3S INSTITUTE
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24-January-2023 (Morning Batch) : JEE Main Paper

PHYSICS

Section - A (Single Correct Answer)

1. A

Sol. Force per unit length between two parallel straight

$$\text{Wire} = \frac{\mu_0 i_1 i_2}{2\pi d}$$

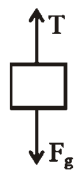
$$\frac{F_1}{F_2} = \frac{\dfrac{\mu_0 (10)^2}{2\pi(5cm)}}{\dfrac{\mu_0 (20)^2}{2\pi\left(\dfrac{5cm}{2}\right)}} = \frac{1}{8}$$

$$\Rightarrow F_2 = 8F_1$$

2. B

Sol. Statement-I

When elevator is moving wi th uni form speed T = Fg



Statement-II

When elevator is going down with increasing speed, its acceleration is downward.

Hence

$$W - N = \frac{W}{g} \times a$$

$$N = W\left(1 - \frac{a}{g}\right)$$ i.e. less than weight.

3. C

Sol. (1) Stopping potential depends on both frequency of light and work function.

(2) Saturation current ∝ intensity of light

(3) Maximum KE depends on frequency

(4) Photoelectric effect is explained using particle theory

4. D

Sol. Acceleration due to gravity at height h

$$g' = \frac{g}{\left[1 + \frac{h}{R}\right]^2}$$

So weight at given height

$$mg' = \frac{mg}{\left[1 + \frac{h}{R}\right]^2} = \frac{18}{\left[1 + \frac{1}{2}\right]^2} = 8N$$

5. D

Sol. Elongation in wire $\delta = \frac{Fl}{AY}$

6. A

Sol. Work done = $P\Delta V$

$= 3 \times 10^5 \times 1600 \times 10^{-6}$

= 480 J

Only 10% of heat is used in work done.

Hence ΔQ = 4800 J

The rest goes in internal energy, which is 90% of heat.

Change in internal energy = 0.9 × 4800 = 4320 J

7. D

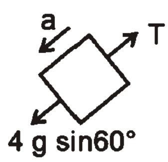
Sol. Modulation index

$$= \frac{\text{Amplitude of modulating signal}}{\text{Amplitude of carrier wave}}$$

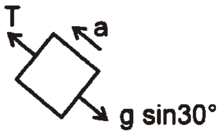
$$\mu = \frac{1}{2}$$

8. B

Sol.



$4g \sin 60° - T = 4a$ .....(1)



$T - g\sin 30° = a$ ....(2)

Solving (A) and (B) we get.

$20\sqrt{3} - T = 4T - 20$

$T = 4\left(\sqrt{3}+1\right)N$

9. B

Sol. Photodiodes are operated in reverse bias as fractional change in current due to light is more easy to detect in reverse bias.

10. A

Sol. Magnetic field vector will be in the direction of $\vec{K} \times \vec{E}$

magnitude of $B = \frac{E}{C} = \frac{K}{\omega}E$

Or $\vec{B} = \frac{1}{\omega}\left(\vec{K} \times \vec{E}\right)$

11. C

Sol. Magnetic field due to current carrying circular loop on its axis is given as

$$\frac{\mu_0 i r^2}{2\left(r^2 + x^2\right)^{3/2}}$$

At centre, x = 0, $B_1 = \frac{\mu_0 i}{2r}$

At x = r, $B_2 = \frac{\mu_0 i}{2 \times 2\sqrt{2}r}$

$\frac{B_1}{B_2} = 2\sqrt{2}$

12. C

Sol. From the given equation $k = 8\ m^{-1}$ and $\omega = 4$ rad/s

Velocity of wave $= \frac{\omega}{k}$

$v = \frac{4}{8} = 0.5$ m/s

13. D

Sol. Equivalent resistance of circuit

$R_{eq} = 3 + 1 + 2 + 4 + 2 = 12\Omega$

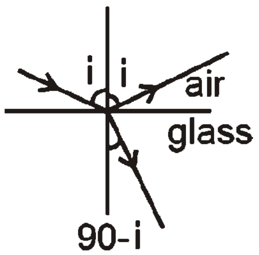
Current through battery $i = \frac{24}{12} = 2A$

$I_4 = \frac{R_5}{R_4 + R_5} \times 2 = \frac{5}{20+5} \times 2 = \frac{2}{5}A$

$I_5 = 2 - \frac{2}{5} = \frac{8}{5}A$

14. B

Sol.



