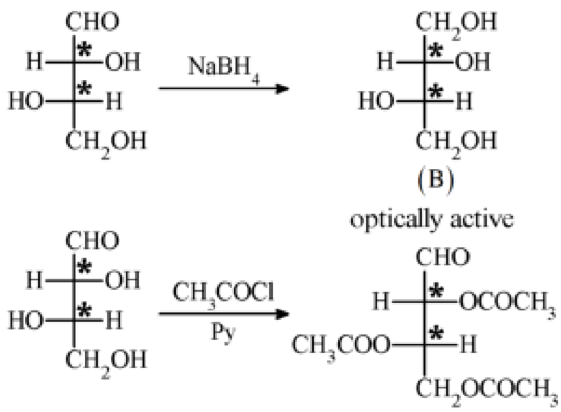
61. B

Sol.



(x) (A)

L-tetrose with two chiral centre



(x) gives positive schiff's test due –CHO group

(x) is L-tetrose.

62. D

Sol. Ethene undergoes addition polymerisation to high density polythene in the presence of catalyst such as $AlEt_3$ and $TiCl_4$ (Ziegler – Natta catalyst) at a temperature of 333 K to 343 K and under a pressure of 6-7 atmosphere.

63. A

Sol. $CH_3COO^- + FeCl_3 \rightarrow Fe(CH_3COO)_3$ or

$$\left[Fe_3(OH)_2(CH_3COO)_6\right]^+$$

Blood red colour

$$\downarrow \Delta$$

$$Fe(OH)_2(CH_3COO)\downarrow$$

Red–brown precipitate

$$2NO_3^- + 4H_2SO_4 + 6Fe^{2+} \rightarrow 6Fe^{3+} + 2NO\uparrow + 4SO_4^{2-} + 4H_2O$$

$$\left[Fe(H_2O)_6\right]^{2+} + NO \rightarrow \underset{\text{Brown}}{\left[Fe(H_2O)_5(NO)\right]^{2+}} + H_2O$$

64. B

Sol. There is a characteristic minimum frequency, or "threshold frequency," for each metal below which the photoelectric effect is not seen. The ejected electrons leave with a specific amount of kinetic energy at a frequency $v > v_0$ with an increase in light frequency of these electron kinetic energies also rise.

65. C

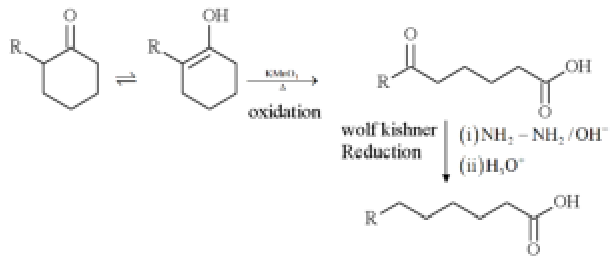
Sol. $AgNO_3 + KI \rightarrow AgI\downarrow + KNO_3$

$$AgI \rightarrow \underset{S}{Ag^+} + \underset{S+0.625}{I^-}$$

AgI is a insoluble salt so concentration Ag+ and F will be negligible.

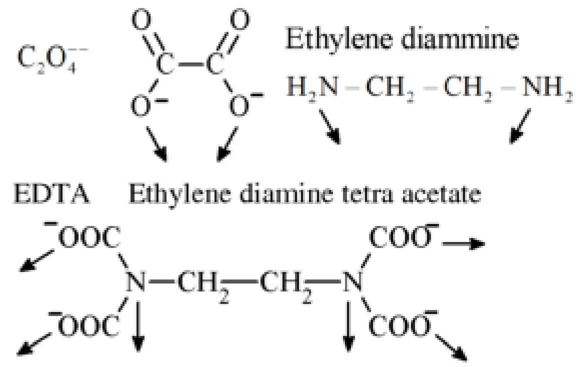
66. D

Sol.



