3S INSTITUTE
JEE ADVANCED | JEE MAIN | NEET | OLYMPIADS | MHT-CET | FOUNDATION

06-April-2023 (Evening Batch) : JEE Main Paper

MATHEMATICS

Section - A (Single Correct Answer)

1. A

Sol. Total number of ways = $6^3 = 216$

Favourable outcomes ${}^6p_3 = 120$

$\Rightarrow$ Probability $=\frac{120}{216}=\frac{5}{9}$

$\Rightarrow$ p = 5, q = 9

$\Rightarrow$ q – p = 4

2. C

Sol. $S_1 = (1999 + 24)^{2022} - (1999)^{2022}$

$\Rightarrow {}^{2022}C_1(1999)^{2021}(24) + {}^{2022}C_2(1999)^{2020}(24)^2 +$ .... soon S_1 is divisible by 8

$S_2 : 13\,(13^n) - 11n - 13$

$13^n = (1 + 12)^n = 1 + 12n + {}^nC_2 12^2 + {}^nC_3 12^3.....$

$13\,(13^n) - 11n - 13 = 145n + {}^nC_2\, 12^2 + {}^nC_3 12^3.....$

If ($n = 144m,\ m \in N$), then it is divisible by 144

For infinite value of n.

3. D

Sol. $\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)^n<\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}}-2^{\frac{1}{5}}\right)\left(2^{\frac{1}{2}}-2^{\frac{1}{7}}\right)$

$---\left(2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}\right)<\left(2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}\right)^n$

$\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)^n<L<\left(2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}\right)^n$

$\lim\limits_{n\to\infty}\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)^n=0$ and $\lim\limits_{n\to\infty}\left(2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}\right)^n=0$

$\Rightarrow \lim\limits_{n\to\infty} L = 0$

4. C

Sol. $\mathrm{Re}(az^2 + bz) = a$

$az^2+bz+a\bar{z}^2+b\bar{z}=2a$

$a(z^2+\bar{a}^2)+b(z+\bar{z})=2a$ ....(1)

$\mathrm{Re}(bz^2 + az) = b$

$bz^2+az+b\bar{z}^2+a\bar{z}=2b$

$b(z^2+\bar{z}^2)+a(z+\bar{z})=2b$ ....(2)

(1) × b – (2) × (a)

$\Rightarrow (b^2-a^2)(z+\bar{z})=0$

$\Rightarrow (z+\bar{z})=0\ (a^2\neq b^2)$

(1) × – (2) × (b)

$\Rightarrow (a^2-b^2)(z+\bar{z})=2(a^2-b^2)\ (a^2\neq b^2)$

$z^2+\bar{z}^2=2$

$\Rightarrow (z+\bar{z})^2-2z\bar{z}=2$

$z\bar{z}=-1$

$\Rightarrow 1 + 1^2 = -1$

$\Rightarrow$ No solution

But when a = –b,

$\mathrm{Re}(az^2 - az) = a$

$\Rightarrow \mathrm{Re}(a(x^2 - y^2 + i2xy) - a(x + iy)) = a$

$\Rightarrow a(x^2 - y^2) - ax = a$

$\Rightarrow x^2 - y^2 - x = 1$

$\Rightarrow x^2 - x - 1 = y^2$

For any real values of y there two values of x, hence infinite complex numbers are possible.

5. A

Sol. $f(x)=\frac{1}{\sqrt{\lceil x\rceil}-x}$

If $x\in I$ $\lceil x\rceil=[x]$ (greatest integer function)

If $x\notin I$ $\lceil x\rceil=[x]+1$

$\Rightarrow f(x)=\begin{cases}\frac{1}{\sqrt{[x]-x}}, & x\in I\\ \frac{1}{\sqrt{[x]+1-x}}, & x\notin I\end{cases}$

$$\Rightarrow f(x)=\begin{cases}\dfrac{1}{\sqrt{-\{x\}}}, & x\in I,\ \text{(does not exist)}\\ \dfrac{1}{\sqrt{1-\{x\}}}, & x\notin I\end{cases}$$

$\Rightarrow$ domain of f(x) = R – I

Now, $f(x)=\dfrac{1}{\sqrt{1-\{x\}}},\ x\notin I$

$\Rightarrow 0 < \{x\} < 1$

$\Rightarrow 0<\sqrt{1-\{x\}}<1$

$\Rightarrow \dfrac{1}{\sqrt{1-\{x\}}}>1$

$\Rightarrow$ Range $(1,\ \infty)$

$\Rightarrow$ A = R – 1

$B=(1,\ \infty)$

So, $A\cap B=(1,\ \infty)-N$

$A\cup B\neq(1,\ \infty)$

$\Rightarrow$ S1 is only correct

6. D

Sol. $(1+\ln x)\dfrac{dx}{dy}-x\ln x=e^{y}$

Let x ln x = t

$(1+\ln x)\dfrac{dx}{dy}=\dfrac{dt}{dy}$

$\dfrac{dt}{dy}-t=e^{y}$

If $=e^{\int -dy}=e^{-y}$

$t.e^{-y}=\int e^{y}e^{-y}dy+c$

$te^{-y} = y + c$

$x \ln x\, e^{-y} = y + c$

$x \ln x = ye^{y} + ce^{y}$

(1, 0) 0 = C

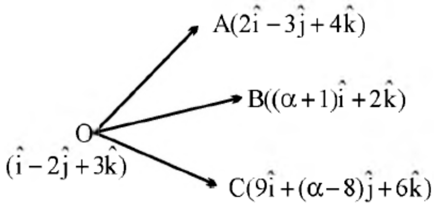
$\Rightarrow x\ln x=ye^{y}$

$\Rightarrow \alpha\ln\alpha=2e^{2}$

$\alpha^{\alpha}=e^{2e^{2}}$

7. D

Sol.



