3S INSTITUTE
JEE ADVANCED | JEE MAIN | NEET | OLYMPIADS | MHT-CET | FOUNDATION

01-February-2023 (Morning Batch) : JEE Main Paper

PHYSICS

Section - A (Single Correct Answer)

1. C

Sol. Based on theory.

2. C

Sol. AB = x

BC = x

2x + CD = 3x

CD = x

$$<v> = \frac{3x}{\frac{x}{v_1}+\frac{x}{v_2}+\frac{x}{v_3}} = \frac{3v_1v_2v_3}{v_2v_3+v_1v_3+v_1v_2}$$

3. C

Sol. geff = g – ω^2 R_e $\sin^2\theta$, $\theta \to$ co-latitude angle

$$g_{eff} = g\left(1-\frac{d}{R_e}\right),\ \text{d here depth}$$

4. C

Sol. Based on theory.

5. B

Sol. Based on theory.

6. A

Sol. $v_{(escape)\ plant} = \sqrt{\frac{2GM_P}{R_P}}$

$$= \sqrt{\frac{2G\left(\frac{M_e}{9}\right)}{\left(\frac{R}{2}\right)}} = \frac{v_e\sqrt{2}}{3} \quad \therefore\ x = 2$$

7. B

Sol. After passing through first sheet

$$I_1 = \frac{I}{2}$$

After passing through second sheet

$$I_2 = I_1\cos^2(45°) = \frac{I}{4}$$

After passing through nth sheet

$$I_n = \frac{I}{2^n} = \frac{I}{64}$$

n = 6

8. C

Sol. $B_P = \left(\frac{\mu_0 i}{4r}+\frac{\mu_0 i}{4\pi r}\right) = \frac{\mu_0 i}{2r}\left(\frac{1}{2}+\frac{1}{2\pi}\right)$

9. B

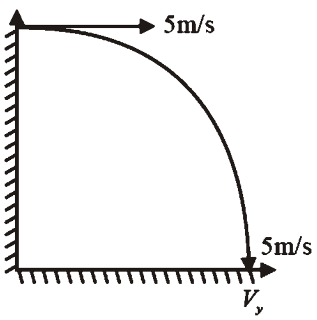
Sol. FM broadcast range is 88MHz to 108MHz

10. B

Sol. $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{70}{70\times10^{-3}}} = 100$ m/s

11. B

Sol.



$$v_y = \sqrt{2gh} = \sqrt{200}$$

$$\upsilon_{net} = \sqrt{25+200} = 15\ \text{m/s}$$

12. B

Sol. $KE = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$

$$\frac{KE_p}{KE_\alpha} = \frac{m_\alpha}{m_p} = 4:1$$

13. D

Sol. $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

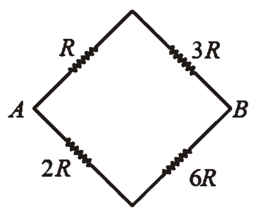
$$TV^{1/2} = T_2(2V)^{1/2}$$

$$T_2 = \frac{T}{\sqrt{2}}$$

$$W = \frac{R(T_1 - T_2)}{\gamma - 1} = \frac{R\left(T - \frac{T}{\sqrt{2}}\right)}{\frac{1}{2}} = RT(2 - \sqrt{2})$$

14. D

Sol. Wheat stone bridge is in balanced condition.



$$\frac{1}{R_{eq}} = \frac{1}{4R} + \frac{1}{8R}$$

$$R_{eq} = \frac{8R}{3}$$

15. D

Sol. Assuming RHS to be $\hat{n}$.

$$\vec{E}_I = \frac{\sigma}{2\epsilon_0}(-\hat{n}) + \frac{\sigma}{2\epsilon_0}(-\hat{n}) = -\frac{\sigma}{\epsilon_0}\hat{n}$$

$$\vec{E}_{II} = 0$$

$$\vec{E}_{III} = \frac{\sigma}{2\epsilon_0}(\hat{n}) + \frac{\sigma}{2\epsilon_0}(\hat{n}) = \frac{\sigma}{\epsilon_0}(\hat{n})$$

16. A

Sol. Initial surface energy = $0.45 \times 4\pi\,(10^{-3})^2$

$$\frac{4}{3}\pi(10^{-3})^3 = 125 \times \frac{4\pi}{3}R_{new}^3$$

$$\therefore 10^{-3} = 5R_{new}$$

$$\therefore R_{new} = \frac{10^{-3}}{5}\,m$$

So, final surface energy

$$= 0.45 \times 125 \times 4\pi\left(\frac{10^{-3}}{5}\right)^2$$

Increase in energy = $0.45 \times 4\pi \times (10^{-3})^2$

= 2.26×10^{-5} J

17. B

Sol. B.E of Helium = $(2m_P + 2m_N - m_{He})\,c^2$

= 28.4 MeV

18. C

Sol. [b] = [V]

$$\left[\frac{a}{b^2}\right] = [P] \quad \therefore \left[\frac{b^2}{a}\right] = \frac{1}{[P]} = \frac{1}{[B]} = [K]$$

19. A

Sol. Basic theory

Translational K.E on average of a molecule is 3/2 KT which is independent of nature, pressure and volume.