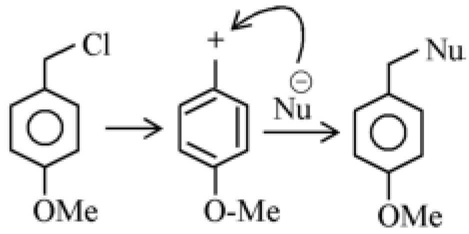
67. A

Sol. NO_2^-, NCS^- are ambidentate ligand



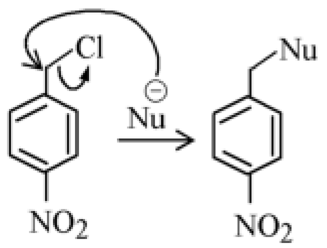
68. C

Sol.



Electron Donating group

S_N1 Mech. : 1st order



Electron withdrawing group

S_N2 Mech : 2nd order

69. C

Sol. First I.E.

F > N > O > C > Be > B > Li

Li – 520 kJ/mol

Be – 899 kJ/mol

B – 801 kJ/mol

C – 1086 kJ/mol

N – 1402 kJ/mol

O – 1314 kJ/mol

F – 1681 kJ/mol

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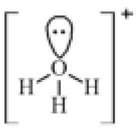
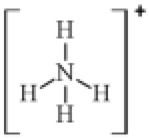
70. A

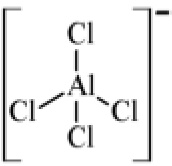
Sol. Molecule/Ion Hybridisation Shape

Molecule/Ion	Hybridisation	Shape	
H_3O^+	sp^3	Pyramidal	
Acelylide	sp	linear	$\bar{C}\equiv\bar{C}$
NH_4^+	sp^3	tetrahedral	
ClO_2^-	sp^3	Bent	

71. A

Sol. Gallous tetrachloro aluminate $Ga^+AlCl_4^-$

$$2Ga + Ga^+GaCl_4^- + 2Al_2Cl_6 \xrightarrow{190^0} 4Ga^+AlCl_4^-$$

Ga^+ 

Structure of $\overset{+}{G}a\overset{-}{A}lCl_4$

Ga is cationic part of salt $GaAlCl_4$.

72. A

Sol. $2Cu_2S + 3O_2 \longrightarrow 2Cu_2O + 3SO_2$

$2Cu_2O + Cu_2S \longrightarrow 6Cu + SO_2$

Blister copper

Due to evolution of SO_2, the solidified copper fomied has a blistered look and is referred to as blister copper.

73. A

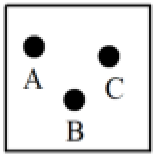
Sol. K^+ – Sodium - Potassium Pump

KCl – Fertiliser

KOH – absorber of CO_2

Li – used in themionuclear reactions

74. A

Sol. 

According to the observation, A is more mobile and interacts with the mobile phase more than C, and C is more drawn to the mobile phase than B.

Hence, the correct order of elution in the silico gel column chromatography is -

B < C < A

75. A

Sol. $[MA_3B_3]$ type of compound exists as facial and meridonial isomer.

76. A

Sol. $$CH_4(g) + \underset{Steam}{H_2O(g)} \xrightarrow[1270\ K]{Ni} CO(g) + 3H_2(g)$$

$$NaNO_2(aq) + NH_4Cl(Aq) \longrightarrow N_2(g) + NaCl(aq) + 2H_2O(\ell)$$

77. C

Sol. Clean water would have BOD value of less than 5 ppm.

Maximum limit of Zn in clean water

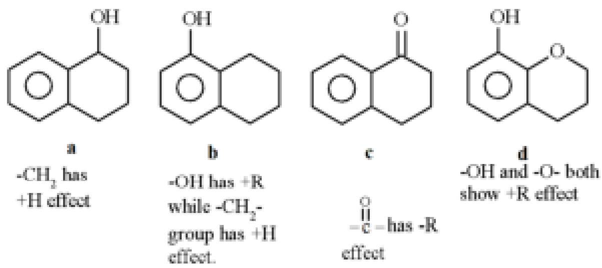
= 5.0 ppm or mg dm^{-3}

Maximum limit of NO_3^- in clean water

= 50 ppm or mg dm^{-3}

78. C

Sol. Benzene becomes more reactive towards EAS when any substituent raises the electron density.



