3S INSTITUTE
JEE ADVANCED | JEE MAIN | NEET | OLYMPIADS | MHT-CET | FOUNDATION

31-January-2023 (Evening Batch) : JEE Main Paper

PHYSICS

Section - A (Single Correct Answer)

1. B
2. D
3. A
4. A
5. D
6. C
7. A
8. D
9. D
10. D
11. B
12. B
13. A
14. C
15. B
16. B
17. A
18. D
19. A
20. C

Section - B (Numerical Value)

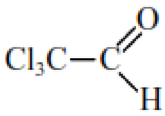
21. 25
22. 48
23. 300
24. 5
25. 5
26. 136
27. 1
28. 55
29. 20
30. 80

CHEMISTRY

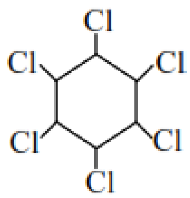
Section - A (Single Correct Answer)

31. B

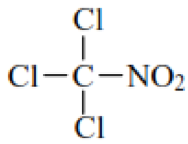
Sol. (1) Chloral



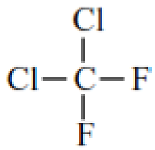
(2) Gammaxene



(3) Chloropicrin



(4) Freon - 12



32. A

Sol.

Indicator	pH range
Methyl orange	3.2 – 4.5
Phenolpthalein	8.3 – 10.5

Methyl orange may be used for a strong acid *vs* strong base and strong acid *vs* weak base titration.

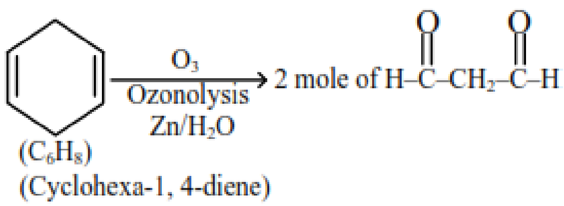
Phenolpthalein may be used for a strong acid *vs* strong base and weak acid *vs* strong base titration.

33. D

Sol. Veronal is neurological medicine, Prontosil is antibiotic, rest all are disinfectants.

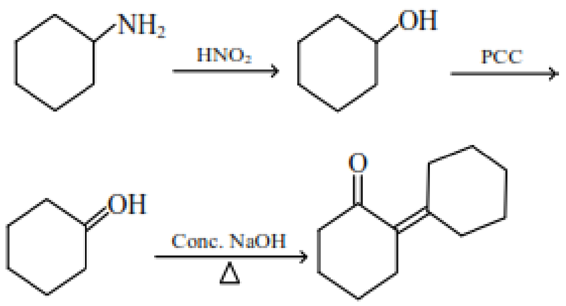
34. D

Sol.



35. B

Sol.



36. C

Sol. (Borax Bead Test)

On treatment with metal salt, boric anhydride forms metaborate of the metal which gives different colours in oxidising and reducing flame.

For example, in the case of copper sulphate, following reactions occur.

$$CuSO_4 + B_2O_3 \xrightarrow[\text{(Oxidising)}]{\text{Non-luminous flame}} \underset{\substack{\text{Cupric metaborate} \\ \text{blue-green}}}{Cu(BO_2)_2} + SO_3$$

Two reactions may take place in reducing flame (Luminous flame).

(i) The blue-green $Cu(BO_2)_2$ is reduced to colourless cuprous metaborate as :

$$2Cu(BO_2)_2 + 2NaBO_2 + C \xrightarrow[\text{flame}]{\text{Luminous}} 2CuBO_2 + Na_2B_4O_7 + CO$$

(ii) Cupric metaborate may be reduced to metallic copper and bead appears red opaque.

$$2Cu(BO_2)_2 + 4NaBO_2 + 2C \xrightarrow[\text{flame}]{\text{Luminous}} 2Cu + 2Na_2B_4O_7 + 2CO$$

37. D

Sol. (A) $\Delta T_b \propto i \times c$

(B) Azeotropic mixtures have same composition in both liquid and vapour phase.

(C) Osmosis always takes place from hypotonic to hypertonic solution.

(D) $M = \dfrac{30 \times 10 \times 1.26}{98} \approx 4.09\ M$

(E) When KI solutions is added to $AgNO_3$ solution, positively charged solution results due to adsorption of Ag^+ ions from dispersion medium.

AgI / Ag^+

Positively charged

38. A

Sol. (i) Formation of tetraacetete with Ac_2O means compound A has four –OH linkage.

Reduction of A with HI gives Isopentane i.e., molecule contains five carbon atom.

39. B

Sol. (A) $n = 3$; $\ell = 0$; $m = 0$; 3s orbital

(B) $n = 4$; $\ell = 0$; $m = 0$; 4s orbital

(C) $n = 3$; $\ell = 1$; $m = 0$; 3p orbital

(D) $n = 3$; $\ell = 2$; $m = 0$; 3d orbital

As per Hund's rule energy is given by $(n + \ell)$ value.

If value of $(n + \ell)$ remains same then energy is given by n only.

40. D

Sol. Extent of back bonding, reduces down the group leading to more Lewis acidic strength.

$BF_3 > BCl_3 > BBr_3 > BI_3$ (extent of back bonding)
(2p – 2p) (2p – 3p) (2p – 4p) (2p – 5p)

$BF_3 < BCl_3 < BBr_3 < BI_3$ (lewis acidic nature)

41. D

Sol. (A) Physisorption = 20 – 40 kJ/mol and Chemisorption = 80 – 240 kJ/mol.

