3S INSTITUTE

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10-April-2023 (Morning Batch) : JEE Main Paper

MATHEMATICS

Section - A (Single Correct Answer)

1. A

Sol. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= (2\hat{i} + 4\hat{j} - 2\hat{k}) - (-2\hat{i} + \hat{j} - 3\hat{k})$

$= 4\hat{i} + 3\hat{j} + \hat{k}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = -2\hat{i} + \hat{j} + 2\hat{k}$

$\overrightarrow{AB} \times \overrightarrow{AC} = 5\hat{i} - 10\hat{j} + 10\hat{k}$

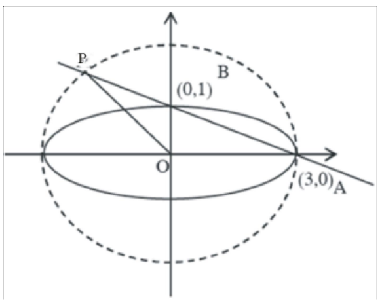
$\overrightarrow{OP} = -\hat{i} - 2\hat{j} + 3\hat{k}$

Projection

$$= \frac{(\overrightarrow{OP}) \cdot (\overrightarrow{AB} \times \overrightarrow{AC})}{\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|} = 3$$

2. C

Sol.



For line AB x + 3y = 3 and circle is $x^2 + y^2 = 9$

$(3 - 3y)^2 + y^2 = 9$

$\Rightarrow 10y^2 - 18y = 0$

$\Rightarrow y = 0, \frac{9}{5}$

$\therefore \text{Area} = \frac{1}{2} \times 3 \times \frac{9}{5} = \frac{27}{10}$

m – n = 17

3. B

Sol. Let $f(x) = \frac{Ax + B}{Cx - A}$

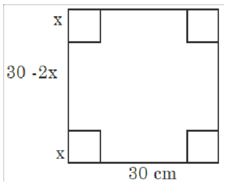
$$f(f(x)) = \frac{A\left(\frac{Ax+B}{Cx-A}\right) + B}{C\left(\frac{Ax+B}{Cx-A}\right) - A} = x$$

$$f\left(f\left(\frac{4}{x}\right)\right) = \frac{4}{x}.$$

$$f(f(x)) + f\left(f\left(\frac{4}{x}\right)\right) = x + \frac{4}{x} \ge 4 \text{ (by A.M.} \ge \text{G.M.)}$$

4. C

Sol.



Volume (V) = x $(30 - 2x)^2$

$$\frac{dV}{dx} = (30 - 2x)(30 - 6x) = 0$$

x = 5 cm

Surface area = 4 × 5 × 20 + (20)2 = 800 cm^2

5. A

Sol. Differentiate the given equation

$\Rightarrow 2xf(x) + x^2 f'(x) - 1 = 4xf(y)$

$\Rightarrow x^2 \frac{dy}{dx} - 2 \cdot xy = 1$

$\Rightarrow \frac{dy}{dx} + \left(-\frac{2}{x}\right) y = \frac{1}{x^2}$

$\text{I.F.} = e^{\int -\frac{2}{x} \ell nx} = \frac{1}{x^2}$

$\therefore y\left(\frac{1}{x^2}\right) = \int \frac{1}{x^4} dx$

$$\Rightarrow \frac{y}{x^2} = \frac{-1}{3x^3} + c$$

$$\Rightarrow y = -\frac{1}{3x^3} + c$$

$$\Rightarrow y = -\frac{1}{3x} + cx^2$$

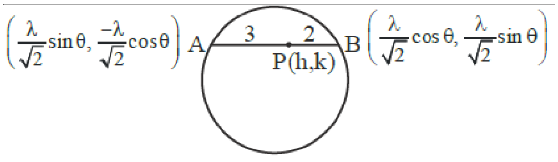
$$\because f(x) = -\frac{1}{3x} + x^2$$

$$f(x) = -\frac{1}{3x} + x^2$$

$18f(3) = 160$

6. D

Sol.



$$h = \frac{\frac{2\lambda}{\sqrt{2}\sin\theta} + 3\times\frac{\lambda}{\sqrt{2}}\cos\theta}{5}$$

$$k = \frac{\frac{-2\lambda}{\sqrt{2}}2\cos\theta + \frac{3\lambda}{\sqrt{2}}\sin\theta}{5}$$

$$h^2 + k^2 = \frac{19\lambda^2}{5}$$

$$r = \frac{\sqrt{19}\lambda}{5}$$

7. D

Sol. $\frac{2z-3i}{2z+i}$ is purely imaginary

$$\therefore \frac{2z-3i}{2z+i} + \frac{2\bar{z}+3i}{2\bar{z}-i} = 0$$

$z = x + iy$

$$\Rightarrow 4x^2 + 4y^2 - 4y - 3 = 0$$

Given that $x + y^2 = 0$

$x^2 + y^2 - y = 3/4$

8. A

Sol. $P = 96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$

$$2P\times\sin\frac{\pi}{33} = 96\times 2\sin\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$$

$$2P\times\sin\frac{\pi}{33} = 6\times\sin\frac{32\pi}{33} = 6\sin\frac{\pi}{33}$$

$P = 3$

9. A

Sol. $|3\ \text{adj}\ (|3A|A^2)| = 3^3\ |\text{adj}\ (54\ A^2)| = 3^3\cdot|54\ A^2|^2$

$= 3^3 \times 54^6 \times |A|^4 = 3^{11} \times 6^{10}$

10. B

Sol. $\frac{dy}{dx} = \frac{1+\left(\frac{y}{x}\right)^2}{2\left(\frac{y}{x}\right)}$