[OA OB OC] = 0

$$\begin{vmatrix}1 & -1 & 1\\ \alpha & 2 & -1\\ 8 & \alpha-6 & 3\end{vmatrix}=0$$

$\Rightarrow \alpha^{2}-2\alpha-8=0$

$\Rightarrow (\alpha-4)(\alpha+2)=0$

$\therefore \alpha=4,\ -2$

8. A

Sol. x + y + z

x + 2y + αz = 10

x + 3y + 5z = β

$$D=\begin{vmatrix}1 & 1 & 1\\ 1 & 2 & \alpha\\ 1 & 3 & 5\end{vmatrix}=1(10-3\alpha)-1(5-\alpha)+1(3-z)$$

= 10 – 3α – 5 + α + 1

= 6 – 2α

For unique solution $6-2\alpha\neq 0\Rightarrow\alpha\neq 0$

9. B

Sol. y = |x – 1| + |x – 2| and y = 3

$\therefore$ Required area $=\dfrac{1}{2}(1+3)\times 2=4$

10. C

Sol. $P^2 = I – P$

$P^{\alpha} + P^{\beta} = \gamma I – 29P,\ P^{\alpha} – P^{\beta} = 5I – 13P$

$P^4 = (I – P)^2 = I – 2P + P^2 = 2I – 3P$

$P^6 = (2I – 3P)(I – P) = 5I – 8P$

$P^8 = (2I – 3P)^2 = 4I – 12P + 9(I – P) = 13I – 21P$

P^8 + P6 = 18I – 29P

P^8 – P6 = 8I – 13P

a = 8; β = 6; γ = 18, δ = 8

α + β + γ – 5 = 8 + 6 + 18 – 8 = 24

11. B

Sol. B .... → 5! = 120

C .... → 5! = 120

I .... → 5! = 120

L .... → 5! = 120

PB .... → 4! = 24

PC .... → 4! = 24

PL .... → 4! = 24

PI .... → 4! = 24

$P \cup BC$ .... $\to 2! = 2$

$P \cup BI$ .... $\to 2! = 2$

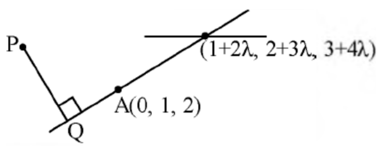
$P \cup BLC$ .... $\to 1! = 1$

$P \cup BLIC$ .... $\to \;\; = 1$

Serial number = 4(120) + 4(24) + 6 = 582

12. D

Sol.



$\overline{AB} \cdot \vec{n}$

$\Rightarrow [(1+2\lambda)\hat{i} + (1+3\lambda)\hat{j} + (1+4\lambda)\hat{k}] \cdot (2\hat{i} + \hat{j} - 3\hat{k})$

$2 + 4\lambda + 1 + 3\lambda - 3 - 12\lambda = 0$

$5\lambda = 0 \Rightarrow \lambda = 0$

Line $\overline{AB},\ \vec{r} = \hat{j} + 2\hat{k} + \mu(\hat{i} + \hat{j} + \hat{k})$

General form : $Q(\mu - 1 + \mu,\ 2 + \mu)$

$\therefore \overrightarrow{PQ} \cdot \overrightarrow{AB} = 0$

$(\mu - 1) + (10 + \mu) + \mu = 0$

$3\mu = -9 \;\Rightarrow \mu = -3$

$\therefore$ distance $= \sqrt{16 + 49 + 9} = \sqrt{74}$

13. B

Sol. Equation of plane P is

(x + y + z – 6) + λ(2x + 3y + 4z + 5) = 0

Plane passes through the point (0, 2, –2)

∴ (2 – 2 – 6) + λ(6 – 8 + 5) = 0

–6 + λ(3) = 0

λ = 2

Equation of plane p is

(x + y + z – 6) + 2(2x + 3y + 4z + 5) = 0

5x + 7y + 9z + 4 = 0

$$d = \left| \frac{5 \times 12 + 7 \times 12 + 9 \times 18 + 4}{\sqrt{5^2 + 7^2 + 9^2}} \right|$$

$$d = \left| \frac{60 + 84 + 162 + 4}{\sqrt{25 + 49 + 81}} \right|$$

$$d = \frac{310}{\sqrt{155}}$$

$$d^2 = \frac{310 \times 310}{155} = 620$$

14. D

Sol. $f(x) + f(\pi - x) = \pi^2$

$$I = \int_0^{\pi} f(x) \sin x\,dx$$

Applying King's Rule

$$I = \int_0^{\pi} f(\pi - x) \cdot \sin(\pi - x)dx$$

$$2I = \int_0^{\pi} [f(x) + f(\pi - x)] \sin x\,dx$$

$$2I = \int_0^{\pi} \pi^2 \sin x\,dx$$

$$2I = \pi^2 \cdot \int_0^{\pi} \sin x\,dx$$

$$2I = \pi^2 \times 2$$

$$I = \pi^2$$

15. B

Sol. $\left(ax^2 + \frac{1}{2} 2bx \right)^{11}$

$$T_{r+1} = {}^{11}C_r (ax^2)^{11-r} \cdot \left(\frac{1}{2bx} \right)^r$$

$$= {}^{11}C_r a^{11-r} \cdot \left(\frac{1}{2b}\right)^r \cdot x^{22-2r-r} = {}^{11}C_r a^{11-r} \cdot \left(\frac{1}{2b}\right)^r \cdot x^{22-3r}$$

$\therefore 22-3r=7$

$r = 5$

Again $\left(ax - \frac{1}{3bx^2}\right)^{11}$

$$T_{r+1} = {}^{11}C_r (ax)^{11-r}\left(-\frac{1}{3bx^2}\right)^2$$

$$= {}^{11}C_r a^{11-r} \cdot \left(\frac{-1}{3b}\right)^r \cdot x^{11-r-2r}$$

$\therefore 11-3r=-7$

$3r = 18$

$r = 6$

Now, $\frac{{}^{11}C_r a^6}{32b^5} = \frac{{}^{11}C_6 \cdot a^5}{3^6 \cdot b^6}$

$729ab = 32$

16. A

Sol. $(p \to q) \vee ((\sim p) \wedge q)$

p	q	$p \to q$	$\sim p \wedge q$	$(p \to q) \vee (\sim p) \wedge p)$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	F	T

