10-April-2023 (Evening Batch) : JEE Main Paper

MATHEMATICS

Section - A (Single Correct Answer)

1. A

Sol. $\int_0^{t^2}(f(t)+x^2)dx=\frac{4}{3}t^3,\ \forall t>0$

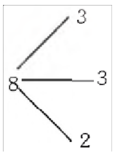
$(f(t^2)+t^4)=2t$

$f(t^2)=2t-t^4$

$t=\frac{\pi}{2}\Rightarrow f\left(\frac{\pi^2}{4}\right)=\frac{2\pi}{2}-\frac{\pi^4}{16}$

$=\pi-\frac{\pi^4}{16}=\pi\left(1-\frac{\pi^3}{16}\right)$

2. B

Sol. 

Ways $=\frac{8!}{3!3!2!2!}\times 3!$

$=\frac{8\times7\times6\times5\times4}{4}$

$=56\times30$

$=1680$

3. D

Sol. $(x=e^{\ln x})$

$\int\left(\left(\frac{x}{e}\right)^{2x}+\left(\frac{e}{x}\right)^{2x}\right)\log_e x\,dx=\int[e^{2(x\ln x-x)}+e^{-2(x\ln x-x)}]\ln x\,dx$

$x\ln x-x=t$

$\ln x\cdot dx=dt$

$\int(e^{2t}+e^{-2t})\,dt$

$\frac{e^{2t}}{2}-\frac{e^{-2t}}{2}+C$

$=\frac{1}{2}\left(\frac{x}{e}\right)^{2x}-\frac{1}{2}\left(\frac{e}{x}\right)^{2x}+C$

$\alpha=\beta=\gamma=\delta=2$

$\alpha+2\beta+3\gamma-4\delta=4$

4. A

Sol. Equation of plane $A(x-1)+B(y-2)+C(z-0)=0$

Put (1, 4, 1) $\Rightarrow 2B+C=0$

Put (0, 5, 1) $\Rightarrow -A+3B+C=0$

Sub : $B-A=0\Rightarrow A=B,\ C=-2B$

$1(x-1)+1(y-2)-2(z-0)=0$

$x+y-2z-3=0$

Image is (α,β,γ) pt $\equiv(1,2,6)$

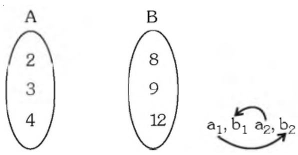
$\frac{\alpha-1}{1}=\frac{\beta-2}{1}=\frac{\gamma-6}{-2}=\frac{-2(1+2-12-3)}{6}$

$\frac{\alpha-1}{1}=\frac{\beta-2}{1}=\frac{\gamma-6}{-2}=4$

$\alpha=5,\ \beta=6,\ \gamma=-2\Rightarrow\alpha^2+\beta^2+\gamma^2$

$=25+36+4=65$

5. A

Sol. 

a_1 divides b_2

Each element has 2 choices

$\Rightarrow 3\times2=6$

Total $=6\times6=36$

6. D

Sol. $|adj\,adj(2A)|=|2A|^{(n-1)^2}$

$=|2A|^4$

$=(2^3|A|)^4$

$=2^{12}|A|^4\Rightarrow 2^{16}$

$$|A| = \frac{1}{5!6!7!} 5!6! \begin{vmatrix} 1 & 6 & 42 \\ 1 & 7 & 56 \\ 1 & 8 & 72 \end{vmatrix}$$

$R_3 \to R_3 \to R_2$

$R_2 \to R_2 \to R_1$

$$|A| = \begin{vmatrix} 1 & 8 & 42 \\ 0 & 1 & 14 \\ 0 & 1 & 16 \end{vmatrix} = 2$$

7. D

Sol.

$$\frac{12\cos\theta + 2}{5} = h \Rightarrow 12\cos\theta = 5h - 2$$

$$\frac{12\sin\theta + 4}{5} = k \Rightarrow 12\sin\theta = 5k - 4$$

Sq. & add :

144 = (5h – 2)2 + (5k – 4)2

$$\left(x - \frac{2}{5}\right)^2 + \left(y - \frac{4}{5}\right)^2 = \frac{144}{25}$$

$$\text{Centre} \equiv \left(\frac{2}{5}, \frac{4}{5}\right) \equiv (\alpha, \beta)$$

$$AC = \sqrt{\left(1 - \frac{2}{5}\right)^2 + \left(2 - \frac{4}{5}\right)^2}$$

$$= \sqrt{\frac{9}{25} + \frac{36}{25}} = \frac{\sqrt{45}}{5} = \frac{3\sqrt{5}}{5}$$

8. D

Sol. P(odd number 7 times) = P(odd number 9 times)

$${}^nC_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{7}\right)^{n-7} = {}^nC_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^{n-9}$$