Let $y = tx$

$$\Rightarrow t + x\frac{dt}{dx} = \frac{1+t^2}{2t}$$

$$\Rightarrow x\frac{dt}{dx} = \frac{1-t^2}{2t}$$

$$\Rightarrow \int\frac{2t}{1-t^2}dt = \int\frac{dx}{x}$$

$$\Rightarrow \ell n\left|1-t^2\right| = \ell nx + \ell nx + \ell nc$$

$$\Rightarrow (1-t^2)(cx) = 1$$

$$\Rightarrow \left(1-\frac{y^2}{x^2}\right)cx = 1$$

$$y(2) = 0 \Rightarrow c = \frac{1}{2}$$

$$\left(1-\frac{y^2}{x^2}\right)\cdot\frac{1}{2}x = 1$$

at $x = 8$

$$\left(1-\frac{y^2}{64}\right)\times\frac{8}{2} = 1$$

$y = \pm 4\sqrt{3}$

11. C

Sol. $\Delta = \begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \alpha \end{vmatrix} = 7(\alpha+5)$

$\Delta_1 = \begin{vmatrix} 5 & -1 & 3 \\ 7 & 2 & -1 \\ \beta & 5 & \alpha \end{vmatrix} = 17\alpha - 5\beta + 130$

$\Delta_2 = \begin{vmatrix} 2 & 5 & 3 \\ 3 & 7 & -1 \\ 4 & \beta & \alpha \end{vmatrix} = -11\beta + \alpha + 104$

$\Delta_3 = \begin{vmatrix} 2 & -1 & 5 \\ 3 & 2 & 7 \\ 4 & 5 & \beta \end{vmatrix} = 7(\beta - 9)$

For infinitely many solutions

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

For $\alpha = 5$ and $\beta = 9$

Hence option (C) is incorrect

12. A

Sol. N = Sum of the numbers when two dice are rolled such that $2N < N!$

$\Rightarrow 4 \le N \le 12$

Probability that $2^N \ge N!$

Now, $P(N=2) + P(N=3) = \frac{1}{36} + \frac{2}{36} = \frac{3}{36} = \frac{1}{12}$

Required probability $= 1 - \frac{1}{12} = \frac{11}{12} = \frac{m}{n}$

$4m - 3n = 8$

13. C

Sol. $P = (3\lambda - 3, \lambda - 2, \lambda - 2\lambda)$

P lies on the plane, $x + y + z = 2$

$\Rightarrow \lambda = 3$

$P = (6, 1, -5)$

$q = \left|\frac{18 - 4 - 60 - 32}{\sqrt{9 + 16 + 144}}\right| = \frac{78}{13} = 6$

$q = 6,\ 2q = 12$

Equation, $x^2 - 18x + 72 = 0$

14. A

Sol. $\sim[(p \vee q) \wedge (q \vee (\sim p)]$

$\Rightarrow \sim(p \wedge q) \vee \sim(q \vee (\sim p))$

$\Rightarrow (\sim p \wedge \sim q) \vee (\sim q \wedge p)$

Applying distribution law

$\Rightarrow \sim q \wedge (\sim p \vee p)$

$\Rightarrow (\sim p \vee p) \wedge (\sim q)$

15. B

Sol. $T_{r+1} = {}^{13}C_r (ax)^{13-r} \left(-\frac{1}{bx^2}\right)^r$

$= {}^{13}C_r (a)^{13-r} \left(-\frac{1}{b}\right)^r x^{13-3r}$

$13 - 3r = 7 \Rightarrow r = 2$

Coefficient of $x^2 = {}^{13}C_2 (a)^{11} \cdot \frac{1}{b^2}$

In the other expansion

$T_{r+1} = {}^{13}C_r (ax)^{13-r} \left(\frac{1}{bx^2}\right)^r$

$13 - 3r = -5 \Rightarrow r = 6$

Coefficient of $x^{-5} = {}^{13}C_6 (a)^7 \cdot \frac{1}{b^6}$

${}^{13}C_2 \frac{a^{11}}{b^2} = {}^{13}C_6 \frac{a^7}{b^6}$

$a^4 b^4 = \frac{{}^{11}C_6}{{}^{10}C_2} = 22$

16. B

Sol. Given, A(2, 4, 6), B(0, –2, –5)

G(2, 1, –1)

Let vertex C(x, y, z)

$\frac{2 + 0 + x}{3} = 2 \Rightarrow x = 4$

$\frac{4 - 2 + y}{3} = 1 \Rightarrow y = 1$

$$\frac{6-5+z}{3}=-1 \Rightarrow z=-4$$

Third vertex, C(4, 1, –4)

Then image of vertex in the plane let image (α, β, γ)

i.e., $\frac{\alpha-4}{1}=\frac{\beta-1}{2}=\frac{\gamma+4}{4}=\frac{-2(4+2-16-11)}{21}$

$\alpha = 6, \beta = 5, \gamma = 4$

$\alpha\beta + \beta\gamma + \gamma\alpha = 30 + 20 + 24 = 74$

17. B

Sol. Given lines

$$\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2} \;\&\; \frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$$

Formula for shortest distance

$$\text{S.D.}=\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}$$

$$=\frac{\begin{vmatrix} 6 & 1 & -8 \\ 1 & -2 & 2 \\ 1 & 2 & 0 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 1 & 2 & 0 \end{vmatrix}}=\frac{54}{6}=9$$

18. C

Sol. $I(x)=\int \frac{e^{\sin x}\cdot \sin 2x}{\text{II}}\cdot\frac{\cos x}{\text{I}}dx-\int e^{\sin^2 x}\cdot \sin x dx$

$$\Rightarrow I(x)=e^{\sin^2 x}-\int(-\sin x)\cdot e^{\sin^2 x}dx-\int e^{\sin^2 x}\cdot \sin x dx$$

$$\Rightarrow I(x)=e^{\sin^2 x}\cdot \cos x+c$$

Put x = 0, c = 0

$$\therefore I\left(\frac{\pi}{3}\right)=e^{\frac{3}{4}}\cdot\cos\frac{\pi}{3}=\frac{1}{2}e^{\frac{3}{4}}$$