$\mu_a \sin i_1 = \mu_g \sin(90 - i_1)$

$\tan\ i_1 = \frac{\mu_g}{\mu_a}$

When going from glass to air

$\tan i_2 = \frac{\mu_a}{\mu_g} = \cot i_1$

Hence $i_2 = \frac{\pi}{2} - i_1$

15. A

Sol. $F = \frac{1}{\left(4\pi\varepsilon_0\right)} \frac{q_1 q_2}{kd^2}\left(\text{in medium}\right)$

$F_{Air} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{d'^2}$

$F = F_{Air}$

$\frac{q_1 q_2}{4\pi\varepsilon_0 kd^2} = \frac{q_1 q_2}{4\pi\varepsilon_0 d'^2}$

$d' = d\sqrt{k}$

16. C

Sol. $^{218}_{84}A \xrightarrow{\alpha} {}^{214}_{82}A_1 \xrightarrow{\beta^-} {}^{214}_{83}A_2 \xrightarrow{\gamma} {}^{214}_{83}A_3$

$^{214}_{83}A_3 \xrightarrow{\alpha} {}^{210}_{81}A_4 \xrightarrow{\beta^+} {}^{210}_{80}A_5 \xrightarrow{\gamma} {}^{210}_{80}A_6$

17. B

Sol. Statement-I

$T_1 = -73°C = 200$ K

$T_2 = 527°C = 800$ K

$$\frac{V_1}{V_2} = \frac{\sqrt{\frac{3RT_1}{M}}}{\sqrt{\frac{3RT_2}{M}}} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{200}{800}} = \frac{1}{2}$$

$V_2 = 2V_1$ (True)

Statement-II

PV = nRT

Translational $KE = \frac{3}{2}nRT$ (False)

18. C

Sol. $H_{max} = \frac{v^2}{2g} = 136$ m

$R_{max} = \frac{v^2}{g} = 2H_{max}$

= 2(136) = 272 m

19. B

Sol. $EMF = \frac{d\phi}{dt} = \frac{BA - 0}{t}$

$A = \pi r^2 = \pi\left(\frac{0.1^2}{\pi}\right) = 0.01$

B = 0.5

$EMF = \frac{(0.5)(0.01)}{0.5} = 0.01V = 10mV$

20. B

Sol. (A) Planck's constant

$h\nu = E$

$h = \frac{E}{\nu} = \frac{M^1L^2T^{-2}}{T^{-1}} = M^1L^2T^{-1}$ (III)

(B) E = qV

$V = \frac{E}{q} = \frac{M^1L^2T^{-2}}{A^1T^1} = M^1L^2T^{-3}A^{-1}$ (IV)

(C) ϕ (work function) = energy

$= M^1L^2T^{-2}$ (I)

(D) Momentum (p) = F.t

$= M^1L^1T^{-2}T^1$

$= M^1L^1T^{-1}$

Section - B (Numerical Value)

21. 40

Sol. $\frac{1}{2} \times 2 \times v^2 = 10000$

$\Rightarrow$ $v^2 = 10000$

$\Rightarrow$ v = 100 m/s

$\Rightarrow$ v = at = a × 5 = 100

$\Rightarrow$ a = 20 m/s^2

F = ma = 2 × 20 = 40 N

22. 5

Sol. $F = -2kx, a = -\frac{2kx}{m}, \omega = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2\times 20}{2}}$

$= \sqrt{20}$ rad/s

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{20}} = \frac{\pi}{\sqrt{5}}$

x = 5

23. 12

Sol. d_0 at 27°C & d_1 at 177°C

$d_1 = d_0(1 + \alpha\Delta T)$

$d_1 - d_0 = 5 \times 1.6 \times 10^{-5} \times 150$ cm

$= 12 \times 10^{-3}$ cm

24. 10

Sol. $\Delta\omega = \frac{R}{L}$

$Q = \frac{\omega_0}{\Delta\omega} = \omega_0 \frac{L}{R}$

$\omega_0 = \frac{1}{\sqrt{3\times 27\times 10^{-6}}} = \frac{1}{9\times 10^{-3}}$

$$\frac{Q}{\Delta\omega}=\frac{\omega_0\frac{L}{R}}{\frac{R}{L}}=\omega_0\frac{L^2}{R^2}=\sqrt{\frac{L}{LC}\frac{L^2}{R^2}}$$

$$=\frac{1}{9\times10^{-3}}\times\frac{9}{100}=10s$$

25. 2

Sol. $R=\rho\frac{l}{A}$, the cross-sectional area is $\pi(b^2-a^2)$

$$R=\rho\frac{l}{\pi\left(b^2-a^2\right)}=\frac{2.4\times10^{-8}\times3.14}{3.14\times\left(4^2-2^2\right)\times10^{-6}}$$

$= 2 \times 10^{-3}\ \Omega \rightarrow n = 2$

26. 120

Sol. $\frac{1}{f_1}=(1.75-1)\left(-\frac{1}{30}\right)$

$\Rightarrow \quad f_1 = -40$ cm

$\frac{1}{f_1}=(1.75-1)\left(\frac{1}{30}\right)\Rightarrow f_2=40$ cm

Image from L_1 will be virtual and on the left of L_1 at focal length 40 cm. So the object for L_2 will be 80 cm from L_2 which is 2f. Final image is formed at 80 cm from L_2 on the right.

