3S INSTITUTE
JEE ADVANCED | JEE MAIN | NEET | OLYMPIADS | MHT-CET | FOUNDATION

01-February-2023 (Evening Batch) : JEE Main Paper

PHYSICS

Section - A (Single Correct Answer)

1. D

Sol. $T_H = 99°C = 99 + 273 = 372$ K.

$$1-\frac{T_C}{T_H}=\frac{1}{3}$$

$$\frac{T_C}{T_H}=\frac{2}{3} \text{ ——— (1)} \Rightarrow T_C=\frac{2}{3}\times 372$$

$= 2 \times 124 = 248$ K

$$1-\frac{T_C+X}{T_H}=\frac{1}{6}$$

$$\frac{5}{6}=\frac{T_C+X}{T_H}$$

$$\frac{5}{6}=\frac{248+X}{372}$$

$248 + X = 5 \times 62$

$X = 310 - 248 = 62$K

2. A

Sol. Potential of a conducting sphere is

$$V=\frac{KQ}{R} \text{ (Solid as well as hollow)}$$

$V_1 = V_2$ and $R_1 = R_2$

$\therefore Q_1 = Q_2$

3. D

Sol. $B_C=\frac{\mu_0 I}{4\pi R}(\pi)$ (B at centre of circular arc)

$$=\frac{\mu_0 I}{4R}=\frac{4\pi\times 10^{-7}\times 3}{4\times\frac{\pi}{10}}$$

$= 3 \times 10^{-6}$ T $= 3\mu$T

4. B

Sol. $\phi=\vec{B}\cdot\vec{A}$

$= BA\cos\theta$

Most suitable ans is 2 [Otherwise ABCD]

5. C

Sol. Modulating Index

$$\mu=\frac{A_m}{A_c}$$

$$\mu_1=\frac{X}{Y}$$

$$\mu_2=\frac{X}{2Y}$$

$$\frac{\mu_1}{\mu_2}=\frac{2}{1}$$

6. A

Sol. At highest point

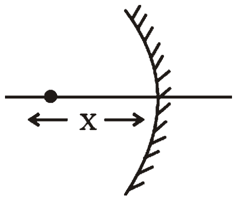
$V_y = 0$

$V_x = u_x = u\cos\theta$

$U_g = mgh$, it is maximum at H_{max}.

7. C

Sol.



by mirror formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v_1}+\frac{1}{-15}=\frac{1}{(-20)}$$

$$\frac{1}{v_1}=-\frac{1}{20}+\frac{1}{15}$$

$$=\frac{-3+4}{60}$$

$v_1 = 60$ cm

$$\frac{1}{v_2}+\frac{1}{(-25)}=\frac{1}{(-20)}$$

$$\frac{1}{v_2} = \frac{-1}{20} + \frac{1}{25} = \frac{-5+4}{100} = \frac{-1}{100}$$

v_2 = – 100 cm

d = 60 + 100 = 160 cm

8. C

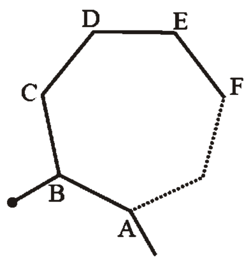
Sol. Tension (F) = mg

$$= 4 \times \frac{10}{4} = 10N$$

= 10^{-4} m = 0.1 mm

9. A

Sol.



Suppose resistance of each arm is r, then r = R/n

$$R_{eq(AB)} = \frac{R_1R_2}{R_1 + R_2}$$

$$\frac{r(n-1)r}{r + (n-1)r}$$

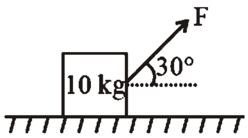
$$= \frac{r(n-1)r}{nr}$$

$$= \frac{n-1}{n} r$$

$$= \frac{(n-1)R}{n^2}$$

10. B

Sol.



N = Mg – F Sin 30°

$$= mg - \frac{F}{2} = 100 - \frac{F}{2} = \frac{200 - F}{2}$$

F Cos 30° = μN

$$\sqrt{3}\frac{F}{2} = 0.25 \times \left(\frac{200 - F}{2}\right)$$

$$4\sqrt{3}F = 200 - F$$

$$F = \frac{200}{4\sqrt{3} + 1} = 25.22$$

11. A

Sol. Error of voltmeter decreases with increase in its resistance.

12. A

Sol. Woks as voltage regulator in reverse bias and as simple P-n junction in forward bias.

13. C

Sol. $T = 2\pi\sqrt{\frac{l}{g}}$

$$T^2 = \frac{4\pi^2}{g} \times l$$

$$T^2 \alpha l$$

14. D

Sol.

$$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G\rho\frac{4}{3}\pi R^3}{R}} = C\sqrt{\rho} \cdot R$$

$$\frac{V_{e_1}}{V_{e_2}} = \frac{R_1}{R_2}\sqrt{\frac{\rho_1}{\rho_2}} = \frac{1}{2}$$

$$\frac{R_1^2}{R_2^2} \times \frac{\rho_1}{\rho_2} = \frac{1}{4}$$

$$\frac{R_1}{R_2} = \frac{1}{3}$$

$$g = \frac{GM}{R^2} = \frac{G\frac{4}{3}\pi R^3 \times \rho}{R^2} C \cdot \rho R$$

$$\frac{g_1}{g_2} = \frac{\rho_1 R_1}{\rho_2 R_2} = \frac{1}{4}\frac{R_2^2}{R_1^2} \times \frac{R_1}{R_2} = \frac{1}{4} \times \frac{R_2}{R_1} = \frac{3}{4}$$