19. A

Sol. $\Rightarrow a^2 + a^2r^2 + a^2r^4 = 33033$

$\Rightarrow a^2 (r^4 + r^2 + 1) = 3 \times 7 \times 112 \times 13 \Rightarrow a = 11$

$\Rightarrow r^4 + r^2 + 1 = 273 \Rightarrow r^4 + r^2 - 272 = 0$

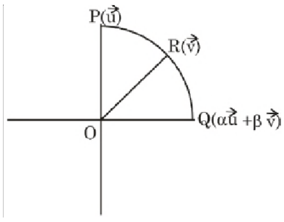
$\Rightarrow (r^2 + 17)(r^2 - 16) = 0$

$\Rightarrow r^2 = 16 \Rightarrow r = \pm 4$

$t_1 + t_2 + t_3 = a + ar + ar^2 = 11 + 44 + 176 = 231$

20. A

Sol.



$$|\vec{u}|=|\vec{v}|=|\alpha\vec{u}+\beta\vec{v}|$$

$$(\vec{u})\cdot(\alpha\vec{u}+\beta\vec{v})=0$$

$$\vec{u}\cdot\vec{v}=|u||v|\cos 45^\circ$$

$$\alpha=-\frac{\beta}{\sqrt{2}}$$

$$=|\alpha\vec{u}+\beta\vec{v}|=r$$

$\alpha = -1, \beta^2 = 2$

Section - B (Numerical Value)

21. 960

Sol. General term $=\frac{10!}{r_1!\cdot r_2!\cdot r_3!}(-1)^{r_2}\cdot(2)^{r_3}x^{r_2+3r_3}$

where $r_1 + r_2 + r_3 = 10$ and $r_2 + 3r_3 = 7$

r_1	r_2	r_3
3	7	0
5	4	1
7	1	2

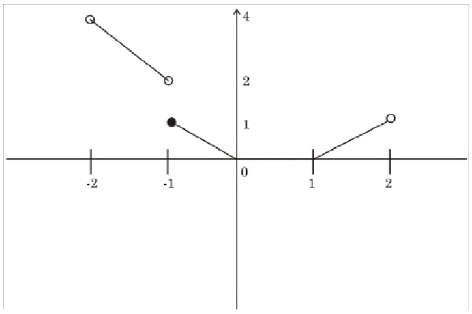
Required coefficient

$$=\frac{10!}{3!\cdot 7!}(-1)^7+\frac{10!}{5!\cdot 4!}(-1)^4(2)+\frac{10!}{7!\cdot 2!}(-1)^1(2)^2$$

$= -120 + 2520 - 1440 = 960$

22. 4

Sol. $f(x)=\begin{cases} x[x] & ,-2<x<0 \\ (x-1)[x] & ,0\le x<2 \end{cases}$



$|f(x)|$ = Remain same

$m = 1, n = 3$

$m + n = 4$

23. 9525

Sol. Required sum = (3 + 8 + 13 + 18 +...........+ 373) – (3 + 18 + 33 +.......+ 363)

$=\frac{75}{2}(3+373)-\frac{25}{2}(3-363)$

= 75 × 188 – 25 × 183

= 9525

24. 32

Sol. General tangent of slope m to the circle $(x-4)^2 + y = 16$ is given by $y=m(x-4)\pm 4\sqrt{1+m^2}$

General tangent of slope m to the parabola y = 4x is given by $y=mx+\frac{1}{m}$

For common tangent $\frac{1}{m}=-4m\pm 4\sqrt{1+m^2}$

$m=\pm\frac{1}{2\sqrt{2}}$

Point of contact on parabola is $(8,\ 4\sqrt{2})$

Length of tangent PQ from $(8,\ 4\sqrt{2})$ on the circle $(x-4)^2 + y^2 = 16$ is equal to

$\sqrt{(8-4)^2+(4\sqrt{2})^2-16}$ is equal to $\sqrt{32}$

PQ^2 is equal to 32

25. 4898

Sol. Digits → 1, 2, 3, 4, 5, 6,7

Total permutations = 7!

Let A = number of numbers containing string 153

Let B = number of numbers containing string 2467

$n(A) = 5! \times 1$ 153   2467

$n(B) = 4! \times 1$ 2467   135

$n(A\cap B)=2!$ 153   2467

$n(A\cup B)=5!+4!-2!=142$

n(neither string 153 nor string 2467)

$=\text{Total}\ -n(A\cup B)$

= 7! – 142 = 4898

26. 8

Sol. $(2a)^{\ln a} = (bc)^{\ln b}$ $2a > 0,\ bc > 0$

$b^{\ln 2} = a^{\ln c}$

ln a (ln 2 + ln a) = ln b (ln b + ln c)

ln2.lnb = lnc.lna

ln 2 = α, ln a = x_1 ln b = y, ln c = z   αy = yz

x (a + x) = y (y + 2)

$\alpha=\frac{xz}{y}$ $(2a)^{\ln a} = (2a)^0$

$x\left(\frac{xz}{y}+x\right)=y(y+z)$

$x^2 (z + y) = y^2 (y + z)$

y + z = 0 or $x^2 = y^2$ ⇒ x = –y

bc = 1 or ab = 1

(I) if bc = 1 ⇒ $(2a)^{\ln a} = 1$ → a=1, a=1/2

$(a,\ b,\ c)=\left(\frac{1}{2},\ \lambda,\ \frac{1}{\lambda}\right),\ \lambda\ne 1,\ 2,\ \frac{1}{2}$

then 6a + 5bc = 3 + 5 = 8

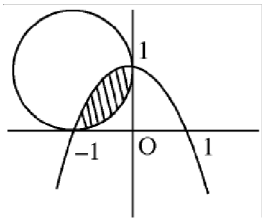
(II) $(a,\ b,\ c)=\left(\lambda,\ \frac{1}{\lambda},\ \frac{1}{2}\right),\ \lambda\ne 1,\ 2,\ \frac{1}{2}$

In this situation infinite answer are possible

So, Bonus.

