3S INSTITUTE

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06-April-2023 (Morning Batch) : JEE Main Paper

MATHEMATICS

Section - A (Single Correct Answer)

1. A

Sol. $5f(x)+4f\left(\frac{1}{x}\right)=\frac{1}{x}+3$ .....(1)

replace $x \to \frac{1}{x}$

$5f\left(\frac{1}{x}\right)+4f(x)=x+3$ .....(2)

Eq. (1) × 5 – eq. (2) × 4

$f(x)=\frac{1}{9}\left(\frac{5}{x}-4x+3\right)$

$I=18\int_1^2 \frac{1}{9}\left(\frac{5}{x}-4x+3\right)dx=10\log_e 2-6$

2. B

Sol. Probability of success $=\frac{1}{9}=p$

Probability of failure $q=\frac{8}{9}$

P(at least 4 success) = P(4 success) + P(5 success)

$= {}^5C_4p^4q+{}^5C_5p^5=\frac{41}{3^{10}}=\frac{123}{3^{11}}$

k = 123

3. D

Sol. $\frac{{}^{27}C_3}{{}^nC_3}=10 \Rightarrow \frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)}$

n = 8

So $(n^2+3n):(n^2-3n+4)=2$

4. B

Sol. $\frac{{}^nC_4 2^{\frac{n-4}{4}}\cdot\left(3^{\frac{-1}{4}}\right)^4}{{}^nC_3{}^{-\left(\frac{n-4}{4}\right)}\cdot\left(2^{\frac{1}{4}}\right)^4}=\frac{\sqrt{6}}{1}$

$\Rightarrow n=10$

So $T_3={}^{10}C_2{}^{\frac{1}{4}8}\cdot 3^{-\frac{1}{4}2}=\frac{45.4}{\sqrt{3}}=60\sqrt{3}$

5. D

Sol. $\vec{a}=\lambda(\vec{b}\times\vec{c})$

$\vec{b}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & -2\\ -1 & 4 & 3\end{vmatrix}=2\hat{i}-\hat{j}+2\hat{k}$

$\vec{d}=\lambda(2\hat{i}-\hat{j}+2\hat{k})$

$\vec{a}\cdot\vec{d}=18$

$\lambda=2$

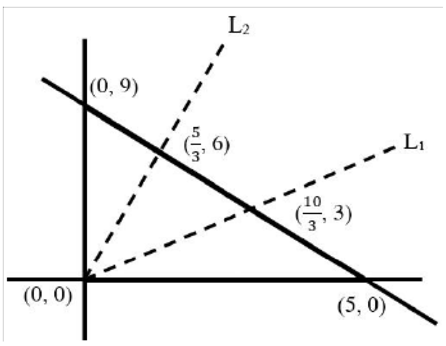
So $\vec{d}=2(2\hat{i}-\hat{j}+2\hat{k})$

$\vec{d}\times\vec{a}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -2 & 4\\ 2 & 3 & 4\end{vmatrix}=-20\hat{i}-8\hat{j}+16\hat{k}$

$\left|\vec{d}\times\vec{a}\right|^2=720$

6. C

Sol.



$m_{L_1} = \frac{3.3}{10} = \frac{9}{10}$

$m_{L_2} = \frac{6.3}{5} = \frac{18}{5}$

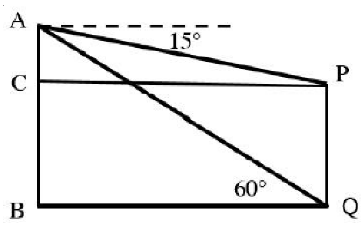
$y = (m_1 + m_2)x$

$y = \frac{9}{2}x$

Point of intersecton with L is $\left(\frac{10}{7}, \frac{45}{7}\right)$

7. A

Sol.



$\tan 60^\circ = \sqrt{3} = \frac{30}{BQ}$

$BQ = 10\sqrt{3}m = CP$

$\tan 15^\circ = 2 - \sqrt{3} = \frac{AC}{CP}$

$AC = 10\sqrt{3}(2-\sqrt{3})$

$\text{Area} = 10\sqrt{3}(60 - 20\sqrt{3}) = 600(\sqrt{3}-1)$

8. D

Sol. $S_{20} = 5 + 11 + 19 + 29 + ...$

Let $T_r = ar^2 + br + c$

$T_1 = a + b + c = 5$

$T_2 = 4a + 2b + c = 11$

$T_3 = 9a + 3b + c = 19$

$a = 1, b = 3, c = 1$

Hence $S_{20} = \sum_{r=1}^{20} r^2 + 3\sum_{r=1}^{20} r + \sum_{r=1}^{20} 1 = 3520$

9. D

Sol. Combine var. $= \frac{n_1\sigma^2 + n_2\sigma^2}{n_1 + n_2} + \frac{n_1 n_2 (m_1 - m_2)^2}{(n_1 + n_2)^2}$

$13 = \frac{15.14 + 15.\sigma^2}{30} + \frac{15.15(12-14)^2}{30 \times 30}$

$13 = \frac{14 + \sigma^2}{2} + \frac{4}{4}$

$\sigma^2 = 10$

10. D

Sol. Let $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$

$A^2 = \begin{bmatrix} p^2 + qr & pq + qs \\ pr + rs & qs + s^2 \end{bmatrix}$

$\Rightarrow p^2 + qr = 1$ (1) $pq + qs = 0 \Rightarrow q(p + s) = 0$ (3)