Correct order : c < a < b < d

79. B

Sol. $Fe_4[Fe(CN)_6]_3$ Prussian Blue-water insoluble

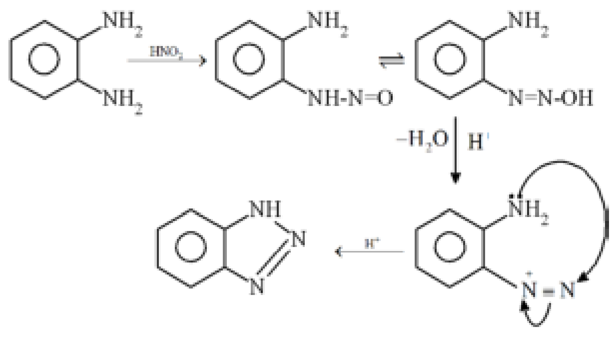
$K_3[Co(NO_2)_6]$ very poorly water soluble

$(NH_4)_3\,[As\,(MO_3O_{10})_4]$ water insoluble ammonium arseno molybdate

$[Fe_3(OH)_2(OAc)_6]Cl$ is water soluble.

80. A

Sol. Orthophenyl amine.



Section - B (Numerical Value)

81. 44

Sol. $CO_{(g)} + H_2O_{(g)} \rightleftharpoons CO_{2(g)} + H_{2(g)}$

t = 0	1 mol	1 mol	0	0
at equ.	1–x	1–x	x	x

at equilibrium 40% by mass water reacts with CO.

$x = 0.4 \qquad 1 - x = 0.6$

$$K_C = \frac{[CO_2][H_2]}{[CO][H_2O]} = \frac{0.4\times0.4}{0.6\times0.6} = 0.44$$

$$K_C \times 10^2 = 44$$

82. 1

Sol. Spin magnetic moment of $[Cr(CN)_6]^{3-}\,(t_{2g}^3 e_g^0)$

$$\mu_1 = \sqrt{3(3+2)} = \sqrt{15}\ \text{BM}$$

Spin magnetic moment of $[Cr(H_2O)_6]^{3+}\,(t_{2g}^3 e_g^0)$

$$\mu_2 = \sqrt{3(3+2)} = \sqrt{15}\ \text{BM}$$

$$\frac{\mu_1}{\mu_2} = \frac{\sqrt{15}}{\sqrt{15}} = 1$$

83. 4

Sol. d = 3 g/cc M = 12 g/mol

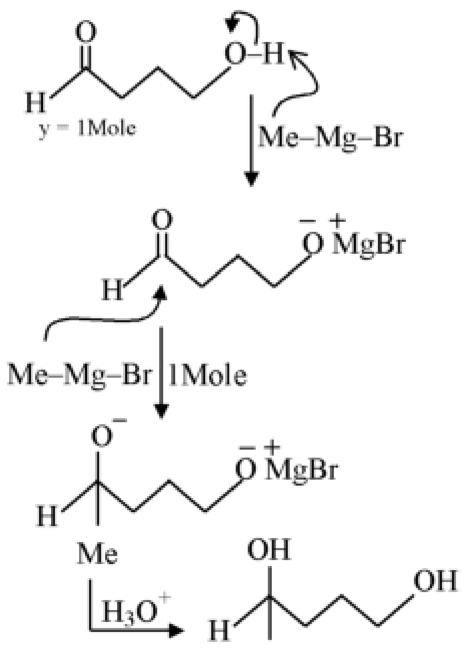
a = 300 pm = 3×10^{-8} cm

$$Z = \frac{d\times N_A \times a^3}{M} = \frac{3\times6.02\times10^{23}\times(3\times10^{-8})^3}{12}$$

$$= 4.06 \approx 4$$

84. 2

Sol.