15. D

Sol. $\Delta E = 13.6Z^2\left[\frac{1}{2^2}-\frac{1}{4^2}\right]eV$

$= 13.6\times(4)^2\left(\frac{1}{4}-\frac{1}{16}\right)eV$

= 13.6 [4 – 1] eV

= 13.6 × 3 = 40.8 eV

16. B

Sol. Impulse = Area under F = t curve

(a) $\frac{1}{2}\times1\times0.5=\frac{1}{4}N.s$

(b) 0.5 × 2 = 1 N.s (maximum)

(c) $\frac{1}{2}\times1\times0.75=\frac{3}{8}N.s$

(d) $\frac{1}{2}\times2\times0.5=\frac{1}{2}N.s$

17. D

Sol. Say dimensional formale of mass is $H^x\ C^y\ G^z$

$M^1 = (ML^2T^{-1})^x\ (LT^{-1})\ (M^{-1}L^3T^{-2})^Z$

$M^1L^0T^0 = M^{x-z}\ L^{2x+y+3z}\ T^{-x-y-2z}$

on comparing both side

x – z = 1

2 x + y + 3z = 0

– x – y – 2z = 0

On solving above equations we get

$x=\frac{1}{2}\quad y=\frac{1}{2}\quad z=\frac{-1}{2}$

18. A

Sol. For isochoric process

$\frac{P}{T}=n\frac{R}{V}=$ constant

$P=\frac{nR}{V}(t+273)$

If P = 0 ⇒ t = – 273°C

19. D

Sol. $< u_E > = < u_B > = 1/2 < u_{total} >$

So $\frac{\langle u_E\rangle}{\langle u_{total}\rangle}=\frac{1}{2}$

20. A

Sol. $K_{max} = hf - hf_0$

For $f = 2f_0$

$\frac{1}{2}mV_1^2 = 2hf_0 - hf_0 = hf_0$

For $f = 5\ f_0$

$\frac{1}{2}mV_2^2 = 5hf_0 - hf_0 = 4hf_0$

$\frac{V_1}{V_2}=\frac{1}{2}$

Section - B (Numerical Value)

21. 200

Sol. u = 20 m/s, S_1 = 500 m, v = 0

By third equation of mation

$0 = (20)^2 - 2a.\ 500 \Rightarrow a = \frac{4}{10}\ m/s^2$

u = 20 m/s, S_2 = 250 m, v = ?

$v^2 = (20)^2 - 2a.\ 250$

$= v = \sqrt{200}$ m/s

x = 200

22. 132

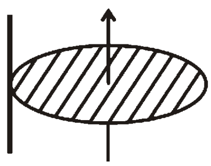
Sol. $F = 5 + 3y^2$

$W = \int_2^5 (5+3y^2)dy$

= 132 J

23. 3

Sol.



$I = I_{cm} + Md^2$

$= \frac{MR^2}{2}+MR^2$

$= \frac{3}{2}MR^2$

x = 3

24. 6

Sol. For A mass number = 34

Total binding energy = 1.2 × 34 = 40.8 MeV

For B mass number = 26

total binding energy = 1.8 × 26 MeV = 46.8 MeV

Difference of BE = 6 MeV

25. 44

Sol. N = 600, A = 70 × 10^{-4} m^2, B = 0.4T

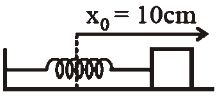
$$\omega = \frac{500 \times 2\pi}{60} = \frac{100\pi}{6} \text{ rad/s}$$

E = NABω sinωt ωt is angle b/w $\vec{A}$ & $\vec{B}$

$$= 600 \times 70 \times 10^{-4} \times 0.4 \times \frac{100\pi}{6} \times \frac{1}{2} = 44 \text{ V}$$

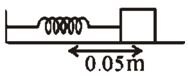
26. 50 or 67

Sol.



$$U_i = \frac{1}{2}kx_0^2$$

$K_i = 0$



$$U_f = \frac{1}{2}k\left(\frac{x_0}{2}\right)^2$$

$K_f = 0.25$ J

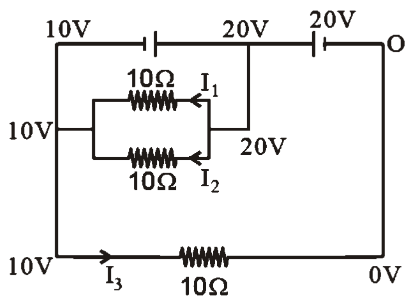
$$\frac{1}{2}kx_0^2 + 0 = \frac{1}{2}k\frac{x_0^2}{4} + 0.25$$

$$\frac{1}{2}kx_0^2\frac{3}{4} = \frac{1}{4}$$

$$\frac{1}{2}k\frac{3}{100} = 1 \Rightarrow k = \frac{200}{3} \text{ N/m} = 67 \text{ N/m}$$

27. 2

Sol.