Not a tautology

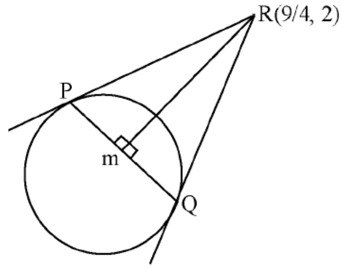
p	q	$q \to p$	$(\sim p) \wedge q$	$(q \to p) \vee (\sim p) \wedge q)$
T	T	T	F	F
T	F	T	F	F
F	T	F	T	T
F	F	T	F	F

Not a contradiction

17. D

Sol. Equation of circle is $x^2 + y^2 - 2x + y - 5 = 0$

$R = \frac{5}{2}$



Let of $PR = QR = \sqrt{S_1}$

$$= \sqrt{\frac{81}{16} + 4 - \frac{2\times 9}{4} + 2 - 5} = \frac{5}{4}$$

Area of triangle $PQR = \frac{RL^3}{R^2+L^2} = \frac{\frac{5}{2}\cdot\frac{125}{64}}{\frac{25}{4}+\frac{25}{16}} = \frac{5}{8}$

18. C

Sol. $V = [\vec{a}\ \vec{b}\ \vec{c}]$

$[\vec{a},\ \vec{b}+\vec{c},\ \vec{a}+2\vec{b}+3c]$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = [\vec{a}\ \vec{b}\ \vec{c}] = 1(3-2)V = V.$$

19. B

Sol. $1^2 - 2^2 + 3^2 - 4^2 + \ldots\ldots (2021)^2 - (2022)^2 + (2023)^2 = 1012\ m^2n$

$= (1-2)(1+2) + (3-4)(3+4) + \ldots\ldots. + (2021 - 2022)(2021 + 2022) + (2023)^2$

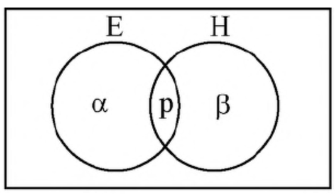
$= (-1)(1+2+3+4+\ldots.+2022) + (2023)^2$

$$= (-1)\cdot\frac{(2022)(2023)}{2} + (2023)^2$$

$= 2023(2023-1011) = 2023 \times 1012\ m^2n = 2023 = 172.7$ m = 17, n = 7 $m^2 - n^2 = 17^2 - 7^2 = 240$

20. C

Sol.



$\alpha + p = 75$ ....(1)

$\beta + p = 40$ ....(2)

$\alpha + \beta + p = 100$ ....(3)

From (1), (2) and (3)

$P = 15,\ \alpha = 60$ and $\beta = 25$

Now equation of ellipse : $25\left(\frac{x^3}{\alpha^2} + \frac{y^2}{\beta^2}\right) = 1$

$\frac{x^2}{144} + \frac{y^2}{25} = 1$

$\Rightarrow e = \frac{\sqrt{119}}{12}$

Section - B (Numerical Value)

21. 0

Sol. Let $f(x) = \frac{x}{(1+x^n)}$, $x \in R - \{-1\},\ n \in N,\ n > 2$

$F^n(x)$ = (fofof .... upto n times)(x).

then $\lim_{n\to\infty} \int_0^1 x^{n-2}(f^n(x))dx$

$f(f(x)) = \frac{x}{(1+2x^n)^{1/n}}$

$f(f(f(x))) = \frac{x}{(1+3x^n)^{1/n}}$

Similar $f^n(x) = \frac{x}{(1+n\cdot x)^{1/n}}$

Now $\lim_{n\to\infty} \int \frac{x^{n-2}\cdot xdx}{(1+n\cdot x^n)^{1/n}} = \lim_{n\to\infty} \int \frac{x^{n-1}}{(1+n\cdot x^n)^{1/n}}$

Now I + nx^n = t

$n^2 \cdot x^{n-1}dx = dt$

$x^{n-1}dx = \frac{dt}{n^2}$

$\Rightarrow \lim_{n\to\infty} \frac{1}{n^2}\int_1^{1+n} \frac{dt}{t^{1/n}}$

$\Rightarrow \lim_{n\to\infty} \frac{1}{n^2}\left[\frac{t^{1-\frac{1}{n}}}{1-\frac{1}{n}}\right]_1^{1+n}$

$\Rightarrow \lim_{n\to\infty} \frac{1}{n(n-1)}\left((1+n)^{\frac{n+1}{n}} - 1\right)$

Now let $n = \frac{1}{h}$

$\Rightarrow \lim_{n\to\infty} \frac{\left(1+\frac{1}{h}\right)^{1-h} - 1}{\frac{1}{h}\frac{(1-h)}{h}}$

Using series expansion.

$\Rightarrow 0$

22. 4

Sol. The value of tan 9° – tan 27° – tan 63° + tan 81°

$\Rightarrow$ tan 9° + cot 9° – tan 27° – cot 27°

$\Rightarrow \frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ}$

$\Rightarrow \frac{2\times 4}{\sqrt{5}-1} - \frac{2\times 4}{\sqrt{5}+1}$

$\Rightarrow 4$

23. 18

Sol. If the lines $\frac{x-1}{2} = \frac{2-y}{-3} = \frac{z-3}{\alpha}$

And $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{\beta}$ intersect

Point on first line (1, 2, 3) and point on second line (4, 1, 0).