$\Rightarrow s^2 + qr = 1$ (2) $pr + rs = 0 \Rightarrow r(p + s) = 0$ (4)

Equation (1) – equation (2)

$p^2 = s^2 \Rightarrow p + s = 0$

Now $3a^2 + 4b^2$

$= 3(p + s)^2 + 4(ps - qr)^2$

$= 3.0 + 4(-p^2 - qr)^2 = 4(p^2 + qr)^2 = 4$

11. C

Sol. $I(x) = \int \frac{x^2(x\sec^2 x + \tan x)}{(x\tan x + 1)^2} dx$

Let $x \tan x + 1 = t$

$I = x^2\left(\frac{-1}{x\tan x + 1}\right) + \int \frac{2x}{x\tan x + 1} dx$

$I = x^2\left(\frac{-1}{x\tan x + 1}\right) + 2\int \frac{x\cos x}{x\sin x + \cos x} dx$

$I = x^2\left(\frac{-1}{x\tan x + 1}\right) + 2\ln|x\sin x + \cos x| + c$

As $I(0) = 0 \Rightarrow C = 0$

$I\left(\frac{\pi}{4}\right) = \ln\left(\frac{(\pi+4)^2}{32}\right) - \frac{\pi^2}{4(\pi+4)}$

12. A

Sol. $(2x - y + z - 3) + \lambda(4x - 3y + 5z + 9) = 0$

$x(2 + 4\lambda) - y(1 + 3\lambda) + z(1 + 5\lambda) - 3 + 9\lambda = 0$

Parallel to the line

$-2(2+4\lambda)-(1+3\lambda)4+(1+5\lambda)5=0$

$5\lambda=3$

$\lambda=\frac{3}{5}$

equation of plane

11x – 7x + 10z + 6 = 0

a + b + c = 14

13. A

Sol. $(p\Rightarrow Q)\wedge(R\Rightarrow Q)$

We known that $P\Rightarrow Q\equiv\sim P\vee Q$

$\Rightarrow(\sim P\vee Q)\wedge(\sim R\vee Q)$

$\Rightarrow(\sim P\wedge\sim R)\vee Q$

$\Rightarrow\sim(P\vee R)\vee Q$

$\Rightarrow(P\vee R)\Rightarrow Q$

14. A

Sol. For $x\le 3$ or $x\ge 5$

$x^2-8x+15-2x+7=0$

$x=5+\sqrt{3}$

For $3<x<5$, $x^2-8x+15+2x-7=0$

x = 4

Hence sum $=9+\sqrt{3}$

15. A

Sol. $\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+....+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$

On rationalising each term

$$\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{\sqrt{a_n}-\sqrt{a_1}}{d}\right)$$

$$\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{(n-1)d}{(\sqrt{a_n}+\sqrt{a_1})d}\right)=1$$

16. A

Sol. $\Delta=\begin{vmatrix}1&1&a\\2&5&2\\1&2&3\end{vmatrix}=0\Rightarrow 11-4-a=0$

a = 7

$$\Delta_1=\begin{vmatrix}b&1&a\\6&5&2\\3&2&3\end{vmatrix}=0\Rightarrow 11b-12-21=0$$

b = 3

2a + 3b = 23

17. B

Sol. $2x^y+3y^x=20$

$$2x^y\left[\frac{y}{x}+(\ln x)y'\right]+3y^x\left[\frac{xy'}{y}+\ln y\right]=0$$

$$y'=\frac{-(12\ln 2+8)}{12+8\ln 2}=-\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$$

18. D

Sol. Eqaution of OP is $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$

$a_1=(0, 0, 0)$ $a_2=(3, 0, 5)$

$b_1=(3, 4, 5)$ $b_2=(0, 0, 1)$

Equation of edge parallel to z axis

$$\frac{x-3}{0}=\frac{y-0}{0}=\frac{z-5}{1}$$

$$S.D=\frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}$$

$$\frac{\begin{vmatrix}3&0&5\\3&4&5\\0&0&1\end{vmatrix}}{\left\|\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&4&5\\0&0&1\end{vmatrix}\right\|}=\frac{3(4)}{|4\hat{i}-3\hat{j}|}=\frac{12}{5}$$

19. A

Sol. Since A, B, C, D are coplanner

Hence $\left[\overrightarrow{BA}\ \overrightarrow{CA}\ \overrightarrow{DA}\right]=0$

$$\begin{vmatrix}4&3&2\lambda-3\\7&5-\lambda&2\lambda-4\\6&0&2\lambda-6\end{vmatrix}=0$$

$\lambda = 2,\ 3$ Hence $\sum_{\lambda=S}(\lambda+2)^2 = 41$

20. B

Sol. $[x]+3+[x]+4 \le 3$

$2[x] \le -4$

$[x] \le -2 \Rightarrow x \in (-\infty,\ -1)$ .....(A)

$$3^x\left(\frac{3\cdot\frac{1}{10}}{1-\frac{1}{10}}\right) < 3^{-3x}$$

$27 < 3^{-3x}$

$-3x > +3$

$x < -1$ ....(B)

A = B

Section - B (Numerical Value)

21. 25

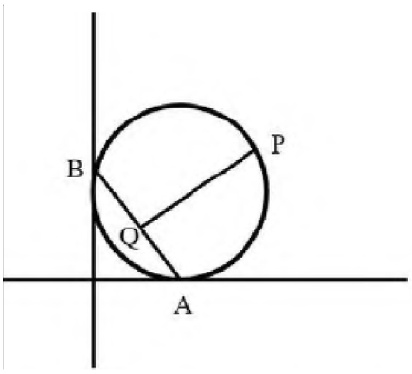
Sol. $f(x) = [a + 13\sin x],\ x \in (0,\ \pi)$

For [n sin x]; Total number of non differentiable points are = 2n – 1 for $x \in (0,\ \pi)$

So number of non differentiable points for [13 sin x] $\Rightarrow$ 25 points

22. 121

Sol.