$$I_1 = I_2 = \frac{20-10}{10} = 1\text{A}$$

I_3 = 1A

$$\left|\frac{I_1 + I_3}{I_2}\right| = 2$$

28. 4

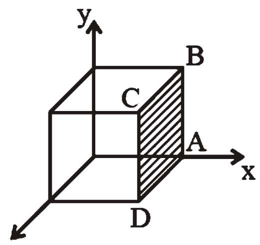
Sol. Fringe shift $= \frac{t(\mu - 1)}{\lambda}B$

$$= \frac{10 \times 10^{-6}(1.2-1)}{5 \times 10^{-7}}B$$

$$= \frac{10^{-5} \times 0.2}{5 \times 10^{-7}} = 4$$

29. 288

Sol.



$$\vec{E} = E_0 x\hat{i}$$

$\phi_{net} = \phi_{ABCD} = E_0$ a.a^2

$$\frac{q_{en}}{\epsilon_0} = E_0 a^3$$

$q_{en} = E_0 \epsilon_0 a^3$

= 4 × 10^4 × 9 × 10^{-12} × 8 × 10^{-6}

= 288 × 10^{-14} C

Q = 288

30. 2

Sol. $A_1V_1 = A_2V_2$

750 × 10^{-4} V_1 = 500 × 10^{-6} × 0.3

$$V_1 = \frac{500 \times 3 \times 10^{-3}}{750} \text{ m/s}$$

= 2 × 10^{-3} m/s

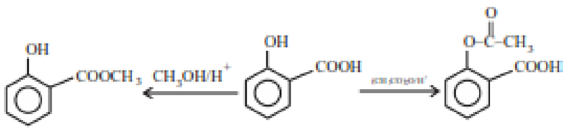
$$\frac{dh}{dt} = -2 \times 10^{-3} \text{ m/s}$$

CHEMISTRY

Section - A (Single Correct Answer)

31. A

Sol.



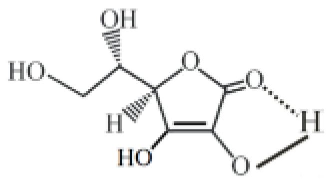
32. D

Sol. (Bond enthalpy order

C – C > Si – Si > Ge – Ge > Sn – Sn)

33. A

Sol. H-bonding stabilised vitamin C

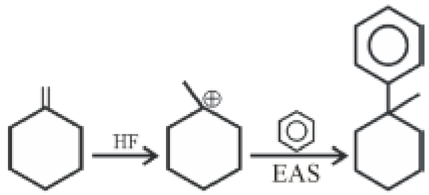


34. C

Sol. It is S_N1 reaction so rate of reaction depends on the concentration of alkyl halide only.

35. A

Sol.



36. C

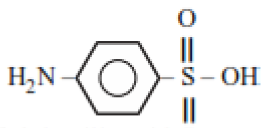
Sol. $[Co(NH_3)_5NO_2]^{2+}$

Two linkage isomers possible

$NO_2 \rightarrow$ Ambidentate ligand

37. D

Sol.



Sulphanilic acid

Does not show esterification test.

Presence of both sulphur and nitrogen give red colour in Lassigne's test.

38. A

Sol. Gypsum is used for making fireproof wall boards.

39. D

Sol. Nessler's Reagent $\rightarrow K_2[HgI_4]$

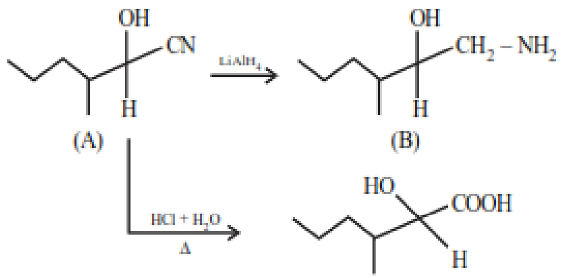
40. A

41. D

Sol. In urea production NH_3 and CO_2 consumed so least responsible for global warming.

42. D

Sol.



43. A

Sol. $2Cu^{+} \rightarrow Cu^{2+} + Cu$

The stability of $Cu^{2+}(aq)$ rather than $Cu^{+}(aq)$, is due to the much more negative $\Delta_{hyd}H$ of $Cu^{2+}(aq)$ than $Cu^{+}(aq)$, which more than compensates for the second ionisation enthalpy of Cu.

44. A

Sol. $K_2S_2O_8(s) + 2D_2O(\ell) \rightarrow 2KDSO_4(aq.) + D_2O_2$

45. C

Sol. $\frac{x}{m} = Kp^{1/n}$

$$\log\frac{x}{m} = \log k + \frac{1}{n}\log P$$

$$Y = 3x + 2.505, \quad \frac{1}{n} = 3, \quad \log K = 2.505$$

46. C

Sol. KOH absorb CO_2.

So its concentration should be checked.

47. D

Sol. K^{+} Cl^{1} Ca^{2+} Sc^{3+}

18 18 18 18

48. A

Sol. $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$

Case 1 : At constant P – volume will increase so reaction will shift in forward direction then answer will be A.

Case 2 : At constant volume no change in active mass so reaction will not shift in any direction then answer will be D.

49. B

Sol. (1) $\Delta_{eg}H$ (Cl) < $\Delta_{eg}H$ (F)

(–345) (– 328) Correct

(2) $\Delta_{eg}H$ (Se) < $\Delta_{eg}H$ (S)

(–195) (– 200) Incorrect

(3) $\Delta_{eg}H$ (I) < $\Delta_{eg}H$ (At)

(–295) (– 270) Correct

(4) $\Delta_{eg}H$ (Te) < $\Delta_{eg}H$ (Po)

(–190) (– 183) Correct

50. D

Sol. According to bent rule more electronegative atom occupy less s-characters so bond length increases. O – H bond will be short than O – F bond due to small size of H than F.