Vector joining both points is $-3\hat{i} + \hat{j} + 3\hat{k}$

Now vector along first line is $2\hat{i} + 3\hat{j} + \alpha\hat{k}$

Also vector along second line is $5\hat{i} + 2\hat{j} + \beta\hat{k}$

Now these three vectors must be coplanar

$$\Rightarrow \begin{vmatrix} 2 & 3 & \alpha \\ 5 & 2 & \beta \\ -3 & 1 & 3 \end{vmatrix} = 0$$

$\Rightarrow 2(6-\beta) - 3(15+3\beta) + \alpha(11) = 0$

$\Rightarrow \alpha - \beta = 3$

Now $\alpha = 3 + \beta$

Given expression $8(3+\beta)\cdot\beta = 8(\beta^2 + 3\beta)$

$$= 8\left(\beta^2 + 3\beta + \frac{9}{4} - \frac{9}{4}\right) = 8\left(\beta + \frac{3}{2}\right)^2 - 18$$

So magnitude of minimum value = 18

24. 400

Sol. If $(20)^{19} + 2(21)(20)^{18} + 3(21)^2(20)^{17} + \ldots + 20(21)^{19} = k(20)^{19}$ then k is

$$20^{19}\left(1 + 2\cdot\left(\frac{21}{20}\right) + 3\left(\frac{21}{20}\right)^2 + \ldots + 20\left(\frac{21}{20}\right)^{19}\right) = k(20)^{19}$$

$$\Rightarrow k = 1 + 2\left(\frac{21}{20}\right) + 3\left(\frac{21}{20}\right)^2 + \ldots + 20\left(\frac{21}{20}\right)^{19} \qquad \ldots(1)$$

$$\Rightarrow k\left(\frac{21}{20}\right) = \frac{21}{20} + 2\cdot\left(\frac{21}{20}\right)^2 + \ldots$$

$$\ldots + 19\left(\frac{21}{20}\right)^{19} + 20\cdot\left(\frac{21}{20}\right)^{20} \qquad \ldots(2)$$

Subtracting equation (2) from (1)

$$\Rightarrow k\left(\frac{-1}{20}\right) = 1 + \frac{21}{20} + \left(\frac{21}{20}\right)^2 + \ldots + \left(\frac{21}{20}\right)^{19} - 20\cdot\left(\frac{21}{20}\right)^{20}$$

$$\Rightarrow \left(\frac{-1}{20}\right) = \frac{1\left(\left(\frac{21}{20}\right)^{20} - 1\right)}{\left(\frac{21}{20} - 1\right)} - 20\cdot\left(\frac{21}{20}\right)^{20}$$

$$\Rightarrow k\left(\frac{-1}{20}\right) = 20\left(\frac{21}{20}\right)^{20} - 20 - 20\cdot\left(\frac{21}{20}\right)^{20}$$

$$\Rightarrow k\left(\frac{-1}{20}\right) = 20\left(\frac{21}{20}\right)^{20} - 20 - 20\cdot\left(\frac{21}{20}\right)^{20}$$

$$\Rightarrow k\left(\frac{-1}{20}\right) = -20$$

$$\Rightarrow k = 400$$

25. 432

Sol. UNIVERSE

Vowels: E, I, U

Consonants: N, V, R, S

$\rightarrow {}^3C_2 \times {}^4C_2 \times 4! = 3 \times 6 \times 24 = 432$

26. 5

Sol. $y = x^5 - 20x^3 + 50x + 2$

$$\frac{dy}{dx} = 5x^4 - 60x^2 + 50 = 5(x^4 - 12x^2 + 10)$$

$$\frac{dy}{dx} = 0 \Rightarrow x^4 - 12x^2 + 10 = 0$$

$$\Rightarrow x^2 = \frac{12 \pm \sqrt{144 - 40}}{2}$$

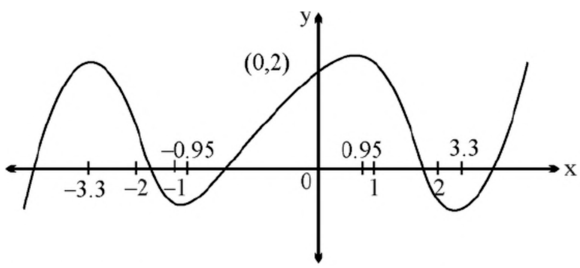
$$\Rightarrow x^2 = 6 \pm \sqrt{26} \Rightarrow x^2 \approx 6 \pm 5.1$$

$$\Rightarrow x^2 \approx 11.1,\ 0.9$$

$$\Rightarrow x^2 \approx 3.3,\ \pm 0.95$$

f(0) = 2, f (1) = + ve, f (2) = –ve

f(–1) = –ve, f (–2) = +ve



27. 2

Sol. For circle :

$|z - z_1|^2 + |z - z_2|^2 = |z_1 - z_2|^2$

$$r = \frac{|z_1 - z_2|}{2} = \frac{|\alpha - \beta|}{2} = \sqrt{\lambda - 1}$$

$$2\lambda = |\alpha - \beta|^2$$

$$|\alpha - \beta| = 2\sqrt{\lambda - 1}$$

$$|\alpha - \beta|^2 = 4\lambda - 4 = 2\lambda$$

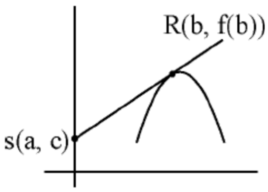
$$\lambda = 2$$

$$\Rightarrow |\alpha - \beta|^2 = 4$$

$$|\alpha - \beta| = 2$$

28. 5

Sol.



Equation of tangent at R(b, f(2)) is

y – f(b) = f'(b)·(x – b)

which passes through (0, c)

$\Rightarrow$ c – f(b) = f'(b)·(–b)

$$\Rightarrow \frac{3}{b} - f(b) = f'(b)\cdot(-b)$$

$$\Rightarrow bf'(b) - f(b) = -\frac{3}{b}$$

$$\Rightarrow \frac{bf'(b) - f(b)}{b^2} = -\frac{3}{b^3}$$

$$\Rightarrow d\left(\frac{f(b)}{b}\right) = -\frac{3}{b^3} \Rightarrow \frac{f(b)}{b} = \frac{3}{2b^2} + \lambda$$