27. 16

Sol. There can be infinitely many parabolas through given points.



$$A=\int_{-1}^{0}(1-x^2)-(x-\sqrt{1-(x+1)^2})dx$$

$$=\int_{-1}^{0}-x^2+\sqrt{1-(x+1)^2}dx$$

$$=\left(-\frac{x^3}{3}+\frac{x+1}{2}=\sqrt{1-(x+1)^2}+\frac{1}{2}\cdot\sin^{-1}\left(\frac{x+1}{1}\right)\right)_{-1}^{0}$$

$$A=\frac{\pi}{4}-\left(\frac{1}{3}\right)$$

$$\therefore 12(\pi-4A)=12\left(\pi-4\left(\frac{\pi}{4}-\frac{1}{3}\right)\right)=16$$

This is possible only when axis of parabola is parallel to Y axis but is not given in question, so it is bonus.

28. 151

Sol. Given mean is = 28

$$\frac{2\times5+3\times15+x\times25+5\times35+4\times45}{14+x}=28$$

x = 6

$$\text{Variance}=\left(\frac{\sum x_i^2f_i}{Ef_i}\right)-(\text{mean})^2$$

$$\text{Variance}=\frac{2\times5^2+3\times15^2+6\times25^2+5\times35^2+4\times45^2}{20}-(28)^2$$

= 151

29. 16

Sol. ${}^nC_2\times{}^{n-2}C_2\times2=840$

$\Rightarrow$ n = 8

Therefore total persons = 16

30. 6

Sol. $-6<n^2-10n+19<6$

$\Rightarrow n^2-10n+25>0$ and $n^2-10n+13<$

$$(n>5)^2>0\ n\in[5-2\sqrt{3},\ 5+2\sqrt{3}]$$

$$n\in R-[5]$$

$$\therefore n\in[1,\ 3,\ 8,\ 3]$$

$\Rightarrow$ n = 2, 3, 4, 6, 7, 8

PHYSICS

Section - A (Single Correct Answer)

31. A

Sol.

$$\frac{\Delta P}{P}\times100\%=\left(2\frac{\Delta a}{a}+3\frac{\Delta b}{b}+\frac{\Delta c}{c}+\frac{1}{2}\frac{\Delta d}{d}\right)\times100\%$$

$$=2(1\%)+3(2\%)+3\%+\frac{1}{2}\times4\%=13\%$$

32. D

Sol. $M=\frac{W}{g}=\frac{200}{10}=20\text{kg}$

Acc. due to gravity at a depth $g'=g\left(1-\frac{d}{R}\right)$

d $\to$ depth from surface

$$d=\frac{R}{2}$$

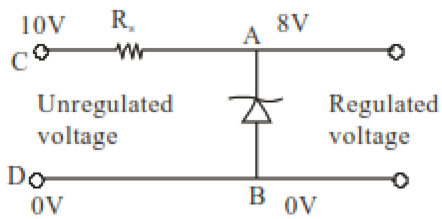
$$g'=g\left(1-\frac{R/2}{R}\right)=\frac{g}{2}=5\ \text{m/s}^2$$

weight = m × g

at depth R/2 = 20 × 5 = 100 N

33. C

Sol.



V_b = 8 volt

V_A-V_B = 8 volt

Current through zener diode,

$$i=\frac{P}{V}=\frac{1.6W}{8V}=0.2A$$

V_C-V_A = 10 – 8 volt

[Note : A zener diode can regulate only if input voltage is ≥ zener breakdown voltage the range of input voltage should be 8 to 10 V so that output voltage remains constant = 8 V]

34. C

Sol. $R = \dfrac{\upsilon^2 \sin 2\theta}{g}$

$R \propto \sin(2\theta)$

$\dfrac{R_1}{R_2} = \dfrac{\sin(2\theta_1)}{\sin(2\theta_2)} = \dfrac{\sin(2\times 15)}{\sin(2\times 45)} = \dfrac{\sin 30^\circ}{\sin 90^\circ}$

$\dfrac{50}{R_2} = \dfrac{1}{2}$

$R_2 = 100m$

35. B

Sol. Given, $A_c = 15$ V

$A_m = 3V$

Maximum amplitude of modulated wave

$A_{max} = A_c + A_m = 15 + 3 = 18V$

Minimum amplitude of modulated wave

$A_{min} = A_c - A_m = 15 - 3 = 12V$

$\therefore \dfrac{A_c + A_m}{A_c - A_m} = \dfrac{18}{12} = \dfrac{3}{2}$

36. C

Sol. $L = mvr,\ r \propto n^2,\ v \propto \dfrac{1}{n}$

$\therefore L \propto n$

Also, $L = \dfrac{nh}{2\pi}$, Bohr orbit is, $L_1 = L = \dfrac{1.h}{2\pi}$

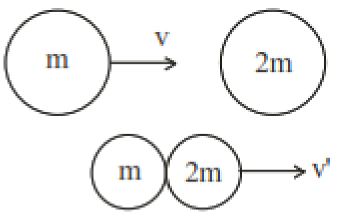
$L_2 = 2\,[L] = 2L$

$L_2 = \dfrac{2h}{2\pi}$

So, change = $L_2 - L_1 = 2L - L = L$

37. C

Sol.