(B) Physisorption is multi-layered and chemisorption is unimolecular layered.

(C) In heterogeneous catalysis, medium and catalyst are in different phases.

(D) Chromatography uses adsorption to purify/ separate mixtures.

42. A

Sol. From Sc to Mn ionization energy is less than that of Mg.

For 3d series :

	Sc	Ti	V	Cr	Mn
IE (KJ/mol)	631	656	650	653	717
	Fe	Co	Ni	Cu	Zn
IE (KJ/mol)	762	758	736	745	906

For 2nd Group :

	Be	Mg	Ca	Sr	Ba	Ra
IE (KJ/mol)	631	656	650	653	717	762

43. B

Sol. Calcium plays important role in neuromuscular function, interneuronal transmission, cell membrane etc.

44. A

Sol. It is used in the synthesis of hydroquinone, tartaric acid and certain food products and pharma-ceuticals (cephalosporin) etc. Restoration of aerobic conditions to sewage wastes etc.

45. A

Sol. We are aware that normally rain water has a pH of 5.6 due to the presence of H^+ ions formed by the reactions of rain water with carbon dioxide present in the atmosphere.

$$H_2O(\ell)+CO_2(g) \rightleftharpoons H_2CO_3(aq)$$

$$H_2CO_3(aq) \rightleftharpoons H^+(aq)+HCO_3^-(aq)$$

46. A

Sol. $C_xH_y+\left(x+\frac{y}{4}\right)O_2 \rightarrow xCO_2+\frac{y}{2}H_2O$

$\frac{y}{2}=4 \qquad \therefore \quad y=8$

$x+\frac{8}{4}=11$

$\therefore \quad x=9$

$\therefore$ Hydrocarbon will be = C_9H_8

47. D

Sol. $C_4H_{11}N$ releases N_2 with HNO_2 i.e., it is primary amine.

After reacting with Hinsberg reagent it forms a compound which is soluble in KOH.

Hence, the amine is primary.

48. B

Sol. Van – Arkel process is used for purification of Ti, Zr, Hf and B.

49. A

Sol. 1. $_{62}Sm : 4f^{6}\ 6s^2$

2. $_{64}Gd : 4f^{7}\ 5d^1\ 6s^2$

3. $_{63}Eu : 4f^{7}\ 6s^2$

4. $_{65}Tb : 4f^{9}\ 6s^2$

5. $_{61}Pm : 4f^{5}\ 6s^2$

50. B

Sol. Duma's method.

The nitrogen containing organic compound, when heated with CuO in a atmosphere of CO_2, yields free N_2 in addition to CO_2 and H_2O.

$$C_xH_yN_z+\left(2x+\frac{y}{2}\right)CuO \rightarrow$$

$$xCO_2+\frac{y}{2}H_2O+\frac{z}{2}N_2+\left(2x+\frac{y}{2}\right)Cu$$

Traces of nitrogen oxides formed, if any, are reduced to nitrogen by passing the gaseous mixture over heated copper gauze.

Section - B (Numerical Value)

51. 480

Sol. $\left(Ti^{+3}(H_2O)_6\right)^{3+}$

$Ti^{+3} : 3d'$

C.F.S.E. $= -0.4\times\Delta_0$

$= -\frac{96\times10^3}{N_0}J$

$\Delta_0=\frac{96\times10^3}{0.4\times6\times10^{23}}$

$\Rightarrow \quad \frac{hc}{\lambda}=\frac{96\times10^3}{0.4\times6\times10^{23}}$

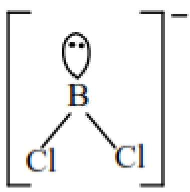
$\lambda=\frac{0.4\times6\times10^{23}\times6.4\times10^{-34}\times3\times10^8}{96\times10^3}$

$=0.48\times10^{-6}m$

$=480\times10^{-9}m$

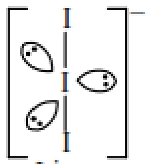
$=480$ nm

52. 5

Sol. (i) 

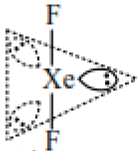
(ii) Cl—Be—Cl

(iii) I_3^-



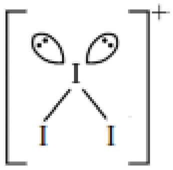
Linear

(iv) XeF_2



Linear

(v)



V - Shape

(vi) CO_2 O = C = O

Linear

(vii C_3O_2 (O=C=C=C=O)

Linear

(viii)

V-

53. 25

Sol. $\Lambda_m = \dfrac{\kappa \times 1000}{M}$

$\Lambda_m = \dfrac{1}{\rho} \times \dfrac{1000}{M}$

$\dfrac{1}{5\times10^{-3}} \times \dfrac{1000}{0.8}$

Ans. $25\times10^4\ \Omega^{-1}cm^{-2}mol^{-1}$

54. 10

Sol. $AgCl(s) \rightarrow \underset{S}{Ag^+}(aq) + \underset{S}{Cl^-}(aq)$

$K_{sp} = S^2 = \left(\dfrac{1.43}{143.4}\times10^{-3}\right)^2 = 10^{-10}$

$-\log K_{sp} = 10$

55. 59

Sol. $M \begin{cases} \rightarrow +2 \longrightarrow x \\ \rightarrow +3 \longrightarrow (0.83 - x) \end{cases}$

$2x + 3(0.83 - x) = 2$

$x = 0.49$

$\%M^{2+} = \dfrac{0.49}{0.83}\times100$

$= 59\%$

56. 227

Sol. $\underset{1\ mol}{2C(s)} + \underset{1.5\ mol}{O_2(g)} \longrightarrow 2CO(g)$

Limiting reagent is carbon. One mole carbon produces one mole CO.