Let equation of circle is $(x - a)^2 + (y - a)^2 = a^2$

which is passing through $P(\alpha,\ \beta)$

then $(\alpha - a)^2 + (\beta - a)^2 = a^2$

$\alpha^2 + \beta^2 - 2\alpha a - 2\beta a + a^2 = 0$

Here equation of AB is x + y = a

Let $Q(\alpha', \beta')$ be foot of perpendicular of P on AB

$$\frac{\alpha'-\alpha}{1} = \frac{\beta'-\beta}{1} = \frac{-(\alpha+\beta-a)}{2}$$

$$PQ^2 = (\alpha'-\alpha)^2 + (\beta'-\beta) = \frac{1}{4}(\alpha+\beta-a)^2 + \frac{1}{4}(\alpha+\beta-a)^2$$

$$121 = \frac{1}{2}(\alpha+\beta-a)^2$$

$242 = \alpha^2 + \beta^2 - 2\alpha a - 2\beta a + a^2 + 2\alpha\beta$

$242 = 2\alpha\beta$

$\Rightarrow \alpha\beta = 121$

23. 171

Sol. 20 distinct oranges distributed among 3 children so that each child gets at least one orange

$= 3^{20} - {}^3C_1\ 2^{20} + {}^3C_2\ 1^{20}$

24. 5

Sol. $x^2 + y^2 - 2y \ge 0$ & $x^2 - 2y \le 0,\ x \ge y$

Hence required area

$$= \frac{1}{2}\times 2\times 2 - \int_0^3 \frac{x^2}{2}dx - \left(\frac{\pi}{4} - \frac{1}{2}\right)$$

$$= \frac{7}{6} - \frac{\pi}{4} \Rightarrow n = 5$$

25. 3

Sol. $3 - x \le y \le \sqrt{9-x^2}$

Points (p, p + 1) lies on y = x + 1

So point of intersection between

y = x + 1 & y = 3 – x is x = 1, y = 2

and point of intersection between

$x + 1 = \sqrt{9-x^2}$ is $x = \frac{-1+\sqrt{17}}{2}$

Hence $p \in \left(1,\ \frac{-1+\sqrt{17}}{2}\right)$

Hence $b^2 + b - a^2 = 3$

26. 2

Sol. (x cos x)dy + (xy sin x + y cos x – 1)dx = 0,

$0 < x < \frac{\pi}{2}$

$$\frac{dy}{dx}+\left(\frac{x\sin x+\cos x}{x\cos x}\right)y=\frac{1}{x\cos x}$$

IF = x sec x

$$y\cdot x\sec x=\int\frac{x\sec x}{x\cos x}dx=\tan x+c$$

Since $y\left(\frac{\pi}{3}\right)=\frac{3\sqrt{3}}{\pi}$

Hence $c=\sqrt{3}$

Hence $\left|\frac{\pi}{6}y''\left(\frac{\pi}{6}\right)+y'\left(\frac{\pi}{6}\right)\right|=|-2|=2$

27. 5005

Sol. $\left(x^4-\frac{1}{x^3}\right)^{15}$

$$T_{r+1}={}^{15}C_r(x^4)^{15-r}\left(\frac{-1}{x^3}\right)^r$$

60 – 7r = 18

r = 6

Hence coeff. of x^{18} = ${}^{15}C_6$ = 5005

28. 18

Sol. A = {1, 2, 3,.....10}

B = {0, 1, 2, 3, 4}

R = {(a, b) ∈ A × A: $2(a-b)^2+3(a-b)$ ∈ B}

Now $2(a-b)^2+3(a-b)=(a-b)(2(a-b)+3)$

⇒ a = b or a – b = –2

When a = b ⇒ 10 order pairs

When a – b = –2 ⇒ 8 order pairs

Total = 18

29. 594

Sol. Let Q(α, β, γ) be the image of P, about the plane 2x – y + z = 9

$$\frac{\alpha-1}{2}=\frac{\beta-2}{-1}=\frac{\gamma-3}{1}=2$$

$\Rightarrow\alpha=5,\ \beta=0,\ \gamma=5$

Then area of triangle PQR is $=\frac{1}{2}\left|\overrightarrow{PQ}\times\overrightarrow{PR}\right|$

Square of area = 594

30. 292

Sol. Equation of tangent at P (1, 3) to the curve $x^2+2x-4y+9=0$ is y –x = 2

Then the point A is (0, 2)

Equation of line passing through P and parallel to the line x – 3y = 6.