So x = 120

27. 110

Sol. $I_{cm}=\frac{2}{5}MR^2$

$I_{PQ} = I_{cm} + md^2$

$I_{PQ}=\frac{2}{5}mR^2+m(10cm)^2$

For radius of gyration

$I_{PQ} = mk^2$

$k^2=\frac{2}{5}R^2+(10cm)^2$

$=\frac{2}{5}(5)^2+100 = 10 + 100 = 110$

$k=\sqrt{110}$ cm

x = 110

28. 1

Sol. For two perpendicular vectors

$\left(a\hat{i}+b\hat{j}+\hat{k}\right)\cdot\left(2\hat{i}-3\hat{j}+4\hat{k}\right)=0$

$2a - 3b + 4 = 0$

On solving, $2a - 3b = -4$

Also given

$3a + 2b = 7$

We get a = 1, b = 2

$\frac{a}{b}=\frac{x}{2}\Rightarrow x=\frac{2a}{b}=\frac{2\times1}{2}$

$\Rightarrow \quad x = 1$

29. 11

Sol. density of nuclei $=\frac{\text{mass of nuclei}}{\text{volume of nuclei}}$

$$\rho=\frac{1.6\times10^{-27}A}{\frac{4}{3}\pi\left(1.5\times10^{-15}\right)^3 A}$$

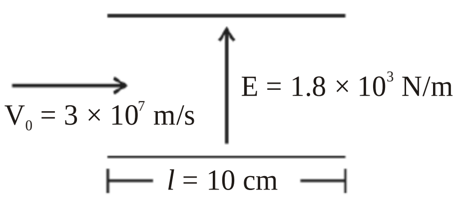
$$=\frac{1.6\times10^{-27}}{14.14\times10^{-45}}=0.113\times10^{18}$$

$\rho_w = 10^3$

Hence $\frac{\rho}{\rho_w}=11.31\times10^{13}$

30. 2

Sol.



$a=\frac{F}{m}=\frac{qE}{m}=\left(2\times10^{11}\right)\left(1.8\times10^3\right)$

$= 3.6 \times 10^{14}$ m/s^2

Time to cross plates $=\frac{d}{v}$

$t=\frac{0.10}{3\times10^7}$

$$y = \frac{1}{2}at^2 = \frac{1}{2}\left(3.6\times10^{14}\right)\left(\frac{0.01}{9\times10^{14}}\right)$$

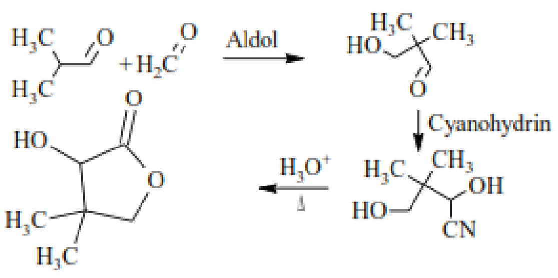
$= 0.2 \times 0.01 = 0.002\text{m} = 2\text{ mm}$

CHEMISTRY

Section - A (Single Correct Answer)

31. A

Sol.



32. C

Sol. The rate of hydrolysis of alkyl chloride improves because of better Nucleophilicity of I^-.

33. C

Sol. According to Fajan's Rule,

A. KF > KI – False ; LiF > KF – True

B. KF < KI – True ; LiF > KF – True

C. $SnCl_4 > SnCl_2$ – True ; CuCl > NaCl – True

D. LiF > KF – True ; CuCl < NaCl – False

E. KF < KI – True ; CuCl > NaCl – True

34. B

Sol. No option is matching the correct answer.

Order should be : C < A < B < D

35. C

Sol. Cr^{+2} : [Ar], $3d^4$, $4s^0$ n = 4, $\mu = \sqrt{4(4+2)} = \sqrt{24}$

= 4.89 BM

Mn^{+2} : [Ar], $3d^5$, $4s^0$ n = 5, $\mu = \sqrt{5(5+2)} = \sqrt{35}$

= 5.91 BM

V^{+2} : [Ar], $3d^3$, $4s^0$ n = 3, $\mu = \sqrt{3(3+2)} = \sqrt{15}$

= 3.87 BM

Ti^{+2} : [Ar], $3d^2$, $4s^0$ n = 2, $\mu = \sqrt{2(2+2)} = \sqrt{8}$

= 2.82 BM

36. A

Sol. Reverberatory furnace : Used for roasting of Copper.