Hence, volume at STP is 227 × 10^{-1} litre.

57. 2

Sol. K, Rb and Cs form stable super oxides but Cs has ionisation enthalpy less than 400 kJ.

58. 173

Sol. $\Delta_r H = \Sigma H_P - \Sigma H_R$

$= (-394 + 4\times-92) - (-105 + (2\times-242))$

$= -173$ kJ/mol

59. 3

Molecules having

Sol. $-\overset{\overset{\large O}{||}}{C} - CH_3$ and $-\overset{\overset{\large OH}{|}}{CH} - CH_3$

gives positive haloform test.

60. 17

Sol. $C = \dfrac{C_0}{2^n} = \dfrac{C_0}{32}$

$n = 5$

$t = 5t_{1/2}$

$= \dfrac{5\times0.693}{20} = \dfrac{0.693}{4}$

$= 0.17325$ min

$= 17.325\times10^{-2}$ min.

MATHEMATICS

Section - A (Single Correct Answer)

61. C

Sol. $\phi'(x)=\frac{1}{\sqrt{x}}[(4\sqrt{2}\sin x-3\phi'(x))\cdot 1-0]-\frac{1}{2}x^{-3/2}$

$\int_{\frac{\pi}{4}}^{x}(4\sqrt{2}\sin t-3\phi'(t))dt,$

$\phi'\left(\frac{\pi}{4}\right)=\frac{2}{\sqrt{\pi}}\left[4-3\phi'\left(\frac{\pi}{4}\right)\right]+0$

$\left(1+\frac{6}{\sqrt{\pi}}\right)\phi'\left(\frac{\pi}{4}\right)=\frac{8}{\sqrt{\pi}}$

$\phi'\left(\frac{\pi}{4}\right)=\frac{8}{\sqrt{\pi}+6}$

62. D

Sol. $2\alpha + 4\beta + 3\gamma = 5$ ....(1)

$2\alpha + 9\beta + 8\gamma = 0$ ....(2)

$10\alpha + 3\beta + 4\gamma = 0$ ....(3)

$8\alpha + 8\beta + 8\gamma = 0$ ....(4)

Subtract (4) from (2)

$-6\alpha + \beta = 0$

$\beta = 6\alpha$ ....(5)

From equation (4)

$8\alpha + 48\alpha + 8\gamma = 0$

$\gamma = -7\alpha$ ....(6)

From equation (1)

$2\alpha + 24\alpha - 21\alpha = 5$

$5\alpha = 5$

$\alpha = 1$

$\alpha = +6, \gamma = -7$

$\therefore\ 6\alpha + 9\beta + 7\gamma$

$= 6 + 54 - 49$

$= 11$

63.A

Sol. $a + 6d = 3,$ .....(1)

$Z = a(a + 3d)$

$= (3 - 6d)(3 - 3d)$

$= 18d^2 - 27d + 9$

Differentiating with respect to d

$\Rightarrow 36d - 27 = 0$

$\Rightarrow d=\frac{3}{4},$ from (1) $a=\frac{-3}{2},$ (Z = minimum)

Now, $S_n=\frac{n}{2}\left(-3+(n-1)\frac{3}{4}\right)=0$

$\Rightarrow n = 5$

Now,

$n! - 4a_{n(n+2)} = 120 - 4(a_{35})$

$= 120 - 4(a + (35 - 1)d)$

$=120-4\left(\frac{-3}{2}+34\cdot\left(\frac{3}{4}\right)\right)$

$=120-4\left(\frac{-6+102}{4}\right)$

$= 120 - 96 = 24$

64. D

Sol. $\sin^{-1}\sin\theta-\left(\frac{\pi}{2}-\sin^{-1}\sin\theta\right)>0$

$\Rightarrow \sin^{-1}\sin\theta>\frac{\pi}{4}$

$\Rightarrow \sin\theta>\frac{1}{\sqrt{2}}$

So, $\theta\in\left(\frac{\pi}{4},\frac{3\pi}{4}\right)$

$\theta\in\left(\frac{\pi}{4},\frac{3\pi}{4}\right)=(a,\ b)$

$b-a=\frac{\pi}{2}=\alpha-\beta$

$\Rightarrow \beta=\alpha-\frac{\pi}{2}$

$\Rightarrow ax^2+\beta x+\sin^{-1}[(x-3)^2+1]+\cos^{-1}[(x-3)^2+1]=0$

$x=3,\ 9\alpha+3\beta+\frac{\pi}{2}+0=0$

$\Rightarrow 9\alpha+3\left(\alpha-\frac{\pi}{2}\right)+\frac{\pi}{2}=0$

$\Rightarrow 12\alpha - \pi = 0$

$\alpha = \frac{\pi}{12}$

65. A

Sol. $(3y^2 - 5x^2)\, y\cdot dx + 2x(x^2 - y^2)\, dy = 0$

$\Rightarrow \frac{dy}{dx} = \frac{y(5x^2 - 3y^2)}{2x(x^2 - y^2)}$

Put y = mx

$\Rightarrow m + x\cdot\frac{dm}{dx} = \frac{m(5-3m^2)}{2(1-m^2)}$

$x\cdot\frac{dm}{dx} = \frac{(5-3m^2)m - 2m(1-m^2)}{2(1-m^2)}$

$\Rightarrow \frac{dx}{x} = \frac{2(m^2-1)}{m(m^2-3)}dm$

$\Rightarrow \frac{dx}{x} = \left(\frac{2}{m} - \frac{\frac{4}{3}}{m} + \frac{\frac{4m}{3}}{m^2-3}\right)dm$