20. C

Sol. $S = ut + \frac{1}{2}at^2$

$$50 = 0 + \frac{1}{2} \times a \times 100$$

a = 1 m/s^2

F – μmg = ma

30 – μ × 50 = 5 × 1

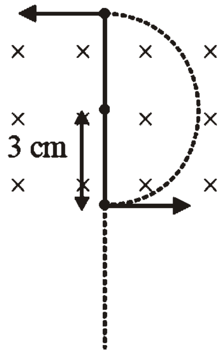
50μ = 25

$$\mu = \frac{1}{2}$$

Section - B (Numerical Value)

21. 144

Sol. $r = \frac{mv}{qB} = \frac{\sqrt{2km}}{qB}, m = \frac{r^2q^2B^2}{2k}$



$$m = \frac{\frac{1}{100} \times \frac{3}{100} \times 2 \times 2 \times 4 \times 10^{-3} \times 4 \times 10^{-3} \times 10^{-12}}{2 \times (100) \times 10^{-6}}$$

= 144×10^{-18} kg

22. 25

Sol. $\frac{E_1}{E_2} = \frac{l_1}{l_2}$

$\frac{1.5}{E_2} = \frac{60}{60+40} = \frac{6}{10} = \frac{3}{5}$

x = 25

23. 40

Sol. $W = \vec{F} \cdot (\vec{r}_f - \vec{r}_i)$

$= (5\hat{i} + 2\hat{j} + 7\hat{k}) \cdot ((5\hat{i} - 2\hat{j} + \hat{k}) - (2\hat{i} + 3\hat{j} - 4\hat{k}))$

W = 40 J

24. 828

Sol. In the ground state energy = – 13.6 eV

So energy

$\frac{-13.6eV}{n^2} = -13.6 + 12.75$

$\frac{-13.6eV}{n^2} = -0.85$

$n = \sqrt{16}$

$\boxed{n = 4}$

Angular momentum $= \frac{nh}{2\pi} = \frac{4h}{2\pi} = \frac{2h}{\pi}$

Angular momentum $= \frac{2}{\pi} \times 4.14 \times 10^{-15}$

$= \frac{828 \times 10^{-17}}{\pi} eVs$

25. 1

Sol. $B_{water} = \frac{-\Delta P}{\left(\frac{\Delta V}{V}\right)} = \frac{-\Delta P}{\frac{0.01}{100}}$

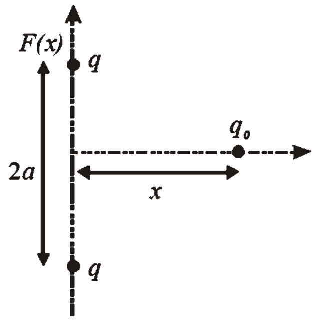
$B_{liquid} = \frac{-\Delta P}{\frac{0.03}{100}}$

$\frac{B_{water}}{B_{liquid}} = 3$

x = 1

26. 2

Sol.



$F = \frac{2Kqq_0x}{(x^2 + a^2)^{3/2}}$

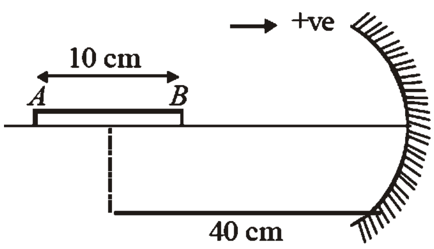
For F to be maximum

$\frac{dF}{dx} = 0$

$x = \frac{a}{\sqrt{2}}$

27. 32

Sol.



U_A = – 45 cm, f = – 20 cm

$V_A = \frac{-45 \times (-20)}{-45 - (-20)} = \frac{-900}{25} = -36$ cm

And U_B = – 35cm

$\therefore V_B = \frac{-35 \times (-20)}{-35 - (-20)} = \frac{700}{-15}$

$\therefore V_A - V_B$ = length of image

$= \left(-36 + \frac{140}{3}\right)$cm

$= \frac{32}{3}$cm

$\therefore$ x = 32

28. 2

Sol. $v = \sqrt{\dfrac{2gh}{1+\dfrac{k^2}{R^2}}}$

Where h = 60 sin 30° = 30 cm

$k^2 = \dfrac{R^2}{2}$

$v = 2\ ms^{-1}$

29. 2

Sol. $KE = PE + \dfrac{PE}{4}$

$KE = \dfrac{5}{4}PE$

$\dfrac{1}{2}m\omega^2\left(A^2 - x^2\right) = \dfrac{5}{4}\times\dfrac{1}{2}m\omega^2x^2$

$\left[v = \omega\sqrt{A^2 - x^2}\right]$

$A^2 - x^2 = \dfrac{5}{4}x^2$

$\dfrac{9x^2}{4} = A^2$

$\boxed{x = \dfrac{2}{3}A}$

$\therefore x = \dfrac{2}{3}\times 3\ cm$

x = 2 cm

30. 40

Sol. To maximize the average rate at which energy supplied i.e. power will be maximum.

So in LCR circuit power will be maximum at the condition of resonance and in resonance condition

$X_L = X_C$

$79.6 = \dfrac{1}{\omega C}$

$\therefore C = \dfrac{1}{2\pi\times 50\times 79.6}$

$\therefore \boxed{C = 40\mu F}$

CHEMISTRY

Section - A (Single Correct Answer)

31. D

Sol. Applying electrical neutrality principle in metal defficiency defect.

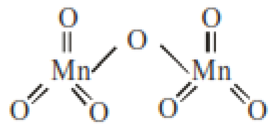
$3A^{2+}$ are replaced by $2A^{3+}$, thus one vacant site per pair of A^{3+} is created.

32. B

Sol. By Haworth structure of mannose.

33. A

Sol.



34. B

Sol. Dehydration of alcohol is directly proportional to the stability of carbocation.