Electrolytic cell : For reactive metal : Al

Blast furnace : Hematite to Pig Iron

Zone Refining furnace: For semiconductors : Si

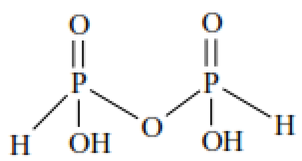
37. C

Sol. According to Henry Moseley $\sqrt{\nu}\ \alpha\ z - b$

So, $n = \dfrac{1}{2}$

38. B

Sol.



Oxyacid having P – H bond can reduce $AgNO_3$ to Ag.

39. D

Sol. $[Co(NH_3)_5Cl]Cl_2$

Oxidation number of Co is +3.

So primary valency is 3.

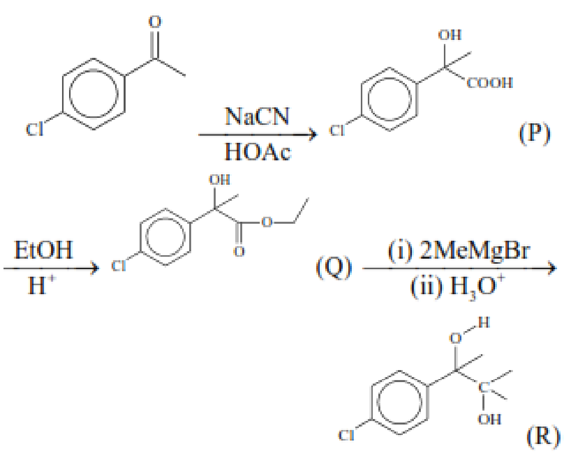
It is an octahedral complex so secondary valency 6 or Co-ordination number 6.

40. D

Sol. $Ni^{+2} + 2DMG \xrightarrow{NH_3(aq)} \underset{\text{Rosy red complex}}{[Ni(DMG)_2]}$

41. B

Sol.



42. A

Sol. Chlorophyll : Mg^{+2} complex

Soda ash : Na_2CO_3

Dentistry, Ornamental work : $CaSO_4$

Used in white washing : $Ca(OH)_2$

43. C

Sol. **Statement I :** For colloidal particles, the values of colligative properties are of small order as compared to values shown by true solutions at same concentration. : True

Statement II : For colloidal particles, the potential difference between the fixed layer and the diffused layer of same charges is called the electrokinetic potential or zeta potential. : True

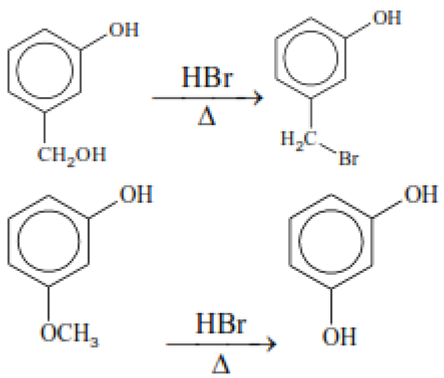
44. A

Sol. $BeO + 2NH_3 + 4HF \longrightarrow (NH_4)_2BeF_4 + H_2O$

$(NH_4)_2BeF_4 \xrightarrow{\Delta} BeF_2 + NH_4F$

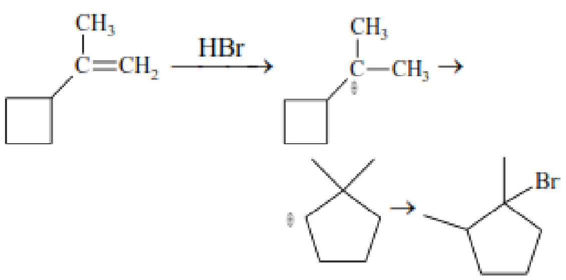
45. D

Sol.



46. D

Sol.



47. B

Sol. Ice > Liquid water > Impure water

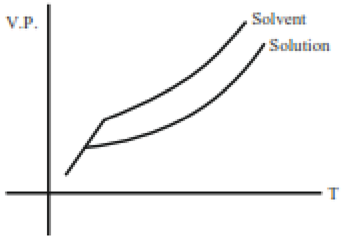
Due to impurity extent of H-Bonding decreases.

48. A

Sol. Fact

49. A

Sol.



Vapour pressure (V.P.) of solvent is greater than vapour pressure (V.P.) of solution.

Only solvent freezes.