Section - B (Numerical Value)

51. 1006

Sol. Bomb calorimeter → const. volume

Heat released

By combustion of 1 mole

$$C_2H_6(\Delta U) = -\frac{20\times 0.5}{0.3}\times 30 = -1000 \text{ kJ}$$

$C_2H_6(g) + 7/2\ O_2(g) \rightarrow 2CO_2(g) + 3H_2O(\ell)$

$\Delta ng = 2 - (2 + 7/2) = -(7/2)$

$\Delta H = \Delta U + \Delta nRT$

$= -1000 - 7/2 \times 8.3 \times 300$ kJ

$= -1000 - 6.225$

$= -1006$ kJ

So heat released = 1006 kJ mol^{-1}

52. 3

Sol. FeS and Cu_2S, present in copper matte.

53. 3

Sol. Chlorodiazepoxide, Veronal, Valium is tranquilizer where as salvarsan is antibiotic.

54. 75

Sol. Assume reaction starts with 1 mole A

$$t_{1/2} = \frac{a}{2k},\ K = \frac{1}{2\times 50}$$

For 75% completion

$$a - \frac{a}{4} = kt$$

$$t = \frac{3}{4}\frac{a}{k} = \frac{3}{4}\times\frac{100}{a} = 75$$

55. 372

Sol. $i = 1 + (n - 1)\alpha$

$i = 1 + 0.2\,(2 - 1) = 1.2$

$\Delta T_f = i\,K_f\,m$

$$\Delta T_f = 1.2 \times 1.86 \times \frac{5\times 1000}{60\times 500}$$

$\Delta t_f = 3.72$

$\Delta T_f = 372 \times 10^{-2}$

56. 139

Sol. 10 ml solute in 90 ml solvent

mass of solute = 10 × 3.2 = 32 g

mass of solvent = 90 × 1.6 g

$$m = \frac{32\times 1000}{160\times 90\times 1.6} = 1.388$$

$m = 138.8 \times 10^{-2} = 139$

57. 14

Sol. $[Ag^+] = 10^{-5}$

$[NO_3^-] = 10^{-5}$

$$[Br^-] = \frac{Ksp}{[Ag^+]} = 4.9\times 10^{-8}$$

$$\Lambda_m = \frac{k}{1000\times M}$$

For Ag^+

$$6\times 10^{-3} = \frac{K_{Ag^+}}{1000\times 10^{-5}}$$

$K_{Ag^+} = 6\times 10^{-5}$

$\Rightarrow 6000\times 10^{-8}$

for Br^-

$$8\times 10^{-3} = \frac{K_{Br^-}}{1000\times 4.9\times 10^{-8}}$$

$K_{Br^-} = 39.2\times 10^{-8}$

for NO_3^-

$$7\times10^{-3}=\frac{K_{NO_3^-}}{1000\times10^{-5}}$$

$$K_{NO_3^-}=7\times10^{-5}$$

$$=7000\times10^{-8}$$

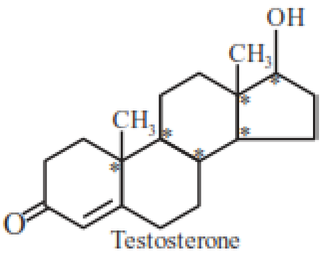
Conductivity of solution,

$\Rightarrow$ (6000 + 7000 + 39.2) × 10^{-8}

$\Rightarrow$ 13039.2 × 10^{-8} S m^{-1}

58. 6

Sol.



59. 6

Sol. $[Mn(H_2O)_6]^{2+}$

$$Mn^{2+}=3d^5$$

$$\mu=\sqrt{5(5+2)}=5.91\text{ BM}$$

60. 4

Sol. $d=\frac{Z\times M}{N_A a^3}$

$$\frac{d_{FCC}}{d_{BCC}}=\frac{\dfrac{4\times M_w}{N_A\times(2)^3}}{\dfrac{2\times M_w}{N_A\times(2.5)^3}}=3.90$$

MATHEMATICS

Section - A (Single Correct Answer)

61. B

Sol. $\sum_{n=1}^{\infty}\frac{2n^2+3n+4}{(2n)!}$

$$\frac{1}{2}\sum_{n=1}^{\infty}\frac{2n(2n-1)+8n+8}{(2n)!}$$

$$\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{(2n-2)!}+2\sum_{n=1}^{\infty}\frac{1}{(2n-1)!}+4\sum_{n=1}^{\infty}\frac{1}{(2n)!}$$

$$e=1+1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+.....$$

$$e^{-1}=1-1+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}+.....$$

$$\left(e+\frac{1}{e}\right)=2\left(1+\frac{1}{2!}+\frac{1}{4!}+......\right)$$

$$e-\frac{1}{e}=\left(1+\frac{1}{3!}+\frac{1}{5!}+.....\right)$$

Now

$$\frac{1}{2}\left(\sum_{n=1}^{\infty}\frac{1}{(2n-2)!}\right)+2\sum_{n=1}^{\infty}\frac{1}{(2n-1)!}+4\sum_{n=1}^{\infty}\frac{1}{(2n)!}$$

$$=\frac{1}{2}\left[\frac{e+\frac{1}{e}}{2}\right]+2\left[\frac{e-\frac{1}{e}}{2}\right]+4\left(\frac{e+\frac{1}{e}-2}{2}\right)$$