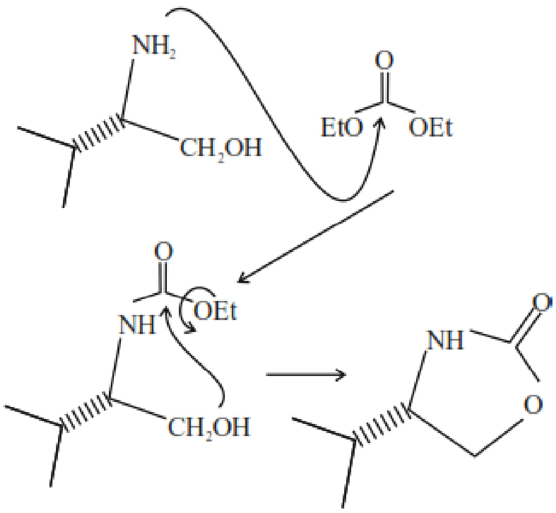
35. A

Sol. Adsorption $\propto$ vanderwaal attraction forces

$Z_c = \dfrac{3}{8}$ for all real gases

36. B

Sol. Initially lone pair electron of $-NH_2$ attack on electrophilic carbon, after then lone pair electron of oxygen attacks leading to formation of cyclic compound.



37. C

Sol. NCERT (Chemistry in every day life)

38. A

Sol. Chlorine oxides, Cl_2O, ClO_2, Cl_2O_6 and Cl_2O_7 are highly reactive oxidising agents and tend to explode.

39. D

Sol. Resonating structure are hypothetical and resonance hybrid is real structure which is weighted average of all the resonating structures.

40. B

Sol. In alcoholic KOH, elimination reaction takes place.

41. D

Sol. Formation of Prussian blue complex takes place.

42. A

Sol. Double salt contain's two or more types of salts. $CuSO_4 \cdot 4NH_3 \cdot H_2O$ and $Fe(CN)_2 \cdot 4KCN$ are complex compounds.

43. C

Sol. $\bar{C}N$ is a strong field ligand so maximum splitting in d orbitals take place.

44. C

Sol. NCERT (Environmental chemistry)

45. C

Sol. From S-block NCERT

46. B

Sol. A. Beryllium oxide is amphoteric in nature.

B. Beryllium carbonate is kept in the atmosphere of CO_2 because it is thermally less stable.

C. Beryllium sulphate is readily soluble in water due to high degree of hydration.

D. Beryllium shows anomalous behaviour due to small size, high ionization energy and high value of ϕ (polarising power).

47. D

Sol. $2C(s) + O_2(g) \rightarrow 2CO(g)$

$\Delta_r S^o$ is +ve, $\Delta_r G^o = \Delta_r H^o - T\Delta_r S^o$; thus slope is negative.

As temperature increases $\Delta_r G^o$ becomes more negative thus it has lower tendency to get decomposed.

48. B

Sol. Incorrect statements are C and D only, correct choice is not available.

49. B

Sol. No pollution occurs by combustion of hydrogen and very low density of hydrogen.

50. C

Sol.

List I	List II
Test	**Functional group / Class of Compound**
(A) Molisch's Test	(II) Carbohydrate
(B) Biuret Test	(I) Peptide
(C) Carbylamine Test	(III) Primary amine
(D) Schiff s Test	(IV) Aldehyde

Section - B (Numerical Value)

51. 364

Sol. $m = \dfrac{1000 \times M}{1000 \times d - M \times M.W \text{ of solute}}$

$= \dfrac{1000 \times 3}{1000 \times 1 - (3 \times 58.5)} = 3.64$

$= 364 \times 10^{-2}$

52. 2

Sol. (A) $V_e = 1000$ m/s ; $h = 6 \times 10^{-34}$ Js ;

$m_e = 9 \times 10^{-31}$ kg

$\lambda = \dfrac{h}{mv} = \dfrac{6 \times 10^{-34}}{9 \times 10^{-31} \times 1000}$

$= 666.67 \times 10^{-9}$ m

$= 666.67$ nm

(B) The characteristic of electrons emitted is independent of the material of the electrodes of the cathode ray tube.

(C) The cathode rays start from cathode and move towards anode.

(D) The nature of the emitted electrons is independent on the nature of the gas present in cathode ray tube.

53. 12

Sol. $HBrO_3$ (Bromic acid)

Ox. State of Br = +5

$HBrO_4$ (per bromic acid)

OX. State of Br = +7

Sum of Ox. State = 12

54. 3

Sol. $MnO_4^- + 8H^+ + 5e^- \rightleftharpoons Mn^{2+} + 4H_2O$

$$E = E^\circ - \frac{0.059}{5}\log\frac{[Mn^{2+}]}{[MnO_4^-][H^+]^8}$$

$$1.282 = 1.54 - \frac{0.059}{5}\log\frac{10^{-3}}{10^{-1}\times[H^+]^8}$$

$$\frac{0.258\times 5}{0.059} = \log\frac{10^{-2}}{[H^+]^8}$$

$\Rightarrow \quad 21.86 = -2 + 8\ pH$

$\therefore \quad pH = 2.98$

$\simeq 3$

55. 2

Sol. As per the language of given question, the best possible isomeric structure is

Ph – CH = CH – O – CH_3 (*cis* and *trans*).

So, the answer is 2.