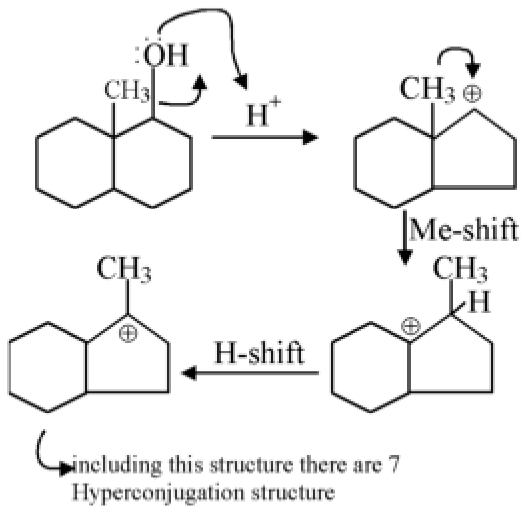


$\therefore$ x = 2 mole

$$\frac{x}{y} = \frac{2}{1} = 2$$

85. 7

Sol.



86. 4

Sol. $Fe_2O_3 + 2Al \longrightarrow Al_2O_3 + 2Fe$

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Molar mass 160g 27g

$$\left(\Delta H_f^0\right)_{reaction} = \left[\left(\Delta H_f^0\right)_{Al_2O_3} + 2\left(\Delta H_f^0\right)_{Fe}\right] - \left[\left(\Delta H_f^0\right)_{Fe_2O_3} + 2\left(\Delta H_f^0\right)_{Al}\right]$$

= [– 1700 + 0] – [– 840 + 0]

= – 860 kJ/mol

Total mass of mixture = Fe_2O_3 + Al (1 : 2 molar ratio)

= 160 + 2 × 27

= 214 g/mol

Heat evolved per gram = $\frac{860}{214} = 4$ kJ/g

87. 36

Sol. Total mass of sugar in mixture of 25% of 200 and 40% of 500 g.

Sugar solution = 0.25 × 200 + 0.40 × 500

= 50 + 200 = 250 g

Total mass of solution = 200 + 500 = 700 g

Mass of sugar in solution = $\frac{250}{700} \times 100 = 35.7\%$

$\approx 36\%$

88. 333

Sol. $KClO_3 + 6FeSO_4 + 3H_2SO_4 \rightarrow KCl + 3Fe_2(SO_4)_3 + 2H_2O$

$$ROR = -\frac{\Delta[KClO_3]}{\Delta t} = \frac{-1}{6}\frac{\Delta[FeSO_4]}{\Delta t}$$

$$= \frac{+1}{3}\frac{\Delta[Fe_2(SO_4)_3]}{\Delta t}$$

$$\frac{\Delta[Fe_2(SO_4)_3]}{\Delta t} = \frac{1}{2}\frac{-\Delta[FeSO_4]}{\Delta t}$$

$$= \frac{1}{2}\frac{(10-8.8)}{30\times 60}$$

$$= 0.333\times 10^{-3}$$

$$= 333\times 10^{-6} \text{ mol litre}^{-1} \text{ sec}^{-1}$$

89. 75

Sol. Isotonic solutions.

$$\pi_{K_2SO_4} = \pi_{Glucose}$$

i × 0.004 × RT = 0.01 × RT

i = 2.5

For K_2SO_4 {for dissociation i = 1 + (n – 1)α}

$$DOD(\alpha) = \frac{i-1}{n-1} = \frac{2.5-1}{3-1} = 0.75$$

% dissociation = 75

90. 2

Sol. $Pb^{2+} + 2e^- \rightarrow Pb$

$\Delta G_1^0 = -2F\,E_1^0$

$Pb^{4+} + 4e^- \rightarrow Pb$

$\Delta G_2^0 = -4F\,E_2^0$

$Pb^{2+} \rightarrow Pb^{4+} + 2e^-$

$\Delta G_3^0 = -2F\,E_3^0$

$\Delta G_3^0 = \Delta G_1^0 - \Delta G_2^0$

$-2F\,E_3^0 = 2F(2n - m)$

$E_3^0 = m - 2n = m - xn$

Hence x = 2

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