$$=\frac{\left(e+\frac{1}{e}\right)}{4}+e-\frac{1}{e}+2e+\frac{2}{e}-4$$

$$=\frac{13}{4}e+\frac{5}{4e}-4$$

62. C

Sol. $0<x<1$

$$2\tan^{-1}\left(\frac{1-x}{1+x}\right)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$\tan^{-1}x=\theta\in\left(0,\ \frac{\pi}{4}\right)\quad\therefore x=\tan\theta$$

$$2\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\theta\right)\right)=\cos^{-1}(\cos2\theta)$$

$$x=\tan\frac{\pi}{8}\quad\therefore x=\sqrt{2}-1\simeq0.414$$

63. A

Sol. $\vec{a}=2\hat{i}-7\hat{j}+5\hat{k}$

$$\vec{b}=\hat{i}+\hat{k}$$

$$\vec{c}=\hat{i}+2\hat{j}-3\hat{k}$$

$$\vec{r}\times\vec{a}=\vec{c}\times\vec{a}\Rightarrow(\vec{r}-\vec{c})\times\vec{a}=0$$

$\therefore \vec{r} = \vec{c} + \lambda\vec{a}$

$\vec{r}\cdot\vec{b} = \vec{c} + \lambda\vec{a}$

$\vec{r}\cdot\vec{b} = 0 \Rightarrow \vec{c}\cdot\vec{b} + \lambda \quad \vec{b}\cdot\vec{a} = 0$

$-2 + \lambda(7) = 0 \Rightarrow \lambda = \frac{2}{7}$

$\therefore \vec{r} = \vec{c} + \frac{2\vec{a}}{7} = \frac{1}{7}(11\hat{i} - 11\hat{k})$

$|\vec{r}| = \frac{11\sqrt{2}}{7}$

64. C

Sol. $A = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$

$A = \begin{bmatrix} \cos 60^o & \sin 60^o \\ -\sin 60^o & \cos 60^o \end{bmatrix}$

If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ Here $\alpha = \frac{\pi}{3}$

$A^2 = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$

$A^{30} = \begin{bmatrix} \cos 30\alpha & \sin 30\alpha \\ -\sin 30\alpha & \cos 30\alpha \end{bmatrix}$

$A^{30} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$A^{25} = \begin{bmatrix} \cos 25\alpha & \sin 25\alpha \\ -\sin 25\alpha & \cos 25\alpha \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

$A^{25} = A$

$A^{25} - A = 0$

65. A

Sol. A : no. on 1st die < no. on 2nd die

A : no. on 1st die = even & no. of 2nd die = odd

C : no. on 1st die = odd & no. on 2nd die = even

n(A) = 5 + 4 + 3 + 2 + 1 = 15

n(B) = 9 n(C) = 9

$n((A \cup B) \cap C) = (A \cap C) \cup (B \cap C)$

= (3 + 2 + 1) + 0 = 6.

66. B

Sol. (i) $p \to (p \wedge (p \to q))$

$(\sim p) \vee (p \wedge (\sim p \vee q))$

$(\sim p) \vee (f \vee (p \wedge q))$

$\sim p \vee (p \wedge q) = (\sim p \vee p) \wedge (\sim p \vee q)$

$= \sim p \vee q$

(ii) $(p \vee q) \to (\sim p \to q)$

$\sim (p \wedge q) \vee (p \vee q) = t$

$\{a,\ b,\ d\} \vee \{a,\ b,\ c\} = V$

Tautology

(iii) $(p \wedge (p \to q)) \to \sim q$

$\sim (p \wedge (\sim p \vee q)) \vee - \sim q = \sim (p \wedge q) \vee \sim q = \sim p \vee \sim q$

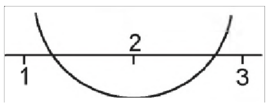
Not tantology

(iv) $p \vee (p \wedge q) = p$

Not tautology.

67. C

Sol. $2x^2 - 8x + k = 0$



f(1) · f(2) < 0& f(2) · f(3) < 0

(k – 6) (k – 8) < 0& (k – 8) (k – 6) < 0

$k \in (6,\ 8)$ $\qquad k \in (6,\ 8)$

integral value of k = 7

68. B

Sol. $f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$

$x = 2 \Rightarrow f(2) + f(-1) = 3$ ....(1)

$x = -1 \Rightarrow f(-1) + f\left(\frac{1}{2}\right) = 0$ ....(2)

$x = \frac{1}{2} \Rightarrow f\left(\frac{1}{2}\right) + f(2) = \frac{3}{2}$ ....(3)

$$(1)+(3)-(2) \Rightarrow 2f(2)=\frac{9}{2}$$

$$\therefore f(2)=\frac{9}{4}$$

69. A

Sol. $P \equiv P_1 + \lambda P_2 = 0$

$(2+\lambda)x+(3+2\lambda)y+(3\lambda-1)z-2-6\lambda=0$

Plane P is perpendicular to $P_3 \therefore \vec{n}\cdot\vec{n}_3=0$

$2(\lambda+2)+(2\lambda+3)-(3\lambda-1)=0$

$\lambda=-8$

$P \equiv -6x-13y-25z+46=0$

$6x + 13y + 25z - 46 = 0$

Dist from (–7, 1, 1)