56. 12

$$x(g) \rightleftharpoons y(g) + z(g) \qquad k_{p_1} = 3$$

Sol.

	x(g)	y(g)	z(g)
Initial moles	n	–	–
at equilibrium	n – αn	αn	αn

$$k_{p_1} = \frac{\left(\frac{\alpha}{1+\alpha}\times p_1\right)^2}{\frac{1-\alpha}{1+\alpha}p_1}$$

$$3 = \frac{\alpha^2\times p_1}{1-\alpha^2}$$

$$A(g) \rightleftharpoons 2B(g) \qquad k_{p_2} = 1$$

	A(g)	2B(g)	
Initial mole	n	–	
at equilibrium	x – αn	2 αn	$p_{total} = p_2$

$$k_{p_2} = \frac{\left(\frac{2\alpha}{1+\alpha}\times p_2\right)^2}{\frac{1-\alpha}{1+\alpha}p_2}$$

$$1 = \frac{4\alpha^2\times p_2}{1-\alpha^2}$$

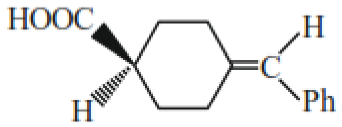
$$\frac{k_{p_1}}{k_{p_2}} = \frac{p_1}{4p_2}$$

$$\frac{3}{1} = \frac{p_1}{4p_2} \qquad \therefore$$

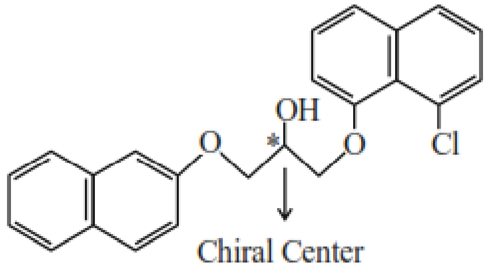
$p_1 : p_2 = 12 : 1$

$x = 12$

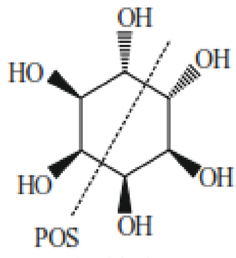
57. 2

Sol. 

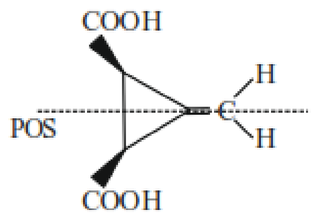
No POS, COS
(Chiral)



No POS, COS
(Chiral)



(Achiral)



(Achiral)

58. 15

Sol. $[A]_t = [A]_0 e^{-kt}$

For A : Let $[A]_t$ be y and $[A]_0$ be x ;

$$k = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{15\,\text{min}}$$

$$y = xe^{-kt}$$

$$= xe^{-\left(\frac{\ln 2}{15}\right)t}$$

For B : $[B]_t = [B]_0\, e^{-kt}$

Let $[B]_t = y$; $[B]_0 = 4x$; $k = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{5\,\text{min}}$

$$y = 4xe^{-\left(\frac{\ln 2}{5}\right)t}$$

$$\Rightarrow \quad xe^{-\left(\frac{\ln 2}{15}\right)t} = 4xe^{-\left(\frac{\ln 2}{5}\right)t}$$

$$e^{t\left(\frac{\ln 2}{5} - \frac{\ln 2}{15}\right)} = 4$$

$$t \times \left[\frac{\ln 2}{5} - \frac{\ln 2}{15}\right] = \ln 4$$

$$t \times \ln 2\left[\frac{1}{5} - \frac{1}{15}\right] = 2\ln 2$$

$$t = 15\,\text{min}.$$

59. 499

Sol.

$2H_2O(g) \rightarrow 2H_2(g) + O_2(g)$	$+(242 \times 2)$ kJ mol^{-1}
$H_2(g) + O_2(g) \rightarrow 2OH$	$+78$ kJ mol^{-1}
$H_2(g) \rightarrow 2H$	$+436$ kJ mol^{-1}
$2H_2O \rightarrow 2H + 2OH$	$+998$ kJ mol^{-1}

$H_2O \rightarrow H + OH \quad 998 \times \frac{1}{2} = +499$ kJ mol^{-1}

60. 3

Sol. $KCl + AgNO_3 \rightarrow AgCl + KNO_3$

KCl: V=25ml; $AgNO_3$: V=20ml, M = 1M

At equivalence point,

mmole of KCl = mmole of $AgNO_3$

= 20 mmole

Volume of solution = 25 ml

Mass of solution = 25 gm

Mass of solvent

= 25 – mass of solute

= 25 – [20 × 10^{-3} × 74.5]

= 23.51 gm

$$\text{Molality of KCl} = \frac{\text{mole of KCl}}{\text{mass of solvent in kg}}$$

$$= \frac{20 \times 10^{-3}}{23.51 \times 10^{-3}} = 0.85$$

i of KCl = 2 (100% ionisation)

$\Delta T_f = i \times K_f \times m$

= 2 × 2 × 0.85

= 3.4

$\simeq 3$

MATHEMATICS

Section - A (Single Correct Answer)

61. B

Sol. $$\lim_{n\to\infty}\left(\frac{1}{1+n} + ... + \frac{1}{n+n}\right) = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n+r}$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\left(\frac{1}{1+\frac{r}{n}}\right)$$

$$= \int_0^1 \frac{1}{1+x}dx = [\ell n(1+x)]_0^1 = \ell n 2$$

62. A

Sol. $\sim(q \vee ((\sim q) \wedge p))$

$= \sim q \wedge \sim((\sim q) \wedge p)$

$= \sim q \wedge (q \vee \sim p)$

$= (\sim q \wedge q) \vee (\sim q \wedge \sim p)$

$= (\sim q \wedge \sim p)$

63. B

Sol. $np + npq = 5,\ np \cdot npq = 6$

$np(1 + q) = 5,\ n^2p^2q = 6$

$n^2p^2(1 + q)^2 = 25,\ n^2p^2q = 6$

$$\frac{6}{q}(1+q)^2 = 25$$

$6q^2 + 12q + 6 = 25q$

$6q^2 - 13q + 6 = 0$

$6q^2 - 9q - 4q + 6 = 0$

$(3q - 2)(2q - 3) = 0$

$$q = \frac{2}{3}, \frac{3}{2}, \ q = \frac{2}{3} \text{ is aceepted}$$

$$p = \frac{1}{3} \Rightarrow n\cdot\frac{1}{3} + n\cdot\frac{1}{3}\cdot\frac{2}{3} = 5$$