Which passes through (1, 3/2)

$$\Rightarrow \frac{3}{2} = \frac{3}{2} + \lambda \Rightarrow \lambda = 0$$

$$\Rightarrow f(b) = \frac{3}{2b}$$

$$f(a) = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{3}{2b} \Rightarrow b = 3$$

$$\Rightarrow c = 1 \Rightarrow Q(3, 1/2)$$

$$\Rightarrow PQ^2 = 2^2 + (1)^2 = 5$$

29. 2

Sol. $e_H = \sqrt{2}$

$$e_E = \frac{1}{\sqrt{2}}$$

Since the curves intersect each other orthogonally
The ellipse and the hyperbola are confocal

$$H: \frac{x^2}{1/2} - \frac{y^2}{1/2} = 1$$

$\Rightarrow$ foci = (1, 0)

For ellipse $a \cdot e_E = 1$

$$\Rightarrow a = \sqrt{2}$$

$$(e_E)^2 = \frac{1}{2} \Rightarrow 1 - \frac{b^2}{a^2} = \frac{1}{2} \Rightarrow \frac{b^2}{a^2} = \frac{1}{2}$$

$$\Rightarrow b^2 = 1$$

$$\text{Length of L.R.} = \frac{2b^2}{a} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

30. 25

Sol.

x_i	f_i	f_ix_i	$f_ix_i^2$
2	4	8	16
4	4	16	64
6	α	6α	36α
8	15	120	960
10	8	80	800
12	β	12β	144β
14	4	56	784
16	5	80	1280

$$N = \sum f_i = 40 + \alpha + \beta$$

$$\sum f_ix_i = 360 + 6\alpha + 12\beta$$

$$\sum f_ix_i^2 = 3904 + 36\alpha + 144\beta$$

$$\text{Mean}(\bar{x}) = \frac{\sum f_ix_i}{\sum f_i} = 9$$

$$\Rightarrow 360 + 6\alpha + 12\beta = 9(40 + \alpha + \beta)$$

$$3\alpha = 3\beta \Rightarrow \alpha = \beta$$

$$\sigma^2 = \frac{\Sigma f_ix_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_ix_i}{\Sigma f_i}\right)^2$$

$$\Rightarrow \frac{3904 + 36\alpha + 144\beta}{40 + \alpha + \beta} - (\bar{x})^2 = 15.08$$

$$\Rightarrow \frac{3904 + 180\alpha}{40 + 2\alpha} - (9)^2 = 15.08$$

$$\Rightarrow \alpha = 5$$

Now, $\alpha^2 + \beta^2 - \alpha\beta = \alpha^2 = 25$

PHYSICS

Section - A (Single Correct Answer)

31. C

Sol. Least count = 0.2 cm

u = (100 ± 0.2) – (80 ± 0.2) = (20 ± 0.4)cm

v = (180 ± 0.2) – (100 ± 0.2) = (80 ± 0.4)cm

From lens formula,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{f}=\frac{1}{80}-\frac{1}{-20}$$

f = 16 cm

$$\text{Also }\frac{\Delta v}{v^2}+\frac{\Delta u}{u^2}=\frac{\Delta f}{f^2}$$

$$\Rightarrow\frac{\Delta f}{f}\times100=\left(\frac{\Delta v}{v^2}+\frac{\Delta u}{u^2}\right)\times f\times100$$

$$\Rightarrow\%f=\left(\frac{0.4}{400}+\frac{0.4}{6400}\right)\times64\times100$$

= 1.70

32. A

Sol. $$I_0=\frac{E_0}{x_c}=\frac{E_0}{\frac{1}{\omega_c}}=E_0\omega_c$$

$$\Rightarrow I_0=36\times120\pi\times150\times10^{-6}$$

$$\Rightarrow I_0=2.03$$

$$\simeq 2A$$

33. C

Sol. (R) is the statement of Pascal's principle & which explains the assertion (S)

34. C

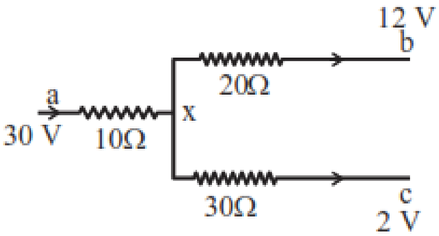
Sol. As medium changes, optical path changes.

$$\text{Also, }\frac{\Delta x}{\lambda}=\frac{\Delta\phi}{2\pi}$$

Hence phase difference changes.

35. A

Sol. Sum of current at junction point will be zero :



$$\frac{x-30}{10}+\frac{x-12}{20}+\frac{x-2}{30}=0$$

$$\Rightarrow x\left(\frac{1}{10}+\frac{1}{20}+\frac{1}{30}\right)=\frac{30}{10}+\frac{12}{20}+\frac{2}{30}$$

$$\Rightarrow x\left(\frac{6+3+2}{60}\right)=\frac{180+36+4}{60}$$

$$\Rightarrow x=\frac{220}{11}=20V$$

$$\therefore\quad \text{Current through } 20\Omega=\frac{x-12}{20}$$

$$=\frac{20-12}{20}=\frac{2}{5}=0.4A$$

36. D

Sol. $$|\langle\vec{v}\rangle|=\frac{|\vec{r}_f-\vec{r}_i|}{\Delta t}$$

$$=\frac{2R\cos\left[\frac{\pi-\theta}{2}\right]}{\frac{2\pi R}{3v}}=3\cos30^\circ$$

$1.5\sqrt{3}$ m/s

Correct option is (D)

37. C

Sol. $eV_s = k_{max}$

$$V_s=\left\{\frac{h}{e}\right\}f+\left\{\frac{-\phi}{e}\right\}$$

Slope is independent of nature of metal

$$\text{slope}(V_s)^{Gold}=\text{slope}(V_s)^{Aluimiun}$$

38. A

Sol. $$C=\sqrt{\frac{\gamma RT}{M}}$$

$$C\propto\frac{1}{\sqrt{M}}$$

$$\frac{C_{H_2}}{C_{O_2}}=\sqrt{\frac{32}{2}}=4:1$$

Correct option (A)

39. D

Sol. $\omega = \frac{2\pi}{3.14} = 2$ rad/s

$\left|\vec{f}_{centrifugal}\right| = \left|-m\vec{a}_{Ref.}\right|$

$= M\omega^2 R = 40$ N

Correct option (D)

40. D

Sol. v = u + at

60 = 10 + 2t

t = 25 sec.