Applying conservation of linear momentum

$\Rightarrow \vec{P}_i = \vec{P}_f$

mv + 2m × 0 = (3m)v′

∴ mv = 3mv′

$v' = \dfrac{v}{3}$

38. D

Sol. For a moving coil galvanometer

BiNA = kθ

$\theta = \left(\dfrac{BNA}{k}\right)i$; Current sensitive $= \dfrac{BNA}{k}$

So, if N is doubled then current sensitivity is doubled.

Voltage sensitivity

$B\dfrac{V}{R}NA = k\theta$

$V = \dfrac{BNA}{Rk}\theta$, as N is doubled R is also doubled.

So, no change in voltage sensitivity.

Hence, option (D) is right.

39. C

Sol. Factual

Type of gases	No. of degrees of freedom
Monoatomic gas	3 T
Diatomic + rigid	3T + 2R
Diatomic + non-rigid	3T+ 2R + 1V
Polyatomic	3T + 3R + More than 1V

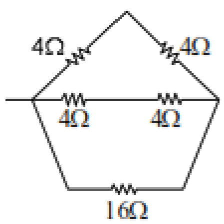
T = Translational degree of freedom

R = Rotational degree of freedom

V = Vibrational degree of freedom

40. B

Sol. The circuit can be reduced to



$\Rightarrow R_{eq} = \frac{16\times 4}{16+4} = \frac{16}{5}\Omega$

$= R_{eq} = 3.2\Omega$

41. D

Sol. Isothermal process, T = constant

PV = nRT = constant

$P_1V_1 = P_2V_2$

$PV = P_A(V/8)$

$P_A = 8P$

Adiabatic process, PV^{γ} = constant

γ for monoatomic gas is 5/3.

$$P_1V_1^{\gamma} = P_2\gamma_2^{\gamma}$$

$$\frac{P_B}{P} = \left(\frac{V_1}{V_2}\right)^{\gamma} = \left(\frac{V}{V/8}\right)^{\frac{5}{3}}$$

$P_B = 32$ P

$$\frac{P_B}{P_A} = \frac{32P}{8P} = 4$$

42. C

Sol. Power will be maximum when impedance is minimum

$$Z = \left[R^2 + (X_L - X_C)^2\right]^{\frac{1}{2}}$$

At resonance, $X_L = X_C$

$Z_{min} = R$

43. C

Sol. $v = \sqrt{\frac{GM}{r}}$

$$v \propto \frac{1}{\sqrt{r}} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{3r}{r}}$$

$= \sqrt{3}:1$

44. B

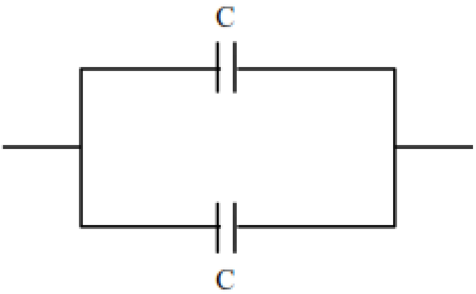
Sol. Pressure in a static liquid will be same at each point on same horizontal level.

$\therefore \quad P = P_{atm} + \rho gh$

As per Pascal law, same pressure applied to enclosed water is transmitted in all directions equally.

45. C

Sol. The circuit can be reduced to



Parallel combination

$C_{eq} = C + C = 2C$

46. C

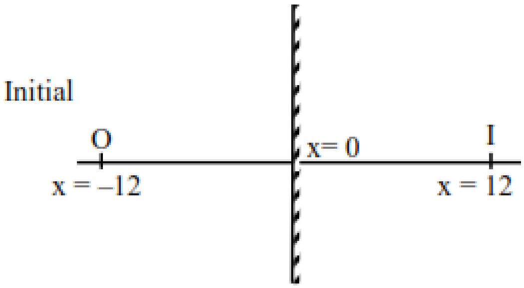
Sol. $E = E_0 \sin(\omega t - kx)$

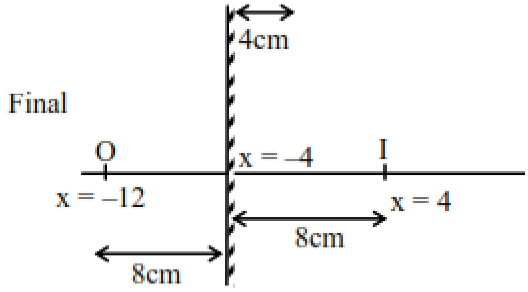
Energy density $\left(\frac{du}{dv}\right) = \varepsilon_0 E_0^2 \sin^2(\omega t - kx)$

$$\frac{\varepsilon_0 E_0^2}{2}\left[1 - \cos(2\omega t - 2kx)\right]$$

47. B

Sol.





$\therefore$ Shifting of image will be 8 cm towards mirror.

48. A

Sol. From K.T.G.

$$v_{RMS} = \sqrt{\frac{3k_B T}{m}}$$

$$v_{RMS} \propto \sqrt{T}$$

and $\frac{h}{mv_{RMS}} = \lambda$ i.e., $\lambda \propto \frac{1}{\sqrt{T}}$

$$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{300}{600}} = \frac{1}{\sqrt{2}}$$

$$\lambda_2 = \frac{\lambda_1}{\sqrt{2}}$$

49. A

Sol. $X = A\sin(\omega t + \delta)$ $V = A\omega\cos(\omega t + \delta)$

$\frac{A}{2} = A\sin(\omega t + \delta)$ $\therefore$ V is +ve, δ must be

At t = 0 in 1^{st} quadrant or 4^{th}

$\sin\delta = \frac{1}{2} \Rightarrow \delta = \frac{\pi}{6}, \frac{5\pi}{6}$ quadrant