$$\frac{3n+2n}{9} = 5$$

$n = 9$

$$\text{So } 6(n+p-q) = 6\left(9 + \frac{1}{3} - \frac{2}{3}\right) = 52$$

64. B

Sol. $$T_r = \frac{(r^2+r+1)(r^2-r+1)}{2(r^4+r^2+1)}$$

$$\Rightarrow T_r = \frac{1}{2}\left[\frac{1}{r^2-r+1} - \frac{1}{r^2+r+1}\right]$$

$$T_1 = \frac{1}{2}\left[\frac{1}{1} - \frac{1}{3}\right]$$

$$T_2 = \frac{1}{2}\left[\frac{1}{3} - \frac{1}{7}\right]$$

$$T_3 = \frac{1}{2}\left[\frac{1}{7} - \frac{1}{13}\right]$$

$\vdots$

$$T_{10} = \frac{1}{2}\left[\frac{1}{91} - \frac{1}{111}\right]$$

$$\Rightarrow \sum_{r=1}^{10} T_r = \frac{1}{2}\left[1 - \frac{1}{111}\right] = \frac{55}{111}$$

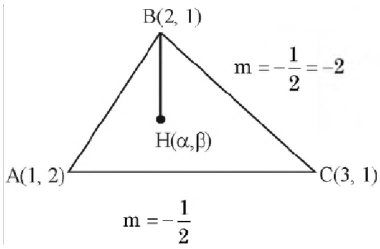
65. B

Sol. $$\sum_{r=1}^{26} \frac{1}{(2r-1)!(51-(2r-1))!} = \sum_{r=1}^{26} {}^{51}C_{(2r-1)} \frac{1}{51!}$$

$$= \frac{1}{51!}\{{}^{51}C_1 + {}^{51}C_3 + \dots + {}^{51}C_3\} = \frac{1}{51!}(2^{50})$$

66. D

Sol.



Here mBH × mAC = –1

$$\left(\frac{\beta-3}{\alpha-2}\right)\left(\frac{1}{-2}\right) = -1$$

$$\beta - 3 = 2\alpha - 4$$

$$\beta = 2\alpha - 1$$

$$m_{AH} \times m_{BC} = -1$$

$$\Rightarrow \left(\frac{\beta-2}{\alpha-1}\right)(-2) = -1$$

$$\Rightarrow 2\beta - 4 = \alpha - 1$$

$$\Rightarrow 2(2\alpha - 1) = \alpha + 3$$

$$\Rightarrow 3\alpha = 5$$

$$\alpha = \frac{5}{3}, \ \beta = \frac{7}{3} \Rightarrow H\left(\frac{5}{3}, \frac{7}{3}\right)$$

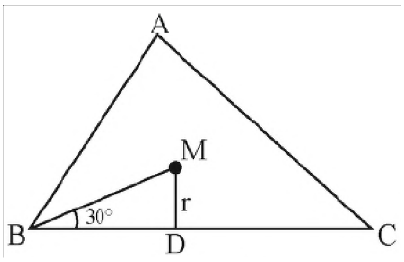
$$\alpha + 4\beta = \frac{5}{3} + \frac{28}{3} = \frac{33}{3} = 11$$

$$\beta + 4\alpha = \frac{7}{3} + \frac{20}{3} = \frac{27}{3} = 9$$

$x^2 - 20x + 99 = 0$

67. D

Sol.



If cos 2A + cos 2B + cos 2C is minimum then A = B = C = 60°

So ΔABC is equilateral

Now in-radias r = 3

So in ΔMBD we have

$$\tan 30^o = \frac{MD}{BD} = \frac{r}{a/2} = \frac{6}{a}$$

$$1/\sqrt{3} = \frac{1}{a} = a = 6\sqrt{3}$$

Perimeter of $\Delta ABC = 18\sqrt{3}$

Area of $\Delta ABC = \frac{\sqrt{3}}{4}a^2 = 27\sqrt{3}$

68. D

Sol. Equation of the pair of angle bisector for the homogenous equation $ax^2 + 2hxy + by^2 = 0$ is given as

$$\frac{x^2y^2}{a-b} = \frac{xy}{h}$$

Here a = 2, h = 1/2 & b = – 3

Equation will become

$$\frac{x^2-y^2}{2-(-3)} = \frac{xy}{1/2}$$

$x^2 - y^2 = 10xy$

$x^2 - y^2 - 10xy = 0$

69. C

Sol. Shortest distance between two lines

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{a_2} = \frac{z-z_1}{a_3} \ \&$$

$$\frac{x-x_2}{b_1} = \frac{y-y_2}{b_2} = \frac{z-z_2}{b_3} \text{ is given as}$$

$$\frac{\begin{vmatrix} x_1-x_2 & y_1y_2 & z_1-z_2 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}}{\sqrt{(a_1b_3-a_3b_2)^2+(a_1b_3-a_3b_1)^2+(a_1b_2-a_2b_1)^2}}$$

$$\frac{\begin{vmatrix} 5-(3) & 2-(-5) & 4-1 \\ 1 & 2 & -3 \\ 1 & 4 & -5 \end{vmatrix}}{\sqrt{(-10+12)^2+(-5+3)^2+(4-2)^2}}$$