${}^nC_7 = {}^nC_9$

$\Rightarrow n = 16$

Required

$$P = {}^{16}C_2 \times \left(\frac{1}{2}\right)^{16}$$

$$= \frac{16 \cdot 15}{2} \times \frac{1}{2^{16}} = \frac{15}{2^{13}}$$

$$\Rightarrow \frac{60}{2^{15}} \Rightarrow k = 60$$

9. B

Sol. g(x) = f(x) + f(1 – x) & f '(x) > 0, x ∈ (0, 1)

g'(x) = f(x) – f '(1 – x) = 0

$\Rightarrow f'(x) = f'(1 - x)$

x = 1 – x

$$x = \frac{1}{2}$$

g'(x) = 0

$$\text{at } x = \frac{1}{2}$$

g''(x) = f ''(x) + f''(1 – x) > 0

g is concave up

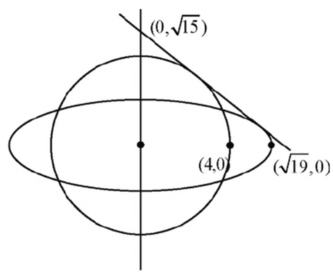
$$\text{hence } \alpha = \frac{1}{2}$$

$$\tan^{-1} 2\alpha + \tan^{-1}\frac{1}{\alpha} + \tan^{-1}\frac{\alpha + 1}{\alpha}$$

$$\Rightarrow \tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$$

10. B

Sol. $\frac{x^2}{19} + \frac{y^2}{15} = 1$



Let tang be

$$y = mx \pm \sqrt{19m^2 + 15}$$

$$mx - y \pm \sqrt{19m^2 + 15} = 0$$

Parallel from (0, 0) = 4

$$\left|\frac{\pm\sqrt{19m^2+15}}{\sqrt{m^2+1}}\right|=4$$

$19m^2 + 15 = 16m^2 + 16$

$3m^2 = 1$

$$m=\pm\frac{1}{\sqrt{3}}$$

$$\theta=\frac{\pi}{6} \text{ with x-axis}$$

Required angle $\frac{\pi}{3}$.

11. C

Sol. $\vec{a}=2\hat{i}+7\hat{j}-\hat{k}$

$\vec{b}=3\hat{i}+5\hat{k}$

$$\vec{d}=\lambda(\vec{a}\times\vec{b})=\lambda\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 7 & -1\\ 3 & 0 & 5\end{vmatrix}$$

$\vec{d}=\lambda(35\hat{i}-13\hat{j}-21\hat{k})$

$\lambda(35+13-42)=12$

$\lambda=2$

$\vec{d}=2(35\hat{i}-13\hat{j}-21\hat{k})$

$(\hat{i}+\hat{j}-\hat{k})(\vec{c}\times\vec{d})$

$$=\begin{vmatrix}-1 & 1 & -1\\ 1 & -1 & 2\\ 70 & -26 & -42\end{vmatrix}=44$$

12. C

Sol. $S_n = 4 + 11 + 21 + 34 + 50 + \ldots\ldots$ n terms

Difference are in A.P.

Let $T_n = an^2 + bn + c$

$T_1 = a + b + c = 4$

$T_2 = 4a + 2b + c = 11$

$T_3 = 9a + 3b + c = 21$

By solving these 3 equations

$$a=\frac{3}{2},\ b=\frac{5}{2},\ c=0$$

So $T_n=\frac{3}{2}n^2+\frac{5}{2}n$

$S_n=\Sigma T_n$

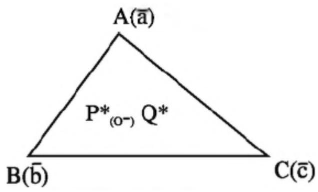
$$=\frac{3}{2}\frac{n(n+1)(2n+1)}{6}=\frac{5}{2}\frac{(n)(n+1)}{2}$$

$$=\frac{n(n+1)}{4}(2n+6)=\frac{n(n+1)(n+3)}{2}$$

$$\frac{1}{60}\left(\frac{29\times30\times32}{2}-\frac{9\times10\times12}{2}\right)=223$$

13. D

Sol.



$$\overline{PA}+\overline{PB}+\overline{PC}=\bar{a}+\bar{b}+\bar{c}$$

$$\overline{PG}=\frac{\bar{a}+\bar{b}+\bar{c}}{3}$$

$$\Rightarrow \bar{a}+\bar{b}+\bar{c}=3\overline{PG}=\overline{PQ}$$

14. D

Sol. $\sim[p\vee(\sim(p\wedge q))]$

$\sim p\wedge(p\wedge q)$

15. A

Sol. Let $9^{\tan^2 x}=P$

$$\frac{9}{P}+P=10$$

$P^2 - 10P + 9 = 0$

$(P - 9)(P - 1) = 0$

$P = 1, 9$

$\tan^2 x = 0,\ \tan^2 x = 1$

$$x=0,\ \pm\frac{\pi}{4}\quad \therefore x\in\left(-\frac{\pi}{2},\ \frac{p}{2}\right)$$

$$\beta=\tan^2(0)+\tan^2\left(+\frac{\pi}{12}\right)\left(-\frac{\pi}{12}\right)$$

$= 0 + 2(\tan 15°)^2$

$2(2-\sqrt{3})^2$

$2(7-4\sqrt{3})$

Than $\frac{1}{6}(14-8\sqrt{3}-14)^2 = 32$

16. A

Sol. $(1 + x)^p (1 - x)^q$

$$\left(1+px+\frac{p(p-1)}{2!}x^2+.....\right)$$

$$\left(1-qx+\frac{p(q-1)}{2!}x^2-.....\right)$$

$p - q = 4$

$$\frac{p(p-1)}{2}+\frac{q(q-1)}{2}-pq=-5$$

$p^2 + q^2 - p - q - 2pq = -10$

$(q + 4)^2 + q^2 - (q + 4) - q - 2(4 + q)q = -10$

$q^2 + 8q + 16 - q^2 - q - 4 - q - 8q - 2q^2 = -10$

$-2q = -22$

$q = 11$

$p = 15$

$2(15) + 3(11)$

$30 + 33 = 63$

17. A

Sol. $\frac{x}{1}=\frac{y-6}{-2}=\frac{z+8}{5}=\lambda$ ....(1)