50. A

Sol. Fact

Section - B (Numerical Value)

51. 10

Sol. Buffer of HOAc and NaOAc

$$pH = pKa + \log \frac{0.1}{0.01}$$

$5 = pKa + 1$

$pKa = 4$

$Ka = 10^{-4}$

$x = 10$

52. 180

Sol. $$M = \frac{5}{40} \times \frac{1000}{450}$$

$$M_1V_1 = M_2V_2$$

$$\left(\frac{5}{40} \times \frac{1000}{450}\right) \times V_1 = 0.1 \times 500$$

$$V_1 = 180$$

53. 492

Sol. $$\frac{1}{(\lambda_1)_P} = R_H Z^2 \left(\frac{1}{9} - \frac{1}{16}\right)$$

$$\frac{1}{(\lambda_2)_P} = R_H Z^2 \left(\frac{1}{9} - \frac{1}{25}\right)$$

$$\frac{(\lambda_2)_P}{(\lambda_1)_P} = \frac{\frac{7}{16 \times 9}}{\frac{16}{25 \times 9}} = \frac{25 \times 7}{16 \times 16}$$

$$(\lambda_2)_P = \frac{25\times 7}{16\times 16}\times 720$$

$$(\lambda_2)_P = 492 \text{ nm}$$

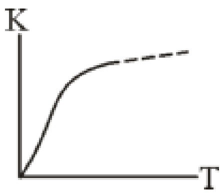
54. 3

Sol. A : $k = Ae^{-\frac{Ea}{RT}}$

As Ea increases k decreases.

B : Temperature coefficient = $\frac{K_{T+10}}{K_T}$

C :



Option (C) is wrong. Δk may be greater or lesser depending on temperature.

D : $\ln k = \ln A - \frac{Ea}{RT}$

55. 917

Sol. $Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$

$$E = 1.33 - \frac{0.059}{6}\log\frac{(0.1)^2}{(10^{-2})(10^{-3})^{14}}$$

$$E = 1.33 - \frac{0.059}{6}\times 42 = 0.917$$

$$E = 917\times 10^{-3}$$

$$x = 917$$

56. 4

Sol. A : $Fe_{0.93}O \rightarrow Fe_2O_3$

$$nf = \left(3 - \frac{200}{93}\right)\times 0.93$$

$$nf = 0.79$$

B : $2x + (0.93 - x)\times 3 = 2$

$$x = 0.79$$

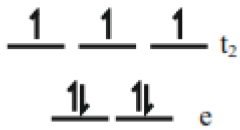
$$Fe^{2+} = 0.79,\ Fe^{3+} = 0.21$$

C : Fact

D : $\%Fe^{2+} = \frac{0.79}{0.93}\times 100 = 85\%;\ Fe^{3+} = 15\%$

57. 7

Sol. $Co^{2+} : 3d^7 4s^0, Cl^- : WFL$



Configuration $e^4\ t_2^3 : m = 4$

Number of unpaired electrons = 3

So, answer = 7

58. 2

Sol. $\Delta G = \Delta H - T\Delta S$

A : ΔG (J mol^{-1}) = $-25 \times 10^3 + 80 \times 300$: –ve

B : ΔG (J mol^{-1}) = $-22 \times 10^3 - 40 \times 300$: –ve

C : ΔG (J mol^{-1}) = $25 \times 10^3 + 300 \times 50$: +ve

D : ΔG (J mol^{-1}) = $22 \times 10^3 - 20 \times 300$: +ve

Processes C and D are non-spontaneous.

59. 25

Sol. Mol. wt. of $C_4N_2H_4O_2$ = 112

$$\%N = \frac{28}{112}\times 100 = 25\%$$

60. 2

Sol. Benzylic and tertiary carbocations are stable.

MATHEMATICS

Section - A (Single Correct Answer)

61. C

Sol. Equation of Plane is

$$= \begin{vmatrix} x-2 & y+3 & z-1 \\ -3 & 4 & -3 \\ 4 & -5 & 4 \end{vmatrix} = 0$$

$x - z - 1 = 0$

Distance of P(7, –3, –4) from Plane is

$$d = \left|\frac{7+4-1}{\sqrt{2}}\right| = 5\sqrt{2}$$

62. B

Sol. $\lim_{t\to 0}(1^{\operatorname{cosec}^2 t} + 2^{\operatorname{cosec}^2 t} + \ldots\ldots + n^{\operatorname{cosec}^2 t})^{\sin^2 t}$

$$= \lim_{t\to 0} n\left(\left(\frac{1}{n}\right)^{\operatorname{cosec}^2 t} + \left(\frac{2}{n}\right)^{\operatorname{cosec}^2 t} + ...+1\right)^{\sin^2 t}$$

= n

63. A

Sol. $\vec{u} = (1, -1, -2), \vec{v} = (2, 1, -1), \vec{v}\cdot\vec{w} = 2$

$\vec{v}\times\vec{w} = \vec{u} + \lambda\vec{v}$ ....(1)

Taking dot with $\vec{w}$ in (1)

$\vec{w}\cdot(\vec{v}\times\vec{w}) = \vec{u}\cdot\vec{w} + \lambda\vec{v}\cdot\vec{w}$

$\Rightarrow 0 = \vec{u}\cdot\vec{w} + 2\lambda$

Taking with $\vec{v}$ in (1)

$\vec{v}\cdot(\vec{v}\times\vec{w}) = \vec{u}\cdot\vec{v} + \lambda\vec{v}\cdot\vec{v}$