$\Rightarrow \int\frac{dx}{x} = \int\frac{\left(\frac{2}{3}\right)}{m} + \int\frac{2}{3}\left(\frac{2m}{m^2-3}\right)dm$

$\Rightarrow \ln|x| = \frac{2}{3}\ln|m| + \frac{2}{3}\ln|m^2-3| + C$

Or, $\ln|x| = \frac{2}{3}\ln\left|\frac{y}{x}\right| + \frac{2}{3}\ln\left|\left(\frac{y}{x}\right)^2 - 3\right| + C$

Put (x = 1, y = 1) : we get $c = -\frac{2}{3}\ln(2)$

$\Rightarrow \ln|x| = \frac{2}{3}\ln\left|\frac{y}{x}\right| + \frac{2}{3}\ln\left|\left(\frac{y}{x}\right)^2 - 3\right| - \frac{2}{3}\ln(2)$

$\Rightarrow \left(\frac{y}{x}\right)\left[\left(\frac{y}{x}\right)^2 - 3\right] = 2\cdot(x^{3/2})$

Put x = 2 to get y(2)

$\Rightarrow y(y^2 - 12) = 4\times2\times2\times2\sqrt{2}$

$\Rightarrow y^3 - 12y = 32\sqrt{2}$

$\Rightarrow |y^3(2) - 12y(2)| = 32\sqrt{2}$

66. A

Sol. $x^2 + y^2 - \frac{(1+a)x}{2} - \frac{(1-a)y}{2} = 0$

Centre $\left(\frac{1+a}{4}, \frac{1-a}{4}\right) \Rightarrow (h, k)$

$P\left(\frac{1+a}{2}, \frac{1-a}{2}\right) \Rightarrow (2h, 2k)$

Equation of chord $\Rightarrow T = S_1$

$\Rightarrow (x-y)\lambda - \frac{2h(x+\lambda)}{2} - \frac{(2k)(y-\lambda)}{2}$

$= 2\lambda^2 - 2h(\lambda) + 2k\lambda$

Now, $\lambda(2h, 2k)$ satisfies the chord

$\therefore (2h2k)\lambda - h(x+\lambda) - k(y-\lambda)$

$\Rightarrow 2\lambda^2 + 4k\lambda - 4h\lambda + h\lambda - k\lambda + hx + ky = 0$

$\Rightarrow 2\lambda^2 + \lambda(3k - 3h) + ky + hx = 0$

$\Rightarrow D > 0$

$\Rightarrow 9(k-h)^2 - 8(ky + hx) > 0$

$\Rightarrow 9(k-h)^2 - 8(2k^2 + 2h^2) > 0$

$\Rightarrow -7k^2 - 7h^2 - 18kh > 0$

$\Rightarrow 7k^2 + 7h^2 + 18kh < 0$

$\Rightarrow 7\left(\frac{1-a}{4}\right)^2 + 7\left(\frac{1+a}{4}\right)^2 + 18\left(\frac{1-a^2}{16}\right) < 0$

$\Rightarrow 7\left[\frac{2(1+a^2)}{16}\right] + \frac{18(1-a^2)}{16} < 0,\ a^2 = t$

$\Rightarrow \frac{7}{8}(1+t) + \frac{18(1-t)}{16} < 0$

$\Rightarrow \frac{14 + 14t + 18 - 18t}{16} < 0$

$\Rightarrow 4t > 32$

$t > 8$ $a^2 > 8$

67. B

Sol. For relation T = $a^2 - b^2 = -I$

Then, (b, a) on relation R

$\Rightarrow b^2 - a^2 = -I$

$\therefore$ T is symmetric

$$S = \left\{(a,\ b)\ :\ a,\ b \in R - \{0\},\ 2 + \frac{a}{b} > 0\right\}$$

If (b, a) $\in S$ then

$2 + \frac{b}{a}$ not necessarily positive

$\therefore$ S is not symmetric

68. A

Sol. $e^{4x} + 8e^{3x} + 13e^{2x} - 8e^x + 1 = 0$

Let $e^x = t$

Now, $t^4 + 8t^3 + 13t^2 - 8t + 1 = 0$

Dividing equation by t^2,

$$t^2 + 8t + 13 - \frac{8}{t} + \frac{1}{t^2} = 0$$

$$t^2 + \frac{1}{t^2} + 8\left(t - \frac{1}{t}\right) + 13 = 0$$

$$\left(t - \frac{1}{t}\right)^2 + 2 + 8\left(t - \frac{1}{t}\right) + 13 = 0$$

Let $t - \frac{1}{t} = z$

$z^2 + 8z + 15 = 0$

$(z + 3)(z + 5) = 0$

$z = -3$ or $z = -5$

So, $t - \frac{1}{t} = -3$ or $t - \frac{1}{t} = -5$

$t^2 + 3t - 1 = 0$ or $t^2 + 5t - 1 = 0$

$$t = \frac{-3 \pm \sqrt{13}}{2} \text{ or } t = \frac{-5 \pm \sqrt{29}}{2}$$

at $t = e^x$ so t must be positive,

$$t = \frac{\sqrt{13} - 3}{2} \text{ or } \frac{\sqrt{29} - 5}{2}$$

So, $x = \ln\left(\frac{\sqrt{13} - 3}{2}\right)$ or $x = \ln\left(\frac{\sqrt{29} - 5}{2}\right)$

Hence two solution and both are negative.