$\frac{x-5}{4}=\frac{y-7}{3}=\frac{z+2}{1}=\mu$ .....(2)

$\frac{x+3}{4}=\frac{y-3}{-3}=\frac{z-6}{1}=\gamma$ ....(3)

Intersection of (1) and (2) "A"

$(\lambda,\ -2\lambda+6,\ 5\lambda-8)$ & $(4\mu+5,\ 3\mu+7,\ \mu-2)$

$\lambda=1,\ \mu=-1$

A(1, 4, –3)

Intersection of (1) & (3) "B"

$(\lambda,\ -2\lambda+6,\ 5\lambda-8)$ & $(6\gamma-3,\ -3\gamma+3,\ \gamma+6)$

$\lambda=3$

$\gamma=1$

B(3, 0, 7)

Mid point of A & B $\Rightarrow$ (2, 2, 2)

Perpendicular distance from the plane

$2x - 2y + z = 14$

$$\Rightarrow\left|\frac{2(2)-2(2)+2-14}{\sqrt{4+4+1}}\right|=4$$

18. C

Sol. $\frac{2z-3i}{qz+2i}\in R$

$$\frac{2(x+iy)-3i}{4(x+iy)+2i}=\frac{2x+(2y-3)i}{4x+(4y+2)1}\times\frac{4x-(4y+2)i}{4x-(4y+2)i}$$

$4x(2y - 3) - 2x(4y + 2) = 0$

$x = 0$ $\qquad$ $x\neq-\frac{1}{2}$

19. B

Sol. $(22)^{2022} + (22)^{2022}$

divided by 3

$(21 + 1)^{2022} + (2022)^{22}$

$= 3k + 1$ $\qquad$ $(\alpha = 1)$

Divided by 7

$(21 + 1)^{2022} + (2023 - 1)^{22}$

$7k + 1 + 1$ $\qquad$ $(\beta = 2)$

$7k + 2$

So $\alpha^2 + \beta^2 \Rightarrow 5$

20. A

Sol. $\sum f_i = 62$

$\Rightarrow 3k^2 + 16k - 12k - 64 = 0$

$\Rightarrow k = \text{or}\ -\frac{16}{3}$(rejected)

$$\mu=\frac{\sum f_i x_i}{\sum f_i}$$

$$\mu=\frac{8+2(15)+3(15)+4(17)+5}{62}=\frac{156}{62}$$

$$\sigma^2=\sum f_i x_i^2-\left(\sum f_i x_i\right)^2$$

$$=\frac{8\times1^2+15\times13+17\times16+25}{62}-\left(\frac{156}{62}\right)^2$$

$$\sigma^2=\frac{500}{62}-\left(\frac{156}{62}\right)^2$$

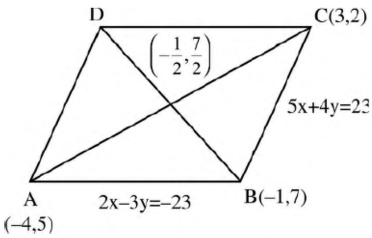
$$\sigma^2+\mu^2=\frac{500}{62}$$

$$[\sigma^2+\mu^2]=8$$

Section - B (Numerical Value)

21. 529

Sol.



A & C point will be (–4, 5) & (3, 2)

mid point of AC will be $\left(-\frac{1}{2},\frac{7}{2}\right)$

equation of diagonal BD is

$$y-\frac{7}{2}=\frac{\frac{7}{2}}{-\frac{1}{2}}\quad\left(x+\frac{1}{2}\right)$$

$\Rightarrow$ 7x + y = 0

Distance of A from diagonal BD

$$=d=\frac{23}{\sqrt{50}}$$

$\Rightarrow\quad 50d^2=(23)^2$

$50d^2=529$

22. 24

Sol. $\Delta=\begin{vmatrix}6\lambda & -3 & 3\\ 2 & 6\lambda & 4\\ 3 & 2 & 3\lambda\end{vmatrix}=0$ (For No Solution)

$$2\lambda(9\lambda^2-4)+(3\lambda-6)+(2-9\lambda)=0$$

$$18\lambda^3-14\lambda-4=0$$

$$(\lambda-1)(3\lambda+1)(3\lambda+2)=0$$

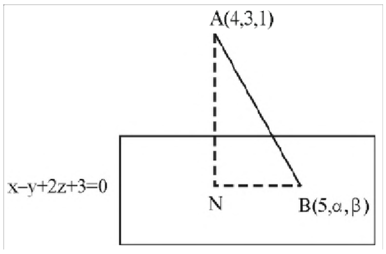
$$\Rightarrow\lambda=1,\ -1/3,\ -2/3$$

$$\text{For each }\lambda,\ \Delta_1=\begin{vmatrix}6\lambda & -3 & 4\lambda^2\\ 2 & 6\lambda & 1\\ 3 & 2 & \lambda\end{vmatrix}\neq0$$

$$12\left(1+\frac{1}{3}+\frac{2}{3}\right)=24$$

23. 7

Sol.