The possible coordinate of B are (4, 4) or (16, 8)

But (4, 4) does not satisfy 2x –3y = 8

Thus the point B is (16, 8)

Then $(AB)^2$ = 292

PHYSICS

Section - A (Single Correct Answer)

31. A

Sol.

$$\frac{\text{Electric energy density}}{\text{Magnetic energy density}}=\frac{\frac{1}{2}\epsilon_0E_{rms}^2}{\left(\frac{B_{rms}^2}{2\mu_0}\right)}$$

$$=\left(\frac{E_{rms}}{B_{rms}}\right)^2\cdot\mu_0\epsilon_0\quad\left[C=\frac{1}{\mu_0\epsilon_0}\right]\quad=\frac{C^2}{C^2}=1$$

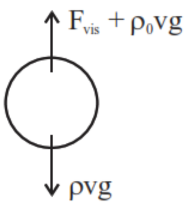
32. B

Sol. Circuit is closed when neither A nor B is closed ⇒ current flows for A = 0 B = 0 when either or both of A & B is closed we get current bypass from switch

Hence it is "NOR" gate

33. A

Sol.



For constant velocity $F_{net}=0$

$F_{vis}+\rho_0vg=\rho vg$

$F_{vis}=(\rho-\rho_0)vg$

34. B

Sol. Collision frequency,

$$f=\frac{V}{\lambda}=\frac{V}{\left(\frac{1}{\sqrt{2}\pi d^2n_v}\right)}=\sqrt{2}\pi d^2vn_v$$

$\therefore \quad f \propto n_v$, n_v is number density

$$\frac{f_1}{f_2} = \frac{n_{v_1}}{n_{v_2}} = \frac{3\times10^{19}}{12\times10^{-19}} = 0.25$$

35. D

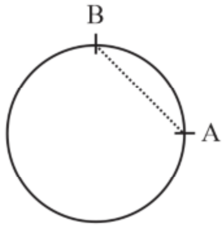
Sol. dQ = dU + dw

$$\frac{dU}{dt} = \frac{dQ}{dt} - \frac{dw}{dt}$$

$$\frac{dU}{dt} = 1000 - 200 = 800 \text{ W}$$

36. A

Sol.



$AB = R\sqrt{2}$

Let instantaneous velocity be v. time,

$$t = \frac{\text{Arc length}}{v} = \frac{2\pi\frac{R}{4}}{v} = \frac{\pi R}{2v}$$

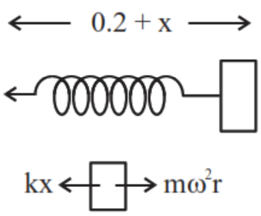
average velocity,

$$\langle v \rangle = \frac{AB}{t} = \frac{R\sqrt{2}(2v)}{\pi R} = \frac{2\sqrt{2}v}{\pi}$$

$$\Rightarrow \quad \frac{V}{\langle V \rangle} = \frac{\pi}{2\sqrt{2}}.$$

37. B

Sol.



Let extension in length of spring be x.

Radius of circle r = 0.2 + x

$Kx = m\omega^2 r$

$$7.5x = \left(\frac{1}{10}\right)(5^2)(0.2 + x)$$

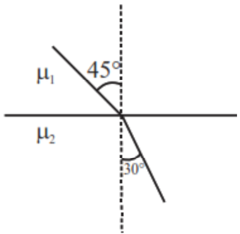
$$\Rightarrow \frac{15}{2}x = \frac{5}{2}\left(x + \frac{1}{5}\right)$$

$$\Rightarrow x = \frac{1}{10}$$

$\therefore \quad$ Tension in spring $= kx = 7.5 \times \frac{1}{10} = 0.75\text{N}$

38. B

Sol.



Snell's law $\mu_1 \sin 45° = \mu_2 \sin 30°$

$$\frac{\mu_1}{\mu_2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{\mu_1}{\mu_2} = \frac{\lambda_2}{\lambda_1} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \lambda_2 = \frac{\lambda_1}{\sqrt{2}}$$

Frequency doesn't change on change in medium.

39. B

Sol. $R = \frac{u^2}{g}\sin 2\theta$

R is maximum for $2\theta = 90°$.

40. C

Sol. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

Differentiating both sides, we get

$$\frac{\Delta R}{R^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}\left[R = \frac{R_1R_2}{R_1 + R_2} = \frac{10\times15}{10+15} = 6\right]$$

$$\Rightarrow \frac{\Delta R}{R} = \left(\frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}\right)R$$

$$= \left(\frac{0.5}{100} + \frac{0.5}{225}\right)6$$

$$= \left(\frac{6\times 0.5}{25}\right)\left(\frac{1}{4} + \frac{1}{9}\right) = \frac{13}{300}$$

$$\frac{\Delta R}{R}\times 100 = \frac{13}{3} = 4.33\%$$

41. C

Sol. $m = \rho \times \frac{4}{3}\pi R^3$

$R \propto m^{\frac{1}{3}} (\rho = \text{ constant})$

Weight $= W \propto g \propto \frac{Gm}{R^2}$

$W \propto \frac{m}{m^{2/3}} \propto m^{1/3}$

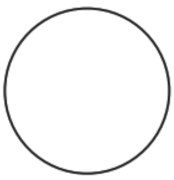
So, $W^1 = (2)^{1/3}$ W

42. D

Sol. At Moon, due to low escape velocity, the rms velocity of molecules is greater than escape velocity. Hence molecules escape and there is no atmosphere at Moon.