$\therefore$ Common solution is $\delta = \frac{\pi}{6}$

50. A

Sol. As slope of B > Slope of A

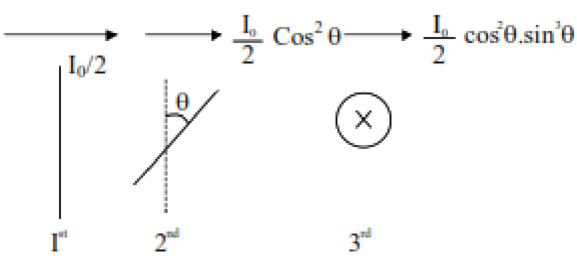
$\therefore V_B > V_A$

Also, $t_B < t_A$

Section - B (Numerical Value)

51. 30, 60

Sol. $I_0 = 32$ w/m^2



$$I_{net} = 3 = \frac{32}{2}\cos^2\theta \cdot \sin^2\theta$$

$$\frac{3}{4} = 4\sin^2\theta \cdot \cos^2\theta = (\sin 2\theta)^2$$

$$\frac{\sqrt{3}}{2} = \sin(2\theta)$$

Hence, $\theta = 30°$ and $60°$

52. + 245 or – 245

Sol. $r_{avg} = 15$ cm

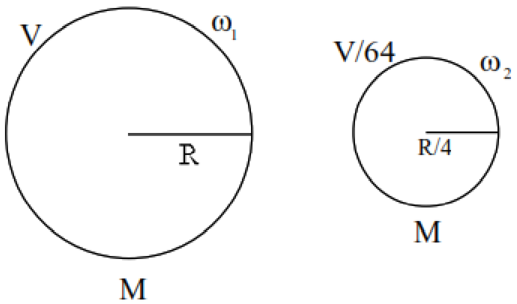
$w_f + w_g = \Delta KE$

$w_f + 10 \times 0.3 = -\frac{1}{2} \times 484$

$w_f = -245$ J

53. 16

Sol. From conservation of angular momentum



$$\frac{2}{5}MR^2\omega_1 = \frac{2}{5}M\left(\frac{R}{4}\right)^2\omega_2$$

$$\Rightarrow MR^2\omega_1 = \frac{MR^2}{16}\omega_2$$

$$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{1}{16} \Rightarrow \frac{T_2}{T_1} = \frac{1}{16} \Rightarrow \frac{T_1}{T_2} = \frac{16}{1} = \frac{t_1}{x}$$

$$\therefore t_1 = 24 \Rightarrow \frac{16}{1} = \frac{24}{t_2} \Rightarrow x = 16$$

54. 6

Sol. I = H

Given, $I = 2.4 \times 10^3$ A/m

$2.4 \times 10^3 = H = ni$

$$n = \frac{N}{l}$$

$$2.4 \times 10^3 = \frac{60}{15 \times 10^{-2}} i$$

$$i = \frac{2.4 \times 15 \times 10}{60} = \frac{36}{6} = 6A$$

55. 18

Sol. F = i l B

$$= \left(\frac{\varepsilon}{R}\right) lB = \left(\frac{vBl}{R}\right) lB = \frac{vB^2 l^2}{R} = \frac{4}{5} \times \left(\frac{15}{100}\right)^2 \times 1^2$$

$$= \frac{4}{5} \times \frac{225}{10^4}$$

$$= \frac{180}{10^4} = 0.018 \text{ N}$$

$= 18 \times 10^{-3}$ N

56. 20

Sol. y(x, t) = 5 sin (6t + 0.003x)

k = 0.003 cm^{-1},

$$\omega = 6 \text{rad/s}, v = \frac{\omega}{k}$$

$$\Rightarrow \frac{6}{0.003 \times 10^2} = 20 \text{ ms}^{-1}$$

= 20 ms^{-1}

57. 15

Sol. $\lambda = 1.5 \times 10^{-5}$ s^{-1}

$$\text{No. of mole } = \frac{1 \times 10^{-6}}{60} = \frac{10^{-7}}{6}$$

No. of atoms = no. of moles × N_A

$$= \frac{10^{-7}}{6} \times 6 \times 10^{23} = 10^{16}$$

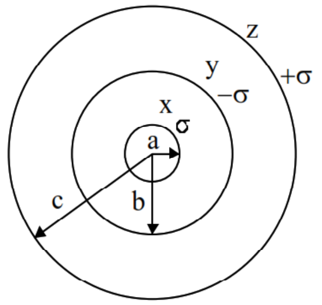
$A = N_0 \lambda e^{-\lambda t}$

For, t = 0, $A = A_0 = N_0\lambda$

$= 1.5 \times 10^{-5} \times 10^{16} = 15 \times 10^{10}$ Bq.

58. 5

Sol.



$q_x = \sigma 4\pi a^2$

$q_y = -\sigma 4\pi b^2$

$q_z = \sigma 4\pi c^2$

Potential x = potential z

$V_x = V_z$

$$\frac{q_x}{4\pi\varepsilon_0 a} + \frac{q_y}{4\pi\varepsilon_0 b} + \frac{q_z}{4\pi\varepsilon_0 c} = \frac{q_x}{4\pi\varepsilon_0 c} + \frac{q_y}{4\pi\varepsilon_0 c} + \frac{q_z}{4\pi\varepsilon_0 c}$$

$$\frac{\sigma 4\pi a^2}{a} - \frac{\sigma 4\pi b^2}{b} + \frac{\sigma 4\pi c^2}{c} = \frac{4\pi\sigma\left[a^2 - b^2 + c^2\right]}{c}$$