$\Rightarrow 0 = (2-1+2) + \lambda(6)$

$\lambda = -\frac{1}{2}$

$\Rightarrow \vec{u}\cdot\vec{w} = -2\lambda = 1$

64. A

Sol. $\sum_{r=0}^{22} {}^{22}C_r \cdot {}^{23}C_r = \sum_{r=0}^{22} {}^{22}C_r \cdot {}^{23}C_{23-r}$

$= {}^{45}C_{23}$

65. A

Sol. $y^2 = 24x$

a = 6 xy = 2

AB = ty = x + $6t^2$ ....(1)

AB = T = S_1

kx + hy = 2hk ....(2)

From (1) and (2)

$\frac{k}{1} = \frac{h}{-t} = \frac{2hk}{-6t^2}$

$\Rightarrow$ then locus is $y^2 = -3x$

Therefore directrix is 4x = 3

66. C

Sol. x + y + z = 1

2x + Ny + 2z = 2

3x + 3y +Nz = 3

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & N & 2 \\ 3 & 3 & N \end{vmatrix}$$

= (N – 2) (N – 3)

For unique solution $\Delta \neq 0$

So $N \neq 2, 3$

$\Rightarrow$ P (system has unique solution) $= \frac{4}{6}$

So k = 4

Therefore sum = 4 + 1 + 4 + 5 + 6 = 20

67. C

Sol. $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right)$

$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{3}$

68. B

Sol. Let P is $\vec{0}$, Q is $\vec{q}$ and R is $\vec{r}$

A is $\frac{2\vec{q}+\vec{r}}{3}$, B is $\frac{2\vec{r}}{3}$ and C is $\frac{\vec{q}}{3}$

Area of ΔPQR is $= \frac{1}{2}|\vec{q}\times\vec{r}|$

Area of ΔABC is $\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$

$\overrightarrow{AB} = \frac{\vec{r}-2\vec{q}}{3}, \overrightarrow{AC} = \frac{-\vec{r}-\vec{q}}{3}$

Area of $\Delta ABC = \frac{1}{6}|\vec{q}\times\vec{r}|$

$\frac{\text{Area }(\Delta PQR)}{\text{Area }(\Delta ABC)} = 3$

69. D

Sol. $A^2 + B = A^2B$

$(A^2 - 1)(B - I) = I$ ....(1)

$A^2 + B = A^2B$

$A^2 (B - I) = B$

$A^2 = B(B - I)^{-1}$

$A^2 = B(A^2 - I)$

$A^2 = BA^2 - B$

$A^2 + B = BA^2$

$A^2B = BA^2$

70. A

Sol. $\frac{dy}{dx}=\frac{1-xy}{x^3}=\frac{1}{x^3}-\frac{y}{x^2}$

$\frac{dy}{dx}+\frac{y}{x^2}=\frac{1}{x^3}$

$\text{If}=e^{\int\frac{1}{x^2}dx}=e^{-\frac{1}{x}}$

$y\cdot e^{-\frac{1}{x}}=-\int e^t\cdot\frac{1}{x^3}dx \text{ (put } -\frac{1}{x}=t\text{)}$

$y\cdot e^{-\frac{1}{x}}=-\int e^t\cdot t dt$

$t=\frac{1}{x}+1+Ce^{\frac{1}{x}}$

Where C is constant

Put $x=\frac{1}{2}$

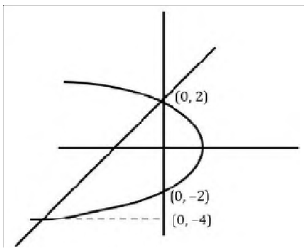
$3-e=2+1+Ce^2$

$C=-\frac{1}{e}$

$y(1)=1$

71. C

Sol.



$y^2+4x=4$

$y^2=-4(x-1)$

$A=\int_{-4}^{2}\left(\frac{4-y^2}{4}-\frac{y-2}{2}\right)dy=9$

72. B

Sol. $\Delta=0=\begin{vmatrix}\alpha^2 & \alpha & 1\\ 1 & 1 & 1\\ a & b & c\end{vmatrix}$

$\Rightarrow\alpha^2(c-b)-\alpha(c-a)+(b-a)=0$

It is singular when $\alpha=1$

$\frac{(a-c)^2}{(b-a)(c-b)}+\frac{(b-a)^2}{(a-c)(c-b)}+\frac{(c-b)^2}{(a-c)(b-a)}$

$\frac{(a-b)^3+(b-c)^3+(c-a)^3}{(a-b)(b-c)(c-a)}$

$=3\frac{(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)}=3$

73. C

Sol. Equation of line

$\frac{x+1}{3}=\frac{y-9}{-4}=\frac{z+16}{12}$

G.P. on line $(3\lambda-1,\ -4\lambda+9,\ 12\lambda-16)$

point of intersection of line & plane

$6\lambda-2-12\lambda+27-12\lambda+16=5$

$\lambda=2$

Point (5, 1, 8)