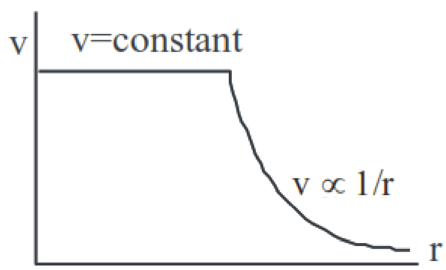
43. A

Sol.



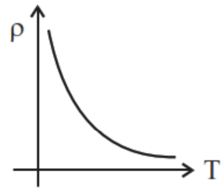
$V_{inside} = \frac{kQ}{R}$

$V_{outside} = \frac{kQ}{r}$



44. B

Sol.



With rise in temperature, number density (n) of electrons and holes increases for semiconductors.
As m, e, τ are constant

$\rho \propto \frac{1}{n} \Rightarrow \rho \propto \frac{1}{T}$ [Rectangular hyperbola]

45. C

Sol.

	Electron	Alpha	Proton
Mass :	$\frac{m}{1840}$	4m	m
Charge :	e	2e	e
Kinetic: energy	4K	2K	K
$\lambda = \frac{h}{\sqrt{2mK}}$	$\frac{h}{\sqrt{2\cdot\frac{m}{1840}\cdot 4k}}$	$\frac{h}{\sqrt{2.4m.2K}}$	$\frac{h}{\sqrt{2mK}}$

$\lambda_\alpha < \lambda_p < \lambda_e$

46. C

Sol. Range, $R = \sqrt{2Rh}$

$R_1 = \sqrt{2Rh_1}$

$h_2 = h_1 + \left(h_1 \times \frac{21}{100}\right) = 1.21h_1$

$\therefore R_2 = \sqrt{2Rh_2} = \sqrt{2R(1.21)h_1} = 1.1\sqrt{2Rh_1}$

$\therefore R_2 = 1.1R_1$

% increase in range

$= \frac{R_2 - R_1}{R_1}\times 100 = \left(\frac{R_2}{R_1} - 1\right)\times 100$

= (1.1 - 1) × 100 = 10%

47. B

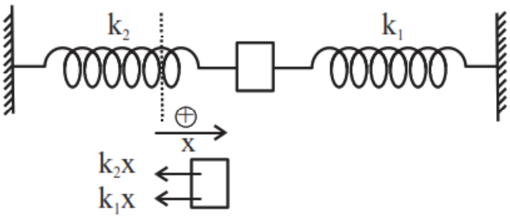
Sol. $\Delta E = \frac{hc}{\lambda} \Rightarrow \lambda \alpha \frac{1}{\Delta E}$

For shortest wavelength, energy gap should be maximum.

So, correct choice is transition from n = 3 to n = 1.

48. C

Sol.



On displacing m to right by x

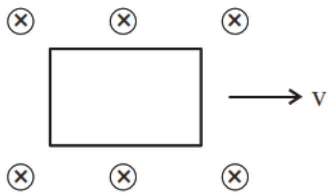
$F = -(k_1x + k_2x) = -(k_1 + k_2)x$

$$a = \frac{F}{m} = -\left(\frac{k_1 + k_2}{m}\right)x = -\omega^2 x$$

$$\therefore \omega = \sqrt{\frac{k_1 + k_2}{m}} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

49. D

Sol.



Moving a coil inside a uniform magnetic field either with uniform or non-uniform speed doesn't changes flux, so, no emf is induced.

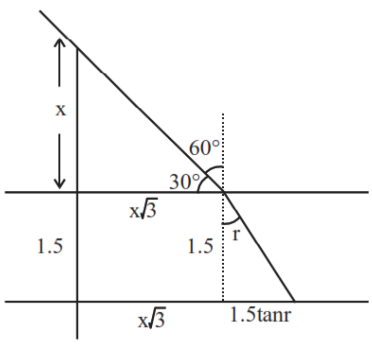
50. C

Sol. $$B = \begin{cases} \dfrac{\mu_0 Ir}{\pi a^2} & r \le a \\ \dfrac{\mu_0 I}{\pi r^2} & r \ge a \end{cases}$$

Section - B (Numerical Value)

51. (50)

Sol.



By Snell's law

$$1\sin 60° = \frac{4}{3}\sin r \to \sin r = \frac{3\sqrt{3}}{8} \to \tan = \frac{3\sqrt{3}}{\sqrt{37}}$$

By the diagram

$$x\sqrt{3} + 1.5\tan r = 2.15$$

$$x\sqrt{3} = 2.15 - 1.5 \times \frac{3\sqrt{3}}{\sqrt{37}}$$

$= 1.241 - 0.739$

$= 0.502$

x = 50 cm

52. (25)

Sol. $R = \rho\frac{l}{A}$ be the initial resistance new resistance

$$R' = \rho\frac{1.2l}{0.96A} = 1.25\rho\frac{l}{A} = 1.25R$$

percentage change $= \dfrac{1.25R - R}{R} \times 100 = 25\%$

53. (2)

Sol. Loss of K.E = work done against retarding force.

$$= \int_0^x madx = \int_0^x m2xdx = mx^2$$

$$= \left(10^{-2}kg\right)x^2 J = \left(\frac{10}{x}\right)^{-2} J$$

So n = 2

54. 628

Sol. Magnetic field B_C at center $= \dfrac{\mu_0 i}{2r}$

$$= \frac{4\pi \times 10^{-7}}{2 \times 0.2} \times \sqrt{2}T$$

Net magnetic field is

$$B_C\sqrt{2} = \frac{4\pi \times 10^{-7} \times \sqrt{2}}{2 \times 0.2} \times \sqrt{2}T = 2\pi \times 10^{-6}T$$