69. B

Sol. $((p \wedge q) \Rightarrow (r \vee q)) \wedge ((p \wedge r) \Rightarrow q)$

We know, $p \Rightarrow q$ is equivalent to

$\sim p \vee q$

$(\sim (p \wedge q) \vee (r \vee q)) \wedge (\sim (p \wedge r)) \vee q))$

$\Rightarrow (\sim p \vee \sim q \vee r \vee q) \wedge (\sim p \vee \sim r \vee q)$

$\Rightarrow (\sim p \vee r \vee t) \wedge (\sim p \vee \sim r \vee q)$

$\Rightarrow (t) \wedge (\sim p \vee \sim r \vee q)$

For this to be tautology, $(\sim p \vee \sim r \vee q)$ must be always true which follows for r = ~p or r = q.

70. A

Sol. Let $y = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$

By cross multiplying

$yx^2 - 8xy + 12y - x^2 - 2x - 1 = 0$

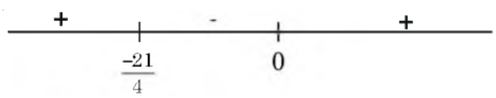
$x^2(y - 1) - x(8y + 2) + (12y - 1) = 0$

Case 1, $y \neq 1$

$D \geq 0$

$\Rightarrow (8y + 2)^2 4(y - 1)(12y - 1) \geq 0$

$\Rightarrow y(4y + 21) \geq 0$



$$y \in \left(-\infty,\ \frac{-21}{4}\right] \cup [0,\ \infty) - \{1\}$$

Case 2, y = 1

$x^2 + 2x + 1 = x^2 - 8x + 12$

$10x = 11$

$x = \frac{11}{10}$ So, y can be 1

Hence $y \in \left(-\infty,\ \frac{-21}{4}\right] \cup [0,\ \infty)$

71. B

Sol. $$\lim_{x\to\infty}\frac{\left(\sqrt{3x+1}+\sqrt{3x-1}\right)^6+\left(\sqrt{3x+1}-\sqrt{3x-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}x^3$$

$$\lim_{x\to\infty}x^3\times\left\{\frac{x^3\left\{\left(\sqrt{3+\frac{1}{x}}+\sqrt{3-\frac{1}{x}}\right)^x+\left(\sqrt{3+\frac{1}{x}}-\sqrt{3-\frac{1}{x}}\right)^6\right\}}{x^6\left\{\left(1+\sqrt{1-\frac{1}{x^2}}\right)^6+\left(1-\sqrt{1-\frac{1}{x^2}}\right)^3\right\}}\right\}$$

$$=\frac{(2\sqrt{3})^6+0}{2^6+0}=3^3=(27)$$

72. C

Sol. Equation of Plane :

$2(x-1)-3(y+1)-6(z+5)=0$

Or $2x-3y-6z=35$

$\Rightarrow$ Required distance =

$$\frac{|2(3)-3(-2)-6(2)-35|}{\sqrt{4+9+36}}$$

$=5$

73. A

Sol. $f(x)=|x^2-x+1|+[x^2-x+1]$; $x\in[-1,2]$

Let $g(x)=x^2-x+1$

$$=\left(x-\frac{1}{2}\right)^2+\frac{3}{4}$$

$\because$ x^2-x+1 and $[x^2-x+2]$

Both have minimum value at $x=1/2$

$\Rightarrow$ Minimum $f(x)=\frac{3}{4}+0$

$$=\frac{3}{4}$$

74. A

Sol. $P: 8x+\alpha_1 y+\alpha_2 z+12=0$

$$L:\frac{x+2}{2}=\frac{y-3}{3}=\frac{z+4}{5}$$

$\because$ P is parallel to L

$\Rightarrow 8(2)+\alpha_1(3)+5(\alpha_2)=0$

$\Rightarrow 3\alpha_1+5(\alpha_2)=-16$

Also y-intercept of plane P is 1

$\Rightarrow \alpha_1=-12$

And $\alpha_2=4$

$\Rightarrow$ Equation of plane P is $2x-3y+z+3=0$

$\Rightarrow$ Distance of line L from Plane P is

$$=\left|\frac{0-3(6)+1+3}{\sqrt{4+9+1}}\right|$$

$=\sqrt{14}$

75. C

Sol. Equation of Plane :

$$(2\hat{i}+a\hat{j}+4\hat{k})\cdot[(x-2)\hat{i}+(y-a)\hat{j}+(z-4)\hat{k}]=0$$

$\Rightarrow 2x+ay+4z=20+a^2$

$$\Rightarrow A\equiv\left(\frac{20+a^2}{2},\ 0,\ 0\right)$$

$$B\equiv\left(0,\ \frac{20+a^2}{a},\ 0\right)$$

$$C\equiv\left(0,\ 0,\ \frac{20+a^2}{4}\right)$$

$\Rightarrow$ Volume of tetrahedron

$$=\frac{1}{6}[\vec{a}\ \vec{b}\ \vec{c}]$$

$$\Rightarrow\frac{1}{6}\left(\frac{20+a^2}{2}\right)\cdot\left(\frac{20+a^2}{a}\right)\cdot\left(\frac{20+a^2}{4}\right)=144$$

$\Rightarrow (20+a^2)^3=144\times48\times a$

$\Rightarrow a=2$

$\Rightarrow$ Equation of plane is $2x+2y+4z=24$

Or $x+y+2z=12$

$\Rightarrow$ (3, 0, 4) Not lies on the Plane $x+y+2z=12$

76. A

Sol.