$$d=\left|\frac{-42+13+25-46}{\sqrt{36+169+625}}\right|=\frac{50}{\sqrt{830}}$$

$$d^2=\frac{50\times 50}{830}=\frac{250}{83}$$

70. A

Sol. $ab<0 \left|\frac{1+ai}{b+i}\right|=1$

$|1+ ai| = |b + i|$

$a^2+1=b^2+1 \Rightarrow a=\pm b \Rightarrow b=-a$ as $ab < 0$

(a, b) lies on $|z - 1| = |2z|$

$|a + ib - 1| = 2|a + ib|$

$(a-1)^2 + b^2 = 4(a^2 + b^2)$

$(a-1)^2 = a^2 = 4(2a^2)$

$1 - 2a = 6a^2$

$\Rightarrow 6a^2 + 2a - 1 = 0$

$$a=\frac{-2\pm\sqrt{28}}{12}=\frac{-1\pm\sqrt{7}}{6}$$

$$a=\frac{\sqrt{7}-1}{6} \;\&\; b=\frac{1-\sqrt{7}}{6}$$

$[a] = 0$

$$\therefore \frac{1+[a]}{4b}=\frac{6}{4(1-\sqrt{7})}=-\left(\frac{1+\sqrt{7}}{4}\right)$$

or $[a = 0]$

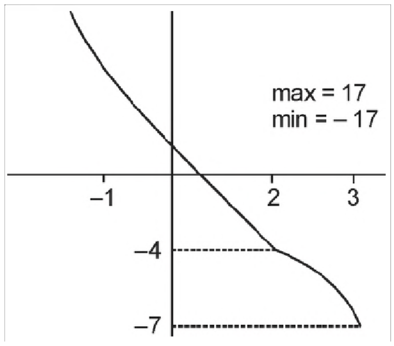
Similarly it is matching with $a=\frac{-1-\sqrt{7}}{6}$

No answer is matching.

71. A

Sol. $f(x) = |x^2 - 5x + 6| - 3x + 2$

$$f(x)=\begin{cases} x^2-8x+8\;; & x\in[-1,\ 2] \\ -x^2+2x-4\;; & x\in[2,\ 3]\end{cases}$$



72. A

Sol. S = {1, 2, 3,.......10}

P(S) = power set of S

$AR,\ B \Rightarrow (A\cap\vec{B})\cup(\vec{A}\cap B)=\phi$

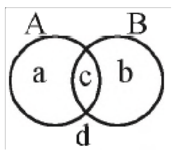
R1 is reflexive, symmetric

For transitive

$(A\cap\vec{B})\cup(\vec{A}\cap B)=\phi\ ;\ \{a\}=\phi=\{b\}\ A=B$

$(B\cap\vec{C})\cup(\vec{B}\cap C)=\phi\ \therefore B=C$

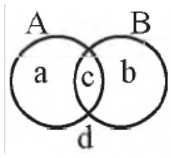
$\therefore$ A = C equivalence.



$R_2 \equiv A\cup\vec{B}=\vec{A}\cup B$

$R_2 \rightarrow$ Reflexive, symmetric

for transitive



$A\cup\vec{B}=\vec{A}\cup B\Rightarrow\{a,\ c,\ d\}=\{b,\ c,\ d\}$

$\{a\}=\{b\}\ \therefore A=B$

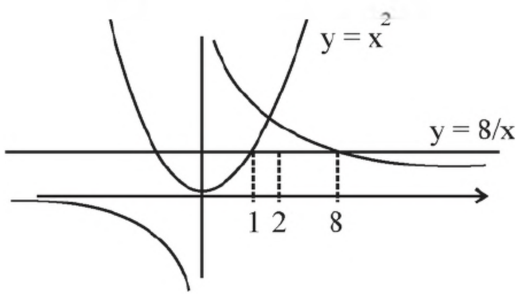
$B\cup\vec{C}=\vec{B}\cup C\Rightarrow B=C$

$\therefore A=C\qquad\therefore A\cup\vec{C}=\vec{A}\cup C$

$\therefore$ Equivalence

73. B

Sol. $xy\le 8\qquad 1\le y\le x^2$



$$\text{Area}=\int_1^2(x^2-1)dx+\int_2^8\left(\frac{8}{x}-1\right)dx$$

$$=\left(\frac{x^3}{3}\right)_1^2+8(\ell nx)_2^8-(x)_1^8$$

$$=\frac{7}{3}+8(2\ell n2)-7$$

$$=16\ell n2-\frac{14}{3}$$

74. A

Sol. $\alpha x=e^{x^{\beta}\cdot y^{\gamma}}$

$$2x^2y\frac{dy}{dx}=1-x\cdot y^2\qquad y^2=t$$

$$x^2\frac{dt}{dx}=1-xt$$

$$\frac{dt}{dx}+\frac{t}{x}=\frac{1}{x^2}\qquad \text{I.F.}=e^{\ell nx}=x$$

$$t(x)=\int\frac{1}{x^2}\cdot x\ dx$$

$$y^2\cdot x=\ell nx+C$$

$$\therefore 2\cdot\ell n2=\ell n2+C$$

$$\therefore C=\ell n2$$

Hence, $xy^2=\ell n2x$

$$\therefore 2x=e^{x\cdot y^2}$$

Hence $\alpha=2,\ \beta=1,\ \gamma=2$

75. D

Sol. $I=\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}}\dfrac{x+\frac{\pi}{4}}{2-\cos 2x}dx$ ....(1)

$x\to -x$

$I=\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}}\dfrac{x+\frac{\pi}{4}}{2-\cos 2x}dx$ ....(2)