$$AN=\sqrt{6}$$

$$5-\alpha+2\beta+3=0$$

$$\Rightarrow\alpha=8+2\beta\qquad\qquad ....(1)$$

N is given by

$$\frac{x-4}{1}=\frac{y-3}{-1}=\frac{z-1}{2}=\frac{-(4-3+2+3)}{1+1+4}$$

$\Rightarrow$ x = 3, y = 4, z = –1

$\Rightarrow$ N is (3, 4, –1)

$$BN=\sqrt{4+(\alpha-4)^2+(\beta+1)^2}$$

$$=\sqrt{4+(2\beta+4)^2+(\beta+1)^2}$$

$$\text{Area of }\Delta ABN=\frac{1}{2}AB\times BN=3\sqrt{2}$$

$$\Rightarrow\frac{1}{2}\times\sqrt{6}\times BN=3\sqrt{2}$$

$$BN=2\sqrt{3}$$

$$\Rightarrow4+(2\beta+4)^2+(\beta+1)^2=12$$

$$(2\beta+4)^2+(\beta+1)^2-8=0$$

$$5\beta^2+18\beta+9=0$$

$$(5\beta+3)(\beta+3)=0$$

$\beta = -3$

$\Rightarrow \alpha = 2$

$\Rightarrow \alpha^2 + \beta^2 + \alpha\beta = 9 + 4 - 6 = 7$

24. 11

Sol. f(x) = (x + 1) (ax + b)

1 = 2a + 2b .....(1)

f '(x) = (ax + b) + a(x + 1)

1 = (3a + b) .....(2)

$\Rightarrow$ b = 1/4, n = 1/4

$$f(x) = \frac{(x+1)^2}{4}$$

$$f'(x) = \frac{x}{2} + \frac{1}{2}$$

$$\alpha + 1 = \frac{(\alpha+1)^2}{4}, \ \alpha > -1$$

$\alpha + 1 = 4$

$\alpha = 3$

normal at (3, 4)

$$y - 4 = -\frac{1}{2}(x - 3)$$

y = 0 x = 8 + 3

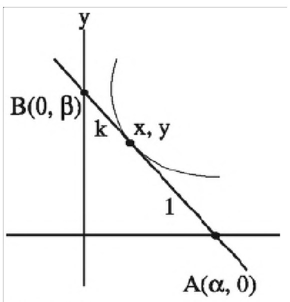
25. 4

Sol. equation of tangent at P(x, y)

$$Y - y = \frac{dy}{dx}(X - x)$$

Y = 0

$$X = \frac{-ydx}{dy} + x$$



$$\frac{k\alpha + 0}{k+1} = x, \ \alpha = \frac{k+1}{k}x$$

$$\frac{k+1}{k}x = -y\frac{dx}{dy} + x$$

$$x + \frac{x}{k} = -y\frac{dx}{dy} + x$$

$$x\frac{dy}{dx} + ky = 0 \qquad \frac{dy}{dx} + \frac{k}{x}y = 0$$

$y \cdot x^k = C$

C = 1

$$100 \cdot \left(\frac{1}{10}\right)^k = 1$$

K = 2

$$\frac{dy}{dx} = \ell n(2x+1)$$

$$y = \frac{(2x+1)}{2}(\ell n(2x+1) - 1) + c$$

$$2 = \frac{1}{2}(0 - 1) + C$$

$$C = 2 + \frac{1}{2} = \frac{5}{2}$$

$$y(1) = \frac{3}{2}(\ell n3 - 1) + \frac{5}{2}$$

$$= \frac{3}{2}\ell n3 + 1$$

$4y(1) = 6\ell n3 + 4$

$4y(1) - 5\ell n3 = 4 + \ell n3$

26. 16

Sol. $\frac{(a-2d)}{4}, \ \frac{(a-d)}{2}, \ a, \ 2(a+d), \ 4(a+2d)$

a = 2

$$\left(\frac{1}{4} + \frac{1}{2} + 1 + 6\right) \times 2 + (-1 + 2 + 8)d = \frac{49}{2}$$

$$2\left(\frac{3}{4} + 7\right) + 9d = \frac{49}{2}$$

$$9d=\frac{49}{2}-\frac{62}{4}=\frac{98-62}{4}=9$$

d = 1

$\Rightarrow a_4 = 4(a + 2d)$

= 16

27. 24

Sol. $f(x)=\sec^{-1}\frac{2x}{5x+3}$

$$\left|\frac{2x}{5x+3}\right|$$

$$\left|\frac{2x}{5x+3}\right|\geq 1\Rightarrow |2x|\geq|5x+3|$$

$$(2x)^2-(5x+3)^2\geq|5x+3|$$

$$(7x+3)(-3x-3)\geq 0$$

$$\begin{array}{ccc} - & + & - \\ \hline -1 & & -\frac{3}{7} \end{array}$$

$$\therefore \text{domain}\left[-1,\frac{-3}{5}\right)\cup\left(\frac{-3}{5},\frac{-3}{7}\right]$$

$$\alpha=-1,\ \beta=\frac{-3}{5},\ \gamma=\frac{-3}{5},\ \delta=\frac{-3}{7}$$

$$3\alpha+10(\beta+\gamma)+21\delta=-3$$

$$-3+10\left(\frac{-6}{5}\right)+\left(\frac{-3}{7}\right)21=-24$$

28. 2664

Sol. 2, 1, 2, 3

$-\ -\ -\ 1\quad \frac{3!}{2!}=3$

$-\ -\ -\ 2\quad 3!=6$

$-\ -\ -\ 3\quad \frac{3!}{2!}=3$

Sum of digits of unit place = 3 × 1 + 6 × 2 + 3 × 3

= 24

$\therefore$ required sum

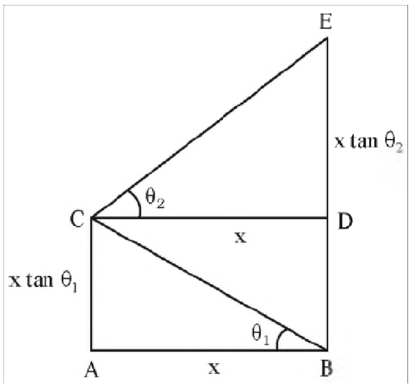
= 24 × 1000 + 24 × 100 + 24 × 10 + 24 × 1