A	B	A + B
$\bar{x}_1=40$	$\bar{x}_2=55$	$\bar{x}=50$
$\sigma_1=\alpha$	$\sigma_2=30-\alpha$	$\sigma^2=350$
$n_1=100$	$n_2=n$	$100+n$

$$\bar{x} = \frac{100 \times 40 + 55n}{100 + n}$$

$5000 + 50n = 4000 + 55n$

$1000 = 5n$

$n = 200$

$$\sigma_1^2 = \frac{\sum x_i^2}{100} - 40^2$$

$$\sigma_2^2 = \frac{\sum x_j^2}{100} - 55^2$$

$$350 = \sigma^2 = \frac{\sum x_i^2 + \sum x_j^2}{300} - (\bar{x})^2$$

$$350 = \frac{(1600\alpha^2) \times 100 + [(30-\alpha)^2 + 3025] \times 200}{300} - (50)^2$$

$$2850 \times 3 = \alpha^2 + 2(30-\alpha)^2 + 1600 + 6050$$

$$8550 = \alpha^2 + 2(30-\alpha)^2 + 7650$$

$$\alpha^2 + 2(30-\alpha)^2 = 900$$

$$\alpha^2 - 40\alpha + 300 = 0$$

$$\alpha = 10,\ 30$$

$$\sigma_1^2 + \sigma_2^2 = 10^2 + 20^2 = 500$$

77. C

Sol. $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$

$\vec{c} = 5\hat{i} - 3\hat{j} + 3\hat{k}$

$(\vec{r} - \vec{c}) \times \vec{b} = 0,\ \vec{r} \cdot \vec{a} = 0$

$\Rightarrow \vec{r} - \vec{c} = \lambda\vec{b}$

Also, $(\vec{c} + \lambda\vec{b}) \cdot \vec{a} = 0$

$\Rightarrow \vec{a} \cdot \vec{c} + \lambda(\vec{a} \cdot \vec{b}) = 0$

$$\therefore \lambda = \frac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}} = \frac{-8}{5}$$

$$\vec{r} = \frac{5(5\hat{i} - 3\hat{i} + 3\hat{k}) - 8(\hat{i} - \hat{j} + 2\hat{k})}{5}$$

$$\vec{r} = \frac{17\hat{i} - 7\hat{j} + \hat{k}}{5}$$

$$|\vec{r}|^2 = \frac{1}{25}(289 + 50)$$

$$25|\vec{r}|^2 = 339$$

78. A

Sol. $2ae = \left|(1+\sqrt{2}) - (1+\sqrt{2})\right| = 2\sqrt{2}$

$ae = \sqrt{2}$

$a = 1$

$\Rightarrow b = 1\ \because e = \sqrt{2} \Rightarrow$ Hyperbola is rectangular

$$\Rightarrow \text{L.R.} = \frac{2b^2}{a} = 2$$

79. A

Sol. After rationalising

$$\int_0^\alpha \frac{x}{\alpha}\left(\sqrt{x+\alpha} + \sqrt{x}\right)$$

$$\int_0^\alpha \frac{1}{\alpha}[(x+\alpha)^{3/2} - \alpha(x+\alpha)^{1/2} + x^{3/2}]$$

$$\frac{1}{\alpha}\left[\frac{2}{5}(x+\alpha)^{5/2}\alpha\frac{2}{3}(x+\alpha)^{3/2} + \frac{2}{5}x^{5/2}\right]\Bigg|_0^\alpha$$

$$= \frac{1}{\alpha}\left(\frac{5}{2}(2\alpha)^{5/2} - \frac{2\alpha}{3}(2\alpha)^{3/2} + \frac{2}{5}\alpha^{5/2} - \frac{2}{5}\alpha^{5/2} + \frac{2}{3}\alpha^{5/2}\right)$$

$$= \frac{1}{\alpha}\left(\frac{2^{7/2}\alpha^{5/2}}{5}\frac{2^{5/2}\alpha^{5/2}}{3} + \frac{2}{3}\alpha^{5/2}\right)$$

$$= \alpha^{3/2}\left(\frac{2^{7/2}}{5} - \frac{2^{5/2}}{3} + \frac{2}{3}\right)$$

$$= \frac{\alpha^{3/2}}{15}\left(24\sqrt{2} - 20\sqrt{2} + 10\right) = \frac{\alpha^{3/2}}{15}\left(4\sqrt{2} + 10\right)$$

Now,

$$\frac{\alpha^{3/2}}{15}\left(4\sqrt{2} + 10\right) = \frac{16 + 20\sqrt{2}}{15}$$

$\Rightarrow \alpha = 2$

80. A

Sol. $Z=\dfrac{i-1}{\cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3}}=\dfrac{i-1}{\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i}$

$=\dfrac{i-1}{\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i}\times\dfrac{\dfrac{1}{2}-\sqrt{\dfrac{3}{2}}i}{\dfrac{1}{2}-\sqrt{3/2}i}=\dfrac{\sqrt{3}-1}{2}+\dfrac{\sqrt{3}+1}{2}i$

Applying polar form,

$r\cos\theta=\dfrac{\sqrt{3}-1}{2}$

Now, $\tan\theta=\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

So, $\theta=\dfrac{5\pi}{12}$

Section - B (Numerical Value)

81. 5040

Sol. $\left(\dfrac{4x}{5}+\dfrac{5}{2x^2}\right)^9$,

Now, $T_{r+1}={}^9C_r\cdot\left(\dfrac{4x}{5}\right)^{9-r}\left(\dfrac{5}{2x^2}\right)^r$