(1) + (2)

$$2I=\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}}\frac{\frac{\pi}{2}}{2-\cos 2x}dx$$

$$I=\frac{\pi}{4}\cdot 2\int_0^{\frac{\pi}{4}}\frac{dx}{2-\cos 2x}dx$$

$$I=\frac{\pi}{4}\cdot 2\int_0^{\frac{\pi}{1}}\frac{(1+\tan^2 x)dx}{2(1+\tan^2 x)-(1-\tan^2 x)}$$

$$I=\frac{\pi}{4}\int_0^1\frac{dt}{3t^2+1}$$

$$\Rightarrow I=\frac{\pi}{2\sqrt{3}}\tan^{-1}\sqrt{3}$$

$$I=\frac{\pi^2}{6\sqrt{3}}$$

76. B

Sol. $9=x_1<x_2<\text{........}<x_7$

9, 9 + d, 9 + 2d,...........9 + 6d

0, d, 2d,........6d

$$\bar{x}_{new} = \frac{21d}{7} = 3d$$

$$16 = \frac{1}{7}(0^2 + 1^2 + ... + 6^2)d^2 - 9d^2$$

$$= \frac{1}{7}\left(\frac{6 \times 7 \times 13}{6}\right)d^2 - 9d^2$$

$16 = 4d^2$

$d^2 = 4$

$d = 2$

$$\bar{x} + x_6 = 6 + 9 + 10 + 9$$

77. A

Sol. $\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$

$\alpha(\alpha^2 - 1) - 1(\alpha - 1) + 1(1 - \alpha) = 0$

$\alpha^3 - 3\alpha + 2 = 0$

$\alpha^2(\alpha - 1) + \alpha(\alpha - 1) - 2(\alpha - 1) = 0$

$(\alpha - 1)(\alpha^2 + \alpha - 2) = 0$

$\alpha = 1, \alpha = -2, 1$

For $\alpha = 1, \beta = 1$

$\Delta = 4$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 3 - 1 - 1 = 1 \qquad \Rightarrow x = \frac{1}{4}$$

$$\Delta_2 = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 2 - 1 = 1 \qquad \Rightarrow y = \frac{1}{4}$$

$$\Delta_3 = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 2 - 1 = 1 \qquad \Rightarrow z = \frac{1}{4}$$

For $\alpha = 2 \Rightarrow$ unique solution

78. A

Sol. $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$

$\vec{b} = \hat{i} - 3\hat{j} + 5\hat{k}$

$$\vec{a} \cdot \hat{b} = \frac{5 - 3 - 15}{\sqrt{35}} = -\frac{-13}{\sqrt{35}}$$

79. C

Sol. $3x^2 - 4y^2 = 36 \qquad 3x + 2y = 1$

$$m = -\frac{3}{2}$$

$$m = +\frac{\sec\theta 3}{\sqrt{12} \cdot \tan\theta}$$

$$\Rightarrow \frac{3}{\sqrt{12}} \times \frac{1}{\sin\theta} = \frac{-3}{2}$$

$$\sin\theta = -\frac{1}{\sqrt{3}}$$

$$(\sqrt{12} \cdot \sec\theta, \ 3\tan\theta)$$

$$\left(\sqrt{12} \cdot \frac{\sqrt{3}}{2}, \ 3 \times \frac{1}{\sqrt{2}}\right) \Rightarrow \left(\frac{6}{\sqrt{2}}, \ \frac{-3}{\sqrt{2}}\right)$$

80. C

Sol. $y' = x^x$

$$y' = x^x(1 + \ell nx)$$

$$y'' = x^x(1 + \ell nx)^2 + x^x \cdot \frac{1}{x}$$

$$y''(2) = 4(1 + \ell n2)^2 + 2$$

$$y'(2) = 4(1 + \ell n2)$$

$$y''(2) - 2y'(2) = 4(1 + \ell n2)^2 + 2 - 8(1 + \ell n2)$$

$$= 4(1 + \ell n2)[1 + \ell n2 - 2] + 2$$

$$= 4((\ell n2)^2 - 1) + 2$$

$$= 4(\ell n2)^2 - 2$$

Section - B (Numerical Value)

81. 81

Sol. Taking single digit $\rightarrow$ 444444 $\quad \frac{6!}{6!} = 1$

Taking two digit $\rightarrow$

(4, 5) 444555 (4, 9) 444999

$\frac{5!}{3!2!}=10$ $\frac{5!}{3!2!}=10$

Taking three digit

$4, 5, 9, 4, 4, 4 \Rightarrow \frac{5!}{3!}=20$

$4, 5, 9, 5, 5, 5 \Rightarrow \frac{5!}{4!}=5$

$4, 5, 9, 9, 9, 9 \Rightarrow \frac{5!}{4!}=5$

$4, 5, 9, 4, 5, 9 \Rightarrow \frac{5!}{2!2!}=30$

Total = 81

82. 105

Sol. ${}^{15}C_2=\frac{15\times 14}{2}=105$

83. 39

Sol. $\frac{a}{e}=8$ .....(1)

ae = 2 .....(2)