= 24 × 1111

Ans. 26664

29. 6

Sol.



$$\sqrt{3}BE=4AB$$

$$Ar(\Delta CAB)=2\sqrt{3}-3$$

$$\frac{1}{2}x^2\tan\theta_1=2\sqrt{3}-3$$

BE = BD + DE

$$=x(\tan\theta_1+\tan\theta_2)$$

$$BE=AB(\tan\theta_1+\cot\theta_1)$$

$$\frac{4}{\sqrt{3}}\tan\theta_1+\cot\theta_1\Rightarrow\tan\theta_1=\sqrt{3},\ \frac{1}{\sqrt{3}}$$

$$\theta_1=\frac{\pi}{6}\qquad\qquad \theta_2=\frac{\pi}{3}$$

$$\theta_1=\frac{\pi}{3}\qquad\qquad \theta_2=\frac{\pi}{6}$$

as $\frac{\theta_2}{\theta_2}$ is largetst $\theta_1=\frac{\pi}{6}\ \theta_2=\frac{\pi}{3}$

$$\therefore x^2=\frac{(2\sqrt{3}-3)\times 2}{\tan\theta_1}=\frac{\sqrt{3}(2-\sqrt{3})\times 2}{\tan\frac{\pi}{6}}$$

$$x^2=12-6\sqrt{3}=(3-\sqrt{3})^2$$

$$x=3-\sqrt{3}$$

Perimeter of ΔCED

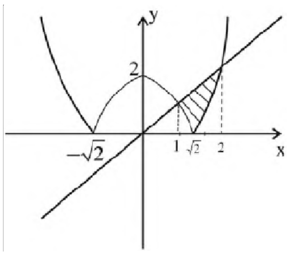
= CD + DE + CE

$$=3\sqrt{3}+(3-\sqrt{3})\sqrt{3}+(3-\sqrt{3})\times 2=6$$

Ans. 6

30. 27

Sol. $\left|x^2-2\right| \le y \le x$



$$A=\int_1^{\sqrt{2}}(x-(2-x^2))dx+\int_{\sqrt{2}}^{2}(x-(x^2-2))dx$$

$$=\left(1-2\sqrt{2}+\frac{2\sqrt{2}}{3}\right)-\left(\frac{1}{2}-2+\frac{1}{3}\right)+\left(2-\frac{8}{3}+4\right)-\left(1-\frac{2\sqrt{2}}{3}+2\sqrt{2}\right)$$

$$=-4\sqrt{2}+\frac{4\sqrt{2}}{3}+\frac{7}{6}+\frac{10}{3}=\frac{-8\sqrt{2}}{3}+\frac{9}{2}$$

$$6A=-16\sqrt{2}+27 \quad \therefore 6A+16\sqrt{2}=27$$

Ans. 27

PHYSICS

Section - A (Single Correct Answer)

31. C

Sol. Average velocity $=\dfrac{x+x}{\dfrac{x}{v_1}+\dfrac{x}{v_2}}=v$

$$\frac{1}{v_1}+\frac{1}{v_2}=\frac{2}{v}$$

32. A

Sol. $t_{1/2}$ = T

$$1\xrightarrow[T]{}\frac{1}{2}\xrightarrow[T]{}\frac{1}{4}\xrightarrow[T]{}\frac{1}{8}$$

$t_{7/8}$ = 3T

33. D

Sol. $$(C_v)_{mix}=\frac{n_1Cv_1+n_2Cv_2}{n_1+n_2}$$

$$(C_v)_{mix}=\frac{2\times\frac{5}{2}R+4\times\frac{3}{2}R}{2+4}=\frac{11R}{6}$$

$$\Delta U=n(C_v)_{mix}RT=6\frac{11R}{6}\times RT=11R$$

34. A

Sol. LC = 0.1mm

Zero Error = 6 × LC = 0.6 mm

Reading = MSR + VSR × LC – Zero Error

= [32mm + (0.1)4mm] - 0.6mm

= 31.8 mm = 3.18 cm

35. B

Sol. Both Statements are correct.

36. C

Sol. $K_{\text{metal sheet}}=\infty,\ t=\dfrac{2d}{3}$

$$C_1=\frac{\epsilon_0 A}{d}$$

$$C_2=\frac{\epsilon_0 A}{d-t+\frac{t}{k}}=\frac{\epsilon_0 A}{d-\frac{2d}{3}+0}=3C_1$$

$$\frac{C_2}{C_1}=3$$

37. B

Sol. Statement I is true due to centrifugal force.

Statement II is incorrect,

At pole g = g_s (no effect)

At equator g = $g_s - r\omega^2\cos^2\lambda = g_s - r\omega^2$

$\therefore$ ($\cos^2\lambda_{maximum}$ at $\lambda = 0°$ i.e. at equator)

Effect is maximum at equator.