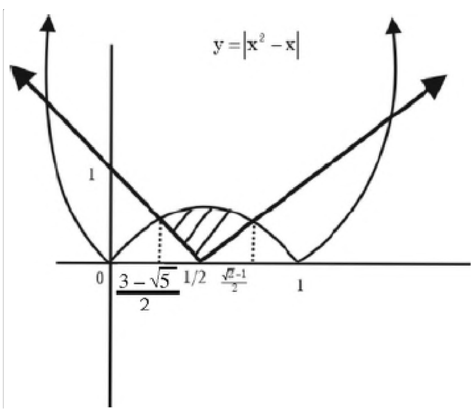
$={}^9C_r\cdot\left(\dfrac{4}{5}\right)^{9-r}\left(\dfrac{5}{2}\right)^r\cdot x^{9-3r}$

Coefficient of x^{-6} i.e., $9-3r=-6\Rightarrow r=5$

So, Coefficient of $x^{-6}={}^9C_5\left(\dfrac{4}{5}\right)^4\cdot\left(\dfrac{5}{2}\right)^5=5040$

82. 125

Sol. $y\ge|2x-1|,\ y\le|x^2-x|$



Both curve are symmetric about $x=\dfrac{1}{2}$ Hence

$A=2\displaystyle\int_{\frac{3-\sqrt{5}}{2}}^{\frac{1}{2}}((x-x^2)-(1-2x))dx$

$A=2\displaystyle\int_{\frac{3-\sqrt{5}}{2}}^{\frac{1}{2}}(-x^2+3x-1)dx=2\left(\dfrac{-x^3}{3}+\dfrac{3}{2}x^2-x\right)_{\frac{3-\sqrt{5}}{2}}^{\frac{1}{2}}$

On solving $6A+11=5\sqrt{5}$

$(6A+11)^2=125$

83. 45

Sol. $\dfrac{(2n+1)!(n-1)!}{(n+2)!(2n-1)!}=\dfrac{11}{21}$

$\Rightarrow\dfrac{(2n+1)(2n)}{(n+2)(n+1)n}=\dfrac{11}{21}$

$\Rightarrow\dfrac{2n+1}{(n+1)(n+2)}=\dfrac{11}{42}$

$\Rightarrow n=5$

$\Rightarrow n^2+n+15=25+5+15=45$

84. 98

Sol. In, $\left(\dfrac{x^{\frac{5}{2}}}{2}-\dfrac{4}{x^{\ell}}\right)^9$

$T_{r+1}={}^9C_r\dfrac{(x^{5/2})^{9-r}}{2^{9-r}}\left(\dfrac{-4}{x^{\ell}}\right)^r$

$=(-1)^r\dfrac{{}^9C_r}{2^{9-r}}4^r x^{\frac{45}{2}-\frac{5r}{2}-lr}$

$=45-5r-21r=0$

$r=\dfrac{45}{5+21}$ .....(1)

Now, according to the equation,

$(-1)^r\dfrac{{}^9C_r}{2^{9-r}}4^r=-84$

$=(-1)^r\,{}^9C_r\,2^{3r-9}=21\times4$

Only natural value of r possible if 3r - 9 = 0 r = 3 and $^9C_3 = 84$

$\therefore$ 1 = 5 from equation (1)

Now, coefficient of $x^{-31} = x^{\frac{45}{2}-\frac{5r}{2}-1r}$ at l = 5, gives r = 5

$\therefore\ {}^9C_5(1)\frac{4^5}{2^4} = 2^\alpha \times \beta$

$= -63 \times 2^7$

$\Rightarrow \alpha = 7,\ \beta = -63$

$\therefore$ value of $|\alpha\ell - \beta| = 98$

85. 3

Sol. $2(\vec{a}\times\vec{b}) = 3(\vec{c}\times\vec{a})$

$\vec{a}\times(2\vec{b}+3\vec{c}) = 0$

$\vec{a} = \lambda(2\vec{b}+3\vec{c})$

$|\vec{a}|^2 = \lambda^2|2\vec{b}+3\vec{c}|^2$

$|\vec{a}|^2 = \lambda^2(4|\vec{b}|^2 + 9|\vec{c}|^2 + 12\vec{b}\cdot\vec{c})$

$31 = 31\lambda^2 \Rightarrow \lambda = \pm 1$

$\vec{a} = \pm(2\vec{b}+3\vec{c})$

$\frac{|\vec{a}\times\vec{c}|}{|\vec{a}\cdot\vec{b}|} = \frac{2|\vec{b}\times\vec{c}|}{2\vec{b}\cdot\vec{b}+3\vec{c}\cdot\vec{b}}$

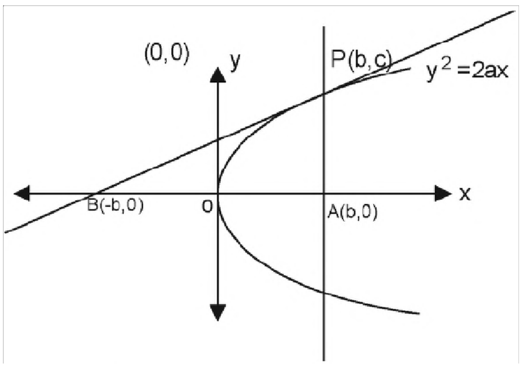
$|\vec{b}\times\vec{c}|^2 = |\vec{b}|^2|\vec{c}|^2 - (\vec{b}\cdot\vec{c})^2 = \frac{3}{4}$

$\frac{|\vec{a}\times\vec{c}|}{|\vec{a}\cdot\vec{b}|} = \frac{2\times\frac{\sqrt{3}}{2}}{2\cdot\frac{1}{4}-\frac{3}{2}} = -\sqrt{3}$

$\left(\frac{\vec{a}\times\vec{c}}{\vec{a}\cdot\vec{b}}\right)^2 = 3$

86. 146

Sol.