$c(a - b + c) = a^2 - b^2 + c^2$

$c(a - b) = a^2 - b^2$

c = a + b

c = 5 cm

59. 100

Sol. Maximum resistance occurs

When all the resisters are connected in series combination

$\therefore$ $R_{max} = 10$ R

Here R = 10 ohm

Minimum resistance occurs

When all the resistance are connected in parallel combination

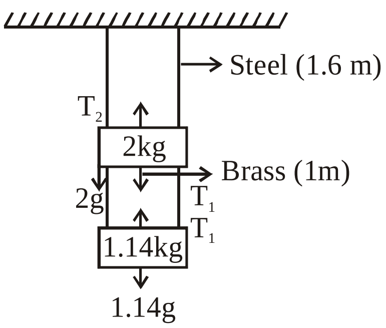
$$R_{min} = \frac{R}{10}$$

$$\therefore \frac{R_{max}}{R_{min}} = 100$$

60. 20

Sol. Tension in steel wire $T^2 = 2g + T_1$

$T_2 = 20 + 11.4 = 31.4$N



Elongation in steel wire $\Delta L = \frac{T_2 L}{Ay}$

$$\Delta L = \frac{31.4 \times 1.6}{\pi\left(0.2 \times 10^{-2}\right)^2 \times 2 \times 10^{11}}$$

$$\Delta L = \frac{16}{2 \times 4 \times 10^{-6} \times 10^{11}}$$

$= 2 \times 10^{-5}$ m $= 20 \times 10^{-6}$ m

CHEMISTRY

Section - A (Single Correct Answer)

61. D

Sol. If any component eluted second then it means that its R_f value is low and its adsorption is stronger.

$$R_f = \frac{\text{distance covered by substance from base line}}{\text{total distance covered by solvent from base line}}$$

62. A

Sol. Prolonged heating will cause oxidation of Fe^{+2} to Fe^{+3}.

63. B

Sol. $CaO + H_2O \rightarrow Ca(OH)_2$

A (less soluble)

$Ca(OH)_2 + CO_2 \rightarrow CaCO_3 + H_2O$

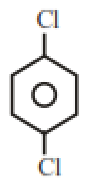
B (insoluble)

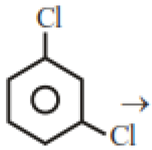
$$\underset{B}{CaCO_3} + H_2O + CO_2 \rightarrow \underset{\text{Soluble}}{Ca(HCO_3)_2}$$

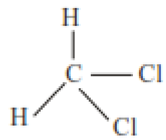
64. C

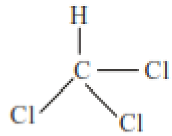
Sol. A Benzene $\rightarrow$ non polar

Anisidine $\rightarrow$ polar

B  $\rightarrow$ Non polar

 $\rightarrow$ polar

C CH_2Cl_2, $\mu_{net} \neq 0$ polar 

$CHCl_3$, $\mu_{net} \neq 0$ polar 

D  $\Rightarrow$ polar

 $\Rightarrow$ non polar

65. A

Sol. Steel plant produces slag from blast furnace. Thermal power plant produces fly ash, Fertilizer industries produces gypsum. Paper mills produces bio degradable waste.

66. A

Sol. (P) Gabriel phthalimide synthesis is used for the preparation of aliphatic primary amines. Aromatic primary amines cannot be prepared by this method.

(Q) 2°-amines reacts with Hinsberg's reagent to give solid insoluble in NaOH.

(R) Aromatic primary amine react with nitrous acid at low temperature (273 - 298 K) to form diazonium salts, which form Red dye with β-Naphthol

67. C

Sol. A $K_2Cr_2O_7$ is used as primary standard. The concentration $Na_2Cr_2O_7$ changes in aq. solution.

B It is less soluble than $Na_2Cr_2O_7$.

68. C

Sol. 2° and 3° structure of proteins are stabilized by hydrogen bonding, disulphide linkages, Van der Waals force of attraction and electrostatic force of attraction.

69. A

Sol. $T_1 = 1270$ K $T_2 = 673$ K

$T_1 > T_2$ on the basis of data

70. A

Sol. (A) The M^{3+}/M^{2+} reduction potential for manganese is greater than iron

(B) $E^o_{Fe^{+3}/Fe^{+2}} = +0.77$

$E^{o}_{Mn^{+3}/Mn^{+2}} = +1.57$

(C) $E^{o}_{Cr^{+3}/Cr^{+2}} = -0.26$

$\therefore \quad Cr^{2\oplus} + H^{\oplus} \longrightarrow Cr^{3\oplus} + \frac{1}{2}H_2$

(D) $V^{2\oplus}$ = 3 unpaired electron

Magnetic Moment = 3.87 B.M

71. D

Sol. Cresol is used as stabilizer.

72. D

Sol. (A) Paramagnetic, High Spin & Tetrahedral

(B) Paramagnetic, High Spin & Octahedral

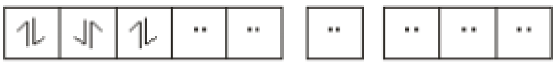
(C) Paramagnetic, High Spin & Octahedral

(D) Diamagnetic, Low Spin & Octahedral

$[Co(NH_3)_6]^{3+}$, CN = 6 (Octahedral)

NH_3 = SFL

$Co^{+3} = [Ar]3d^6$



Diamagnetic & Low spin complex.

73. A

Sol. Target equation,

$$C(\text{graphite}) + \frac{1}{2}O_{2(g)} \rightarrow CO_{(g)} \quad \text{... (i) } \Delta H$$

$$C(\text{graphite}) + O_{2(g)} \rightarrow CO_{2(g)} \quad \text{... (ii)}$$

$\Delta H_1 = -y$ kJ/mole

$$CO_{2(g)} \rightarrow CO_{(g)} + \frac{1}{2}O_{2(g)} \quad \text{... (iii)}$$

$\Delta H_2 = \frac{x}{2}$ kJ/mole

eq. (i) = eq.(ii) + eq (iii)

$$\therefore \quad \Delta H = \frac{x}{2} - y = \frac{x-2y}{2}$$

74. A

Sol. Sodium superoxide is not stable.