$$\frac{\begin{vmatrix} 8 & 7 & 3 \\ 1 & 2 & -3 \\ 1 & 4 & -5 \end{vmatrix}}{\sqrt{(2)^2+(2)^2+(2)^2}}$$

$$= \frac{|18(-10)+12)-7(-5+3)+3(4-2)|}{\sqrt{4+4+4}}$$

$$= \frac{16+14+6}{\sqrt{12}} = \frac{36}{\sqrt{12}} = \frac{36}{2\sqrt{3}}$$

$$= \frac{18}{\sqrt{3}} = 6\sqrt{3}$$

70. D

Sol. $\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$

$$(\lambda+2)\begin{vmatrix} 1 & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$$(\lambda+2)[1(\lambda^2-1)1(\lambda-1)+(1-\lambda)] = 0$$

$$(\lambda+2)[(\lambda^2 2\lambda+1)] = 0$$

$$(\lambda+2)(\lambda-1)^2 = 0 \Rightarrow \lambda = -2, \ \lambda = 1$$

at $\lambda = 1$ system has infinite solution, for inconsistent $\lambda = -2$

so $\sum(|-2|^2 + |-2|) = 6$

71. B

Sol. Let $(\sqrt{3}+\sqrt{2})^{x^2-4} = t$

$$t + \frac{1}{t} = 10$$

$$\Rightarrow t = +5+2\sqrt{6}, \ 5-2\sqrt{6}$$

$$\Rightarrow (\sqrt{3}+\sqrt{2})^{x^2-4} = 5+2\sqrt{6}, \ 5-2\sqrt{6}$$

$$\Rightarrow x^2 4 = 2, \ -2 \text{ or } x^2 = 6, 2$$

$$\Rightarrow x = \pm\sqrt{2}, \ \pm\sqrt{6}$$

72. B

Sol. $\cos^{-1}(2x) - 2\cos^{-1}\sqrt{1-x^2} = \pi$

$\cos^{-1}(2x)\cos^{-1}(2(1-x^2)-1) = \pi$

$\cos^{-1}(2x)\cos^{-1}(1-2x^2) = \pi$

$-\cos^{-1}(1-2x^2) = \pi\cos^{-1}(2x)$

Taking cos both sides we get

$\cos(-\cos^{-1}(1-2x^2)) = \cos(\pi - \cos^{-1}(2x))$

$1 - 2x^2 = -2x$

$2x^2 - 2x - 1 = 0$

On solving, $x = \frac{1-\sqrt{3}}{2}, \frac{1+\sqrt{3}}{2}$

As x = [–1/2, 1/2], $x = \frac{1+\sqrt{3}}{2} =$ rejected

So $x = \frac{1-\sqrt{3}}{2} \Rightarrow x^2 - 1 = -\sqrt{3}/2$

$= 2\sin^{-1}(x^2-1) = 2\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \frac{-2\pi}{3}$

73. D

Sol. $\sqrt{(x-2)^2 + y^2} = 2\sqrt{(x-3)^2 + y^2}$

$= x^2 + y^2 - 4x + 4 = 4x^2 + 4y^2 - 24x + 36$

$= 3x^2 + 3y^2 - 20x + 32 = 0$

$= x^2 + y^2 - 4x + 4 = 4x^2 + 4y^2 24x + 36$

$= 3x^2 + 3y^2 - 20x + 32 = 0$

$= x^2 + y^2 \frac{20}{3}x + \frac{32}{3} = 0$

$= (\alpha, \beta) = \left(\frac{10}{3}, 0\right)$

$\gamma = \sqrt{\frac{100}{9} - \frac{32}{3}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$

$3(\alpha, \beta, \gamma) = 3\left(\frac{10}{3} + \frac{2}{3}\right)$

= 12

74. A

Sol. Here I.F. = sec x

Then solution of D.E :

y(sec x) = x tan x – ln(sec x) + c

Given y(0) = 1 $\Rightarrow$ c = 1

$\therefore$ y(sec x) = x tan x – ln(sec x) + 1

At $x = \frac{\pi}{6}$, $y = \frac{\pi}{12} + \frac{\sqrt{3}}{2}\ln\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$

75. A

Sol. Check for reflexivity :

As $3(a-a) + \sqrt{7} = \sqrt{7}$ which belongs to relation so relation is reflexive

Check for symmetric :

Take $a = \frac{\sqrt{7}}{3}$, $b = 0$

Now (a, b) $\in$ R but (b, a) $\notin$ R

As $3(b-a) + \sqrt{7} = 0$ which is rational so relation is not symmetric.

Check for Transitivity :

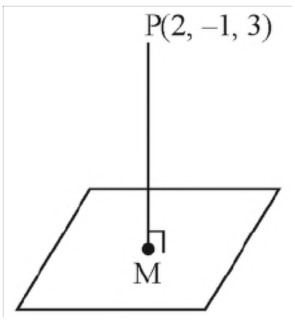
Take (a, b) as $\left(\frac{\sqrt{7}}{3}, 1\right)$

& (b, c) as $\left(1, \frac{2\sqrt{7}}{3}\right)$

So now (a, b) $\in$ R & (b, c) $\in$ R but (a, c) $\notin$ R which means relation is not transitive.