Distance $=\sqrt{36+64+576}=26$

74. A

Sol. $pq^2=\log_x\lambda$

$qr=\log_y\lambda$

$p^2r=\log_z\lambda$

$\log_y x=\frac{qr}{pq^2}\frac{r}{pq}$ ....(1)

$\log_x z=\frac{pq^2}{p^2r}=\frac{q^2}{pr}$ .....(2)

$\log_z y=\frac{p^2r}{qr}=\frac{p^2}{q}$ .....(3)

$3,\ \frac{3r}{pq},\ \frac{3p^2}{q},\ \frac{7q^2}{pr}$ in A.P.

$\frac{3r}{pq}-3=\frac{1}{2}$

$r=\frac{7}{6}pq$ ....(4)

$r = pq + 1$

$pq = 6$ .....(5)

$r = 7$ .....(6)

$$\frac{3p^2}{q} = 4$$

After solving p = 2 and q = 3

75. B

Sol. $(1-\sqrt{3}i)^{200} = 2^{199}(p+iq)$

$$2^{200}\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)^{200} = 2^{199}(p+iq)$$

$$2\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = p + iq$$

$$p = -1,\ q = -\sqrt{3}$$

$$\alpha = p + q + q^2 = 2 - \sqrt{3}$$

$$\beta = p - q + q^2 = 2 + \sqrt{3}$$

$$\alpha + \beta = 4$$

$$\alpha \cdot \beta = 1$$

equation $x^2 - 4x + 1 = 0$

76. D

Sol. Reflexive : (a, a) $\Rightarrow$ gcd of (a, a) = 1

Which is not true for every a $\in$ Z.

Symmetric :

Take a = 2, b = 1 $\Rightarrow$ gcd (2, 1) = 1

Also 2a = 4 $\neq$ b

Now when a = 1, b = 2 $\Rightarrow$ gcd (1, 2) = 1

Also now 2a = 2 = b

Hence a = 2b

$\Rightarrow$ R is not Symmetric

Transitive :

Let a = 14, b = 19, c = 21

gcd (a, b) = 1

gcd (b, c) = 1

gcd (a, c) = 7

Hence not transitive

$\Rightarrow$ R is neither symmetric nor transitive.

77. A

Sol. The compound statement

$(\sim(P\wedge Q)) \vee ((\sim P)\wedge Q) \Rightarrow ((\sim P)\wedge(\sim Q))$ is equivalent to

(1) $((\sim P)\vee Q)\wedge((\sim Q)\vee P)$

(2) $(\sim Q)\vee P$

(3) $((\sim P)\vee Q)\wedge(\sim Q)$

(4) $(\sim P)\vee Q$

78. B

Sol. Continuity of $f(x): f(0^+) = h^2\cdot\sin\frac{1}{h} = 0$

$$f(0^-) = (-h)^2\cdot\sin\left(\frac{-1}{h}\right) = 0$$

f(0) = 0

f(x) is continuous

$$f'(0^+) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = \frac{h^2\cdot\sin\left(\frac{1}{h}\right)-0}{h} = 0$$

$$f'(0^-) = \lim_{h\to 0}\frac{f(0-h)-f(0)}{-h} = \frac{h^2\cdot\sin\left(\frac{1}{-h}\right)-0}{-h} = 0$$

f(x) is differentiable.

$$f'(x) = 2x\cdot\sin\left(\frac{1}{x}\right) + x^2\cdot\cos\left(\frac{1}{x}\right)\cdot\frac{-1}{x^2}$$

$$f'(x) = \begin{cases} 2x\cdot\sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$\Rightarrow$ f '(x) is not continuous (as $\cos\left(\frac{1}{x}\right)$ is hight oscillating at x = 0)

79. D

Sol. $x^2 - 4x + [x] + 3 = x[x]$

$\Rightarrow x^2 - 4x + 3 = x[x] - [x]$

$\Rightarrow (x-1)(x-3) = [x]\cdot(x-1)$

$\Rightarrow$ x = 1 or $x - 3 = [x]$

$\Rightarrow x - [x] = 3$

$\Rightarrow \{x\} = 3$ (Not Possible)

Only one solution $x = 1$ in $(-\infty, \infty)$

80. C

Sol. $\Omega =$ sample space

A = be an event

If $P(A) = 0 \Rightarrow A = \phi$

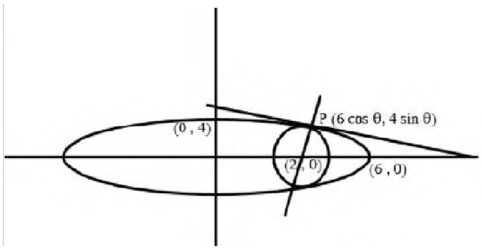
If $P(A) = 1 \Rightarrow A = \Omega$

Then both statement are true

Section - B (Numerical Value)

81. 118

Sol.