75. D

Sol.

(A) Nylon-2-nylon-6
Biodegradable polymer and polyamides (II)

(B) Buna-N $\rightarrow$ Butadiene acrylonitrile rubber $\rightarrow$ synthetic rubber (III)

(C) Urea-formaldehyde resin $\rightarrow$ Thermosetting polymer (I)

(D) Dacron $\rightarrow$ Polyester polymer of ethylene glycol and terephthalic acid (IV)

76. D

Sol. Number of moles of $O_2 = \frac{2.8375}{22.7} = 0.125$

$\Rightarrow$ Number of molecules = 0.125 N_A

$= 7.525 \times 10^{22}$

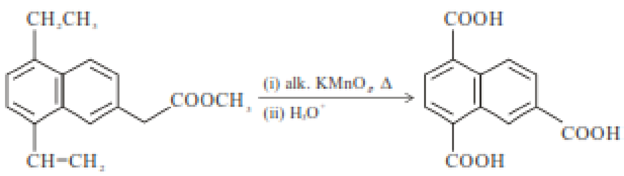
77. A

Sol. Adsorption is exothermic process due to decrease in surface energy.

Micelle formation is endothermic.

78. D

Sol. $KMnO_4$ oxidises benzylic carbon containing atleast one α-hydrogen atom to –COOH.



79. C

Sol. $PhO^{-}Na^{+} + Me\text{–}Br \xrightarrow{S_N2} Ph\text{–}O\text{–}Me$

80. B

Sol. All are correct

(A) S_N2 reaction decreases with increase in steric crowding.

(B) S_N1 reaction increases with stability of carbocation.

(C) EAS reaction decreases with decrease in electron density.

(D) Presence of electron withdrawing group at *ortho* and *para*-position to a halogen in haloarene increase nucleophilic aryl substitution.

Section - B (Numerical Value)

81. 3

Sol. (A) is correct

(B) for equilibrium $r_f = r_b$

$\Rightarrow$ (B) is correct

(C) at equilibrium the value of parameters become constant of a given temperature and not equal

$\Rightarrow$ (C) is incorrect

(D) for a given solid solute and a liquid solvent solubility depends upon temperature only

$\Rightarrow$ (D) is correct

82. 3

Sol. N_3^- linear

NO_2^- bent

I_3^- linear

O_3 bent

SO_2 bent

83. 2

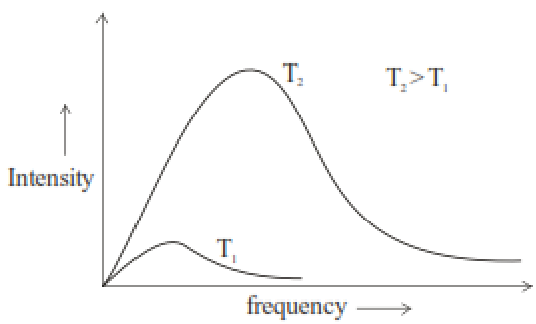
Sol. $\frac{1}{t_{1/2}} = \frac{1}{3} + \frac{1}{12} = \frac{4+1}{12} = \frac{5}{12}$

$t_{1/2} = \frac{12}{5}\text{min.} = 2.4$

Ans. is 2.

84. 0

Sol. A blackbody can emit and absorb all the wavelengths in electromagnetic spectrum $\Rightarrow$ (A) is correct.



$\Rightarrow$ (B), (C), (D) correct

Ans (0)

85. 5

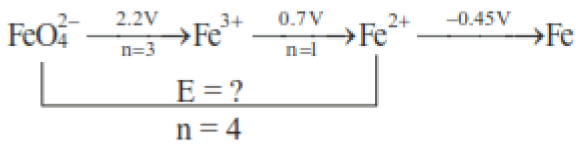
Sol. $Na_2O + H_2O \rightarrow 2NaOH$

$Cl_2O_7 + H_2O \rightarrow 2HClO_4$

$1 + 4 = 5$

86. 1825

Sol.



$4 \times E = 3 \times 2.2 + 1 \times 0.7$

$E = \frac{7.3}{4} = 1.825\text{ V} = 1825 \times 10^{-3}\text{ V}$

87. 30

Sol. $i = 1 + \alpha$ (for HA)

$= 1.3$

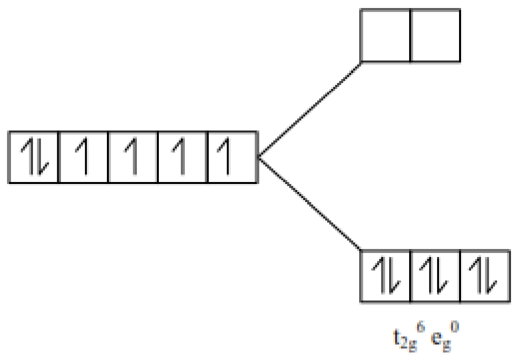
$\%\text{ increase} = \frac{(\Delta T_f)_{obs} - (\Delta T_f)_{cal}}{(\Delta T_f)_{cal}}$

$= \frac{K_f \times i \times m - K_f \times m}{K_f \times m} \times 100$

$= \frac{i-1}{1} \times 100 = 30\%$

88. 3

Sol. $K_4[Fe(CN)_6]$



$Fe^{+2} = [Ar]3d^6$

$CN^- = SFL$

t_{2g} contain 6 electron so it become 3 pairs.

89. 1567

Sol. $P_1V_1 = P_2V_2$

$940.3 \times 100 = P_2 \times 60$

$P_2 = 1567$ mm of Hg

90. (1)

Sol. $IF_5 = 1$ lone pair

$IF_7 = 0$ lone pair

$1 + 0 = 1$

❒ ❒ ❒