38. C

Sol. $$\frac{mv^2}{2R}=\frac{GMm}{(2R)^2}\Rightarrow v=\sqrt{\frac{GM}{2R}}=\sqrt{\frac{Rg}{2}}$$

$$T=\frac{2\pi(2R)}{v}=\frac{4\pi R\sqrt{2}}{\sqrt{Rg}}=\sqrt{32R}$$

39. B

Sol. Fact

40. B

Sol. $\dfrac{E}{B}=C$

$E = BC$

$= 6 \times 10^{-7} \times 3 \times 10^{8} = 18 \times 10$

$E = 180\ Vm^{-1}$

41. D

Sol. (1) $\Delta Q = 0$

(2) $\Delta Q = \Delta U + \Delta W$

$\Rightarrow \Delta U = -\Delta W$

adiabatic compression ($V\downarrow$)

$\Delta W = -ve \Rightarrow \Delta U = +ve$

$\Delta U \uparrow \Rightarrow T \uparrow$

$\Delta U \neq 0$

42. C

Sol. $I_{net} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$$= I_0 + I_0 + 2I_0\cos\frac{\pi}{3} = 2I_0 + 2I_0 \times \frac{1}{2} = 3I_0$$

$$I_{net} = I_0 + I_0 + 2I_0\cos 90° = 2I_0$$

$$\text{Ratio} = \frac{3}{2}$$

43. C

Sol. $eV_0 = h\nu - \phi$

$0 = h\nu - \phi$

$\phi = h\nu$

$= 66 \times 10^{-34} \times 5 \times 10^{14}$

$= 33 \times 10^{-20}$ J

$$\phi = \frac{33 \times 10^{-20}}{1.6 \times 10^{-19}} = 2.07\ eV$$

44. B

Sol. $y = \sin\omega t + \cos\omega t$

$$y = \sin\omega t + \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\Delta\phi = \frac{\pi}{2}$$

$$A_{net} = \sqrt{1^2 + 1^2 + 2 \times 1 \times 1 \times \cos(\Delta\phi)}$$

$$A_{net} = \sqrt{2}$$

45. B

Sol. Move in curve path

$i = neAV_d$

46. B

Sol. $\Delta L = \dfrac{FL}{AY}$

$$\frac{\Delta L_A}{\Delta L_B} = \frac{A_B}{A_A}\frac{Y_B}{Y_A} = 12$$

47. C

Sol. $H_{max} = \dfrac{u^2 \sin^2\theta}{2g}$

$$\frac{H_1}{H_2} = \frac{\sin^2\theta_1}{\sin^2\theta_2} = \frac{1}{3}$$

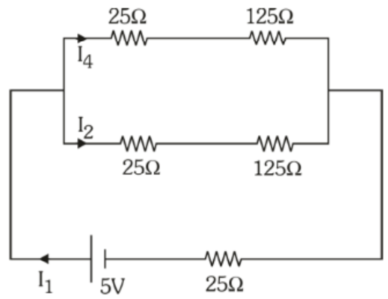
48. D

Sol. Bandwidth $= 2f_m$

$= 2 \times 3$ kHz $= 6$ kHz

49. D

Sol.



$$R_{eq} = \frac{150 \times 150}{300} + 25 = 100\ \Omega$$

$$I_1 = \frac{5}{10} = 0.05A$$

$$I_2 = I_4 = \frac{0.05}{2} = 0.025A$$

$$\frac{I_1}{I_2} = 2$$

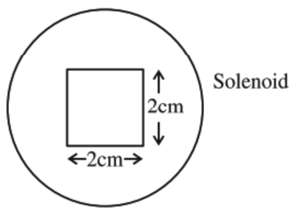
50. A

Sol. After some time both force becomes equal.

Section - B (Numerical Value)

51. 44

Sol.



$B_{\text{due to solenoid}} = \mu_0 nI$

$\Phi_{\text{through square}} = \mu_0 nI \times A$ (A = Area)

$$\text{Emf} = \mu_0 nA \times \frac{dI}{dt}$$

$= \mu_0 n A \times I_0\, \omega \cos \omega t$

Emf amplitude $= \mu_0 n A \times I_0 \omega$

$$= 4\pi \times 10^{-7} \times \frac{50}{10^{-2}} \times 4 \times 10^{-4} \times 2.5 \times 700$$

$= 44 \times 10^{-4}$ V

52. 20

Sol. $\therefore$ $T = 3.14 = \pi$

$$T = \pi = \frac{2\pi}{\omega} \Rightarrow \omega = 2$$

$F_{max} = m\, a_{max}$

$= m (A\omega^2) = mA (2)^2 = 5 \times 1 \times 4 = 20$ N

53. 3668

Sol. For lowest wavelength of Lyman series

$$\frac{1}{\lambda} = RZ^2 \left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = RZ^2$$

For lowest wavelength of Balmer series

$$\frac{1}{\lambda'} = RZ^2 \left[\frac{1}{2^2} - \frac{1}{\infty^2}\right] = \frac{RZ^2}{4}$$

$$\lambda' = \frac{4}{RZ^2} = 4 \times 917 = 3668\text{Å}$$

54. 80

Sol. By equation of continuity

$A_1V_1 = A_2V_2$

$$V_2 = \frac{2 \times 4}{10 \times 10^{-2}} = 80 \text{ cm/s}$$

55. 2

Sol. $$B_{\text{at O}} = \frac{\mu_0 I}{4R} = \frac{4\pi \times 10^{-7} \times 14}{4 \times 2.2 \times 10^{-2}}$$

$= 2 \times 10^{-4}$ T

56. 34

Sol. From Ist lens $\frac{1}{v} + \frac{1}{6} = \frac{1}{24}$

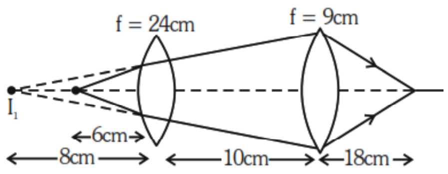
$$\frac{1}{v} = \frac{1}{24} - \frac{1}{6} = -\frac{1}{8}$$