$8e=\frac{2}{e}$

$e^2=\frac{1}{4}\Rightarrow e=\frac{1}{2}$

a = 4

$b^2 = a^2(1 - e^2)$

$=16\left(\frac{3}{4}\right)=12$

$\frac{x\cos\theta}{4}+\frac{y\sin\theta}{2\sqrt{3}}=1$

$\sin\theta=\frac{1}{2}$

$\theta=30^o$

$P(2\sqrt{3}, \sqrt{3})$

$Q\left(\frac{8}{\sqrt{3}}, 0\right)$

$(3PQ)^2 = 39$

84. 16

Sol. $y^2 = 8x + 4y + 4$

$(y - 2)^2 = 8(x + 1)$

$y^2 = 4ax$

a = 2, X = x + 1, Y = y – 2

focus (1, 2)

y – 2 = m (x – 1)

Put (3, 0) in the above line

m = –1

Length of focal chord = 16

85. 13

Sol. $I=\int_0^{\pi}\frac{5^{\cos x}(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x)}{1+5^{\chi osx}}dx$

$I=\int_0^{\pi}\frac{5^{-\cos x}(1+\cos x\cos 3x+\cos^2 x+\cos^2 x\cos 3x)}{1+5^{-\cos x}}dx$

$2I=\int_0^{\pi}(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x)dx$

$\not{2}I=\not{2}\int_0^{\frac{\pi}{2}}(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x)dx$

$I=\int_0^{\frac{\pi}{2}}(1+\sin x(-\sin 3x)+\sin^2 x\sin^3 x\sin 3x)dx$

$2I=\int_0^{\frac{\pi}{2}}(3+\cos 4x+\cos^3 x\cos 3x-\sin^3 x\sin 3x)dx$

$2I=\int_0^{\frac{\pi}{2}}3+\cos 4x+\left(\frac{\cos 3x+3\cos x}{4}\right)\cos 3x-\sin 3x\left(\frac{3\sin x-\sin 3x}{4}\right)dx$

$2I=\int_0^{\frac{\pi}{2}}\left(3+\cos 4x+\frac{1}{4}+\frac{3}{4}\cos 4x\right)dx$

$2I=\frac{13}{4}\times\frac{\pi}{2}+\frac{7}{4}\left(\frac{\sin 4x}{4}\right)_0^{\frac{\pi}{2}}\Rightarrow I=\frac{13\pi}{16}$

86. 4

Sol. $T_6 = {}^mC_5(10-3^x)^{\frac{m-5}{2}}\cdot(3^{x-2})=21$ ....(1)

${}^mC_1$, ${}^mC_2$, ${}^mC_3$ are in A.P.

$2\cdot{}^mC_2 = {}^mC_1 + {}^mC_1$

Solving for m, we get

m = 2 (rejected), 7

Put in equation (1)

$$21\cdot(10-3^x)\frac{3^x}{9}=21$$

$3^x = 3^0, 3^2$

x = 0, 2

Sum of the squares of all possible values of x = 4

87. 1

Sol. $T_{r+1} = {}^{23}C_r\cdot\left(x^{\frac{2}{3}}\right)^{22-r}\cdot(\alpha)^x,\ x^{-3r}$

$$= {}^{22}C_r\cdot x^{\frac{44}{3}-\frac{2x}{3}3r}(\alpha)^r$$

$$\frac{44}{3}=\frac{11r}{3}$$

r = 4

$${}^{22}C_4\cdot\alpha^4=7315$$

$$\frac{22\times21\times20\times19}{24}\cdot\alpha^4=7315$$

$\alpha=1$

88. 321

Sol. 3, 7, 11, 15,..............., 399 d_1 = 4

2, 5, 8, 11,................., 359 d_2 = 3

2, 7, 12, 17,........, 197 d_3 = 5

LCM (d_1, d_2, d_3) = 60

Common terms are 47, 107, 167

Sum = 321

89. 6

Sol. Given Equation is not equation of plane as yz is present. If we consider y is g then answer would be 6.

Normal vector of plane $=3\hat{i}-\hat{j}-2\hat{k}$

Plane : $3x-y2-z+\lambda=0$

Point (3, –2, 5) satisfies the plane

$\lambda=-1$

3x – y – 2z = 1

$\alpha\beta y=6$

90. 10

Sol. Plane : 8x + y + 2z = 0

Given line AB: $\frac{x-2}{5}=\frac{y-4}{10}=\frac{z+3}{4}=\lambda$

Any point on line (5λ + 2, 10λ + 4, –4λ – 3)

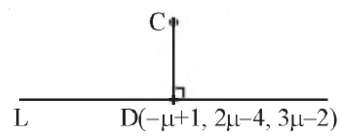
Point of intersection of line and plane

8(5λ + 2) + 10λ + 4 – 8λ – 6 = 0

$$\lambda=-\frac{1}{3}$$

$$C\left(\frac{1}{3},\ \frac{2}{3},\ -\frac{5}{3}\right)$$

$$L:\frac{x-1}{-1}=\frac{y+4}{2}=\frac{z+2}{3}=\mu$$



$$\overrightarrow{CD}-\left(-\mu+\frac{2}{3}\right)\hat{i}+\left(2\mu-\frac{14}{3}\right)\hat{j}+\left(3\mu-\frac{1}{3}\right)\hat{k}$$

$$\left(-\mu+\frac{2}{3}\right)(-1)+\left(2\mu\frac{14}{3}\right)2+\left(3\mu-\frac{1}{3}\right)3=0$$

$$\mu=\frac{11}{14}$$

$$\overrightarrow{CD}=\frac{-5}{42},\ \frac{-130}{42},\ \frac{85}{42}$$

Direction ratios → (–1, –26, 17)

|a + b + c| = 10

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