Correct option (D)

41. D

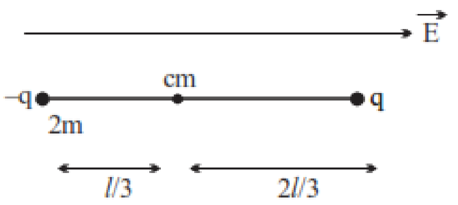
Sol. Planets revolve in elliptical paths around sun. Thus their linear speed is not constant

42. C

Sol. In forward biased condition, diffusion of majority charge carriers takes place from p-side to n-side which constitute the diffusion current.

43. C

Sol.



If released, it will oscillate about centre of mass.

For small 'θ'

$\tau = -\,PE\cdot\theta$

$\Rightarrow \left[2m\frac{l^2}{9} + m\frac{4l^2}{9}\right]\alpha = -qlE\cdot\theta$

$\Rightarrow \frac{2ml^2}{3}\alpha = -qlE\cdot\theta \Rightarrow \alpha = -\frac{3qE}{2ml}\theta$

$\omega = \sqrt{\frac{3qE}{2ml}}$

44. D

Sol. $U_E = \frac{1}{2}\epsilon_0 E^2, U_B = \frac{B^2}{2\mu_0}$

45. D

Sol. $V_{rms} = \sqrt{\frac{3RT}{M}}$

$\Rightarrow V_{rms} \alpha \sqrt{T}$

Increasing temperature 4 times, rms speed gets doubled.

46. C

Sol. To convert galvanometer into ammeter low resistances should be added into parallel & for voltmeter conversion, a very high resistance should be added in series.

47. C

Sol. using average rate of Newton's law of cooling

$\frac{T_1 - T_2}{t} = K\left(\frac{T_1 + T_2}{2} - T_s\right)$

Given $\frac{60-40}{7} = K(50-10)$

....(i)

$\& \frac{40-T}{7} = K\left(\frac{40+T}{2} - 10\right)$

....(ii)

From (i) & (ii)

T = 28°C

48. A

Sol. $U = \frac{1}{2}m\omega^2 r^2$

$F = -\frac{dv}{dr} = -m\omega^2 r$

Now $m\omega^2 r = \frac{mv^2}{r} \Rightarrow v = \omega r$ ....(i)

$\&\ mvr = \frac{nh}{2\pi}$ ....(ii)

From (i) & (ii)

$m\omega r^2 = \frac{nh}{2\pi}$

$\Rightarrow r \propto \sqrt{n}$

49. B

Sol. Modulation index $= \frac{A_m}{A_c} = 0.6$

Minimum amplitude of modulated wav

$= A_c - A_m = 3$

$\therefore \quad A_C - 0.6\,A_c = 3 \Rightarrow 0.4\,A_c = 3$

$$A_c = \frac{3}{0.4} = \frac{15}{2} = 7.5V$$

$A_m = 0.6\,A_c = 4.5$ V

$\therefore$ Maximum amplitude $= A_c + A_m$

$= 7.5 + 4.5 = 12V$

$\therefore$ Correct option is (B)

50. B

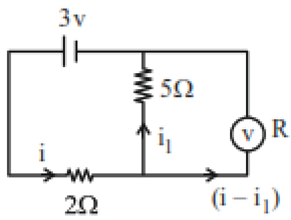
Sol. $$\Rightarrow g' = \frac{gR^2}{r^2} = \frac{gR^2}{\left(R + \frac{R}{4}\right)^2} = \frac{16g}{25}$$

$\therefore$ Weight $= \frac{16}{25} \times 100 = 64N$

Section - B (Numerical Value)

51. 20

Sol. $$i_1 = \frac{2V}{5\Omega} = \frac{2}{5}A$$



$$i = \frac{1V}{2\Omega} = \frac{1}{2}A$$

$\therefore$ Current through voltmeter $= i - i_1$

$$= \frac{1}{2} - \frac{2}{5} = \frac{5-4}{10} = \frac{1}{10}A$$

$\therefore$ For voltmeter

$$2 = \left(\frac{1}{10}\right)R \Rightarrow R = 20\Omega$$

52. 11

Sol. Tensile stress, $\sigma = \frac{F}{A} = \frac{4mg}{\pi D^2}$

$$\therefore m = \frac{\pi D^2 \sigma}{4g}$$

$$= \frac{22}{7} \times \frac{\left(14 \times 10^{-3}\right)^2 \times 7 \times 10^5}{4 \times 9.8} = 11 \text{ kg}$$

53. 5

Sol. $$c = \frac{\varepsilon_0 A}{(d - c)} = \frac{\varepsilon_0 \times 200 \times 10^{-4}}{4 \times 10^{-3}}$$

$\therefore x = 5$

The situation is equivalent to a conducting slab placed between the plates

54. 5

Sol. For ring $I = mR_1^2 = mK_1^2$

$\therefore$ Radius of gyration $K_1 = R_1$

For solid sphere

$$I' = \frac{2}{5}m'R_2^2 = m'K_2^2$$

$\therefore$ Its radius of gyration $= K_2 = \sqrt{\frac{2}{5}}R_2$

$\because K_1 = K_2$

$$\therefore R_1 = \sqrt{\frac{2}{5}}R_2$$

$$\therefore \frac{R_1}{R_2} = \sqrt{\frac{2}{5}}$$

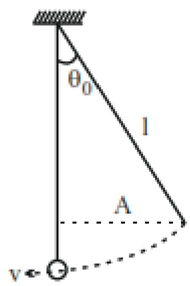
$\therefore \quad x = 5$

55. 99

Sol. $$\sin\theta_0 = \frac{A}{l} = \frac{10}{100} = \frac{1}{10}$$

From conservation of energy

$$\frac{1}{2}mv^2 = mgl(1 - \cos\theta)$$



Maximum tension occurs at mean position.