Equation of normal of ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$ at any point $P(6\cos\theta,\ 4\sin\theta)$ is

$3\sec\theta x - 2\operatorname{cosec}\theta y = 10$ this normal is also the normal of the circle passing through the point (2, 0) So,

$6\sec\theta = 10$ or $\sin\theta = \frac{4}{5}$ so point $P = \left(\frac{18}{5}, \frac{16}{5}\right)$

So the largest radius of circle

$$r = \frac{\sqrt{320}}{5}$$

So the equation of circle $(x-2)^2 + y^2 = \frac{64}{5}$

Passing it through $(1, \alpha)$

Then $\alpha^2 = \frac{59}{5}$

$10\alpha^2 = 118$

82. 1012

Sol. using result

$$\sum_{r=0}^{n} r^2\ {}^{n}C_r = n(n+1)\cdot 2^{n-2}$$

Then $\sum_{r=0}^{2023} r^2\ {}^{2023}C_r = 2023 \times 2024 \times 2^{2021}$

$= 2023 \times \alpha \times 2^{2022}$ So,

$\Rightarrow \alpha = 1012$

83. 22

Sol. $12\int_0^3 |x^2 - 3x + 2|\,dx$

$$= 12\int_0^3 \left|\left(x-\frac{3}{2}\right)^2 - \frac{1}{4}\right| dx$$

If $x - \frac{3}{2} = t$

$dx = dt$

$$= 24\int_0^{3/2} \left|t^2 - \frac{1}{4}\right| dt$$

$$= 24\left[-\int_0^{1/2}\left(t^2-\frac{1}{4}\right)dt + \int_{1/2}^{3/2}\left(t^2-\frac{1}{4}\right)dt\right] = 22$$

84. 60

Sol. Even digits occupy at even places

$$\frac{4!}{2!2!} \times \frac{5!}{2!3!} = 60$$

85. 5

Sol. $|x|^2 - 2|x| + |\lambda - 3| = 0$

$|x|^2 - 2|x| + |\lambda - 3| - 1 = 0$

$(|x|-1)^2 + |\lambda - 3| = 1$

At $\lambda = 3$, x = 0 and 2

at $\lambda = 4$ or 2, then

x = 1 or –1

So maximum value of $x + \lambda = 5$

86. 546

Sol. For at most two language courses

$= {}^5C_2 \times {}^7C_3 + {}^5C_1 \times {}^7C_4 + {}^7C_5 = 546$

87. 7

Sol. Equation of tangent at point $P(4\cos\theta,\ 3\sin\theta)$ is

$\frac{x\cos\theta}{4}+\frac{y\sin\theta}{3}=1$ So A is $(4\sec\theta,\ 0)$ and point B is $(0,\ 3\operatorname{cosec}\theta)$

Length AB = $\sqrt{16\sec^2\theta+9\operatorname{cosec}^2\theta}$

$=\sqrt{25+16\tan^2\theta+9\cot^2\theta}\geq 7$

88. 2

Sol. $I=\frac{8}{\pi}\int_0^{\frac{\pi}{2}}\frac{(\cos x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$ .....(1)

Using $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

$I=\frac{8}{\pi}\int_0^{\frac{\pi}{2}}\frac{(\sin x)^{2023}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$ ......(2)

Adding (1) and (2)

$2I=\frac{8}{\pi}\int_0^{\frac{\pi}{2}}1\,dx$

I = 2

89. 14

Sol. Shortest distance between the lines

$$=\frac{\begin{vmatrix}4 & 2 & -14\\ 3 & 2 & 2\\ 3 & -2 & 0\end{vmatrix}}{\left|\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 2 & 2\\ 3 & -2 & 0\end{vmatrix}\right|}$$

$$=\frac{16+12+168}{\left|-4\hat{i}+6\hat{j}-12\hat{k}\right|}=\frac{196}{14}=14$$

90. 12

Sol. T_4 = 500 where a = first term,

r common ratio $=\frac{1}{m},\ m\in N$

$ar^3 = 500$

$\frac{a}{m^3}=500$

$S_n-S_{n-1}=ar^{n-1}$

$S_6 > S_5 + 1$

and $S_7-S_6<\frac{1}{2}$

$S_6 - S_5 > 1$ $\quad\quad$ $\frac{a}{m^6}<\frac{1}{2}$

$ar^5 > 1$ $\quad\quad$ $m^3 > 10^3$

$\frac{500}{m^2}>1$ $\quad\quad$ m > 10 .....(2)

$m^2 < 500$ .....(1)

From (1) and (2) m = 11, 12, 13...... , 22

So number of possible values of m is 12

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