$v = -8$ cm

From IInd lens $\frac{1}{v} + \frac{1}{18} = \frac{1}{9}$

$$\frac{1}{v} = \frac{1}{18}$$

$v = 18$

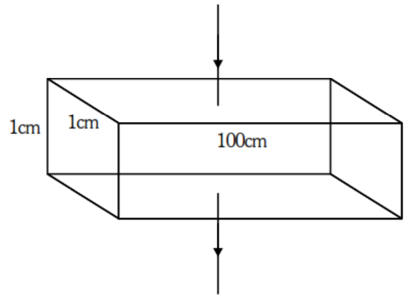


So distance between object and its image

$= 6 + 10 + 18 = 34$ cm

57. 3

Sol.



$$R = \rho \frac{l}{A} = \frac{3 \times 10^{-7} \times (1 \times 10^{-2})}{100 \times 1 \times 10^{-4}} = 3 \times 10^{-7}\,\Omega$$

58. 3

Sol. $\tau = \vec{r} \times \vec{F}$

Where $\vec{r} = -2\hat{i} + 3\hat{j}$ $= P(-2\hat{j} - 3\hat{i}) = P(-3\hat{i} - 2\hat{j})$

$\Rightarrow$ So a = – 3, b = – 2

$$\frac{a}{b} = \frac{3}{2}$$

59. 48

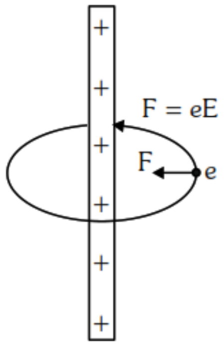
Sol. $P_{max} = F_{max} \times v$

$F_{max} = 1400\, g + \text{friction} = 14000 + 2000 = 16000$

$P_{max} = 16000 \times 3 = 48000$ W = 48KW

60. 8

Sol.



$$eE = \frac{mV^2}{r}$$

$$e \cdot \frac{2K\lambda}{r} = \frac{mV^2}{r} = \sqrt{\frac{e \cdot 2k\lambda}{m}}$$

$$= \sqrt{\frac{1.6\times10^{-19}\times2\times9\times10^{9}\times2\times10^{-8}}{9\times10^{-31}}}$$

= 8 × 106 m/s

CHEMISTRY

Section - A (Single Correct Answer)

61. A

Sol. Ozonolysis of alkene, gives aldehyde, ketone and carboxylic acid.

62. D

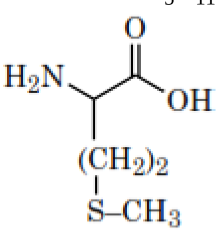
Sol. Assertion & Reason are correct and Reason is correct explanation.

$\because$ Successive I.E. always increases.

63. D

Sol. $X \xrightarrow{Na_2O_2} Y \xrightarrow{BaCl_2} \underset{[BaSO_4]}{Z}$

Methionine: $C_5H_{11}NO_2S$



64. D

Sol. Assertion is correct.

$H_2C_2O_4 \cdot 2H_2O$

$$M = \frac{3.15\times1000}{126\times250}$$

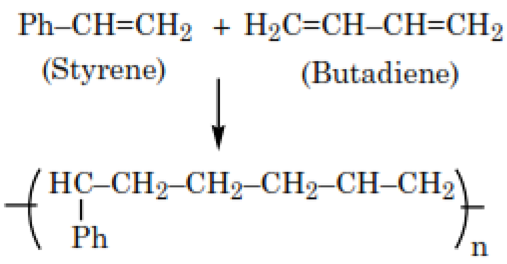
$$= \frac{12.6}{126} = 0.1$$

Reason is correct.

It is used as a fact in explanation of assertion.

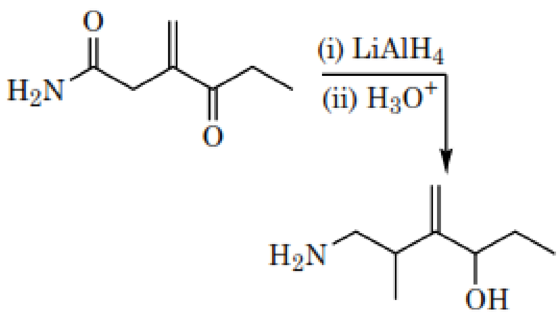
65. B

Sol.



66. A

Sol.



67. C

Sol. Fe^{3+} coagulation negatively charged sol blood.

68. B

Sol. Saponification: Alkaline hydrolysis.

69. B

Sol. Stability order of cations is : C < A < B < D

70. C

Sol. FCC

$$a\sqrt{2} = 4r$$

$$r = \frac{a\sqrt{2}}{4}$$

$$\Rightarrow a = 2\sqrt{2}r$$

BCC

$$4r = a\sqrt{3}$$

71. D

Sol. Washing soda : $Na_2CO_3 \cdot 10H_2O$

Soda ash : Na_2CO_3

72. D

Sol. Respiration, is a natural process, so balance of CO_2 and O_2 not disturbed by respiration.

73. B

Sol. A. $[Fe(CN)_6]^{3-}$ n = 1

B. $[FeF_6]^{3-}$ n = 5

C. $[CoF_6]^{3-}$ n = 4

D. $[Cr(oxalate)_3]^{3-}$ n = 3

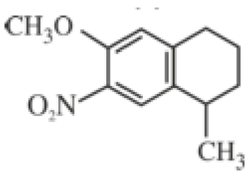
E. $[Ni(CO)_4]$ n = 0

74. A

Sol. E > C > D > A > B

75. D

Sol.