As P(b, c) lies on parabola so $c^2 = 2ab$ ....(1)

Now equation of tangent to parabola $y^2 = 2ax$ in point from is $yy_1 = 2a\frac{(x+x_1)}{2}$, $(x_1, y_1) = (b, c)$

$\Rightarrow$ yc = a(x + b)

For point B, put y = 0, now x = –b

So, area of ΔPBA, $\frac{1}{2}\times AB\times AP = 16$

$\Rightarrow \frac{1}{2}\times 2b\times c = 16$

$\Rightarrow$ bc = 16

As b and c are natural number so possible values of (b, c) are (1, 16), (2, 8), (4, 4), (8, 2) and (16, 1)

Now from equation (1) $a = \frac{c^2}{2b}$ and $a \in N$, so values of (b, c) are (1, 16), (2, 8) and (4, 4) now values of are 128, 16 and 2.

Hence sum of value of a is 146.

87. 6925

Sol. Separating odd placed and even placed terms we get

$S = (1.1^2 + 3.5^2 + ...15.(29)^2) - (2.3^2 + 4.7^2 + ... + 14.(27)^2$

$S = \sum_{n=1}^{8}(2n-1)(4n-3)^2 - \sum_{n=1}^{7}(2n)(4n-1)^2$

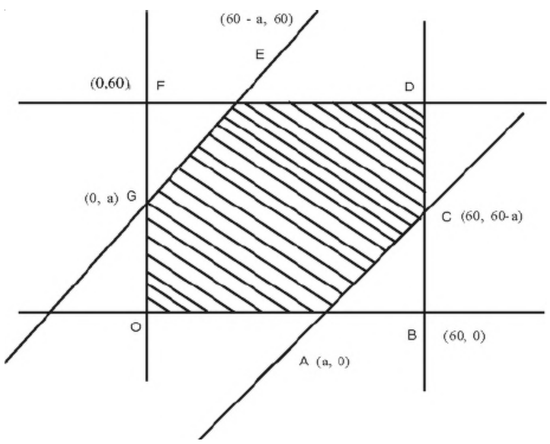
Applying summation formula we get

$= 29856 - 22904 = 6952$

88. 10

Sol. $|x - y| < a \Rightarrow -a < x - y < a$

$\Rightarrow$ x – y < a andx – y > – a



$$P(A)=\frac{ar(OACDEG)}{(OBDF)}$$

$$=\frac{ar(OBDF)-ar(ABC)-ar(EFG)}{ar(OBDF)}$$

$$\Rightarrow \frac{11}{36}=\frac{(60)^2-\frac{1}{2}(60-a)^2-\frac{1}{2}(60-a)^2}{3600}$$

$\Rightarrow$ 1100 = 3600 – $(60 – a)^2$

$\Rightarrow$ $(60 – a)^2$ = 2500 $\Rightarrow$ 60 – a = 50

$\Rightarrow$ a = 10

89. 204

Sol. As given a + b + c + d = 3 or 5 or 7 or 11

if sum = 3

$(1 + x + x^2 +.......+ x^4)^4 \to x^3$

$(1 – x^5)^4 (1 – x)^{–4} \to x^3$

$\therefore\ {}^{4+3-1}C_3 = {}^6C_3 = 20$

If sum = 5

$(1 – 4x^5)\ (1 – x)^{–4} \to x^5$

$\Rightarrow\ {}^{4+5-1}C_5 - 4x^{4.4+0-1}C_0 = {}^8C_5 - 4 = 52$

If sum = 7

$(1 – 4x^5)(1 – x)^{–4} \to x^7$

$\Rightarrow\ {}^{4+5-1}C_4 - {}^{4.4+0-1}C_0 = {}^8C_5 - 4 = 52$

If sum = 11

$(1 – 4x^5 + 6x^{10})(1 – x)^{–4} \to x$

$\Rightarrow\ {}^{4+11-1}C_{11} - 4\cdot{}^{4+6-4}C_6 + 6\cdot{}^{4+1-1}C_1$

$= {}^{14}C_{11} - 4\cdot{}^9C_6 + 6.4 = 364 - 336 + 24 = 52$

$\therefore$ Total matrices = 20 + 52 + 80 + 52 = 204

90. 5

Sol. $|Adj(2Adj(2A^{–1}))|$

$= |2Adj(Adj(2a^{–1}))|^{n-1}$

$= 2^{n(n-1)}\ |Adj(2A^{–1})|^{n-1}$

$= 2^{n(n-1)}\ |(2A^{–1})|^{n-1\ (n-1)}$

$= 2^{n(n-1)}2^{n(n-1)(n-1)}\ |A^{–1}|^{(n-1)(n-1)}$

$$=2^{n(n-1)+n(n-1)(n-1)}\frac{1}{|A|^{(n-1)^2}}$$

$$=\frac{2^{n(n-1)+n(n-1)(n-1)}}{2^{(n-1)^2}}$$

$$=2^{n(n-1)+n(n+1)^2-(n-1)^2}$$

$$=2^{(n-1)(n^2-n+1)}$$

Now, $2^{(n-1)(n^2-n-1)^2}$

$2^{(n-1)(n^2-n+1)} = 2^{84}$

So, n = 5

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