76. D

Sol.



eq. of line PM $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-3}{1} = \lambda$

any point on line $=(\lambda+2,\ 2\lambda-1,\ -\lambda+3)$

for point 'm' $(\lambda+2)+2(2\lambda-1)-(3-\lambda)=0$

$$\lambda=\frac{1}{2}$$

Point $m\left(\frac{1}{2}+2,\ 2\times\frac{1}{2}-1,\ \frac{-1}{2}+3\right)$

$=\left(\frac{5}{2},\ 0,\ \frac{5}{2}\right)$

For Image $Q(\alpha,\ \beta,\gamma)$

$\frac{\alpha+2}{2}=\frac{5}{2},\ \frac{\beta-1}{2}=0,$

$\frac{\gamma+3}{2}=\frac{5}{2}$

$Q:(3,1,2)$

$$d=\left|\frac{3(3)+2(1)+2+29}{\sqrt{3^2+2^2+1^2}}\right|$$

$$d=\frac{42}{\sqrt{14}}=3\sqrt{14}$$

77. A

Sol. $C_1\to C_1+C_2+C_3$

$$f(x)=\begin{vmatrix}2+\sin 2x & \cos^2 x & \sin 2x\\ 2+\sin 2x & 1+\cos^2 x & \sin 2x\\ 2+\sin 2x & \cos^2 x & 1+\sin 2x\end{vmatrix}$$

$$f(x)=(2+\sin 2x)\begin{vmatrix}1 & \cos^2 x & \sin 2x\\ 1 & 1+\cos^2 x & \sin 2x\\ 1 & \cos^2 R & 1+\sin 2x\end{vmatrix}$$

$$R_2\to R_2-R_1$$

$$R_3\to R_3-R_1$$

$$f(x)=2+\sin 2x\begin{vmatrix}1 & \cos^2 x & \sin 2x\\ 0 & 1 & 0\\ 0 & 0 & 1\end{vmatrix}$$

= (2 + sin 2x) (1) = 2 + sin 2x

$$=\sin 2x\in\left[\frac{\sqrt{3}}{2},\ 1\right]$$

Hence $2+\sin 2x\in\left[2+\frac{\sqrt{3}}{2},\ 3\right]$

78. B

Sol. $f(x)=2x+\tan^{-1}x$ and $g(x)=\ln\left(\sqrt{1+x^2}+x\right)$

and $x\in[0,\ 3]$

$$g'(x)=\frac{1}{\sqrt{1+x^2}}$$

Now, $0\le x\le 3$

$0\le x^2\le 9$

$1\le 1+x^2\le 10$

So, $2+\frac{1}{10}\le f'(x)\le 3$

$\frac{21}{10}\le f'(x)\le 3$ and $\frac{1}{\sqrt{10}}\le g'(x)\le 1$

option (D) is incorrect

From above, $g'(x)<f'(x)\forall x\in[0,\ 3]$

Option (A) is correct $f'(x)$ and $g'(x)$ both positive so f(x) and g(x) both are increasing

So, max (f(x) at x = 3 is $6+\tan^{-1}3$

Max (g(x) at x = 3 is $\ln(3+\sqrt{10})$

And $6+\tan^{-1}3>\ln(3+\sqrt{10})$

Option (B) is correct

79. A

Sol. a + b = 16 .....(1)

$$\sigma^2=\frac{\sum x_1^2}{5}-\left(\frac{\sum x}{5}\right)^2$$

$$8=\frac{1^2+3^2+5^2+a^2+b^2}{5}-25$$

$a^2+b^2=130$ ....(2)

by (1), (2)

a = 7, b = 9

or a = 9, b = 7

80. D

Sol. $\frac{dy}{dx}+\frac{x+a}{y-2}=0$

$\frac{dy}{dx}=\frac{x+a}{2-y}$

(2 – y) dy = (x + a) dx

$2y\frac{-y}{2}=\frac{x^2}{2}+ax+c$

$a+c=-\frac{1}{2}$ as y(1) = 0

$X^2 + y^2 + 2ax - 4y - 1 - 2a = 0$

$\pi r^2 = 4\pi$

$r^2 = 4$

$4=\sqrt{a^2+4+1+2\alpha}$

$(a + 1)^2 = 0$

$P,\ Q=(0,\ 2\pm\sqrt{3})$

Equation of normal at P, Q are $y-2=\sqrt{3}(x-1)$

$y-2=-\sqrt{3}(x-1)$

$R=\left(1-\frac{2}{\sqrt{3}},\ 0\right)$

$S=\left(1+\frac{2}{\sqrt{3}},\ 0\right)$

$RS=\frac{4}{\sqrt{3}}=4\frac{\sqrt{3}}{3}$

Section - B (Numerical Value)

81. 754

Sol. $a_1 + a_2 + a_3 + a_4 = 50$

$\Rightarrow$ 32 + 6d = 50

$\Rightarrow$ d = 3

and, $a_{n-3} + a_{n-2} + a_{n-1} + a_n = 170$

$\Rightarrow$ 32 + (4n – 10).3 = 170

$\Rightarrow$ n = 14

$a_7 = 26,\ a_8 = 29$

$\Rightarrow a_7 \cdot a_8 = 754$

82. 11

Sol. A(2, 6, 2) B(–4, 0, λ), C(2, 3, –1) D(4, 5, 0)

$\text{Area}=\frac{1}{2}\left|\overrightarrow{BD}\times\overrightarrow{AC}\right|=18$

$\overrightarrow{AC}\times\overrightarrow{BD}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & -3 & -3\\ 8 & 5 & -\lambda\end{vmatrix}$

$=(3\lambda+15)\hat{i}-\hat{j}(-24)+\hat{k}(-24)$

$\overrightarrow{AC}\times\overrightarrow{BD}=(3\lambda+15)\hat{i}+24\hat{j}-24\hat{k}$