$= 200\pi \times 10^{-8}T$

$= 2 \times 314 \times 10^{-8}$ T $= 628 \times 10^{-8}$ T

55. (420)

Sol. Frequency of reflected sound $=\left(\frac{v+v_e}{v-v_e}\right)f_0$

$$f=\left(\frac{330+15}{330-15}\right)\times f_0$$

$$=\frac{345}{315}f_0$$

$$\frac{345}{315}f_0-f_0=40$$

$$\frac{30}{315}f_0=40$$

$$f_0=\frac{4\times315}{3}=420\text{Hz}$$

56. (425)

Sol. $r_n=r_0\frac{n^2}{z}\rightarrow r_n=0.51\times\frac{25}{3}\text{Å}=4.25\times10^{-10}\text{m}$

$= 425\times10^{-12}$ m

57. (25)

Sol. $\text{Strain}=\frac{\text{Stress}}{Y}=\frac{\frac{62.8\times10^3}{\pi\times(0.02)^2}}{2\times10^{11}}$

$$=\frac{62.8\times10^3}{3.14\times4\times10^{-4}\times2\times10^{11}}=2.5\times10^{-4}=25\times10^{-5}$$

58. (240)

Sol. $v_p = 12\times10^3$ volts

$v_s = 120$ volts

$p_s = 60$ KW $= v_s\times i_s$

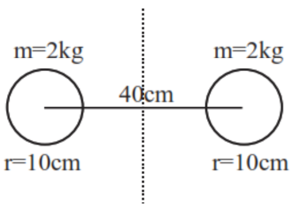
$$i_s=\frac{60\times10^3}{120}=5\times10^2\text{A}$$

$$R_L=\frac{v_s}{i_s}=\frac{120}{5\times10^2}=24\times10^{-2}=240\times10^{-3}\Omega$$

$= 240$ mΩ

59. (176)

Sol.



$$I=2\left(I_{cm}+md^2\right)$$

$$=2\left(\frac{2}{5}mr^2+md^2\right)$$

$$=\frac{4}{5}\times2\times(0.1)^2+2(2)(0.20)^2$$

$$=\frac{8}{5}\times10^{-2}+16\times10^{-2}$$

$= (1.6 + 16)\times10^{-2}$

$= 17.6\times10^{-2}$

$$I=176\times10^{-3}\text{ kg m}^2$$

60. (3)

Sol. For $x=\frac{d}{3}$

$$C_1=\frac{\epsilon_0 A}{\left(\frac{d/3}{k}+\frac{2d}{3}\right)}=\frac{\epsilon_0 A}{\frac{d}{12}+\frac{2d}{3}}$$

$$=\frac{\epsilon_0 A}{d}\times\left(\frac{12}{9}\right)$$

$$C_1=\frac{4}{3}\frac{\epsilon_0 A}{d}=2\mu\text{F}$$

for $x=\frac{2d}{3}$

$$C_2=\frac{\epsilon_0 A}{\left(\frac{2d/3}{k}+\frac{d}{3}\right)}=\frac{\epsilon_0 A}{d}\times2$$

$$\Rightarrow\frac{6}{4}\times2=3\mu\text{F}$$

CHEMISTRY

Section - A (Single Correct Answer)

61. A

Sol. Y : CCP $\Rightarrow$ 4Y

X = 1/3 THV = 1/3 × 8 $\Rightarrow$ 8/3x

$\therefore$ Formula : $X_{8/3}Y_4$ or X_2Y_3

62. D

Sol. Nitrogen detection by lassaigne's method

$Na + C + N \rightarrow NaCN$

$6NaCN + FeSO_4 \rightarrow Na_4[Fe(CN)_6] + Na_2SO_4$

$Na_4[Fe(CN)_6] + Fe^{3+} \rightarrow Fe_4[Fe(CN)_6]_3$ (Prussian blue)

Sulphur detection by Sodium nitroprusside

$Na_2[Fe(CN)_5NO] + Na_2S \rightarrow Na_4[Fe(CN)_5NOS]$

[Purple]

Phosphorus detection by ammonium molybdate

$Na_3PO_4 + 3HNO_3 \rightarrow H_3PO_4 + 3NaNO_3$

$H_3PO_4 + 12(NH_4)_2 MoO_4 + 21HNO_3 \rightarrow (NH_4)_3PO_4.12MoO_3 + 21NH_4NO_3 + 12H_2O$

(canary yellow)

Halogen give specific coloured ppt with $AgNO_3(aq)$

$NaCl + AgNO_3(aq) \rightarrow AgCl + NaNO_3$

(White)

$NaBr + AgNO_3(aq) \rightarrow AgBr + NaNO_3$ (Pale yellow)

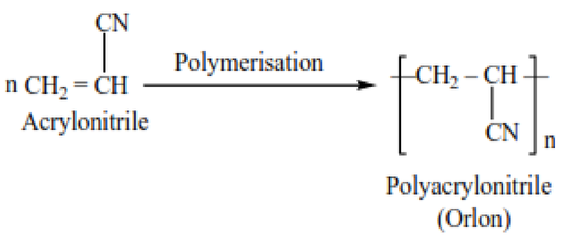
$NaI + AgNO_3(aq) \rightarrow AgI + NaNO_3$ (Yellow)

63. A

Sol. Factual

64. A

Sol.