$$\therefore T - mg = \frac{mv^2}{l}$$

$$\Rightarrow T = mg + \frac{mv^2}{l}$$

$\therefore$ T = mg + 2mg (1 – cos θ)

$$= mg\left[1 + 2\left(1 - \sqrt{1 - \sin^2\theta}\right)\right]$$

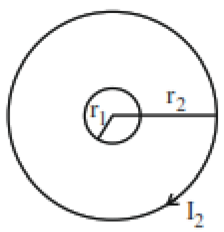
$$= \frac{250}{1000} \times 9.8\left[3 - 2\left(1 - \frac{1}{200}\right)\right] = \frac{99}{40}$$

$\therefore$ x = 99

56. 4

Sol. r_1 = 1cm, N_1 = 10

r_2 = 1000cm, N_2 = 200



$\phi_{1.2}$ = MI_2

$$N_2\vec{B}_2 \cdot N_1\vec{A}_1 = MI_2$$

$$\Rightarrow N_1N_2\frac{\mu_0 I_2}{2r_2} \cdot \pi r_1^2 = MI_2$$

57. 462

Sol. d = 2.5 mm, D = 150 cm

Fringe width $\beta = \frac{\lambda D}{d}$

Let n^{th} bright tringle of λ_1 match with m^{th} bright triangle of λ_2

$\Rightarrow$ $n\beta_1 = n\beta_2$

$$\Rightarrow n\lambda_1 = m\lambda_2 \Rightarrow \frac{n}{m} = \frac{\lambda_2}{\lambda_1} = \frac{5500}{7000}$$

$$\Rightarrow \frac{n}{m} = \frac{11}{14}$$

Distance where bright fringe will match

$$= n\beta_1 = \frac{11 \times 7000\text{Å} \times 150\text{cm}}{0.25\text{cm}} = 462 \times 10^{-5}$$

58. 375

Sol. Let V_1 and V_2 are velocity just before and just after hitting the floor.

$$\frac{V_1}{V_2} = 4 \Rightarrow V_1 = 4V_2$$

$$KE_{before} = \frac{1}{2}mV_1^2$$

$$KE_{after} = \frac{1}{2}mV_2^2 = \frac{1}{2}\frac{m \cdot V_1^2}{16}$$

$$\Delta KE = \frac{1}{2}mV_1^2\left(\frac{1}{16} - 1\right) = \frac{-15}{32}mV_1^2$$

$$\% \text{ change } = \frac{\Delta KE}{KE_{before}} \times 100\%$$

$$= \frac{-15}{6} \times 100 = \frac{-375}{4}\%$$

59. 16

Sol. Binding energy of system $= \frac{ke^2}{2r}$ joule and

$$\frac{ke^2}{2r} = 12.8 \text{ ev}$$

$$\frac{9 \times 10^9 \times \left(1.6 \times 10^{-19}\right)^2}{2r} = 12.8 \times 1.6 \times 10^{-19}$$

$$\Rightarrow r = \frac{9 \times 10^9 \times 1.6 \times 10^{-19}}{12.8 \times 2}$$

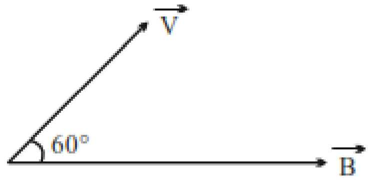
$$\Rightarrow r = \frac{9 \times 10^{-10}}{16}$$

60. 40

Sol. $B = \frac{\pi}{2} \times 10^{-3}$

$$K.E. = \frac{1}{2}mV^2$$

$$\Rightarrow V = \sqrt{\frac{2KE}{m}}$$



Pitch = v cos 60° × time period of one rotation

$$= v\cos 60° \times \frac{2\pi m}{eB}$$

$$= \sqrt{\frac{2\times 2\times 1.6\times 10^{-19}}{1.6\times 10^{-27}}} \times \cos 60° \times \frac{2\pi \times 1.6\times 10^{-27}}{1.6\times 10^{-19}\times \frac{\pi}{2}\times 10^{-3}}$$

$$= 2\times 10^{4}\times \frac{1}{2}\times 4\times 10^{-5}$$

= 4 × 10^{-1} m = 40 cm

CHEMISTRY

Section - A (Single Correct Answer)

61. A

Sol. Hydration enthalpy $\propto \frac{1}{\text{size}}$

Down the group as size increases hydration enthalpy decreases.

Order : $Be^{2+} > Mg^{+2} > Ca^{+2} > Sr^{+2} > Ba^{+2}$

62. A

Sol. IUPAC name of $K_3[Co(C_2O_4)_3]$ is

Potassium trioxalatocobaltate(III)

63. D

Sol. Factual

64. D

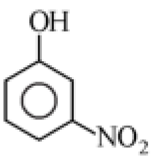
Sol. Nessler reagent is – $K_2[HgI_4]$

65. B

Sol. In solid state $BeCl_2$ as polymer, in vapour state it fomi chloro-bridged dimer while above 1200 K it is monomer.

66. A

Sol. Strongest acid from the following is



—NO_2 group has more EWG nature so more acidic.

67. C

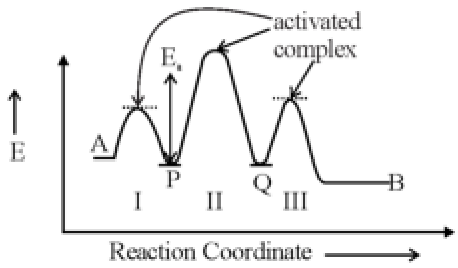
Sol. As electropositive character increases basic character of oxide increases.

$$\underbrace{B_2O_3}_{\text{acidic}} < \underbrace{Al_2O_3 < Ga_2O_3}_{\text{amphoteric}} < \underbrace{In_2O_3 < Tl_2O_3}_{\text{basic}}$$

68. B

Sol. Step with highest activation energy is RDS, so step II is RDS

No. of activated complex = 3



P and Q are intemiediates

(Number of intemiediates = 2)

69. A

Sol. Low oxidation state of metals can stabilized by synergic bonding so ligand has to be π-acceptor.