76. B

Sol. A-IV, B-I, C-II, D-III

(A) $[Ti(H_2O)_6]^{2+}$

$Ti^{2+} \Rightarrow 3d^2\ 4s^0$

$t_{2g}\ e^- = 2$

$e_g\ e^- = 0$

CFSE = $[-0.4 \times 2 + 0.6 \times 0]\ \Delta_o$

$= -0.8\ \Delta$

(B) $[V(H_2O)_6]^{2+}$

$V^{2+} \Rightarrow 3d^3\ 4s^0$

$t_{2g}\ e^- = 3$

$e_g\ e^- = 0$

CFSE = $[-0.4 \times 3 + 0.6 \times 0]\ \Delta_0$

$= -1.2\ \Delta_0$

(C) $[Mn(H_2O)_6]^{3+}$

$Mn^{3+} \Rightarrow 3d^4\ 4s^0$

$t_{2g}\ e^- = 3$

$e_g\ e^- = 1$

CFSE = $[-0.4 \times 3 + 0.6 \times 1]\ \Delta_0$

$= -0.6\ \Delta_0$

(D) $[Fe(H_2O)_6]^{3+}$

$Fe^{3+} \Rightarrow 3d^5\ 4s^0$

$t_{2g}\ e^- = 3$ $e_g = 2$

CFSE = $[-0.4 \times 3 + 0.6 \times 2]\ \Delta_0$

$= 0\ \Delta_0$

77. D

Sol. Both A and R are true and R is the correct explanation of A.

Due to mass difference in isotopes of hydrogen, these have different physical property.

78. D

Sol. 16g CH_4 = 1 mole CH_4 contains $10 \times 6.02 \times 10^{23}$ electrons

$= 60.2 \times 10^{23}$

1g H_2 = 0.5 mole H_2 gas occupy 11.35 litre volume at STP

1 mole of N_2 = 28 g

0.5 mole of SO_2 = 32 g

79. C

Sol. On moving from top to bottom metallic character increases while on moving from left to right metallic decreases.

K > Ca > Be

80. B

Sol. at 600°C,

$FeO + C \longrightarrow Fe + CO_2$

Section - B (Numerical Value)

81. 3

Sol. $A_{(g)} \rightleftharpoons 2B_{(g)} + C_{(g)}$

t = 0 450

time t 450 - x 2x x

$P_T = P_A + P_B + P_C$

720 = 450 – x + 2x + x

2x = 270

x = 135

Fraction of A decomposed $= \dfrac{135}{450} = 0.3 = 3 \times 10^{-1}$

So, x = 3

82. 3

Sol. In faintly alkaline medium,

$MnO_4^- + 3e^- + 2H_2O \longrightarrow MnO_2 + 4OH^-$

No. of electrons gained = 3

83. 4

Sol. A $\rightarrow$ Endothermic (Atomisation)

B $\rightarrow$ Endothermic (Atomisation)

C $\rightarrow$ Endothermic (Vapourisation)

D $\rightarrow$ Exothermic (Combustion)

E $\rightarrow$ Endothermic (Dissolution)

84. 4

Sol. H_2O, CO, N_2, NO, has two lone pair of electrons.

85. 2

Sol. $6XeF_4 + 12H_2O \rightarrow 2XeO_3 + 4Xe + 24HF + 3O_2$

in XeO_3, Oxidation state of Xe = +6

in XeF_4, Oxidation state of Xe = +4

So difference in oxidation state = 2.

86. 40535

Sol. $\because \quad \pi = CRT$

$$\pi = \frac{n}{V}RT$$

$$\pi = \frac{\omega}{V}\frac{RT}{M}$$

$$M = \frac{\omega RT}{\pi \times V}$$

$$M = \frac{0.63 \times 0.083 \times 300}{1.29 \times 10^{-3} \times 300 \times 10^{-3}}$$

M = 40535 gm/moL

87. 4

Sol. $\mu = \sqrt{n(n+2)}BM$

$$4.90 = \sqrt{n(n+2)}$$

$$n = 4$$

88. 1

Sol. $\Delta E = R_H Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$1.47 \times 10^{-17} = 2.18 \times 10^{-18} \times 9\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$$

$$\frac{1.47}{1.96} + \frac{3}{4} = \frac{1}{n^2} - \frac{1}{(n+1)^2}$$

So, n = 1.

89. 66

Sol. $\wedge_m = \frac{k}{C} \times 1000$

Given, $k = 5 \times 10^{-5}\ S\ cm^{-1}$

$C = 0.0025\ M$

$$\wedge_m = \frac{5 \times 10^{-5} \times 10^3}{0.0025} = \frac{5 \times 10^{-2}}{2.5 \times 10^{-3}}$$

$$= 20\ S\ cm^2\ mol^{-1}$$

$$\alpha = \frac{20}{400} = \frac{1}{20}$$

$$K_a = \frac{C\alpha^2}{1-\alpha} = \frac{0.0025 \times \frac{1}{20} \times \frac{1}{20}}{\frac{19}{20}}$$

$$= \frac{0.0025}{19 \times 20} = 6.6 \times 10^{-6}$$

$$= 66 \times 10^{-7}$$

90. 1

Sol. (A) For zero order $t_{1/2} = \frac{[A]_0}{2K}$ as concentration decreases half life decreases

(Correct statement)

(B) If order w.r.t. that reactant is zero then it will not affect rate of reaction.

(Correct statement)

(C) Order can be fractional but molecularity can not be

(Incorrect statement)

(D) For zero order reaction unit is mol L^{-1} s^{-1} and for second order reaction unit is mol^{-1} Ls^{-1}

(Correct statement)

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