65. B

Sol. Cl has the most negative ΔH_{eg} among all the elements and Ne has the most positive ΔH_{eg}.

66. C

Sol. Factual

67. D

Sol. Photochemical smog occurs in warm, dry and sunny climate. The main components come from the action of sunlight on unsaturated hydrocarbon and nitrogen oxides produced by automobiles and factories.

68. D

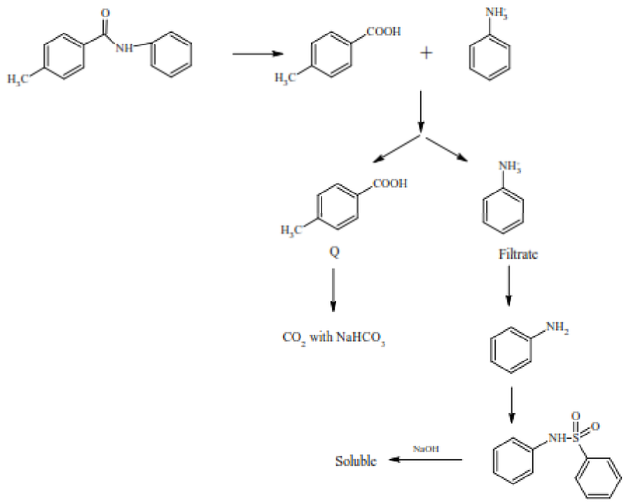
Sol. Factual

69. D

Sol. Factual

70. B

Sol.



71. D

Sol. HVZ reactions = Br_2 / red P

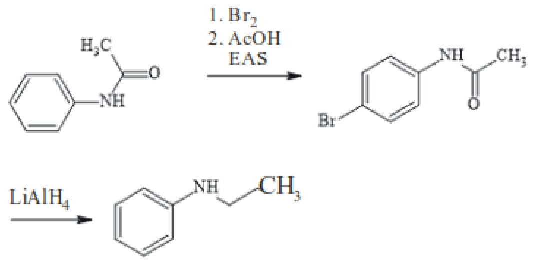
Iodoform reaction = $NaOH + I_2$

Etard reaction = (i) CrO_2Cl_2, CS_2 (ii) H_2O

Gatterman-Koch Reaction = CO, HCl, Anhydrous, $AlCl_3$

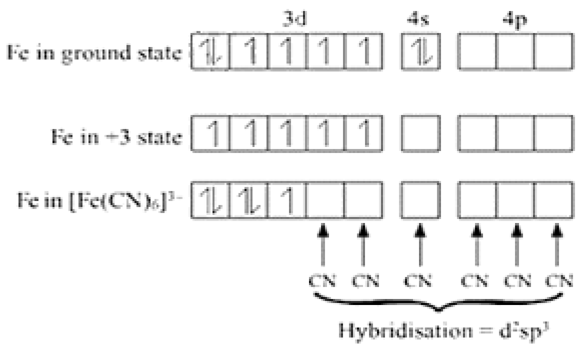
72. D

Sol.



73. A

Sol. $[Fe(CN)_6]^{3-}$



Unpaired electron = 1

$\mu = \sqrt{n(n+2)} = \sqrt{1 \times 3} = 1.74$ B.M.

$[Fe(H_2O)_6]^{3+}$ No pairing because H_2O is WFL

Number of unpaired electrons = 5, μ = 5.92 BM

Assertion is true, Reason is true but not correct explanation.

74. D

Sol. Factual

75. A

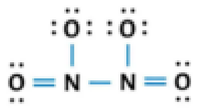
Sol. $2\left[\overset{+1}{Au}(CN)_2\right]^- + \overset{0}{Zn}(s) \longrightarrow 2\overset{0}{Au} + \left[\overset{+2}{Zn}(CN)_4\right]^{-2}$

Zn displaced Au^+

Reduction and Oxidation both are taking place.

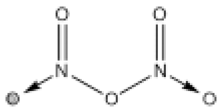
76. D

Sol. N_2O_4



NO_2

N_2O_5



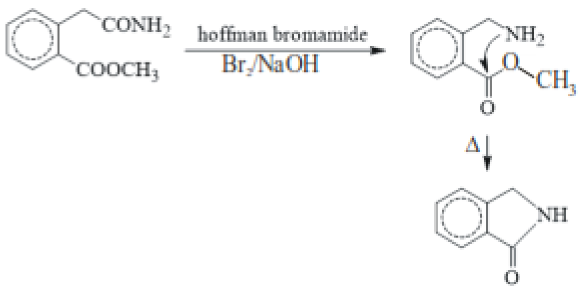
N_2O

77. D

Sol. Factual

78. C

Sol.