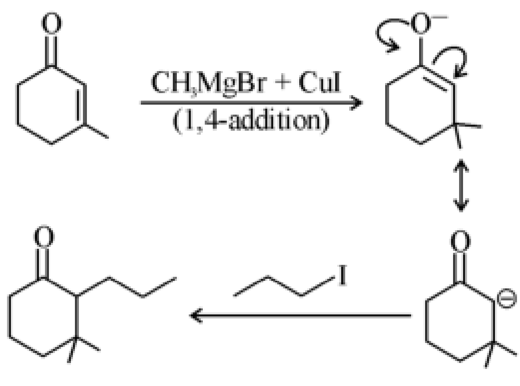
70. B

Sol. In neutral or weakly alkaline solution oxidation state of Mn changes by 3 unit

$$\overset{+7}{Mn}O_4^{-1} \rightarrow \overset{+4}{Mn}O_2$$

71. A

Sol.



72. C

Sol. ∵ $Pb(NO_3)_2$ is a soluble colourless compound so it cannot be used in confirmatory test of Pb^{+2} ion.

73. A

74. D

Sol. Statement-I – Morphine relieves in pain and produce sleep (incorrect)
Statement-II – Correct

75. C

Sol. $N_1v_1 = N_2v_2$
$\Rightarrow 0.02\ v_1 = 0.02 \times 10$
$\Rightarrow v_1 = 10$ ml

76. D

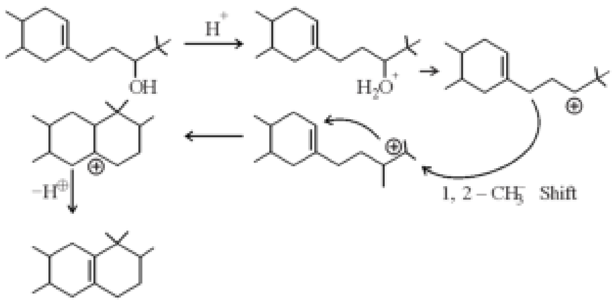
Sol. Brine is aq. Solution of NaCl
$NaCl_{(aq)} \rightarrow Na^+ + Cl^-$
Cathode reaction,
$$2H_2O + 2e^- \xrightarrow{\text{reduction}} H_{2(g)} + 2OH^-$$
Anode reaction,
$$2Cl^- \xrightarrow{\text{oxidation}} Cl_{2(g)} + 2e^-$$
So HCl will not form during electrolysis.

77. C

Sol. Pesticides → D.D.T and Aldrin

78. B

Sol.



79. B

Sol. Li, Cs reacts vigorously with water.
Br_2 changes in vapour state in boiling water (BP = 58°C)
Ga reacts with water above 100°C (MP = 29°C, BP = 2400°C)

80. C

Sol. $(r_3)_H = \dfrac{a_0n^2}{Z} = a_0 \times 3^2 = 9a_0$

$2\pi r = n\lambda$

$\Rightarrow 2\pi \times 9a_0 = 3\lambda$

$\Rightarrow \lambda = 6\pi a_0$

Section - B (Numerical Value)

81. 4

Sol. In ice each water molecule is hydrogen bonded with four other water molecules.

82. 48

Sol.

	PCl_3	+	Cl_2	$\rightleftharpoons$	PCl_5
	0.2M		(0.1+0.2)M		0.4M
Eq^m.	0.2–x		0.3–x		0.4+x

$$\frac{(0.4+x)}{(0.2-x)(0.3-x)} = 20$$

$\Rightarrow x \simeq 0.086$
$[PCl_5]_{eq} = 0.486\ M = 48.6 \times 10^{-2}\ M$

83. 4

Sol. π = icRT
A, B, C and D are isotonic pairs.

84. 3

Sol. Metal having lower SRP than 0.97 V will be oxidised by NO_3^-.

85. 5

Sol. Paints, milk, hair cream, froth, soap lather.

86. 4

Sol. XeF_4, BrF_4^{-1}, $[Cu(NH_3)_4]^{+2}$, $[PtCl_4]^{-2}$ has square planar shape.

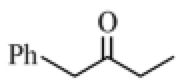
87. 278

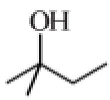
Sol. $2C_{(s)} + 3H_{2(g)} + \frac{1}{2}O_{2(g)} \rightarrow C_2H_5OH_{(\ell)}$

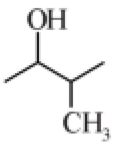
$= 2 \times (-393.5) + 3(-241.8) - (-1234.7)$

$= -277.7$ kJ/mol

88. 4

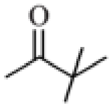
Sol. (a) 

(b) 

(c) 

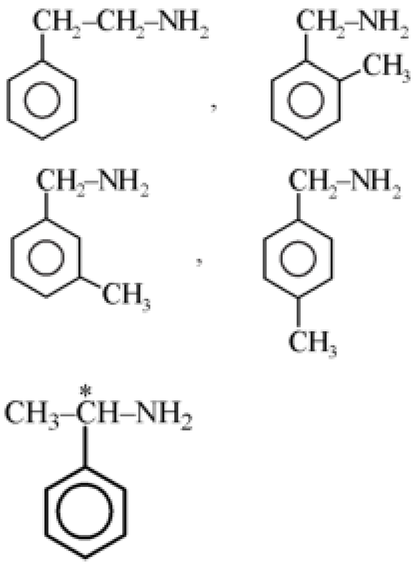
(Positive for iodoform)

(d) Ph–CH(OH)–CH_3 (Positive for iodoform)

(e)  (Positive for iodoform)

(f)  (Positive for iodoform)

89. 5

Sol. 

(d + l)

90. 3

Sol. Cubic, tetragonal and orthorhombic have body centered unit cell.

❐ ❐ ❐