$=\sqrt{(3\lambda+15)^2+(24)^2+(24)^2}=36$

$=\lambda^2+10\lambda+9=0$

$=\lambda=-1,\ -9$

$|\lambda|\le 5\Rightarrow\lambda=-1$

$5-6\lambda=5-6(-1)=11$

83. 514

Sol. Divisible by 2 $\to$ 450

Divisible by 3 $\to$ 300

Divisible by 7 $\to$ 128

Divisible by 2 & 7 $\to$ 64

Divisible by 3 & 7 $\to$ 43

Divisible by 2 & 3 $\to$ 150

Divisible by 2, 3 & 7 $\to$ 21

$\therefore$ Total numbers = 450 + 300 – 150 – 64 – 43 + 21 = 514

84. 29

Sol. $(21 + 2)^{200} + (21 - 2)^{200}$

$\Rightarrow 2[{}^{100}C_0 21^{200} + 200C_2\ 21^{198}\cdot 2^2 + \ldots + {}^{200}C_{198}\ 21^2.$

$2^{198} + 2^{200}]$

$\Rightarrow 2[49\ I_1 + 2^{200}] = 49I_1 + 2^{201}$

Now, $2^{201} = (8)^{67} = (1 + 7)^{67} = 49I_2 + {}^{67}C_0\ {}^{67}C_1\cdot 7 = 49I_2 + 470 = 49I_2 + 49 \times 9 + 29$

$\therefore$ Remainder is 29

85. 63

Sol. $\int (x^{20}+x^{13}+x^6)(2x^{21}+3x^{14}+6x^7)^{1/7}dx$

$2x^{21} + 3x^{14} + 6x^7 = t$

$42(x^{20} + x^{13} + x^6)\,dx = dt$

$$\frac{1}{42}\int_0^{11} t^{\frac{1}{7}}dt = \left(\frac{t^{\frac{8}{7}}}{\frac{8}{7}} \times \frac{1}{42}\right)_0^{11}$$

$$= \frac{1}{48}\left(t^{\frac{8}{7}}\right)_0^{11} = \frac{1}{48}(11)^{8/7}$$

$l = 48$, $m = 8$, $n = 7$

$l + m + n = 63$

86. 14

Sol. $f(x) = x^2 + g'(1)x + g''(2)$

$f'(x) = 2x + g'(1)$ $f''(x) = 2$

$g(x) = f(1)\,x^2 + x\,[2x + g'(1)] + 2$

$g'(x) = 2f(1)\,x + 4x + g'(1)$

$g''(x) = 2f(1) + 4$

$g''(x) = 0$

$2f(1) + 4 = 0$

$f(1) = -2$

$-2 = 1 + g'(1) = g'(1) = -3$

So, $f'(x) = 2x - 3$

$f(x) = x^2 - 3x + c$

$c = 0$

$f(x) = x^2 - 3x$

$g(x) = -3x + 2$

$f(4) - g(4) = 14$

87. 3501

Sol. $[\vec{u}\ \vec{v}\ \vec{w}] = u \cdot (\vec{v} \times \vec{w})$

$\min.(|u||\vec{v} \times \vec{w}|\cos\theta) = -\alpha\sqrt{3401}$

$\Rightarrow \cos\theta = -1$

$|u| = \alpha$ (Given)

$|\vec{v} \times \vec{w}| = \sqrt{3401}$

$$\vec{v} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 2 & -3 \\ 2\alpha & 1 & -1 \end{vmatrix}$$

$\vec{v} \times \vec{w} = \hat{i} - 5\alpha\hat{j} - 3\alpha\hat{k}$

$|\vec{v} \times \vec{w}| = \sqrt{1 + 25\alpha^2 + 9\alpha^2} = \sqrt{3401}$

$34\alpha^2 = 3400$

$\alpha^2 = 100$

$\alpha = 10$ (as $\alpha > 0$)

So $\vec{u} = \lambda(\hat{i}5\alpha\hat{j} - 3\alpha\hat{k})$

$\vec{u} = \sqrt{\lambda^2 + 25\alpha^2\lambda^2 + 9\alpha^2\lambda}$

$\alpha^2 = \lambda^2(1 + 25\alpha^2 + 9\alpha^2)$

$100 = \lambda^2(1 + 34 \times 100)$

$\lambda^2 = \frac{100}{3401} = \frac{m}{n}$

88. 50400

Sol. Vowels : A, A, A, I, I, O

Consonants : S, S, S, S, N, N, T

$\therefore$ Total number of ways in which vowels come together

$$= \frac{\lfloor 8}{\lfloor 4 \lfloor 2} \times \frac{\lfloor 6}{\lfloor 3 \lfloor 2} = 50400$$

89. 62

Sol. $A = \int_{-1}^{0}(x^2 - 3x)dx + \int_0^2 (3x - x^2)dx$

$$\Rightarrow A = \left.\frac{x^3}{3} - \frac{3x^2}{2}\right|_{-1}^{0} + \left.\frac{3x^2}{2} - \frac{x^3}{3}\right|_0^2$$

$$\Rightarrow A = \frac{11}{6} + \frac{10}{3} = \frac{31}{6}$$

$\therefore 12A = 62$

90. 1

Sol. $\frac{dy}{dx} + y = k$

$y \cdot e^x = k \cdot e^x + c$

$f(0) = e^{-2}$

$\Rightarrow c = e^{-2} - k$

$\therefore y = k + (e^{-2} - k)e^{-x}$

now $k = \int_0^2 (k + (e^{-2} - k)e^{-x})dx$

$\Rightarrow$ k = $e^{-2} - 1$

$\therefore$ y = $(e^{-2} - 1) + e^{-x}$

f(2) = $2e^{-2} - 1$, f(0) = e^{-2}

2f(0) – f(2) = 1

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