79. A

Sol. $A_2B_3(aq.) \rightleftharpoons 2A^{3+}_{(aq.)} + 3B^{2-}_{(aq)}$

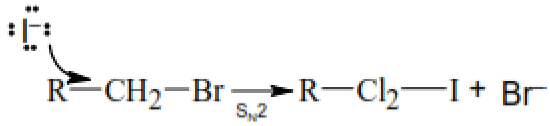
$c(1-\alpha) \quad 2c\alpha \quad 3c\alpha$

$$K_{eq} = \frac{[A^{3+}]^2[B^{2-}]^3}{[A_2B_3]} = \frac{4c^2\alpha^2 \times 27c^3\alpha^3}{c(1-\alpha)}$$

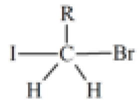
$$K_{eq} = \frac{108c^5\alpha^5}{c} \quad \alpha = \left(\frac{K_{eq}}{108c^4}\right)^{\frac{1}{5}}$$

80. A

Sol. This is finkelstein reaction



Transition state



Clearly, the transition state is less polar than free anions. Br^- and I^-

Acetic acid is protic which does not support S_N2

Acetone does not solvate anion

Br^- gets precipitated and hence can not compete with I^-

So only A is correct

Section - B (Numerical Value)

81. 7

Sol. $\lambda_d = \frac{h}{mv} = \frac{h}{\sqrt{2mKE}}$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9 \times 10^{-31} \times 4.5 \times 10^{-29}}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{9^2 \times 10^{-60}}}$$

$$= \frac{6.6 \times 10^{-34}}{9 \times 10^{-30}} = \frac{6.6}{9} \times 10^{-4} = 7.3 \times 10^{-5} m$$

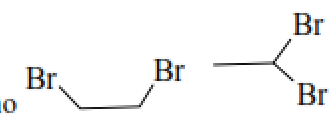
Therefore Ans = 7

82. 9

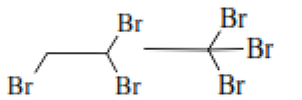
Sol. $CH_3 - CH_3 + Br_2$ (Excess) $\xrightarrow{hv}$

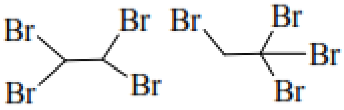
Monobromo

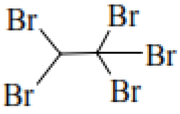
Dibromo

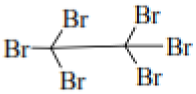


Tribromo



Tretrabromo 

Pentabromo 

Hexabromo 

83. 2

Sol. For, Spontaneous process dG < 0

For, Equilibrium dG = 0

For, Nonspontaneous process dG > 0

$\therefore$ A Wrong

B Correct

C Correct

D Wrong

84. 1111

Sol. $$\frac{P^0 - P_s}{P_s} = \frac{n_{solute}}{n_{solvent}} = \frac{\frac{x}{60}}{\frac{1000}{18}} = \frac{P^0 - 0.75P^0}{0.75P^0}$$

$$\Rightarrow x = \frac{10000}{9} = 1111 \text{ gm}$$

85. 10

Sol. $\Delta G^0 = \Delta H^0 - T\Delta S$

$\Rightarrow$ $\Delta G^0 = (-54070 - 10 \times 298)$

Also, $\Delta G^0 = (-2.303\ RT \log K)$

$\Rightarrow$ $(-54070 - 10 \times 298)$

$= (-2.303 \times 8.134 \times 298 \log K)$

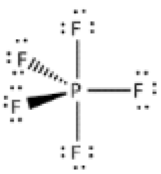
$\Rightarrow$ $\log K = 10$

86. 3

Sol. PF_5

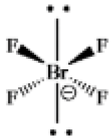
sp^3d (0 lone pair)

Trigonal bipyramidal



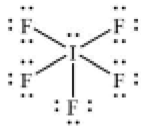
BrF_4^-,

sp^3d^2 (2 lone pair)

 square planar

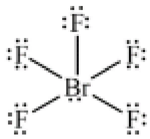
IF_5

sp^3d^2 (1 lone pair)

 square pyramidal

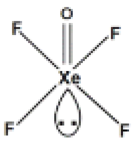
BrF_5

sp^3d^2 (1 lone pair)

 square pyramidal

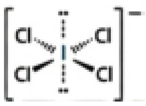
$XeOF_4$

sp^3d^2 (1 lone pair)

 square pyramidal

ICl_4^-

sp^3d^2 (2 lone pair)

 square planar

87. 4

Sol. $[M(en)(SCN)_4]$

$\nearrow S = C = N^- \nwarrow$

Ambidentate ligand means two ligand site, so ambidentate ligand is SCN^-.

88. 2

Sol. $K = Ae^{-\frac{E_a}{RT}}$

$K_1 = Ae^{-\frac{(E_a)_1}{RT}}$

$$K_2 = Ae^{-\frac{(E_a)_2}{RT}}$$

$$\frac{K_2}{K_1} = e^{\frac{(E_a)_1-(E_a)_2}{RT}}$$

$$\log\frac{K_2}{K_1} = \frac{(E_a)_1-(E_a)_2}{2.3RT}$$

$$= \frac{(41.4-30)\times 1000}{2.3\times 8.3\times 300} = 1.99$$

$$= 2$$

89. 1

Sol. $3BaCl_2 + 2Na_3PO_4 \rightarrow Ba_3(PO_4)_2 + 6NaCl$

5 **2**

Na_3PO_4 is limiting reagent.

2 mole Na_3PO_4 gives 1 mole of $Ba_3(PO_4)_2$

90. 6

Sol. $(NH_4)_3\ PO_4 \cdot 12MoO_3$

Let X = oxidation state of Mo in MoO_3

X + (–2) × 3 = 0

X = + 6

❒ ❒ ❒